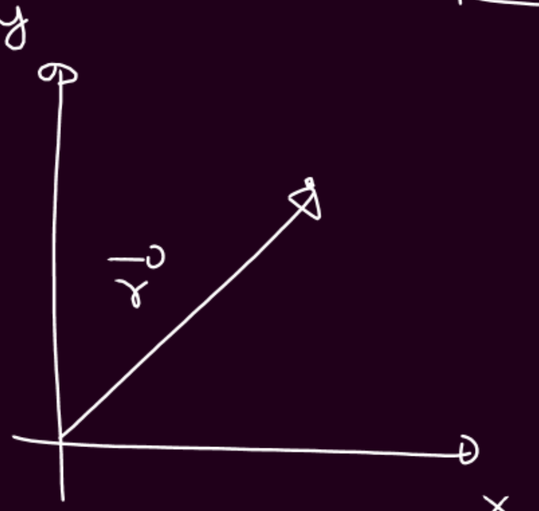
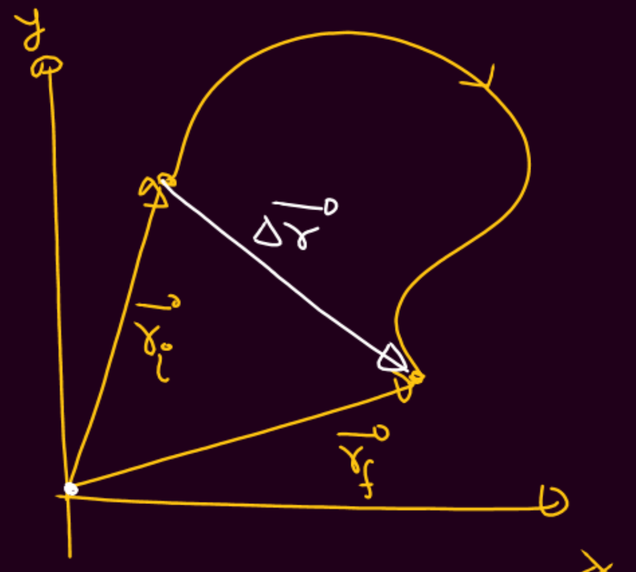
Kinematics

Rest & Motion → Always wrt. foR (observer)

① Position $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$





② Distance → Actual length of path

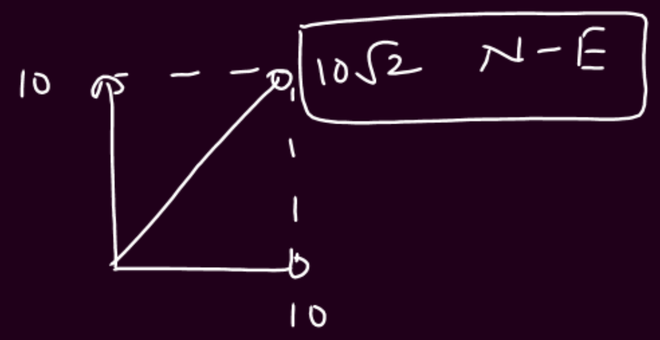
③ Displacement $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

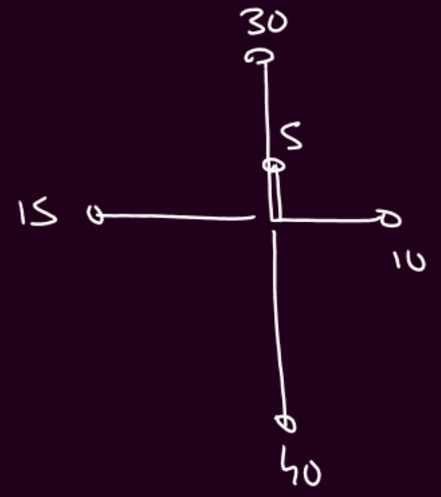
$\Rightarrow$ Distance $\geq$ |Displ.|

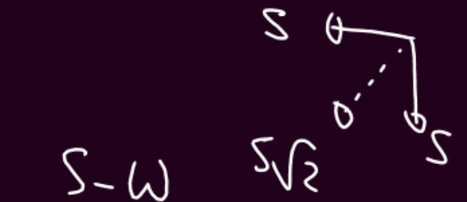
$\Rightarrow$ Dist. = |Displ.| (unidirection)

Q. 10 m N
10 m E

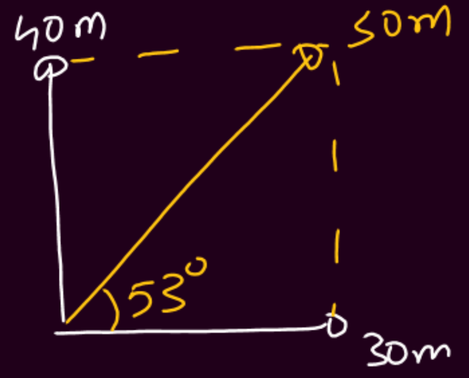


Q. 30m N
10m E
15m W
40m S
5m N





Q.



$\tan\theta = \frac{40}{30} = \frac{4}{3}$

$\theta = 53^\circ$

50m 53° N of E

Q. 10m E i
6m N j
8m vertically up k

$r = \sqrt{10^2 + 6^2 + 8^2} = 10\sqrt{2}$ m

$$\text{Avg. Speed} = \frac{\text{Distance}}{\text{time}}$$

① Equal interval of time

$$v_{av} = \frac{v_1 + v_2}{2} \qquad v_{av} = \frac{v_1 + v_2 + v_3}{3}$$

② Equal Distance

$$v_{av} = \frac{2 v_1 v_2}{v_1 + v_2} \qquad v_{av} = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}}$$

$$v_{av} = \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}}$$

$$\text{Avg. Vel.} = \frac{\text{Displacement}}{\text{time}}$$

Dist. $\geq |$Disp.$|$

Avg. Sp. $\geq |$Avg. Vel.$|$

When motion is unidirection $\Rightarrow$ Dist. $= |$Disp.$|$

Avg Sp. $= |$Avg Vel.$|$

Q. 10 km/hr for 2 hrs
30 km/hr for 3 hrs

$$v_{av} = \frac{(10 \times 2) + (30 \times 3)}{5}$$

$$= \frac{110}{5} = 22 \text{ km/hr}$$

Q. 60 km/hr for 2 km
30 km/hr for next 2 km

$$v_{av} = 2\left(\frac{60}{3}\right) = 40 \text{ km/hr}$$

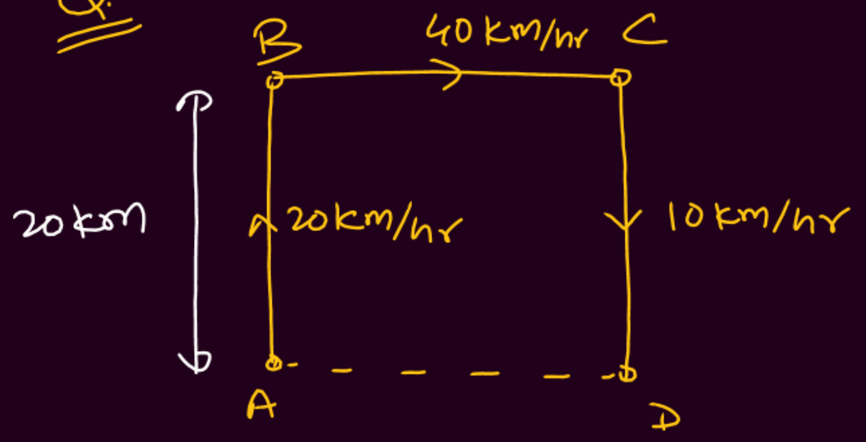
Q. 10 km/hr for 10 min
30 km/hr for 10 min

$$v_{av} = \frac{10+30}{2} = 20 \text{ km/hr}$$

Q. 10 km/hr for 10 min
30 km/hr for 10 min
40 km/hr for 10 min

$$v_{av} = \frac{10+30+40}{3} = \frac{80}{3} \text{ km/hr}$$

Q.



For A to D

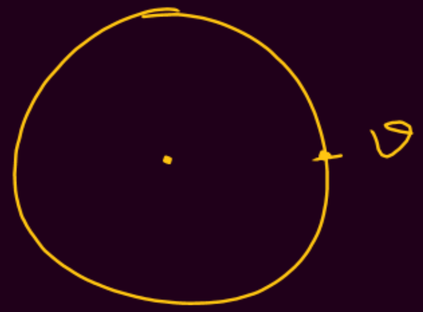
① Avg. Speed

$$v_{av} = \frac{3}{\frac{1}{20} + \frac{1}{40} + \frac{1}{10}}$$

$$= \frac{3 \times 40}{2 + 1 + 4} = \frac{120}{7} \text{ km/hr}$$

② Avg Vel. $= \frac{20}{1 + 0.5 + 2} = \frac{20}{3.5} = \frac{40}{7}$ km/hr

$$v_{av} = \frac{20 + 20 + 20}{1 + 0.5 + 2} = \frac{60}{3.5} = \frac{120}{7} \text{ km/hr}$$



Avg. Vel.

① Round Trip:

Avg. vel. = 0

② Semicircular path

$$\text{Avg vel.} = \frac{2R}{\left(\frac{\pi R}{v}\right)} = \frac{2v}{\pi}$$

Uniform Motion → const. velocity → { Speed, Dir }

→ path s-l. line

→ Dist. = Disp

→ Avg Sp = Avg vel = Inst. sp = Inst. vel

→ Acc. = 0

Instantaneous Vel. → Rate of change of position

3D

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

1D

$$v = \frac{dx}{dt}$$

Non-Uniform / Accelerated Motion → vel. is changing

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

Acc. = Rate of change of vel. wrt time

Avg. Acc. $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

Q. $x = (3t^2 - 2t + 1)$ m

① **Initial vel.**

$$v = \frac{dx}{dt} = 6t - 2$$

$t = 0$ $\quad \boxed{v = -2 \text{ m/s}}$

② **Avg. vel. in 2s**

$$v_{av} = \frac{\Delta x}{\Delta t}$$

$t = 0 \quad x = 1$

$t = 2 \quad x = 12 - 4 + 1 = 9$

$$v_{av} = \frac{\text{Disp.}}{\text{time}} = \frac{9-1}{2} = 4 \text{ m/s}$$

③ **Vel. at 2s**

$$v = 6t - 2$$

$$v = 6 \times 2 - 2 = 10 \text{ m/s}$$

④ **Acc.** $\quad a = \frac{dv}{dt} = 6 \text{ m/s}^2$

Q. $v = (t^2 - 4t + 3)$ m/s

① Find a at $t = 2s$

$$a = 2t - 4$$

$$a = 2 \times 2 - 4 = 0$$

② Find vel. when $a = 0$

$$a = 2t - 4 = 0$$

$$t = 2s$$

$$v = (2)^2 - 4(2) + 3$$

$$= -1 \text{ m/s}$$

$v = t^2 + 2$

Find a at $t = 1s$

$$a = \frac{dv}{dt} = 2t$$

$$a = 2(1) = 2\ m/s^2$$

$v = x^2 + 2$

Find a at $x = 1m$

$$a = v\left(\frac{dv}{dx}\right)$$

$$a = (x^2 + 2)(2x)$$

$$a = (1^2 + 2)(2(1))$$

$$a = 6\ m/s^2$$

Position (x)

↓ Diff

Vel. $v = \frac{dx}{dt}$

↓ Diff

acc. $a = \frac{dv}{dt} = v\frac{dv}{dx}$

Displ. $\Delta x = \int v\,dt$

(change in velo.) $v - u = \int a\,dt$

$\frac{v^2 - u^2}{2} = \int a\,dx$

Q. $a = kt$

initial vel is u

Find v ?

$$v - u = \int_0^t kt \, dt$$

$$v - u = \left[\frac{kt^2}{2}\right]_0^t$$

$$v - u = \frac{kt^2}{2} - 0$$

$$v = u + \frac{kt^2}{2}$$

Q. $v = (3t^2 - 4t + 1)$ m/s

Disp. in 2s ?

$$\Delta x = \int_0^2 (3t^2 - 4t + 1)\, dt$$

$$= \left[3\frac{t^3}{3} - 4\frac{t^2}{2} + t\right]_0^2$$

$$= \left[t^3 - 2t^2 + t\right]_0^2$$

$$= (8 - 8 + 2) - 0$$

$$\underline{\Delta x = 2m}$$

Q $x = 2t^2$ $y = 3t$

① Vel. at $t = 1s$

$v_x = 4t$ $v_y = 3$

$t=1$ $\vec{v} = 4\hat{i} + 3\hat{j}$

$v = \sqrt{4^2+3^2} = 5\ m/s$

② Acc.

$a_x = 4$ $a_y = 0$

$a = 4\ m/s^2$ in x-dir

Q. $x = bt^4 + at^3 + ct^2 + dt + e$

① Initial vel.

$v = 4bt^3 + 3at^2 + 2ct + d$

$t=0$ $v = d$

② Initial Acc.

$Acc. = 12bt^2 + 6at + 2c$

$t=0$ $Acc. = 2c$

Uniformly Acc. Motion $\Rightarrow$ Acc. is constant.

Equations of Motion [a = const.]

① $v = u + at$

② $S = ut + \frac{1}{2}at^2$

③ $v^2 = u^2 + 2aS$

④ $S_n = u + \frac{a}{2}(2n-1)$

[n^{th} Sec (n-1) to n]

Q. $a = 5\ m/s^2$

$u = 2\ m/s$

① Vel at $t = 2s$

$$v = u + at$$

$$v = 2 + (5)(2) = 12\ m/s$$

② Disp in 3s

$$S = ut + \frac{1}{2}at^2$$

$$= 2(3) + \frac{1}{2}(5)(3)^2$$

$$= 6 + 22.5 = \underline{28.5\ m}$$

③ Disp in 3rd sec

$$S_n = u + \frac{a}{2}(2n-1)$$

$$S_n = 2 + \frac{5}{2}(2\times 3 - 1)$$

$$\underline{S_n = 14.5\ m}$$

Q. 10 m/s to 20 m/s in 5s

① $v = u + at$

$20 = 10 + a(5)$

$a = 2\ m/s^2$

② $S = ut + \frac{1}{2}at^2$

$= 10(5) + \frac{1}{2}(2)(5)^2$

$= \underline{75m}$

Q) a = const

10 m in 3rd sec

20 m in 5th sec

Disp. in 4 sec?

$$10 = u + \frac{5a}{2}$$

$$20 = u + \frac{9a}{2}$$

$$10 = 0 + \frac{4a}{2}$$

$$\boxed{a = 5\ m/s^2}$$

$$10 = u + \frac{5 \times 5}{2}$$

$$u = 10 - 12.5 = -2.5\ m/s$$

$$S = ut + \frac{1}{2}at^2$$

$$S = (-2.5)(4) + \frac{1}{2}(5)(4)^2$$

$$S = -10 + 40 = 30m$$

Stopping Distance

$$0 = u^2 - 2aS$$

$$\boxed{S = \frac{u^2}{2a}}$$

$$\frac{S_2}{S_1} = \left(\frac{u_2}{u_1}\right)^2$$

Deceleration → (-ve) Acc.
→ vel. ↓

Retardation → Slowing Down
→ Speed ↓

Q. 50 km/hr in 100m

100 km/hr in ?

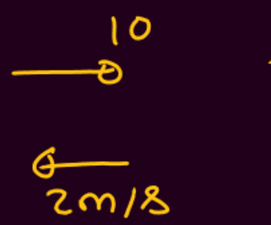
$$100 = \frac{50^2}{2a}$$

$$S = \frac{100^2}{2a}$$

$$S = 4(100) = \underline{\underline{400\,m}}$$

Pg-49

(3)



$v = u + at$

$-2 = 10 + a(4)$

Pg-63

(40)

$v = (3x^2 - 4x)$

$a = v\frac{dv}{dx} = (3x^2 - 4x)(6x - 4)$

Pg-71

(2)

$\left[x = (t+5)^{-1}\right]^3$

$\left[v \propto (t+5)^{-2}\right]^{3/2}$

$a \propto (t+5)^{-3}$

$a \propto x^3$

$a \propto v^{3/2}$

(6)

$(5+3)(4)+5$

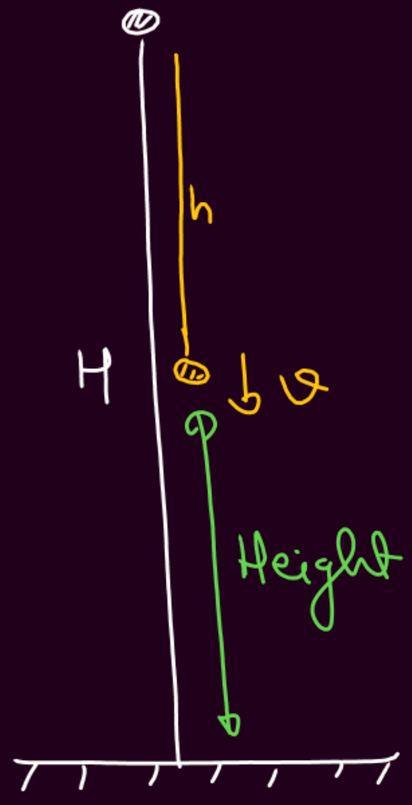
Motion under Gravity (free fall)

— small heights g = const

— neglecting Air resistance

① "Dropped" from H

$u = 0$



① $v = gt = \sqrt{2gh}$

② Height $= H - h = H - \frac{1}{2}gt^2$

③ $h = \frac{1}{2}gt^2$

④ Time of flight $\quad T = \sqrt{\frac{2H}{g}}$

⑤

t	x
t	3x
t	5x

$v = 0 + gt$

$v^2 = 0 + 2gh$

$h = 0 + \frac{1}{2}gt^2$

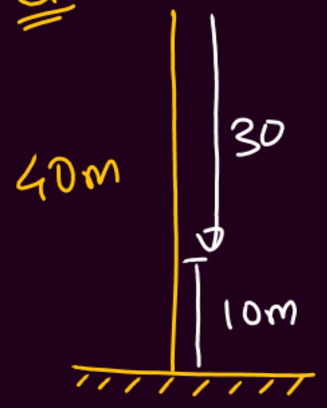
Q.

① $T = \sqrt{\frac{2 \times 80}{10}} = 4\,s$

② $v = \sqrt{2 \times 10 \times 80}$

$= 40\ m/s$

Q.



① $t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{2 \times 30}{10}}$

$t = \sqrt{6}\,s$

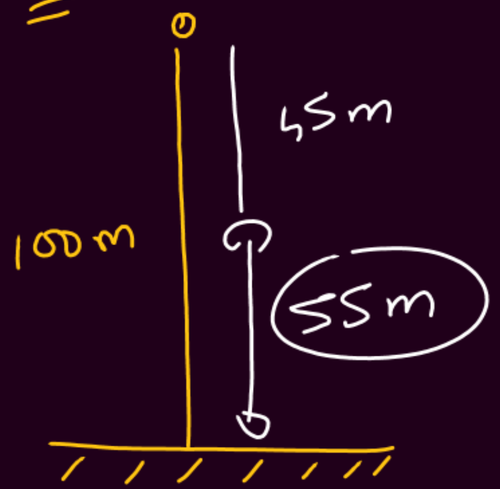
② Height after 1s

$= H - \frac{1}{2} g t^2$

$= 40 - \frac{1}{2}(10)(1)^2$

$= 35m$

Q. Dropped



$30 = \sqrt{2 \times 10 h}$

$900 = 20h$

$h = 45m$

Q. Dropped

Reaches gr. in 6s

Find H and v

① $H = \frac{1}{2} g T^2$

$H = \frac{1}{2}(10)(6)^2 = 180m$

② $v = gT = 10 \times 6 = \underline{60 m/s}$

Q. Dropped H reaches gr in T

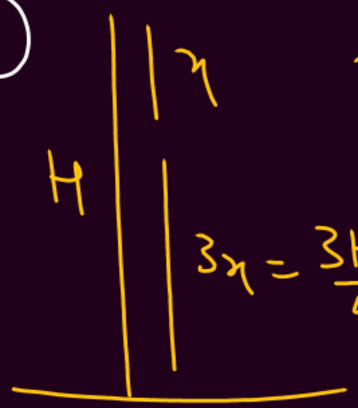
find height $T/2$?

① $H = \frac{1}{2} g T^2$

Height $= H - \frac{1}{2} g \left(\frac{T}{2}\right)^2$

$= H - \frac{H}{4} = \frac{3H}{4}$

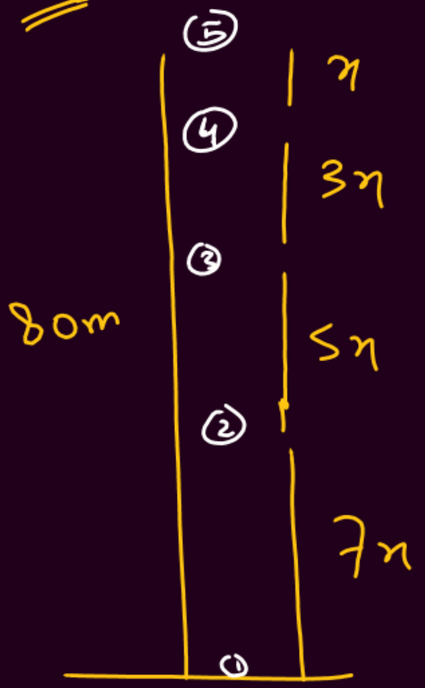
②



$x + 3x = H$

$x = \frac{H}{4}$

Q.



$x + 3x + 5x + 7x = 80$

$16x = 80$

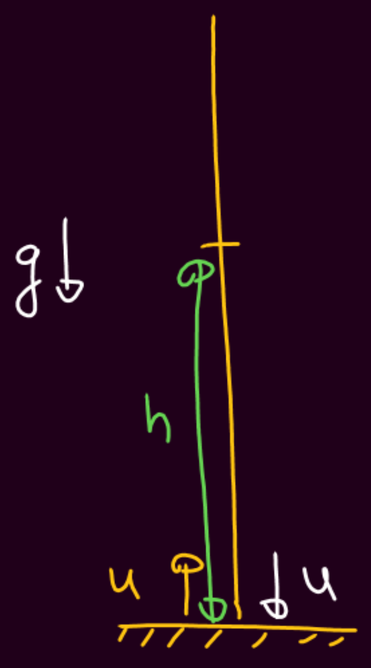
$x = 5m$

② $7x = 7(5) = 35m$

③ $5x + 7x = 12(5) = 60m$

④ $3x + 5x + 7x = 15(5) = 75m$

② Projected Up.



① Max Height $\boxed{H = \frac{u^2}{2g}}$

② Time of flight $\boxed{T = \frac{2u}{g}}$

③ $t_{up} = t_{down} = \frac{u}{g}$

④ $v = u - gt$

⑤ height $= h = ut - \frac{1}{2}gt^2$

⑥ $v^2 = u^2 - 2gh$

$0 = u^2 - 2gH$

$0 = uT - \frac{1}{2}gT^2$

$0 = u - g\,t_{up}$

⑦ At max height

$v = 0$

$a = g\downarrow$

Q

① $T = \frac{2u}{g} = \frac{2 \times 40}{10} = 8s$

② $H = \frac{u^2}{2g} = \frac{(40)^2}{2 \times 10} = 80m$

③ Speed after 2s $\quad v = u - gt$

$v = 40 - (10 \times 2)$

$= 20m$

④ height after 3s

$h = ut - \frac{1}{2}gt^2$

$= (40 \times 3) - \frac{1}{2}(10)(3)^2 = 75m$

Q

if $T = 10s$

① $10 = \frac{2(u)}{10}$

$\boxed{u = 50 m/s}$

② $H = \frac{u^2}{2g} = \frac{(50)^2}{2 \times 10} = 125m$

Q.

if max height 45m

① $45 = \frac{u^2}{2 \times 10}$

$u^2 = 900$

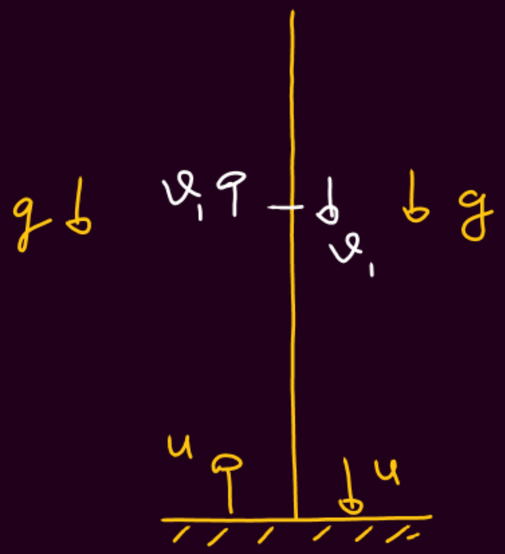
$u = 30$

$T = \frac{2u}{g} = \frac{2 \times 30}{10} = 6s$

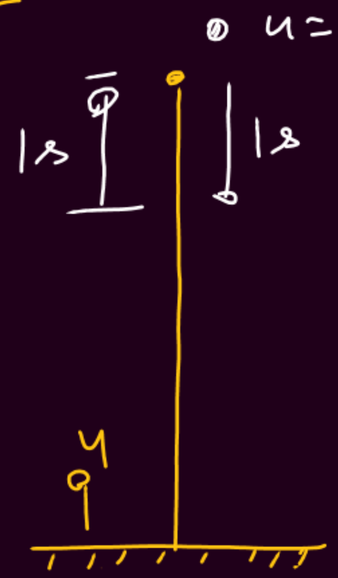
② $T = 2\sqrt{\frac{2 \times 45}{10}}$

$T = 6s$

Symmetry

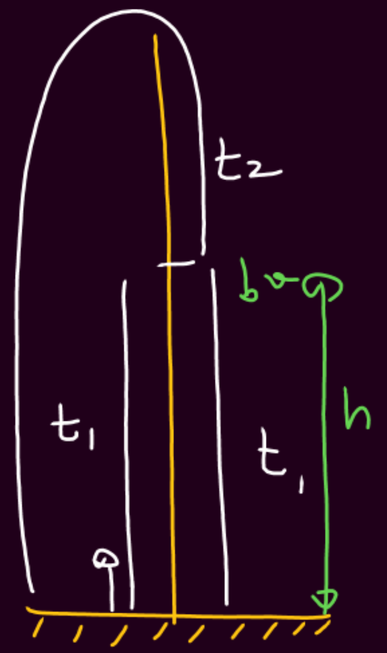


Q.



$$\frac{1}{2}gt^2 = \frac{1}{2}g(1)^2 = 5m$$

Case



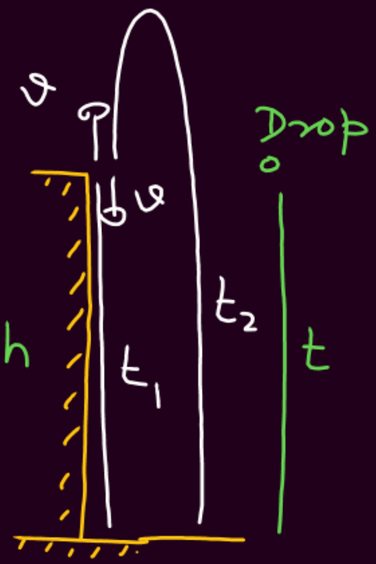
① $T = t_1 + t_2$

② $u = g\left(\frac{t_1+t_2}{2}\right)$

③ $H = \frac{1}{2} g \left(\frac{t_1+t_2}{2}\right)^2$

④ $h = \frac{1}{2} g t_1 t_2$

Case



① $h = \frac{1}{2} g t_1 t_2$

② $t = \sqrt{t_1 t_2}$

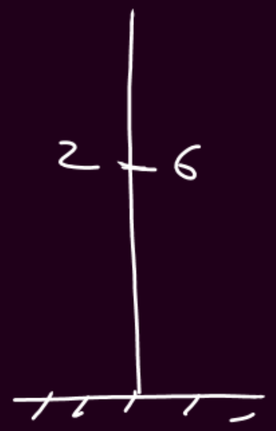
Q. 2s & 6s

① $T = 2 + 6 = 8s$

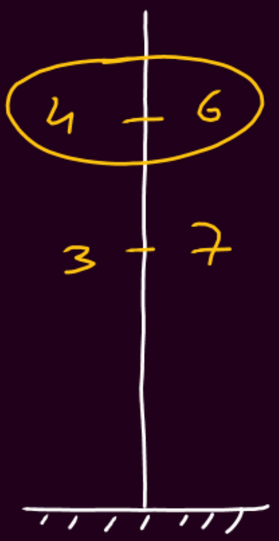
② $u = 10\left(\frac{2+6}{2}\right) = 40\ m/s$

③ $H = \frac{1}{2}\ 10\left(\frac{2+6}{2}\right)^2 = 80m$

④ $h = \frac{1}{2}(10)(2)(6) = 60m$



Q.

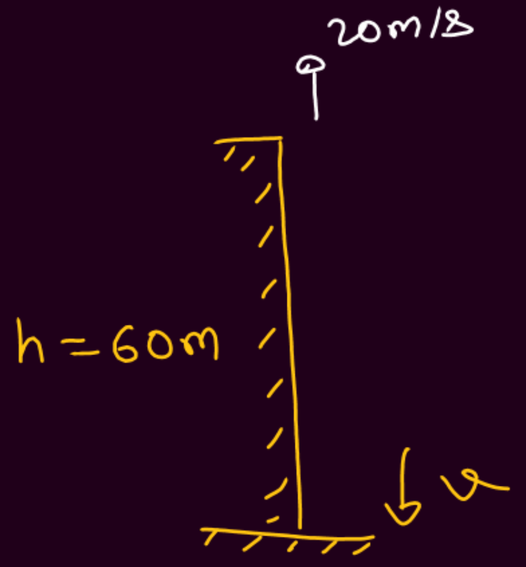


① $T = 4 + 6 = 10s$

② $u = 10\left(\frac{4+6}{2}\right) = 50\ m/s$

③ $H = \frac{1}{2}(10)\left(\frac{4+6}{2}\right)^2 = 125m$

Projection from Height



① $H = h + \frac{u^2}{2g}$

$H = 60 + \frac{(20)^2}{2(10)} = 80m$

② $v^2 = u^2 + 2as$

$v^2 = u^2 + 2gh$

$v^2 = 20^2 + 2(10)(60) = 400 + 1200 = 1600$

$v = 40\ m/s$

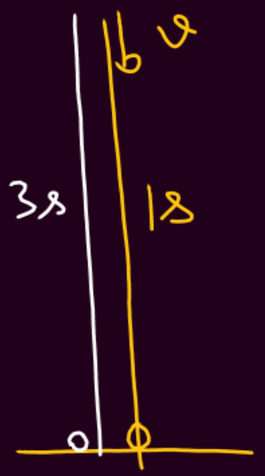
Q. An object is dropped
Another dropped after 2s
Dist. bet^n them
3s after later is dropped

1st $\quad h_1 = \frac{1}{2} g (5)^2 = 125\,m$

2nd $\quad h_2 = \frac{1}{2} g (3)^2 = 45\,m$

$80\,m$

Q. An object is dropped
Another object is thrown $\downarrow v$
2 sec after 1st
Both reaches go. simultaneously
3 sec after 1st. $v = ?$



$h = \frac{1}{2} g (3)^2 = v(1) + \frac{1}{2} g (1)^2$

$45 = v + 5$

$\boxed{v = 40\,m/s}$

Air Resistance

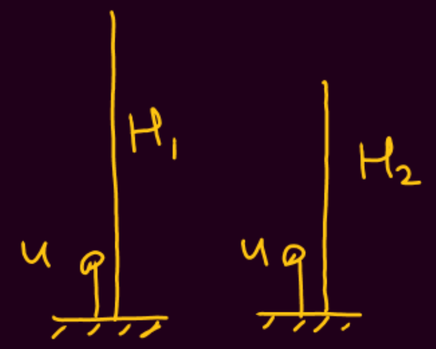
steel ⊙ paper ⊙

① Vacuum / Moon

Both reaches gr. Simultaneously

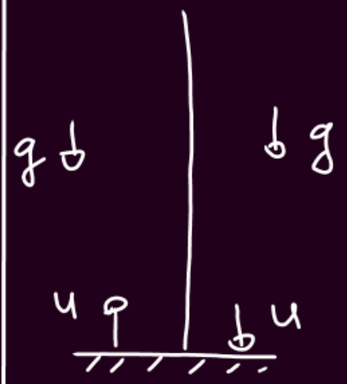
② Air

steel reaches gr first



$H_1 > H_2$

Vacuum



$t_{up} = t_{down}$

Air

① $a_1 = g + \frac{F_{air}}{m}$

$a_2 = g - \frac{F_{air}}{m}$

② $t_{up} < t_{down}$

③ $v_1 > v_2$

Q. Car (Rest) → $a = 4\,m/s^2$

Bus → $40\,m/s$

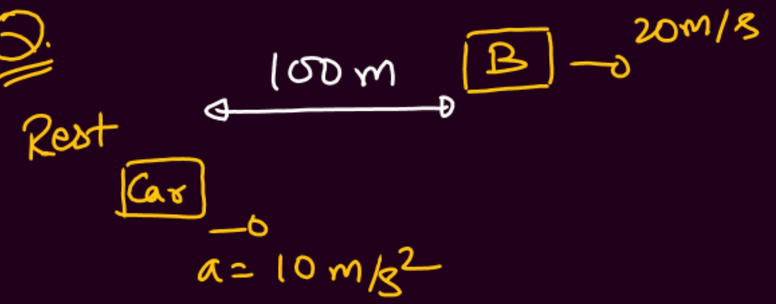
(1) $S_{car} = S_{Bus}$

$$0 + \frac{1}{2}(4)t^2 = 40t$$

$$\boxed{t = 20s}$$

(2) $S_{car} = \frac{1}{2}(4)(20)^2 = \underline{800m}$

Q.



time when car overtakes the Bus?

$$\frac{1}{2}(10)t^2 = (20t) + 100$$

$$5t^2 - 20t - 100 = 0$$

$$t^2 - 4t - 20 = 0$$

$$t = \frac{4 \pm \sqrt{4^2 + 4(20)}}{2(1)}$$

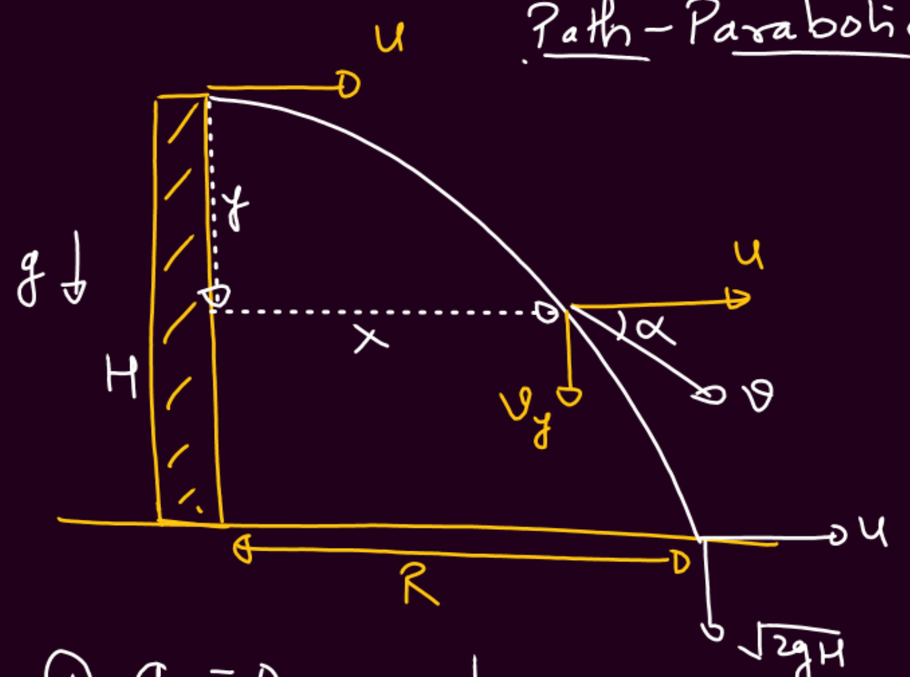
$$t = \frac{4 \pm 4\sqrt{1+5}}{2}$$

$$t = \frac{4 + 4\sqrt{6}}{2} = \underline{\underline{(2+2\sqrt{6})\ s}}$$

Projectile Motion

① Horizontal Projectile

Path - Parabolic



① $a_x = 0$

② $v_x = u = \text{const.}$

③ $x = ut$

① $a_y = g \downarrow$

② $u_y = 0$

③ $v_y = gt = \sqrt{2gy}$

④ $y = \frac{1}{2}gt^2$

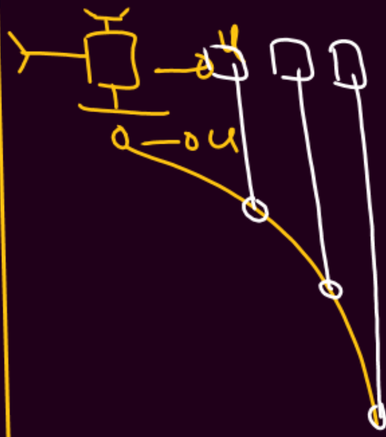
① $T = \sqrt{\frac{2H}{g}}$ ② $y = \frac{gx^2}{2u^2}$

③ $R = u\sqrt{\frac{2H}{g}} = uT$

④ $\tan\alpha = \frac{v_y}{v_x} = \frac{gt}{u} = \frac{\sqrt{2gy}}{u}$

⑤ Height $= H - \frac{1}{2}gt^2$

⑥ Speed $v^2 = u^2 + 2gy$



$v_x = u$

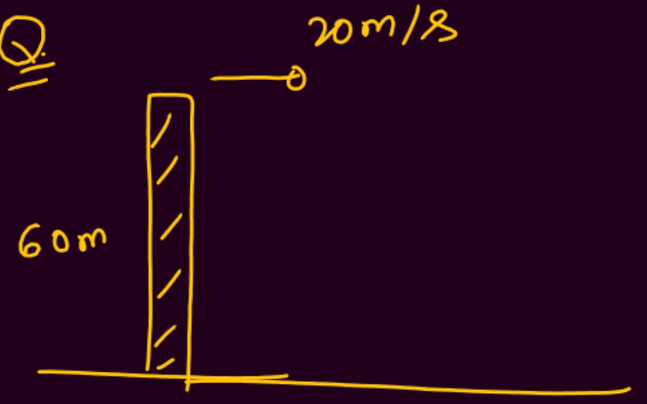
$v_y = gt = \sqrt{2gy}$

$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{u^2 + 2gy}$

$v = \sqrt{u^2 + (gt)^2}$

Q.



① $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times 60}{10}} = 2\sqrt{3}\ s$

② $R = uT = 20(2\sqrt{3}) = 40\sqrt{3}\ m$

③ $v = \sqrt{20^2 + 2(10)(60)}$

$= 40\ m/s$

④ Speed after 2s

$v_x = u = 20$

$v_y = gt = 10 \times 2 = 20$

$v = \sqrt{20^2 + 20^2} = 20\sqrt{2}\ m/s$

⑤ $t = ?$ when $\alpha = 30^\circ$

$\tan\alpha = \frac{gt}{u}$

$\frac{1}{\sqrt{3}} = \frac{10t}{20}$

$\boxed{t = \frac{2}{\sqrt{3}}\ s}$

Q. $80y = x^2$

$$y = \frac{x^2}{80}$$

$$y = \frac{gx^2}{2u^2}$$

$$\frac{g}{2u^2} = \frac{1}{80}$$

$$\frac{800}{2} = u^2$$

$$u^2 = 400$$

$$u = 20 \text{ m/s}$$

Q $R = 40m$

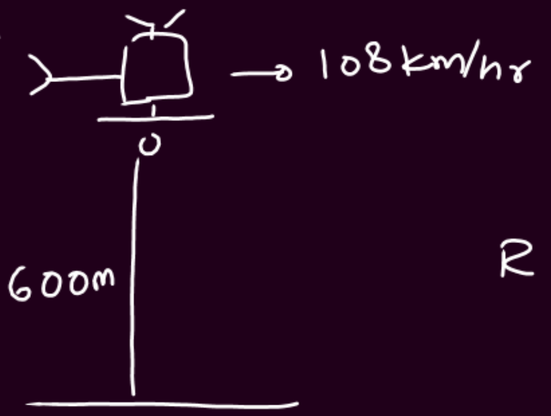
$H = 80m$

$$R = u\sqrt{\frac{2H}{g}}$$

$$40 = u\sqrt{\frac{2\times 80}{10}}$$

$$u = 10 \text{ m/s}$$

Q.



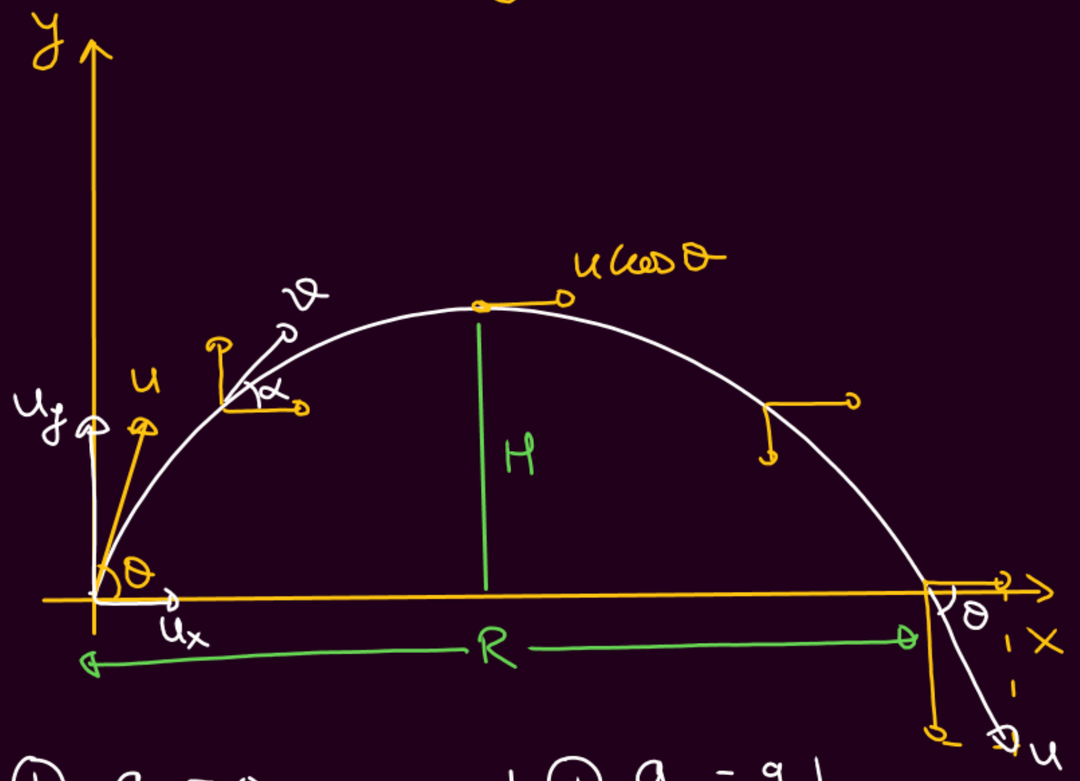
$R = ?$

$$R = \left(108 \times \frac{5}{18}\right)\sqrt{\frac{2\times 600}{10}}$$

$$R = 30\left(2\sqrt{30}\right)$$

$$R = 60\sqrt{30} \text{ m}$$

② Oblique Projectile



① $a_x = 0$

② $v_x = u_x = u\cos\theta$

③ $x = (u\cos\theta)t$

① $a_y = g \downarrow$

② $v_y = u_y - gt$

③ $h = y = u_y t - \frac{1}{2}gt^2$

④ $v_y^2 = u_y^2 - 2gh$

① $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

② $H = \frac{u_y}{2g} = \frac{u^2\sin^2\theta}{2g}$

③ $R = u_x T = \frac{2u_x u_y}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$

$$R = \frac{u^2\sin(2\theta)}{g}$$

④ $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

⑤ $\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$

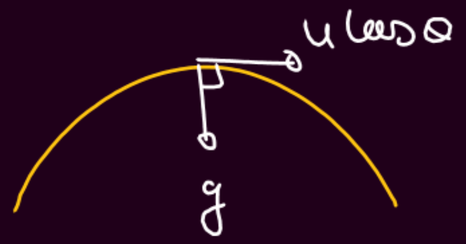
⑥ At max. height

① vel./speed → min

② $u_y = 0$

③ $v = u\cos\theta$

④ $\vec{v} \perp \vec{a}$



⑦ $R_{90-\theta} = R_\theta$ for same u

⑧ R is max. at 45°

$$R_{max} = \frac{u^2}{g}$$

$\theta = 45°$ $\quad H = \frac{R_{max}}{4}$

⑨ $R\tan\theta = 4H$

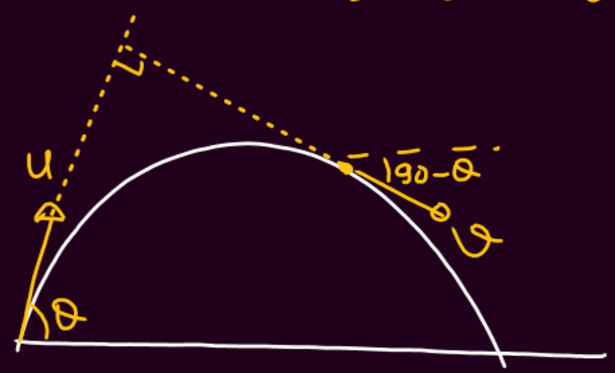
⑦ $H_\theta + H_{90-\theta} = \frac{u^2}{2g}$

$$\frac{u^2\sin^2\theta}{2g} + \frac{u^2\cos^2\theta}{2g}$$

⑩

$v\cos\alpha = u\cos\theta$

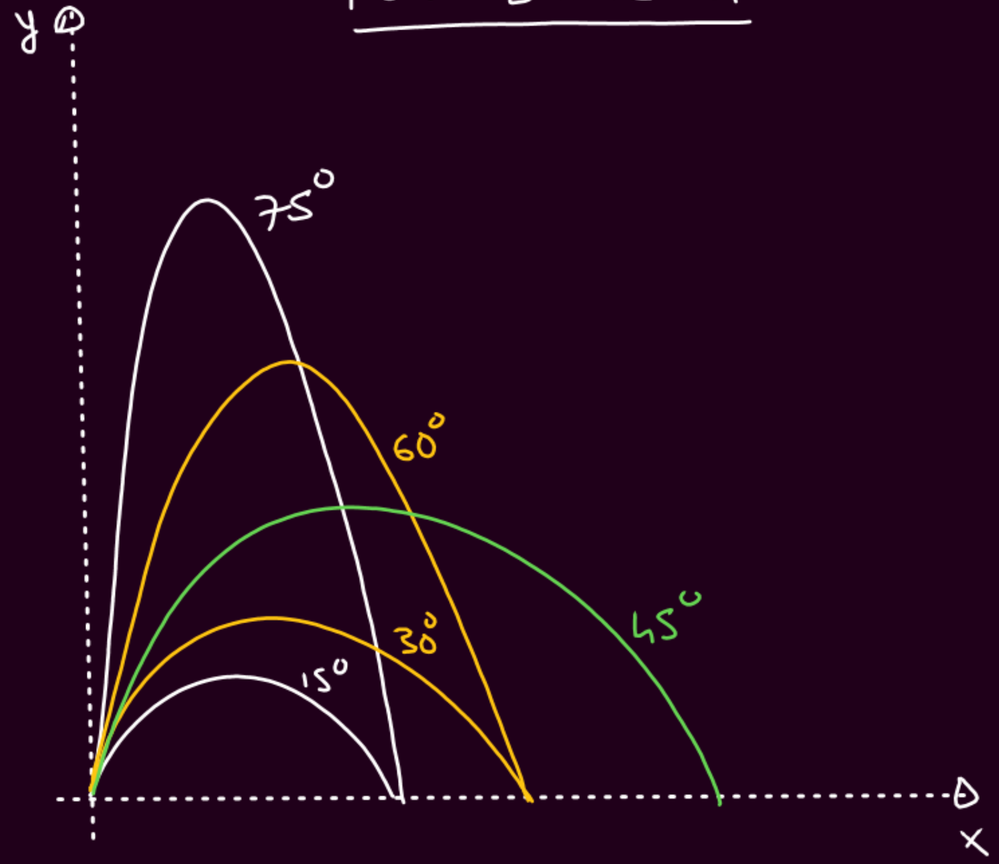
⑪ $u\cos\theta = v\cos(90°-\theta)$



$$v = u\sin\theta$$

(12)

For same u



① θ ↑ T ↑

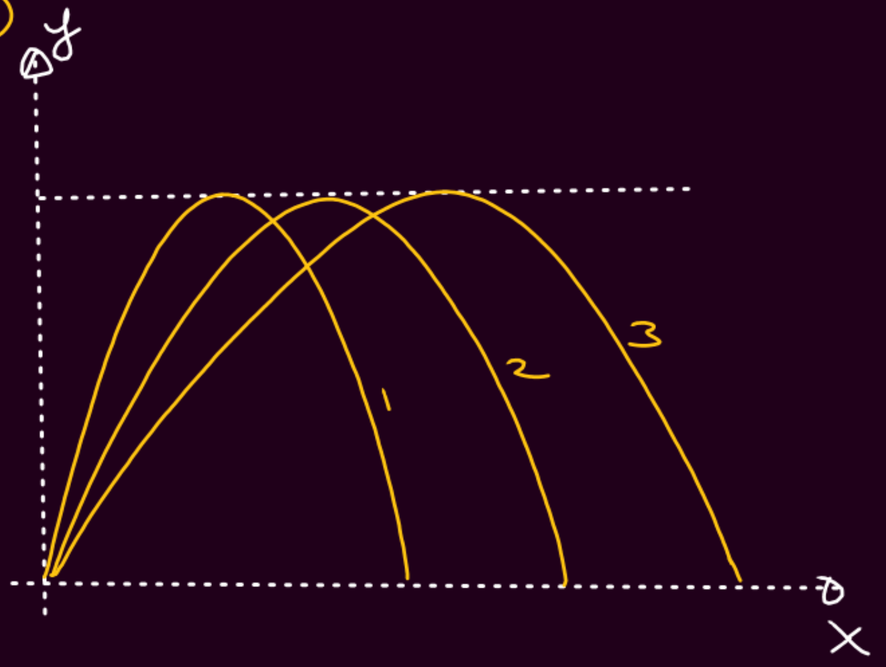
② θ ↑ H ↑

③ θ ↑ R ↑

then max at 45°

then ↓

(13)



(1) $H_1 = H_2 = H_3$

(2) $u_{y_1} = u_{y_2} = u_{y_3}$

(3) $T_1 = T_2 = T_3$

(4) $R_1 < R_2 < R_3$

(5) $u_{x_1} < u_{2x} < u_{3x}$

(6) $u_1 < u_2 < u_3$

$H = \frac{u_y^2}{2g}$

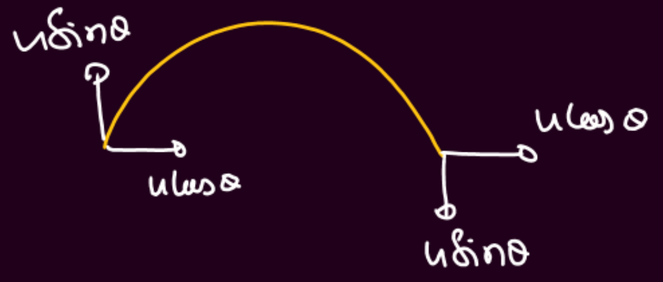
$T = \frac{2u_y}{g}$

$R = u_x T$

$u = \sqrt{u_x^2 + u_y^2}$

(14) For Entire Journey

- Avg. Vel. = $u\cos\theta$
- change in vel. = $-2u\sin\theta$
- change in Sp. = 0



(15) Max. distance

$\theta = 45^\circ$

$$R_{max} = \frac{u^2}{g}$$

(16) Max. possible Height

$\theta = 90^\circ$

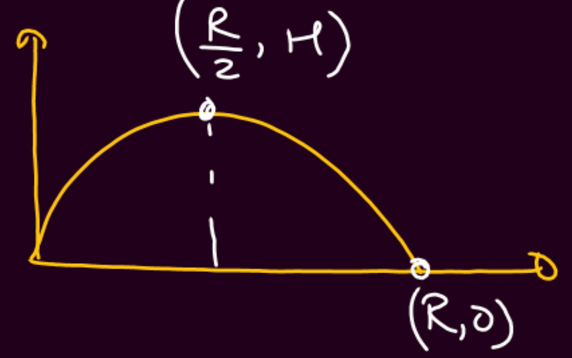
$$H_{max} = \frac{u^2}{2g}$$

(17) $y = ax - bx^2$

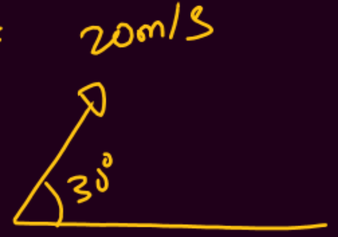
(1) $\tan\theta = a$

(2) $$R = \frac{a}{b} = \frac{\text{Coeff of } x}{\text{Coeff of } x^2}$$

(3) $$H = \frac{a^2}{4b}$$



Q



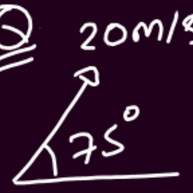
① $T = \dfrac{2(20)\left(\frac{1}{2}\right)}{10} = 2s$

② $H = \dfrac{(20)(20)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{2(10)} = 5m$

③ $R = \dfrac{(20)(20)\sqrt{3}/2}{10} = 20\sqrt{3}\,m$

④ $u_{min} = u\cos 30^\circ = 20\left(\sqrt{3}/2\right)$
$= 10\sqrt{3}\,m/s$

Q



$R = \dfrac{u^2 \sin(2\theta)}{g} = \dfrac{(20)^2 \sin(150^\circ)}{10} = 20m$

Q

$R = \dfrac{2(20)^2 \sin 37^\circ \cos 37^\circ}{10} = \dfrac{(2)(20)(20)\,3\times 4}{10 \quad 5\times 5}$

$R = \dfrac{192}{5}\,m$

Q $\vec{u} = 10\hat{i} + 15\hat{j}$

$R = \dfrac{2u_x u_y}{g} = \dfrac{2(10)(15)}{10} = 30m$

Q: $u = 20\ m/s$

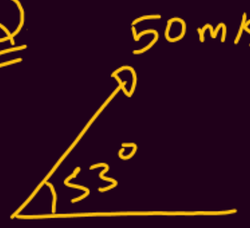
① Max Height

$$H_{max} = \frac{u^2}{2g} = \frac{(20)^2}{2(10)} = 20\ m$$

② Max. dist

$$R_{max} = \frac{u^2}{g} = \frac{(20)^2}{10} = 40\ m$$

Q



① sp. at max height $= 50 \cos 53^\circ = 30\ m/s$

② Sp. when 30°

$$v \cos 30^\circ = 50 \cos 53^\circ$$

$$v \frac{\sqrt{3}}{2} = 50 \times \frac{3}{5}$$

$$v = 20\sqrt{3}\ m/s$$

③ Sp. when vel. is $\perp^r$ to initial vel.

$$v = 50 \cot 53^\circ = 50\left(\frac{3}{4}\right) = 37.5\ m/s$$

Q $H = 20m$ when $\theta = 60^\circ$

① R

$$R \tan 60^\circ = 4(20)$$

$$R = \frac{80}{\sqrt{3}} \, m$$

② T

$$T = 2\left(\frac{40}{\sqrt{3}}\right)\frac{\sqrt{3}}{2} \times \frac{1}{10} = 4s$$

③ u

$$20 = \frac{u^2 \sin^2 60^\circ}{2(10)}$$

$$20 = u\sqrt{3}/2$$

$$\boxed{u = \frac{40}{\sqrt{3}} \, m/s}$$

Q $H = 30m$ at 60°

$H = ?$ at 45°

$$30 = \frac{u^2 \sin^2 60^\circ}{2g}$$

$$H = \frac{u^2 \sin^2 45^\circ}{2g}$$

$$\frac{H}{30} = \frac{\sin^2 45^\circ}{\sin^2 60^\circ} = \frac{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}} = \frac{1}{2}\frac{1}{\frac{3}{4}} = \frac{2}{3}$$

$$H = 20m$$

Q. $40y = 30x - x^2$

$y = \frac{3}{4}x - \frac{x^2}{40}$

① $\tan\theta = \frac{3}{4}$

$\boxed{\theta = 37^\circ}$

② $R = \frac{3/4}{1/40} = 30m$

③ $H = \frac{(3/4)^2}{4 \times \frac{1}{40}} = \frac{9}{16} \times 10 = \frac{45}{8} m$

Q. $x = 20t$

$y = 10t - 5t^2$

$x = u_x t$

$y = u_y t - \frac{1}{2} g t^2$

$u_x = 20 \quad u_y = 10$

$\frac{g}{2} = 5$

$g = 10$

① $u = \sqrt{u_x^2 + u_y^2}$

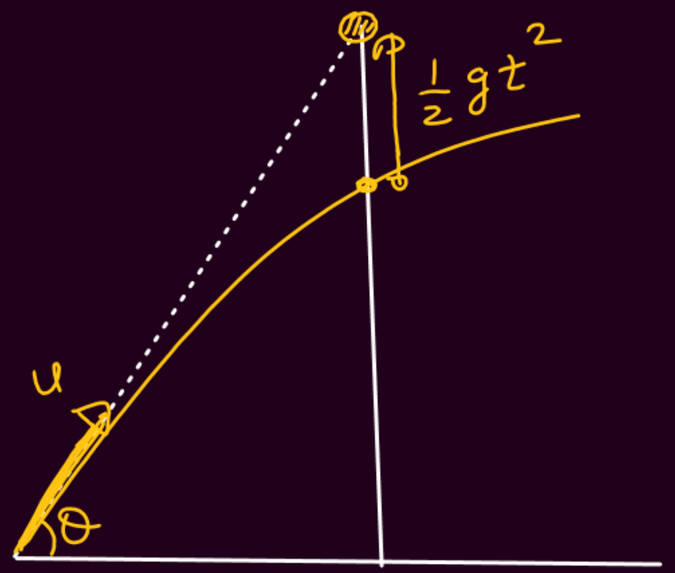
$= \sqrt{20^2 + 10^2} = 10\sqrt{5}$ m/s

② $T = \frac{2u_y}{g} = \frac{2(10)}{10} = 2s$

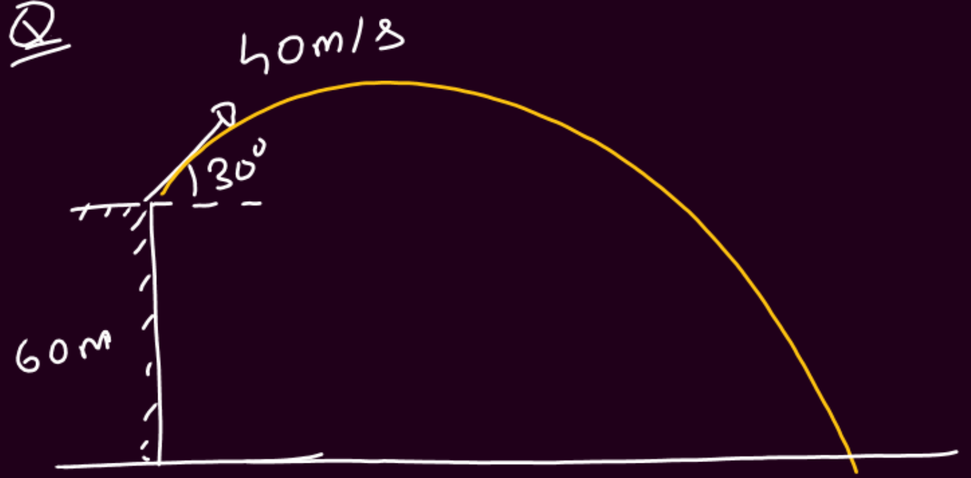
③ $H = \frac{u_y^2}{2g} = \frac{(10)^2}{2(10)} = 5m$

④ $R = \frac{2u_x u_y}{g} = \frac{2(20)(10)}{10} = 40m$

Short fall



Q



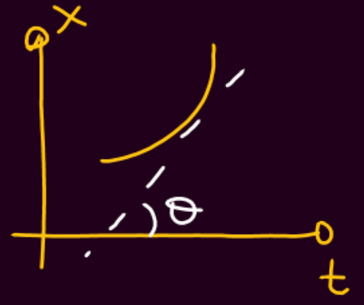
① $H = 60 + \dfrac{(40)^2 \sin^2 30^\circ}{2(10)} = 80m$

② $v = \sqrt{u^2 + 2gh} = \sqrt{(40)^2 + 2(10)\,60)}$

$= 20\sqrt{7}$ m/s

Graphs

① Position - time

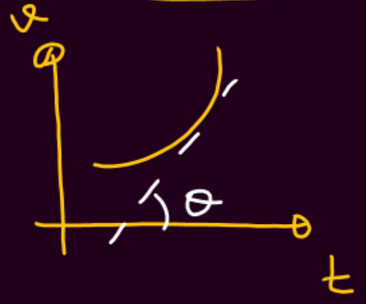


$$\text{slope} = \frac{dx}{dt} = v$$

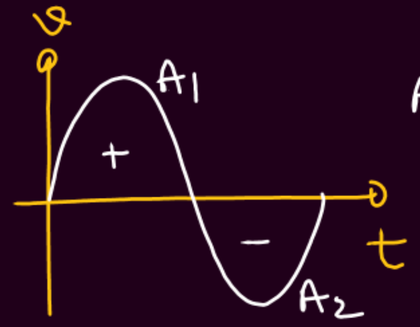
$a = (+)$

$a = (-)$

② Vel. - time



$$\text{slope} = \frac{dv}{dt} = a$$

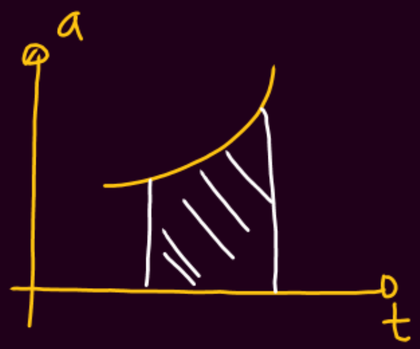


Area = Displacement

$$\text{Disp.} = A_1 - A_2$$

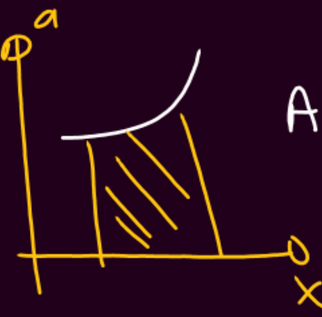
$$\text{Dist.} = A_1 + A_2$$

③ Acc. - time



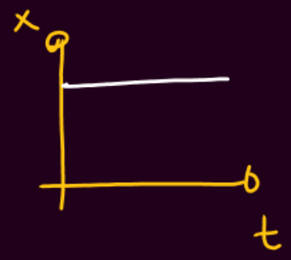
Area = change in vel.

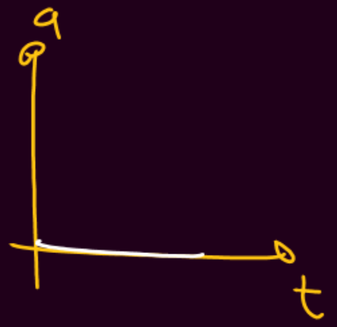
④ Acc. - position



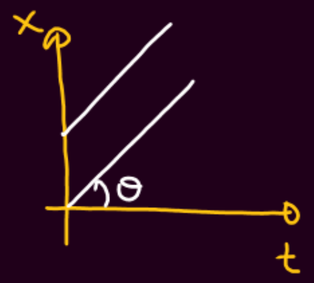
$$\text{Area} = \frac{v^2 - u^2}{2}$$

① Rest

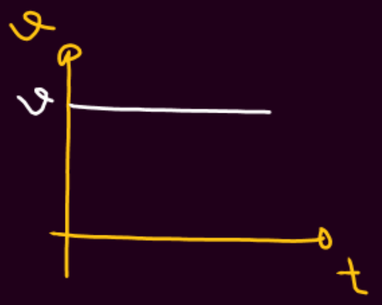


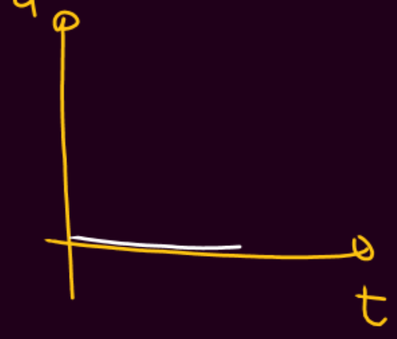


② Uniform Motion.



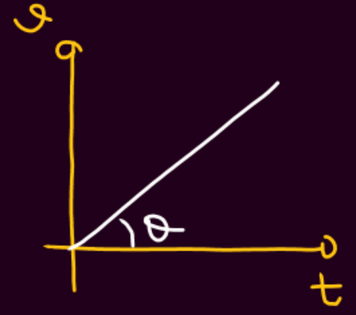
$\tan\theta = v$



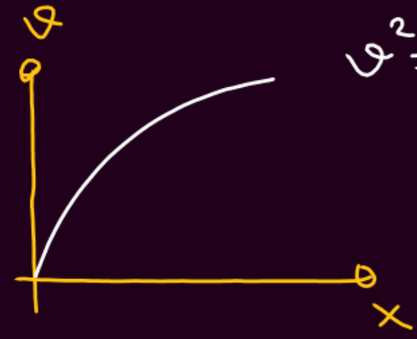


③ Uniformly Acc. Motion

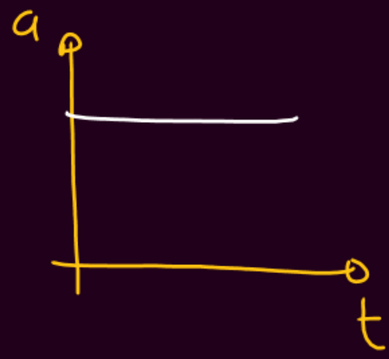
$x = \frac{1}{2}at^2$



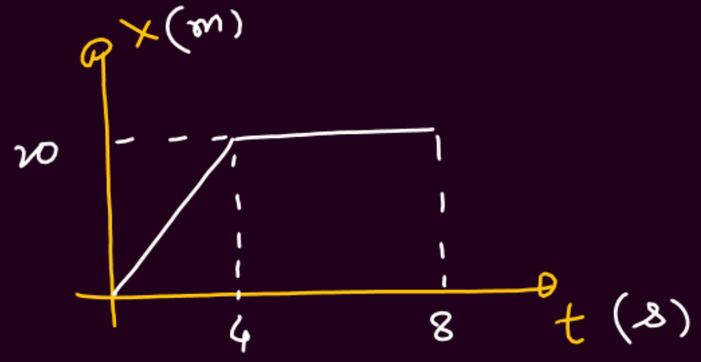
$\tan\theta = a$



$v^2 = 2ax$



Q



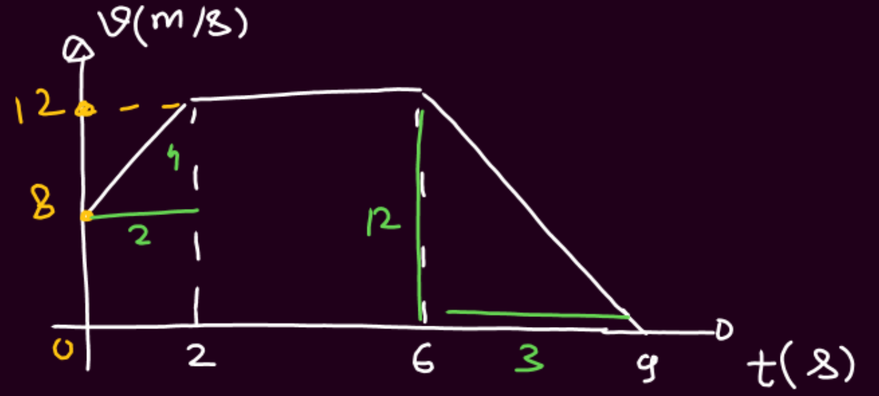
① vel. at $t = 2s$

$$\text{slope} = \frac{20}{4} = 5\ m/s$$

② vel. at $t = 5s$

$$\text{slope} = 0$$

Q

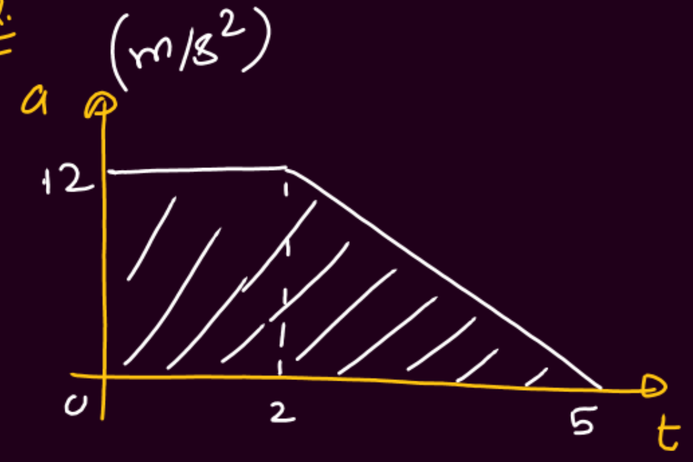


$t = 1s$ $\quad a = \frac{4}{2} = 2\ m/s^2$

$t = 4s$ $\quad a = 0$

$t = 8s$ $\quad a = -\frac{12}{3} = -4\ m/s^2$

Q.



① $t = 2s$

Area = $v - 10 = (12 \times 2)$

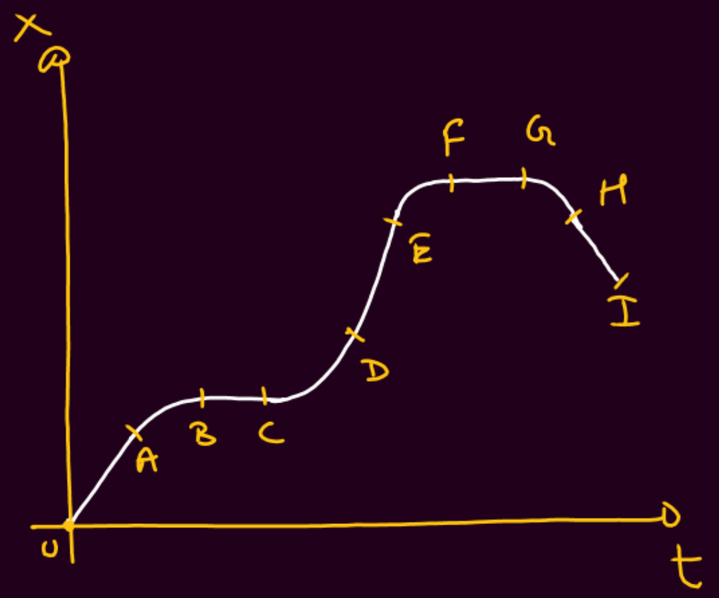
$v = 34$ m/s

if initial vel. = 10 m/s

② $t = 5s$

Area = $v - 10 = (12 \times 2) + \left(\frac{1}{2} \times 3 \times 12\right)$

$v = 52$ m/s



① If $\vec{v}$ & $\vec{a}$ in same dir

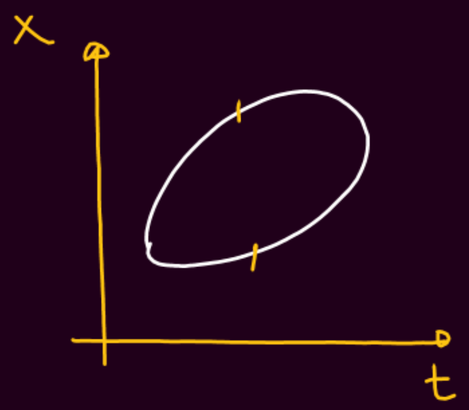
speed ↑

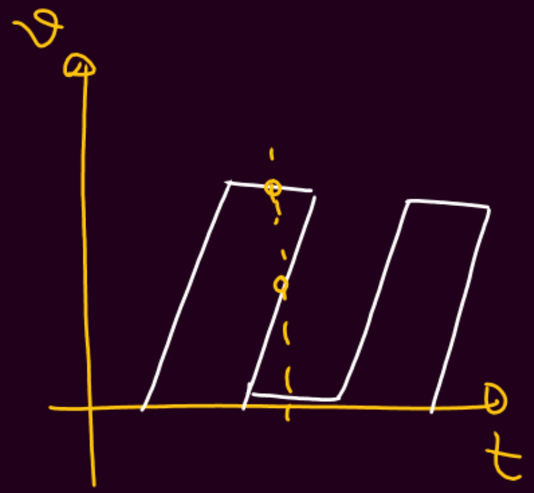
② If $\vec{v}$ & $\vec{a}$ in opp dir

speed ↓

	Velocity ⊕/⊖/0	Acc. ⊕/⊖/0	Vel. ↑ ↓ Const.	Speed ↑ ↓ Const
OA	+	0	Const.	Const
AB	+	−	↓	↓
BC	0	0	Const.	Const
CD	+	+	↑	↑
DE	+	0	Const	Const
EF	+	−	↓	↓
FG	0	0	Const	Const.
GH	−	−	↓	↑
HI	−	0	Const.	Const.

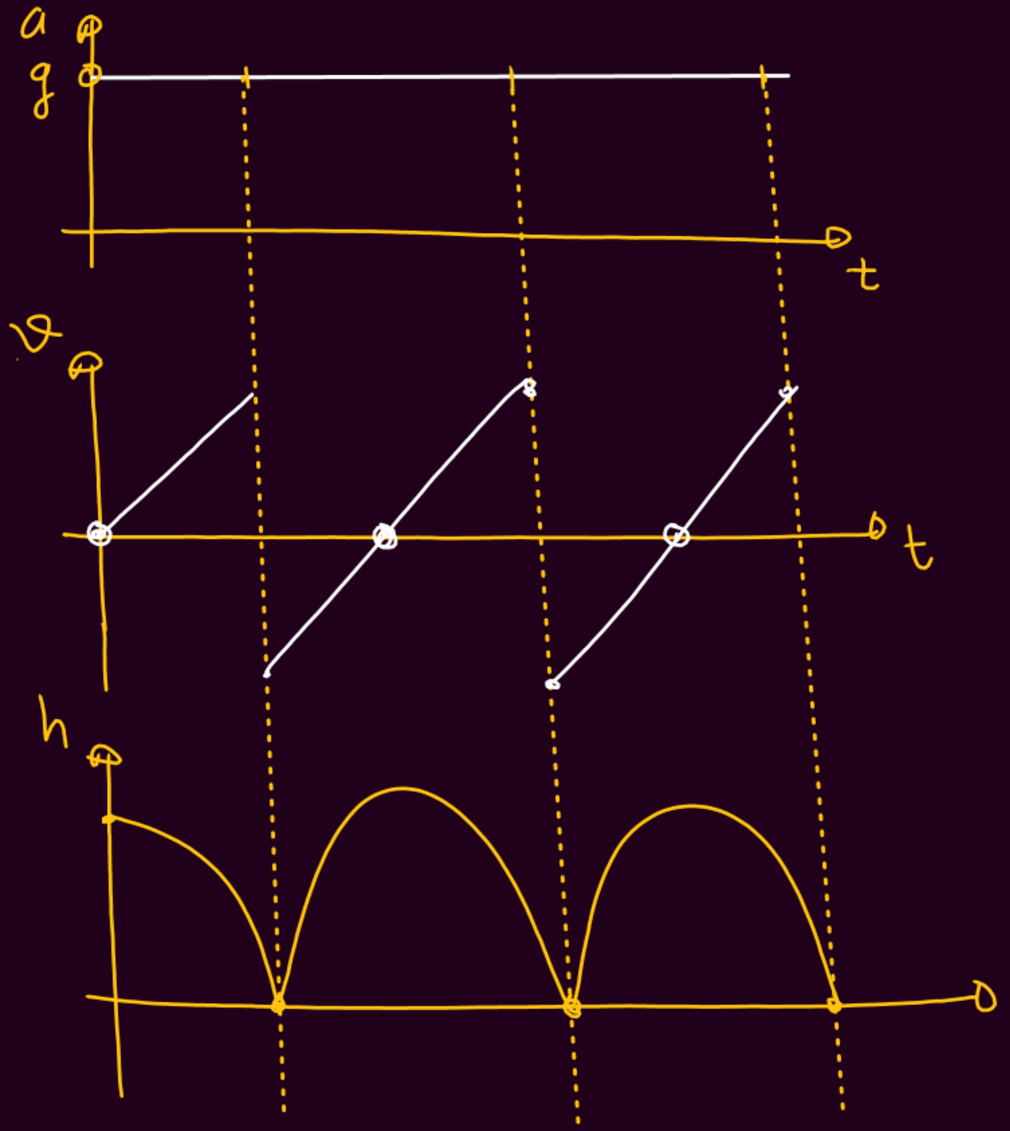
Some graphs are not possible



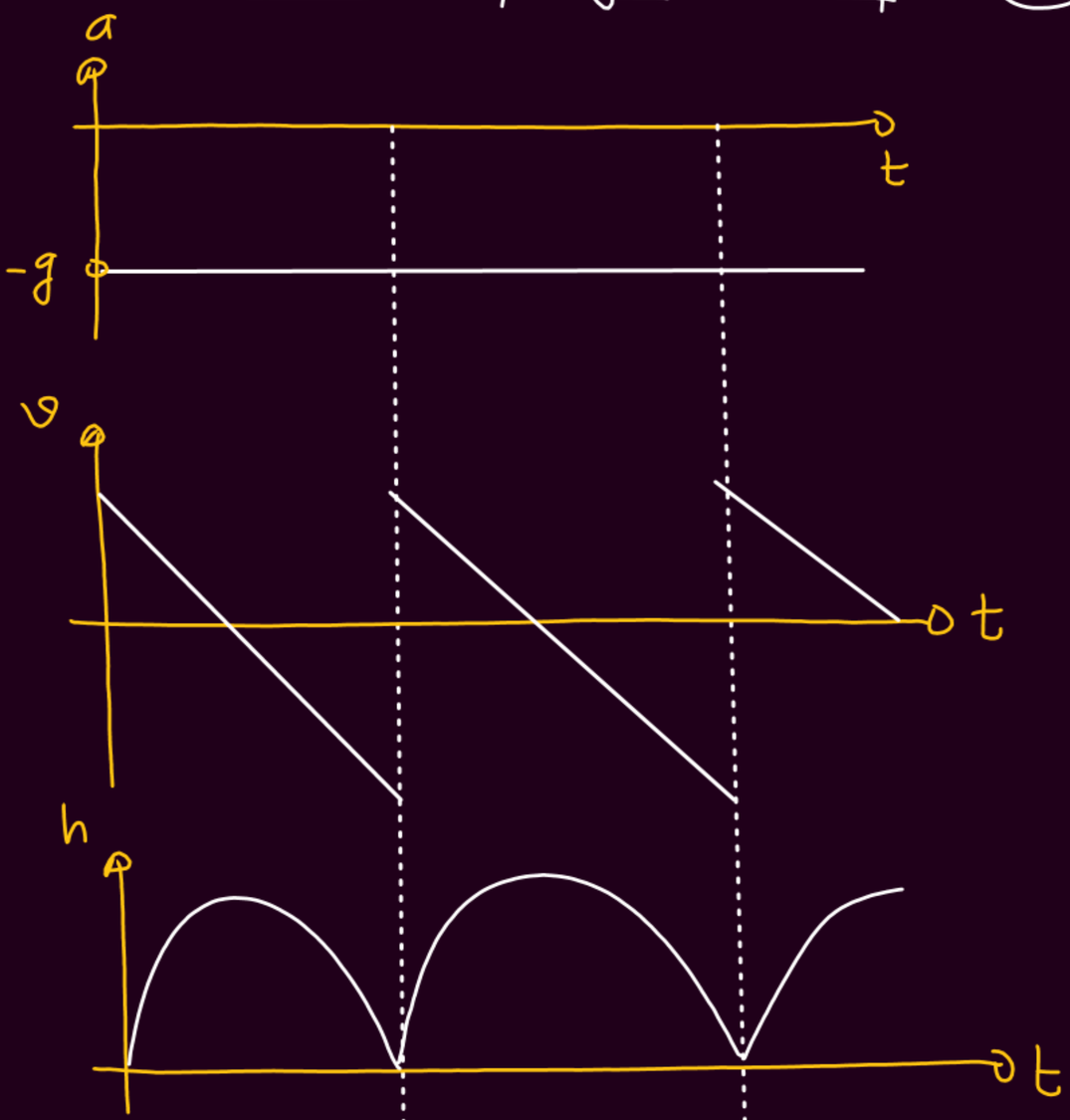


Motion Under gravity

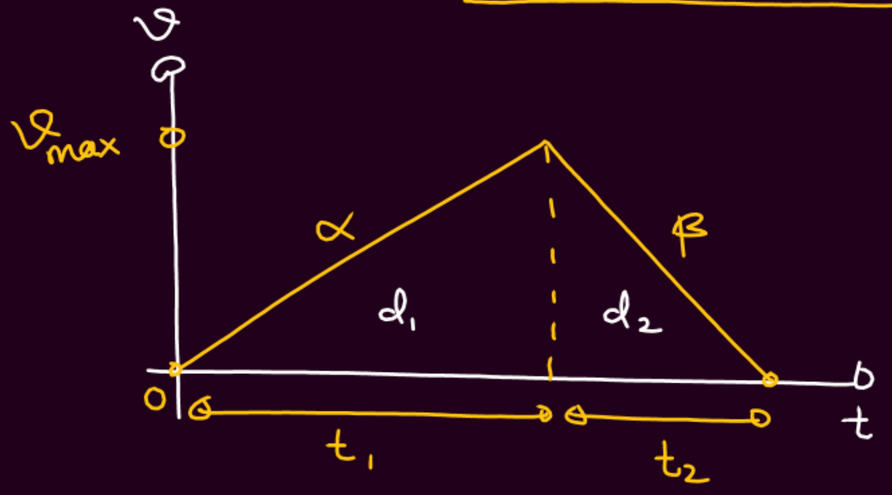
① object is dropped (Elastic) (d+)



② object is projected Up (q1)



Case Tram Min time bet^{n} Two stations



① $v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right) t_{min}$

② $d = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right) t_{min}^2$

③ $v_{max}^2 = 2\left(\frac{\alpha\beta}{\alpha+\beta}\right) d$

④ Avg Vel. $= \frac{v_{max}}{2}$

⑤ $d = \frac{v_{max}}{2} t_{min}$

⑥ $\frac{d_1}{d_2} = \frac{t_1}{t_2} = \frac{\beta}{\alpha}$

Q. Tram $\alpha = 4\ m/s^2$

$\beta = 6\ m/s^2$

$d = 2\ km$

① $t_{min} = ?$

$$d = \frac{1}{2}\frac{\alpha\beta}{\alpha+\beta} t_{min}^2$$

$$2000 = \frac{4\times 6}{2\times 10} t_{min}^2$$

$$t_{min}^2 = \frac{20\times 2000}{24}$$

$$t_{min} = \frac{200}{2\sqrt{6}} = \frac{100}{\sqrt{6}}\ sec$$

② $v_{max} = ?$

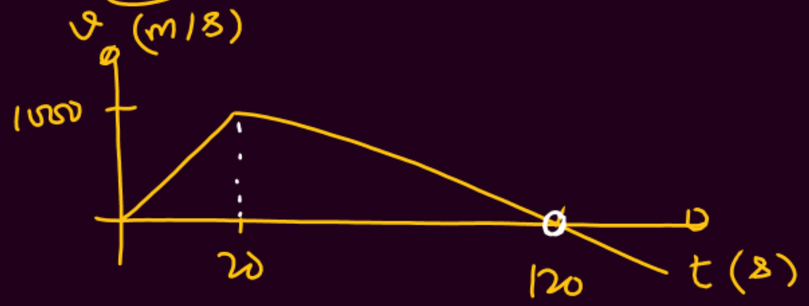
$$v_{max} = \frac{\alpha\beta}{\alpha+\beta} t_{min}$$

$$= \frac{(4)(6)}{10} \frac{100}{\sqrt{6}}$$

$$v_{max} = 40\sqrt{6}\ m/s$$

Pg-63

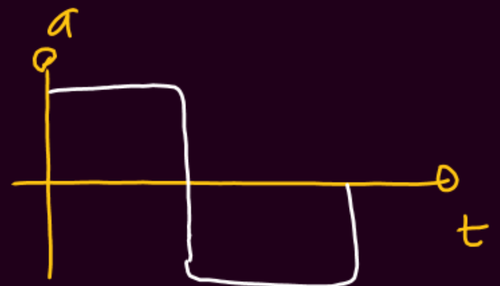
(33)



$$H_{max} = \text{Area} = \frac{1}{2}(120)\,1050$$

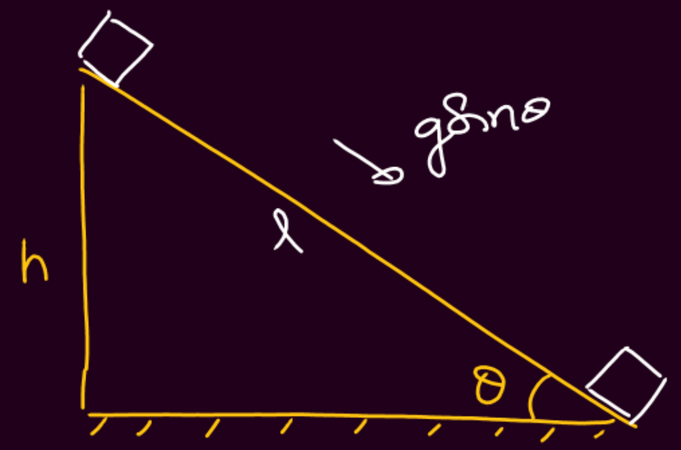
$$= 60 \text{ km}$$

(22)



Pg-52

(7)

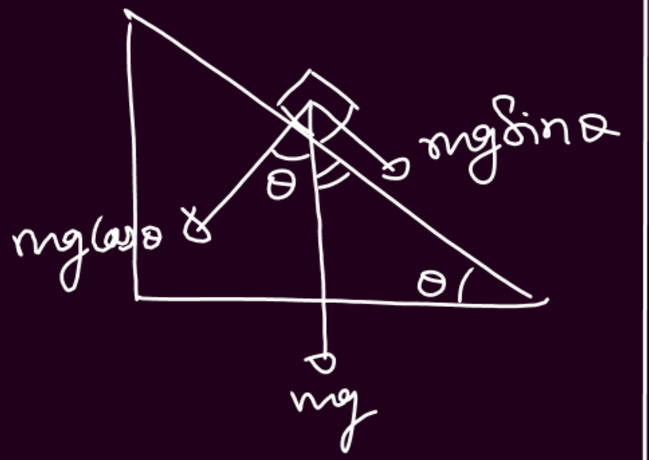


$$S = \frac{1}{2} a t^2$$

$$\frac{h}{\sin\theta} = l = \frac{1}{2}(g\sin\theta)t^2$$

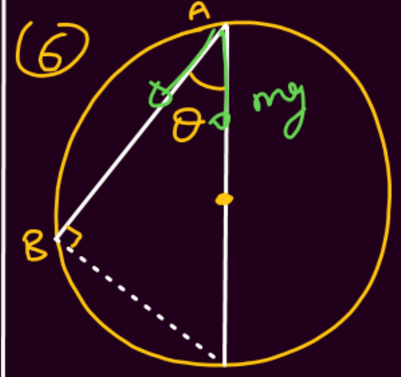
$$\frac{2h}{g\sin^2\theta} = t^2$$

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$



$$l\sin\theta = h$$

(6)



$$S = \frac{1}{2} a t^2$$

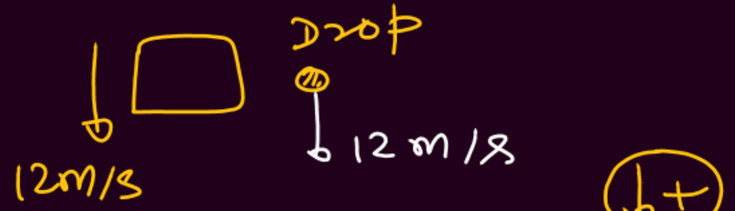
$$(2R\cos\theta) = \frac{1}{2}(g\cos\theta)t^2$$

$$\frac{4R}{g} = t^2$$

$$t = 2\sqrt{\frac{R}{g}}$$

Pg-51

(1)



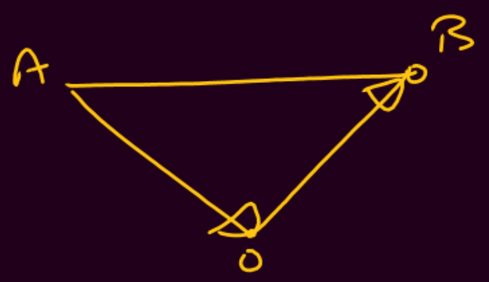
(b+)

$$S = ut + \frac{1}{2}at^2$$

$$S = 12(10) + \frac{1}{2}(10)(10)^2$$

$$= 620\text{m}$$

Relative Motion



$$\vec{AB} = \vec{AO} + \vec{OB}$$

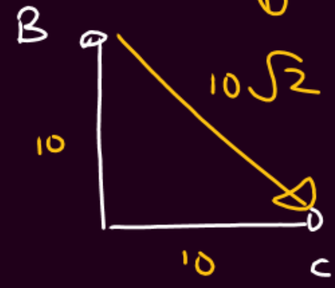
$$\checkmark \quad \vec{v}_{AB} = \vec{v}_{AO} + \vec{v}_{OB}$$

$$\checkmark \quad \vec{v}_{AB} = \vec{v}_{AO} - \vec{v}_{BO}$$

Q $v_{car} = 10\ m/s$ East

$v_{Bus} = 10\ m/s$ North

$\vec{v}_{CB} = ?$



$10\sqrt{2}$ S-E

$$\vec{v}_{CB} = \vec{v}_C - \vec{v}_B$$

Relative in 1D

Relative [Same dir → Subtract; Opp dir → Add]

①

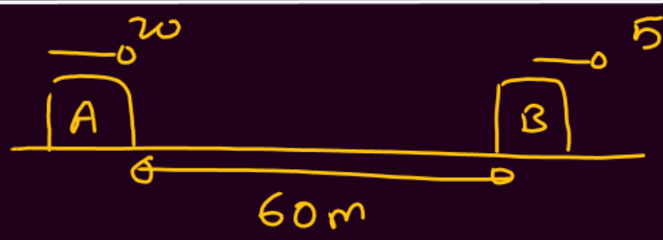
$v_{BA} = v_B - v_A$

$v_{BA} = 20 - 10 = 10\ m/s$

②

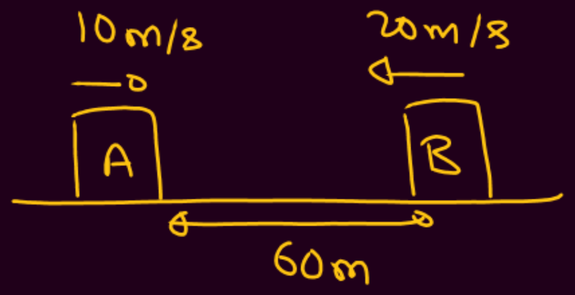
$v_{BA} = v_B - v_A$

$v_{BA} = (-20) - 10 = -30\ m/s$

Q. 

wrt B

$t_{meet} = \frac{S_{rel}}{v_{rel}} = \frac{60}{20-5} = 4\ s$

Q. 

$t_{meet} = \frac{S_{rel}}{v_{rel}} = \frac{60}{10+20} = 2\ s$

Case

$$v_m + v_e = (v_m + v_e)$$

$$\frac{d}{t_1} + \frac{d_2}{t_2} = \frac{d}{t}$$

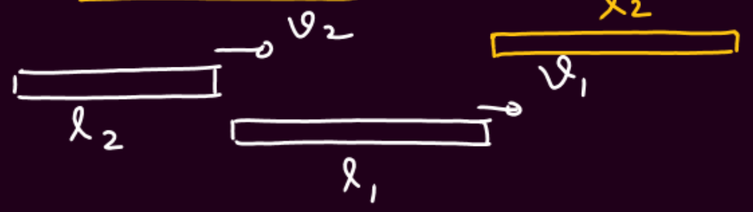
① Man is walking $\Rightarrow t_1$

② Man is standing on moving Escalator (Platform) $\Rightarrow t_2$

③ Man is walking on moving Escalator $\Rightarrow$ $\boxed{t = \frac{t_1 t_2}{t_1 + t_2}}$

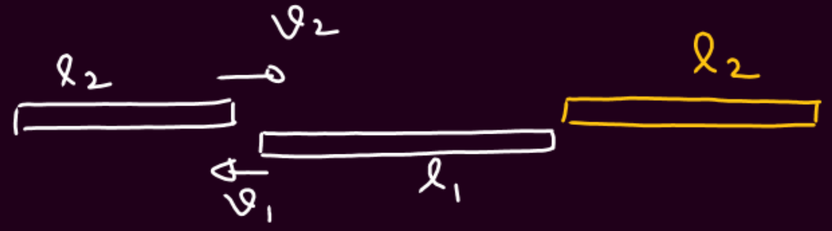
Crossing of Trains

① Same dir



$$t = \frac{l_1 + l_2}{v_2 - v_1}$$

② Opp dir



$$t = \frac{l_1 + l_2}{v_1 + v_2}$$

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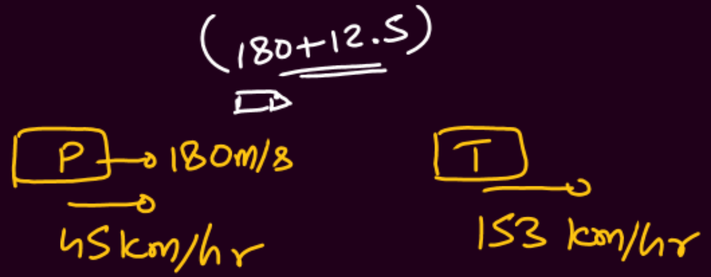
(4)



Muzzle vel. = $v_{bullet,\,car}$

$45 \times \frac{5}{18} = 12.5\ m/s$

$153 \times \frac{5}{18} = 42.5\ m/s$

$v_{Bullet,\,Theif} = (180 + 12.5) - (42.5) = \underline{150\ m/s}$

Case: Coin & Lift

$\downarrow a$

$\downarrow v_0$

$$a_{coin,gr} = g\downarrow$$

$$a_{cl} = a_c - a_l$$
$$= g - (-a)$$
$$= g + a$$

① Drop

$$S_{cl} = u_{cl}t + \frac{1}{2}a_{cl}t^2 \quad (\downarrow +)$$

$$h = 0 + \frac{1}{2}(g+a)t^2$$

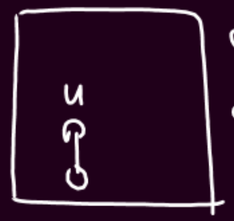
② Projected Up.

$\downarrow v_0 = \text{const.}$

$$S_{cl} = u_{cl}t + \frac{1}{2}a_{cl}t^2$$

$$0 = ut + \frac{1}{2}(g-0)t^2$$

$$t = \frac{2u}{g}$$



$\downarrow a$

$\downarrow v_0$

$$S_{cl} = u_{cl}t + \frac{1}{2}a_{cl}t^2$$

$$0 = ut + \frac{1}{2}(g-(-a))t^2$$

$$t = \frac{2u}{g+a}$$

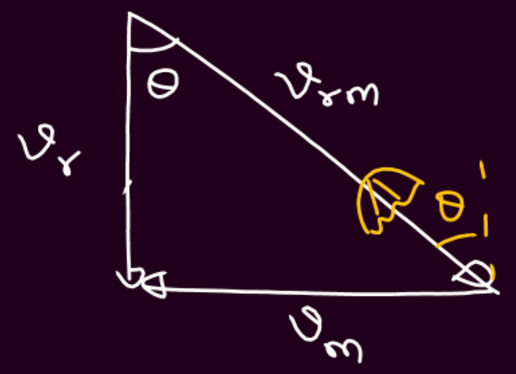
Q

$u = 10\ m/s$

if coin reaches floor in 1s then find $a_2 = ?$

Rain-man

① Rain is falling vertically

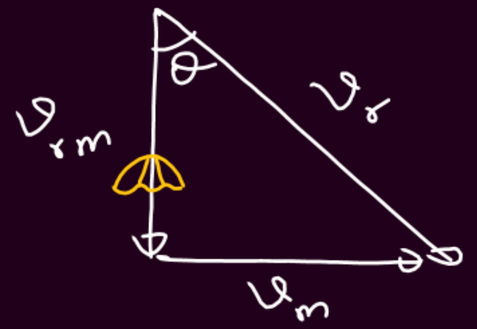


① $v_{rm}^2 = v_r^2 + v_m^2$

② $\tan\theta = \frac{v_m}{v_r}$

② Rain is falling at θ to vertical

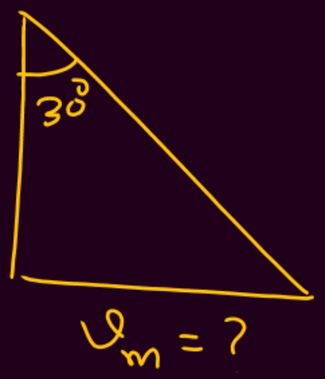
"But appears to be vertical"



① $v_r^2 = v_{rm}^2 + v_m^2$

② $\sin\theta = \frac{v_m}{v_r}$

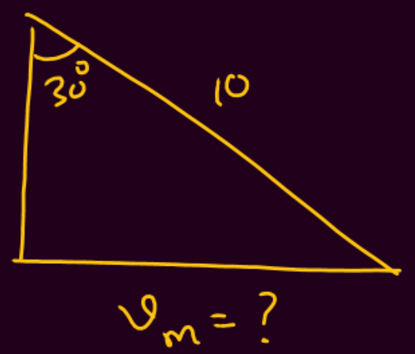
Q.



$$\tan 30^\circ = \frac{v_m}{5}$$

$$v_m = \frac{5}{\sqrt{3}} \text{ km/hr}$$

Q. Ram 10 km/hr at 30°



$$\sin 30^\circ = \frac{v_m}{10}$$

$$v_m = 5 \text{ km/hr}$$

Case Whenever 2 objects are in Air (free fall)

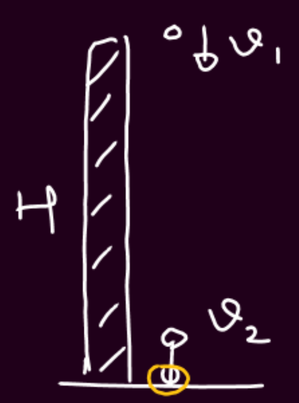
① Both $g \downarrow$

② $a_{rel} = 0$

③ v_{rel} = Const.

④ $S_{rel} = v_{rel}\, t$

⑤ Path of 1 wrt 2 will be st. line



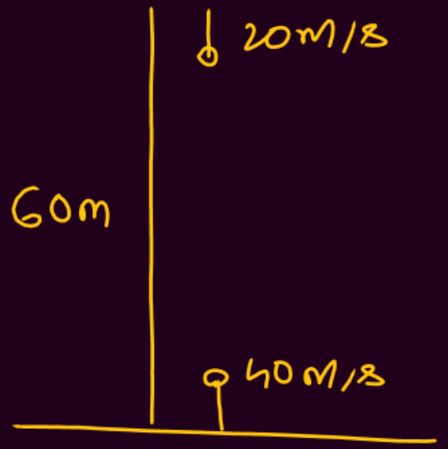
When & where

① $t = \dfrac{S_{rel}}{v_{rel}} = \dfrac{H}{v_1 + v_2}$

② $h = v_2 t - \frac{1}{2} g t^2$

Q

"Simultaneously"



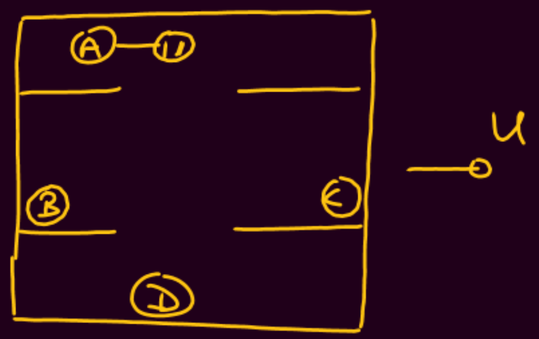
① $t = \frac{60}{20 + 40} = 1s$

② $h = 40(1) - \frac{1}{2}(10)(1)^2 = \underline{\underline{35m}}$

Case

① Dropped from Helicopter / Tram

path of object wrt HC is vertically down

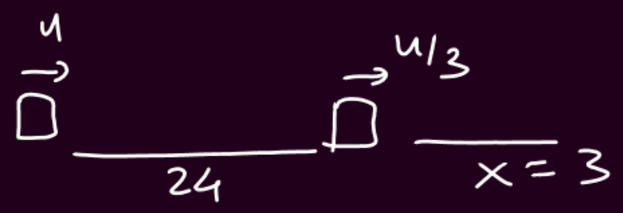
② 

① u = const. D

② Acc. [u ↑] B

③ Retard [u ↓] C

(21)



(1) $\left(\frac{u}{3}\right)^2 = u^2 - 2a(24)$

(2) $0 = \left(\frac{u}{3}\right)^2 - 2ax$

$u^2 = g(2ax)$

$2ax = g(2ax) - 24(2a)$

$24 = 8x$

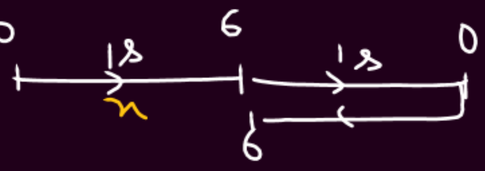
$x = 3$

(16) $\frac{S_n}{S_{n+1}} = \frac{0 + \frac{a}{2}(2n-1)}{0 + \frac{a}{2}[2(n+1)-1]} = \frac{2n-1}{2n+1}$

(8) $v = \beta x^{-2n}$

$a = v\frac{dv}{dx} = \left(\beta x^{-2n}\right)\left(-2n\beta x^{-2n-1}\right)$

(13)



$a = \frac{6-0}{1}$

$x = \frac{1}{2}(6)(1)^2 = 3m$

Avg vel. $= \frac{x}{3} = \frac{3}{3} = 1\ m/s$

Avg. sp. $= \frac{3x}{3} = x = 3\ m/s$

(34) $ut - \frac{1}{2}gt^2 = u(t-4) - \frac{1}{2}g(t-4)^2$

$u[t-(t-4)] = \frac{g}{2}[t^2 - (t-4)^2]$

$u(4) = \frac{g}{2}(2t-4)(4)$

$98 = \frac{9.8}{2}(2t-4)$

$20 = 2t - 4$

$t = 12\text{s}$

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(43)

$v_{rel}^2 = v^2 - 2\alpha x \geq 0$

$v^2 \geq 2\alpha x$

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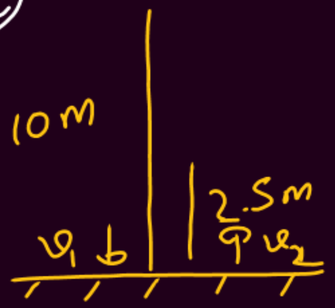
(2)



$v_1 = \sqrt{2(10)(10)} = 10\sqrt{2}$

$v_2 = \sqrt{2(10)(2.5)} = 5\sqrt{2}$

$a_{av} = \frac{\Delta v}{\Delta t} = \frac{15\sqrt{2}}{0.01}$

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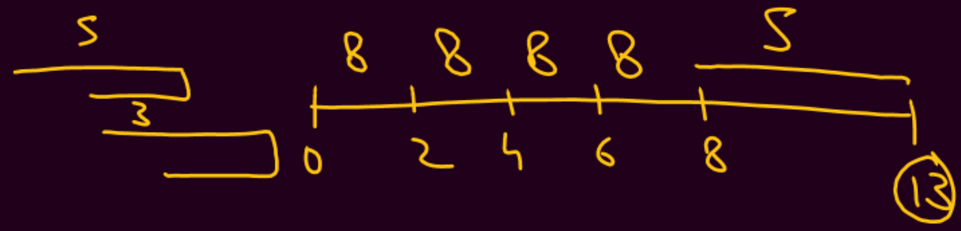
(9)

$u = g\left(\frac{t_1 + t_2}{2}\right)$

(6)