Differentiation (Rate of change)

1) $\frac{d}{dx}(k) = 0$

2) $\frac{d}{dx}(x) = 1$

3) $\frac{d}{dx}(x^n) = n x^{n-1}$

4) $\frac{d}{dx}(\sin x) = \cos x$

5) $\frac{d}{dx}(\cos x) = -\sin x$

6) $\frac{d}{dx}(e^x) = e^x$

7) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

8) $\frac{d}{dx}(k y) = k \frac{d}{dx}(y)$

9) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

10) $\frac{d}{dx}(ax+b)^n = a\, n(ax+b)^{n-1}$

11) $\frac{d}{dx} \sin(ax+b) = a \cos(ax+b)$

12) $\frac{d}{dx} \cos(ax+b) = -a \sin(ax+b)$

13) $\frac{d}{dx} e^{ax+b} = a\, e^{ax+b}$

14) $\frac{d}{dx} \log_e(ax+b) = \frac{a}{ax+b}$

Integration (Summation)

Indefinite Integration

1) $\int dx = x + c$

2) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$(n \neq -1)$

3) $\int ky\, dx = k \int y\, dx$

4) $\int (u \pm v) dx = \int u\, dx \pm \int v\, dx$

5) $\int k\, dx = kx + c$

6) $\int e^x dx = e^x + c$

7) $\int \sin x\, dx = -\cos x + c$

8) $\int \cos x\, dx = \sin x + c$

9) $\int \frac{1}{x} dx = \log_e x + c$

10) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

11) $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$

12) $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$

13) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$

14) $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log_e(ax+b) + C$

Definite Integration

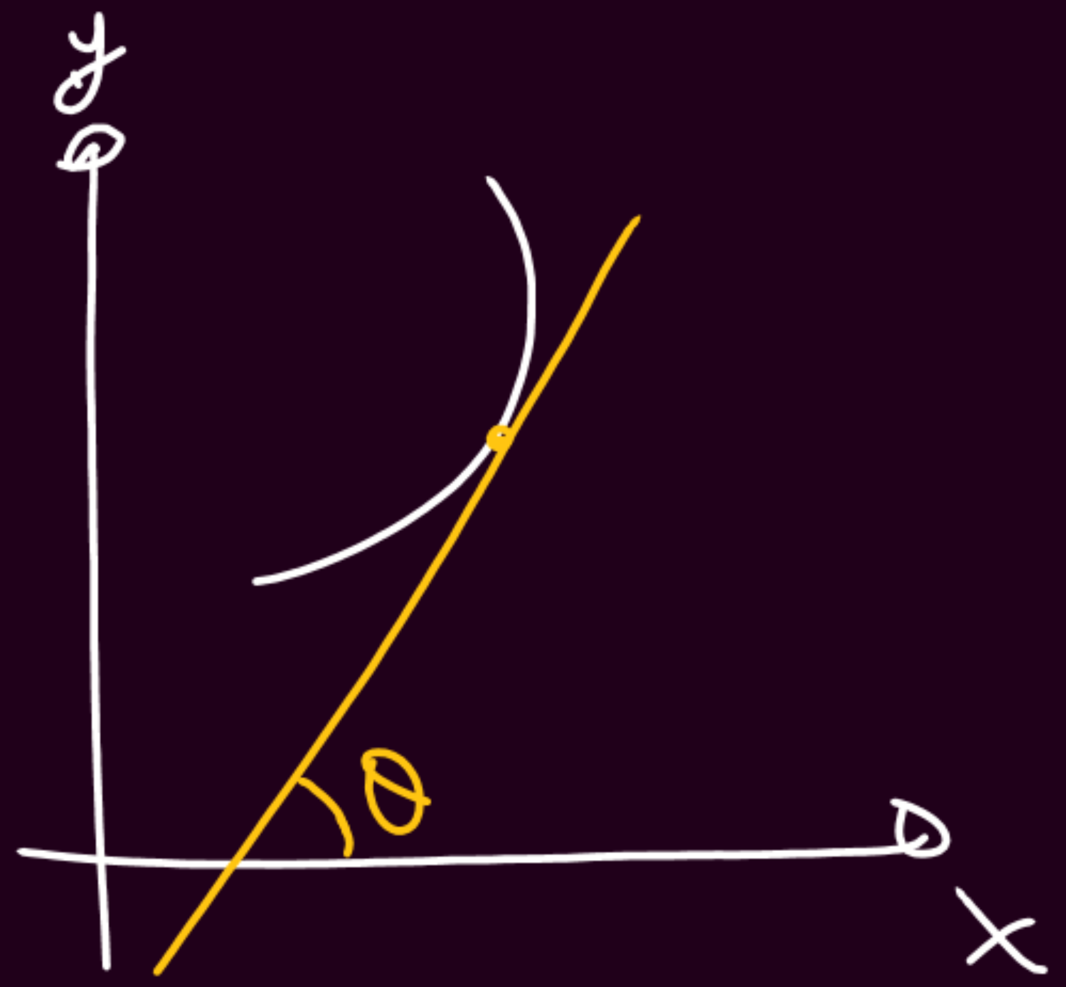
$$\int_{x_1}^{x_2} y\,dx$$

$x_2 \rightarrow$ upper limit

x_1 lower limit

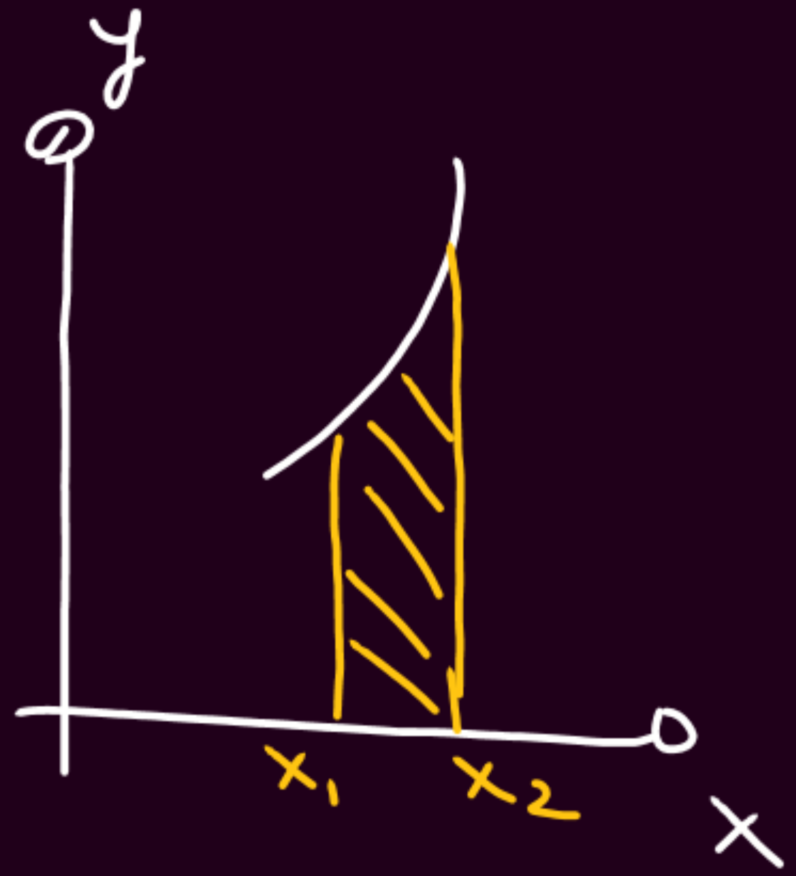
Graphical Interpretation

① Differentiation

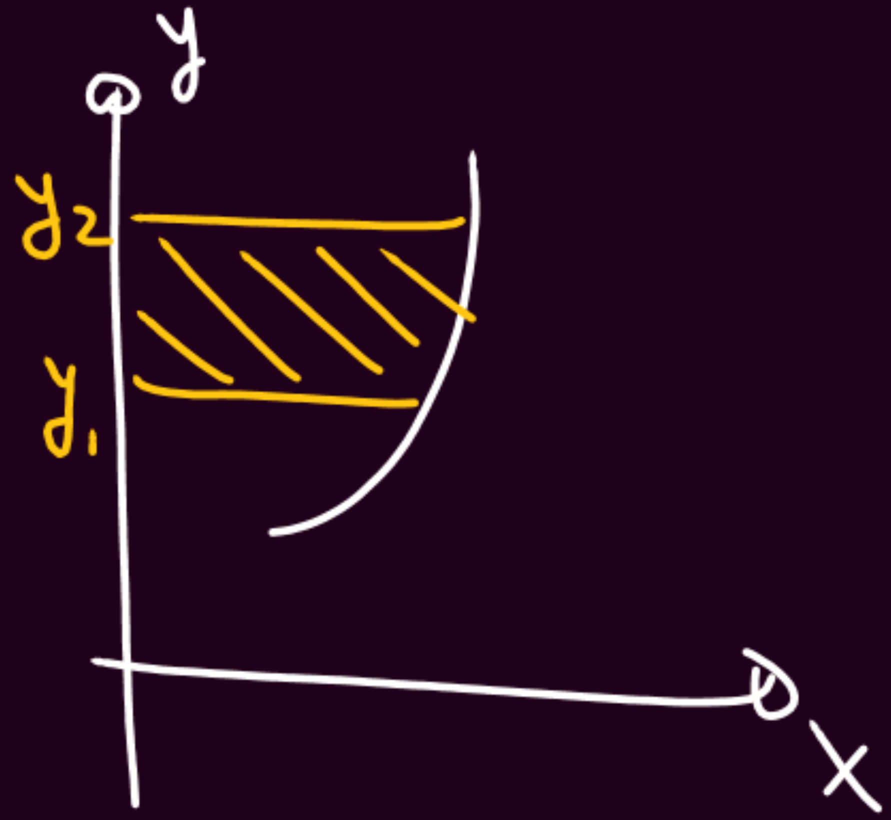


$\frac{dy}{dx}$ = slope of the tangent = $\tan\theta$

② Integration → Area

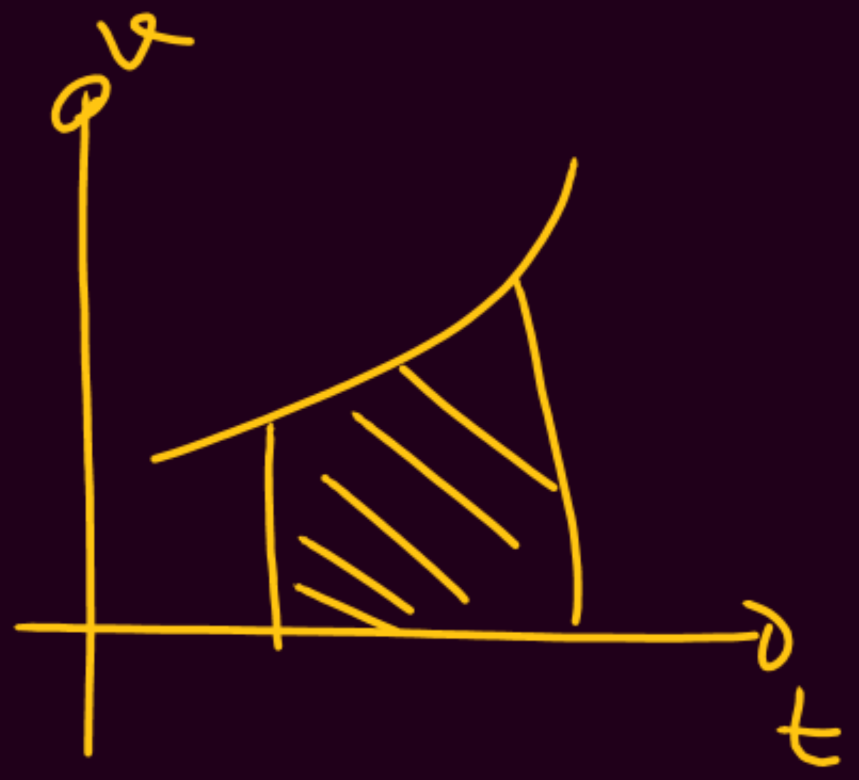


$\int y\,dx$ = Area

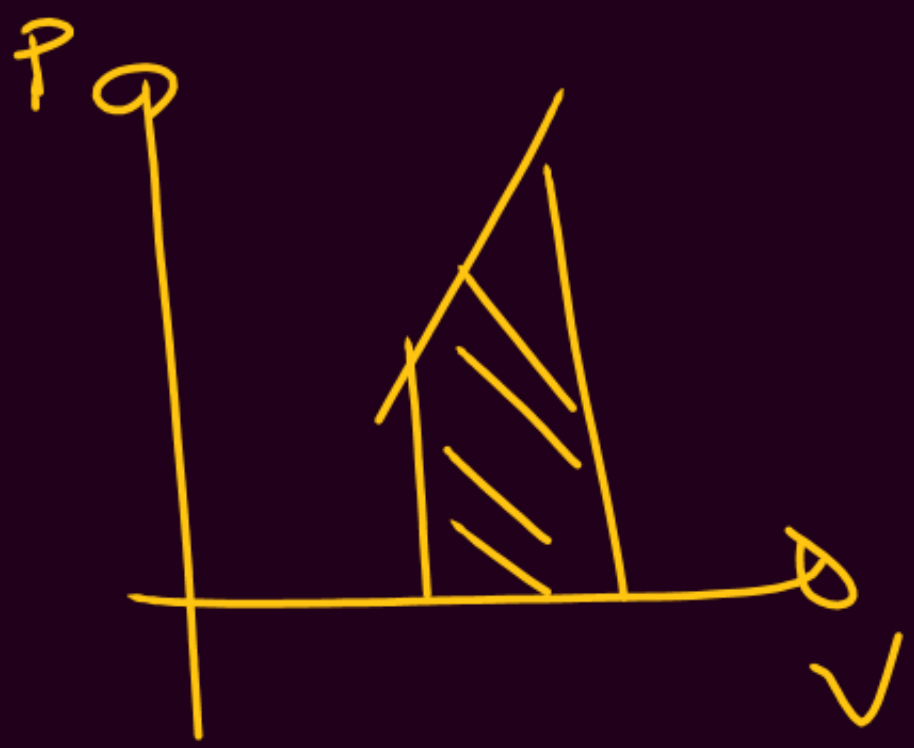


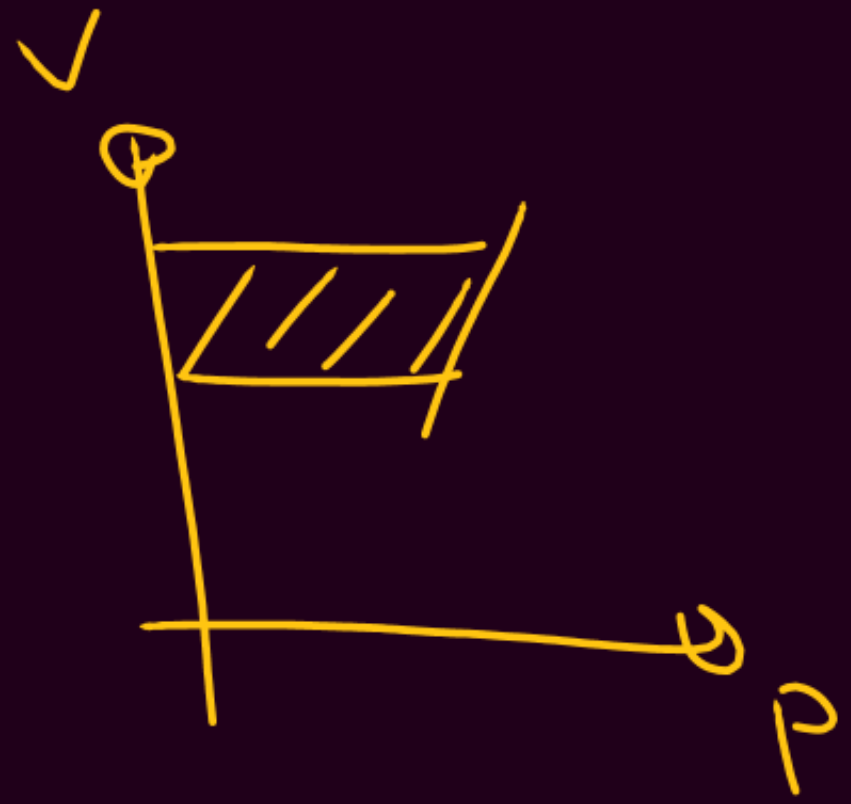
$\int x\,dy$ = Area

$\int v\,dt$



$\int P\,dV$





$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}(x^5) = 5x^4$$

$$\frac{d}{dx}(x^9) = 9x^8$$

$$\frac{d}{dx}(x^{-2}) = -2x^{-3}$$

$$\boxed{-2-1=-3}$$

$$\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{d}{dx}(x^{-3}) = -3x^{-4} = -\frac{3}{x^4}$$

$$\frac{d}{dx}(3x^2) = 3(2x) = 6x$$

$$\frac{d}{dx}(4x^3) = 4(3x^2) = 12x^2$$

$$\frac{d}{dx}(x^2+x+1) = 2x + 1 + 0$$

$$\frac{d}{dx}(x^3 - x^2) = 3x^2 - 2x$$

$$\frac{d}{dx}\left(x^2 - 2x + 4\right) = 2x - 2(1) + 0 = 2x - 2$$

$$\frac{d}{dx}\left(3x^2 - 4x + 5\right) = 3(2x) - 4(1) + 0 = 6x - 4$$

$$\frac{d}{dx}\left(6x^3 - 3x^2 + 4x - 6\right) = 6\left(3x^2\right) - 3(2x) + 4(1) - 0 = 18x^2 - 6x + 4$$

$$\frac{d}{dx}\left(\sin x - \cos x\right) = \cos x - (-\sin x) = \cos x + \sin x$$

$$\frac{d}{dt}\sin\left(\omega t + \pi/3\right) = \omega \cos\left(\omega t + \pi/3\right)$$

$$\frac{d}{dt}\cos\left(\omega t - \pi/6\right) = -\omega \sin\left(\omega t - \pi/6\right)$$

$$\frac{d}{dx}(2x+3)^9 = (2)\,9(2x+3)^8$$

$$\frac{d}{dx}(3-4x)^6 = (-4)\,6(3-4x)^5$$

$$\int x^7\,dx = \frac{x^8}{8} + C$$

$$\int x^{-3}\,dx = \frac{x^{-2}}{-2} + C$$

$$\int x\,dx = \frac{x^2}{2} + C$$

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\int x^3\,dx = \frac{x^4}{4} + C$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

$$\int 3x^2\,dx = \cancel{3}\,\frac{x^3}{\cancel{3}} + C = x^3 + C$$

$$\int 4x^2\,dx = 4\,\frac{x^3}{3} + C$$

$$\int (x - x^2 + x^3)\,dx = \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} + C$$

$$\int (1 - x + x^3)\,dx = x - \frac{x^2}{2} + \frac{x^4}{4} + C$$

$$\int (4x^2 - 2x)\,dx = 4\frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} + C$$

$$\int 4\,dx = 4x + C$$

$$\int (3 - 2x)\,dx = 3x - \cancel{2}\frac{x^2}{\cancel{2}} + C$$

$$\int (4x^3 - 3x^2 + 4x + 5)\,dx$$

$$= \cancel{4}\frac{x^4}{\cancel{4}} - \cancel{3}\frac{x^3}{\cancel{3}} + \overset{2}{\cancel{4}}\frac{x^2}{\cancel{2}} + 5x + C$$

$$= x^4 - x^3 + 2x^2 + 5x + C$$

$$\int_1^2 x\,dx = \left[\frac{x^2}{2}\right]_1^2 = \left(\frac{2^2}{2}\right) - \left(\frac{1^2}{2}\right) = \frac{3}{2}$$

$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{2^3}{3} - 0 = \frac{8}{3}$$

$$\int_1^2 4x^3\,dx = \left[\frac{\cancel{4}x^4}{\cancel{4}}\right]_1^2 = \left[x^4\right]_1^2 = \left(2^4\right) - \left(1^4\right) = 16 - 1 = 15$$

$$\int_1^2 (3-2x)\,dx = \left[3x - x^2\right]_1^2 = \left[3(2) - 2^2\right] - \left[3 - 1\right]$$

$$= (6-4) - (2) = \underline{\underline{0}}$$