Wave Motion

→ Disturbance

→ Energy & Momentum

① Mechanical Wave → Req Med.

→ Sound | water wave | String

② Electromagnetic Wave (EMW)

↳ Doesn't Req. med

Light / Radiation

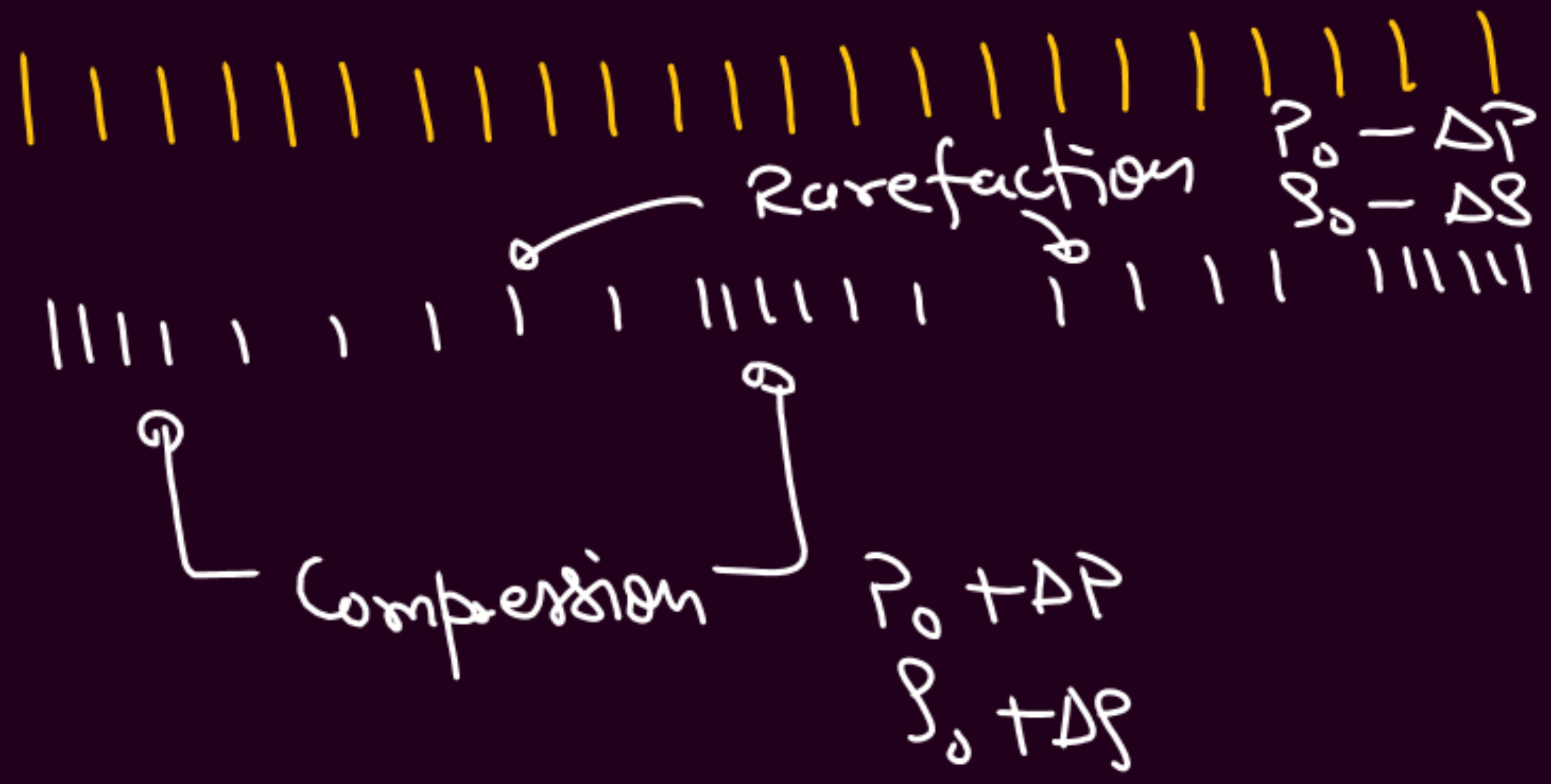
→ λ

① Transverse Wave

→ EMW (all)

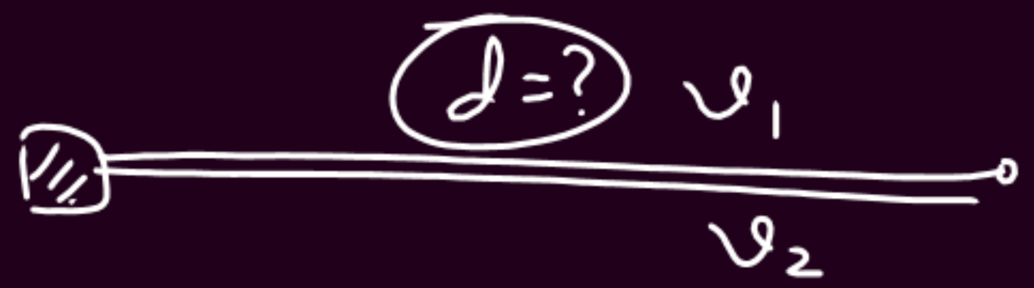
→ String

② Longitudinal Wave → Sound



- → Water Ripples → Trans & long [particle → Elliptical]
- → Earthquack → — " —

Epicenter [Scorpion]

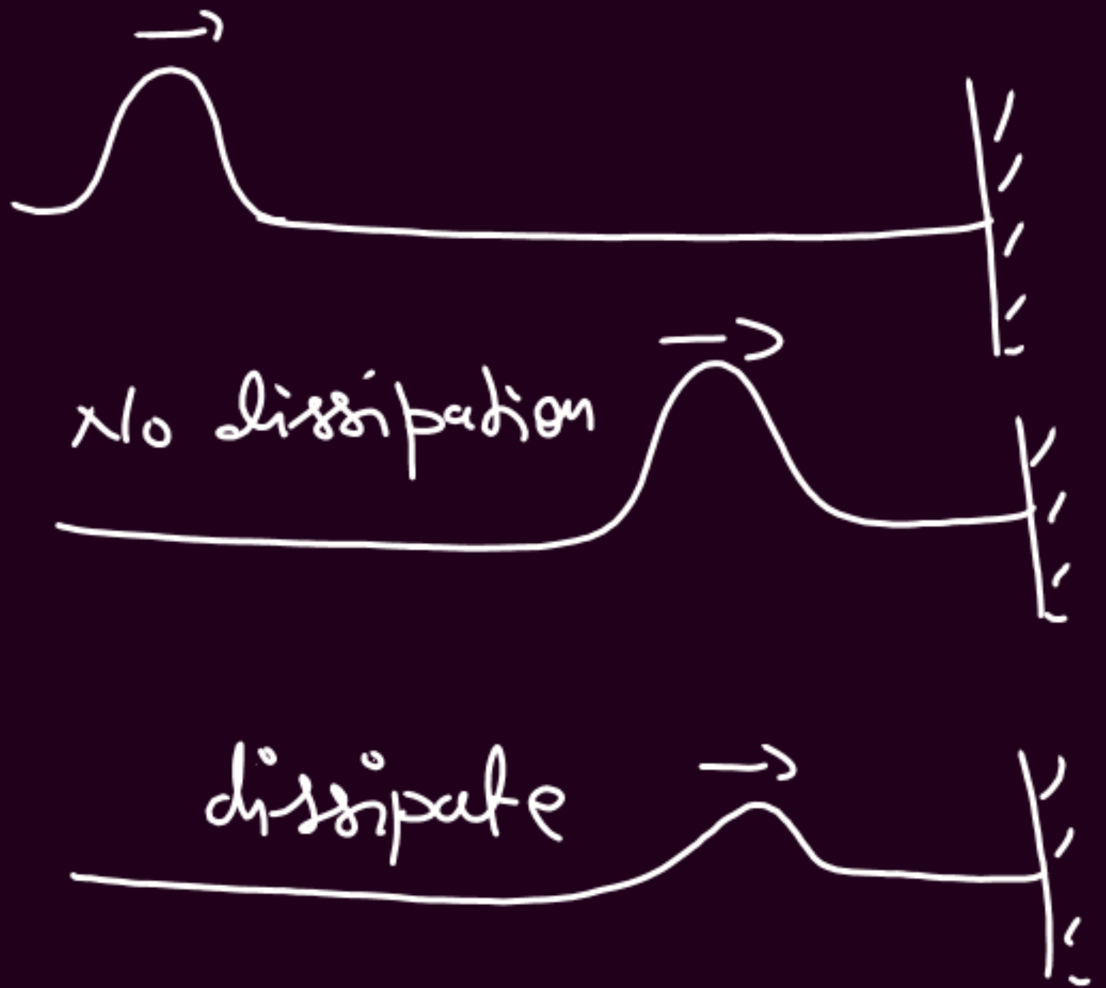


$$\boxed{\Delta t = \frac{d}{v_1} - \frac{d}{v_2}}$$

→ Mechanical Waves

- → Elasticity
- → Inertia

Assumptions
- → Isotropic
- → No dissipation



Wavelength (λ)

Wave Equation

$$y = f\boxed{\left(t - \frac{x}{v}\right)} \quad (-ve)\ x\text{-dir}$$

$$y = f\left(t + \frac{x}{v}\right) \quad (+ve)\ x\text{-dir}$$

$$(at - bx)$$

$$a\left(t - \frac{b}{a}x\right)$$

$$y = f(at - bx)$$

$$y = f(at + bx)$$

Wave speed $v = \dfrac{\lambda}{T} = \lambda f = \dfrac{\text{coeff of 't'}}{\text{coeff of 'x'}}$

Wave

- Continuous
- Bound

① $y = A\sin(\omega t - kx + \phi)$

② $y = A\cos(\omega t - kx + \phi)$

Continuous

③ $y = Ae^{-(at \pm bx)^2} = \dfrac{A}{e^{(\ )^2}}$

④ $y = \dfrac{A}{B + (at \pm bx)^2}$

wave pulse

$\boxed{e = 2.72}$

Not Wave

1) $y = (at + bx)$

2) $y = (at + bx)^2$

3) $y = e^{(at + bx)}$

4) $y = e^{-(at + bx)}$

5) $y = \log(at + bx)$

6) $y = \dfrac{A}{B + (at + bx)}$

7) $y = \dfrac{A}{B - (at + bx)^2}$

$$y = (10\,\boxed{cm})\sin\left(\pi t - \frac{\pi}{2}x + \frac{\pi}{3}\right)$$

$\left\{\begin{array}{l} t \text{ is in 's'} \\ x \text{ — " — } \boxed{\text{'m'}} \end{array}\right.$

$$\boxed{v = \frac{\pi}{(\pi/2)}} = 2 \text{ m/s}$$

$\left\{\begin{array}{l} \boxed{2} \text{ cm/s} \\ \underline{\underline{200 \text{ cm/s}}} \end{array}\right.$

$$y = Ae^{-(3x+2t)^2}$$

$$v = \frac{2}{3} \quad \underline{\underline{\text{units}}}$$

Plane Progressive Simple Harmonic Wave

$$y = A \sin(\underbrace{\omega t} - \underbrace{kx} + \phi)$$

$$y = A \cos(\omega t - kx + \phi)$$

A = Amplitude

ω = Ang. freq.

k = Propagation const
[Angular wave no.]

f = freq.

$$\boxed{v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k} = \frac{\text{Coeff of } t}{\text{Coeff of } x}}$$

$$\omega T = 2\pi$$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

$$k\lambda = 2\pi$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

$\lambda \rightarrow 1$ wave

$1 \rightarrow \frac{1}{\lambda}$ = wave no. = no. of waves per unit length

$$\boxed{\frac{\Delta \phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{\Delta t}{T}}$$

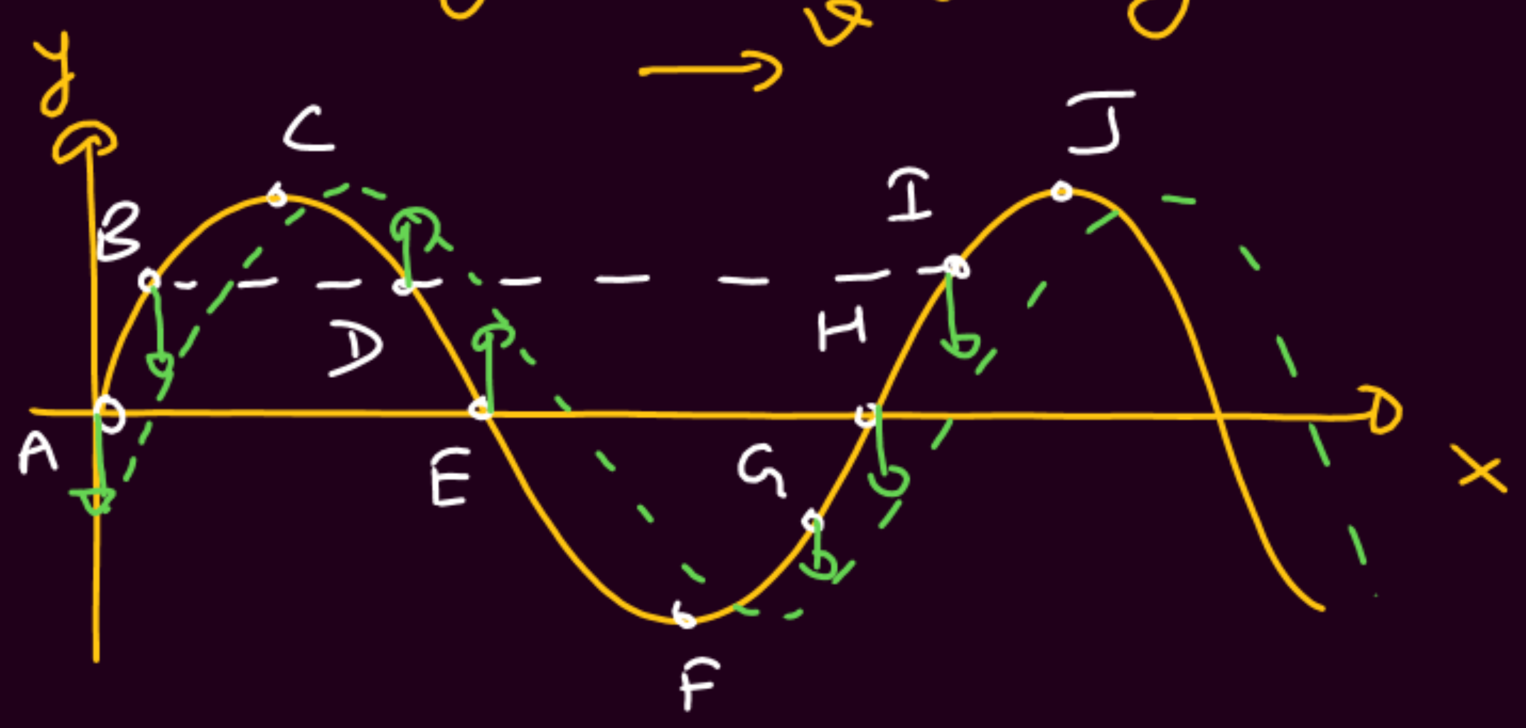
$T \,/\, \lambda \,/\, 2\pi$

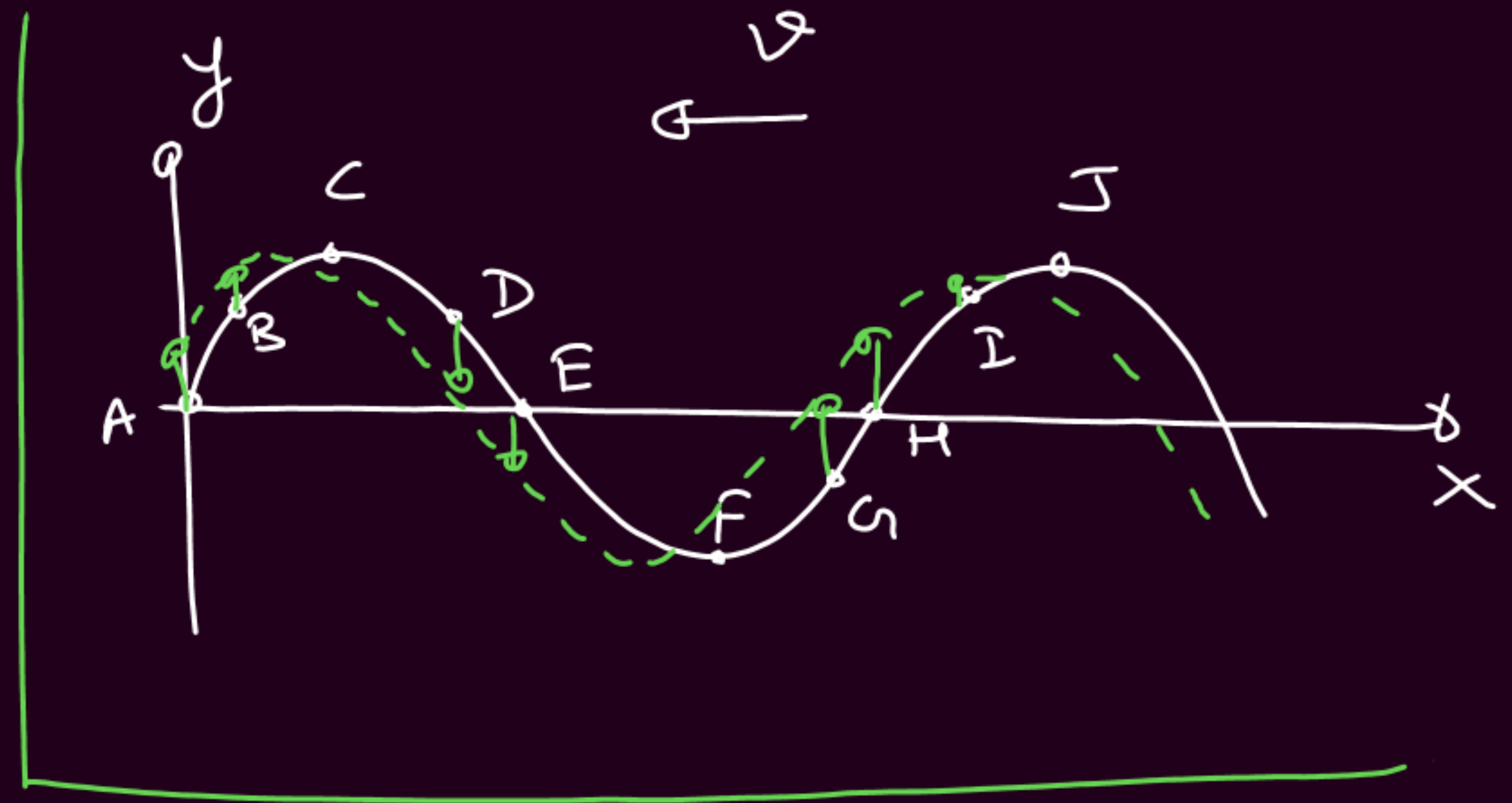
$\Delta \phi$ = phase diff.

Δx = path —"—

Δt = time —"—

Leading & lagging





→ A leads C by $\pi/2$

→ C leads F by π

[C & F are out of phase]

→ H lags C by $3\pi/2$

→ B leads I by 2π

(B and I are in same phase)

Dir of vel

Upward	D/E
Downward	A/B/G/H/I
Inst. Rest	C/F/J

$$y = A\sin(\omega t - kx + \phi)$$

① wave speed $v = \frac{\omega}{k}$

② Max. particle speed $v_{p_{max}} = \omega A$

③ Max particle Acc. $a_{p_{max}} = \omega^2 A$

Extra

④ $\boxed{v_p = -v(\text{slope})}$

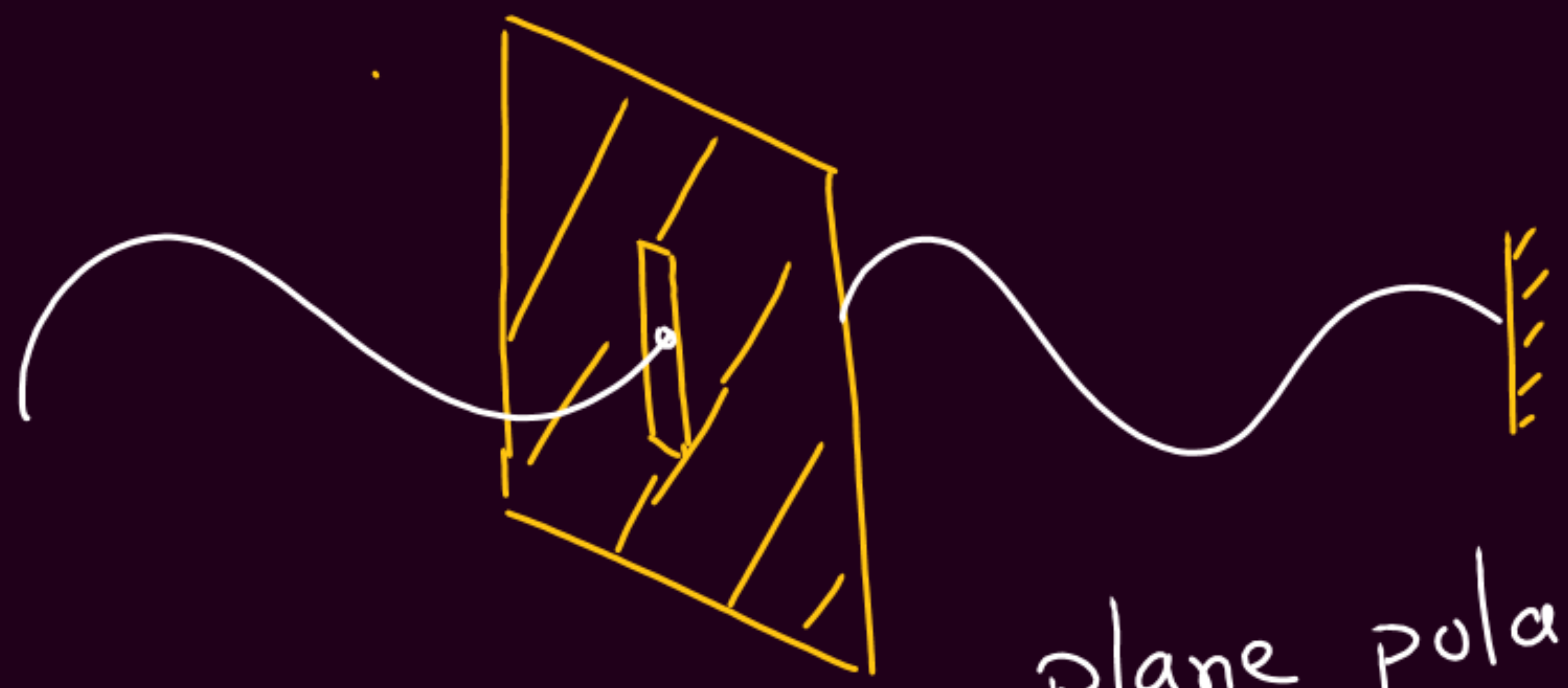
SHM

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Wave

$$\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$$

Polarization [only for transverse wave]



plane polarisation

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1, 3, 4, 5,

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1 to 12

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1 to 12

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1, 3, 8, 9

Q $y = (10\,cm)\sin\left(\frac{\pi}{2}t - 2\pi x + \frac{\pi}{3}\right)$ $\left[\begin{matrix} x \text{ is in } m \\ t \text{ is in } s \end{matrix}\right]$

(1) $A = 10\,cm$

(2) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/2} = 4s$

(3) $\lambda = \frac{2\pi}{K} = \frac{2\pi}{2\pi} = 1m$

(4) $v = \frac{\omega}{K} = \frac{\pi/2}{2\pi} = \frac{1}{4}\,m/s$

(5) $v_{p\,max} = \omega A = \frac{\pi}{2}(10) = 5\pi\,cm/s$

(6) Disp. of particle at $x = 0.5m$, $t = 4s$

$y = 10cm \sin\left(\overset{2\pi}{\frac{\pi}{2}(4)} - \overset{\pi}{2\pi(0.5)} + \frac{\pi}{3}\right)$

$y = 10cm \sin\left(\underset{180^\circ}{\pi} + \underset{60^\circ}{\frac{\pi}{3}}\right)$

$y = -(10\,cm)\sin(\pi/3)$

$y = -10(\sqrt{3}/2) = \underline{\underline{-5\sqrt{3}\,cm}}$

$$y = A\sin(\omega t - kx + \phi) = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right)$$

Q $$y = A\sin\left(\pi t - \frac{\pi}{2}x + \frac{\pi}{6}\right)$$

$$\cancel{\omega} A = \frac{\cancel{\omega}}{K}$$

$$A = \frac{1}{K} = \frac{1}{\pi/2} = \frac{2}{\pi}$$

$$y = 5 \sin \frac{\pi}{2}\left(\frac{t}{2} - \frac{x}{3}\right) \quad cm$$

$\left\{ \begin{array}{l} x \text{ is in } m \\ t \text{ is in } s \end{array} \right.$

① $A = 5\,cm$

② $f = \frac{\omega}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8}\ Hz$

③ wave no. $= \frac{1}{\lambda} = \frac{1}{12}\ m^{-1}$

④ wave speed $= \frac{\pi/4}{\pi/6} = 3/2\ m/s$

$\lambda = \frac{2\pi}{K} = \frac{2\pi}{\pi/6}$

$\lambda = 12\ m$

Extra

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$

1) $l << R$ $\left(\frac{1}{l} >> \frac{1}{R}\right)$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

2) $l >> R$ (Pendulum of infinite length)

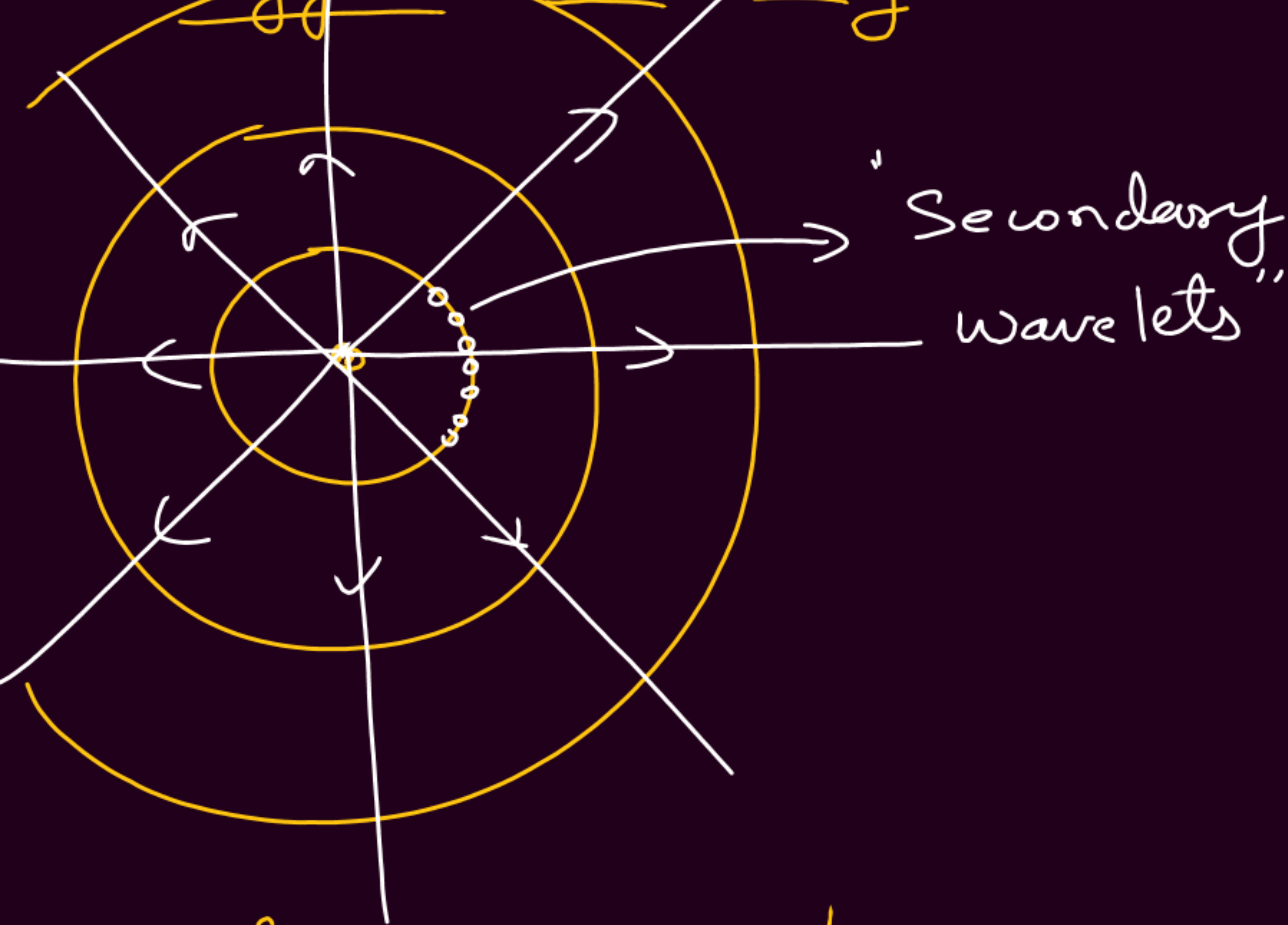
$$T = 2\pi\sqrt{\frac{R}{g}} \simeq 84.6 \text{ min}$$

3) $l = R$

$$T = 2\pi\sqrt{\frac{R}{2g}} \simeq \frac{84.6}{\sqrt{2}} = 84.6(0.7)$$

$$\boxed{\frac{500}{\sqrt{2}} \simeq 500(0.7)}$$

Huygen Wave Theory



Wavefronts ⇒ collection / locus

Energy / Power / Intensity

SHM $E = \frac{1}{2} m \omega^2 A^2$

Wave

Energy Density $= \frac{1}{2} \rho \omega^2 A^2$

$$\text{Intensity} = \frac{\text{Energy}}{(\text{Area})(\text{time})} = \frac{\text{Power}}{\text{Area}}$$

$$\boxed{I = \frac{1}{2} \rho \omega^2 A^2 v}$$

Heavy Rope → [free End]

$$\left\{ \sqrt{gx} = \sqrt{2as} \quad 2a = g \right\}$$

$$\left\{ l = \frac{1}{2}\left(\frac{g}{2}\right)t^2 \right\}$$

① $v = \sqrt{gx}$

② $a = \frac{g}{2}$

③ $t = 2\sqrt{\frac{l}{g}}$

$v = \sqrt{\frac{T}{\mu}}$

$$\boxed{v = f\lambda}$$

① $\frac{T_A}{T_B} = \frac{m_1}{m_1+m_2}$

② $\frac{f_A}{f_B} = \frac{1}{1}$

③ $\frac{v_A}{v_B} = \sqrt{\frac{m_1}{m_1+m_2}}$

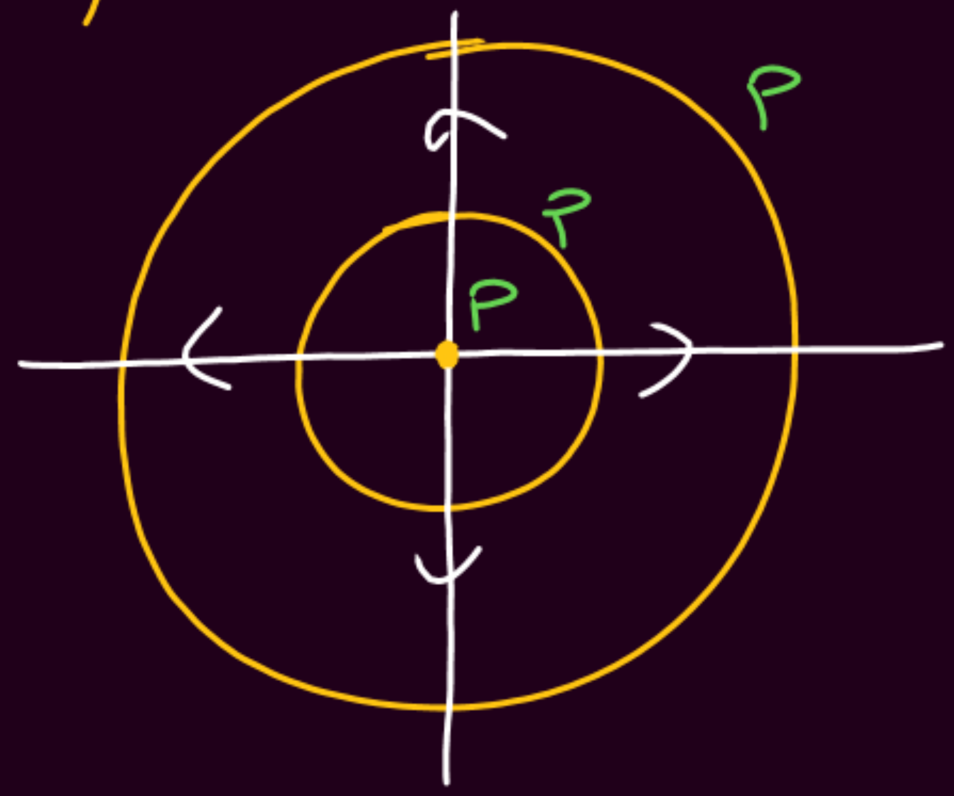
$\frac{v_A}{v_B} = \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{m_1}{m_1+m_2}}$

Wavefront

$$I = \frac{P}{\text{Area}}$$

$$\boxed{I \propto A^2}$$

Spherical/Point Source



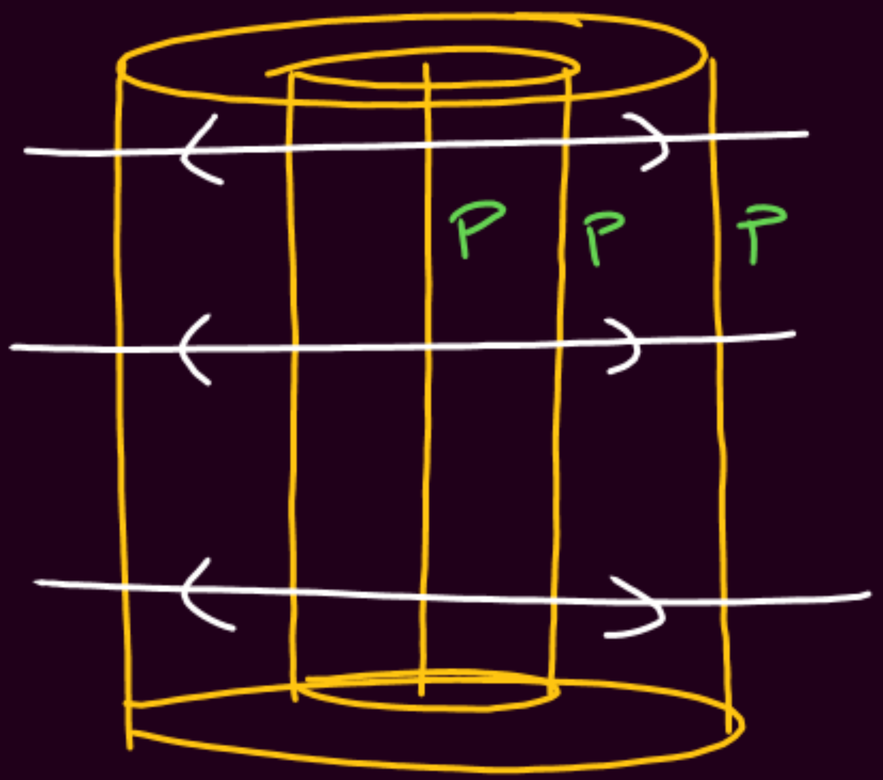
spherical

$$I = \frac{P}{4\pi r^2}$$

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{A_2}{A_1} = \frac{r_1}{r_2}$$

Line Source



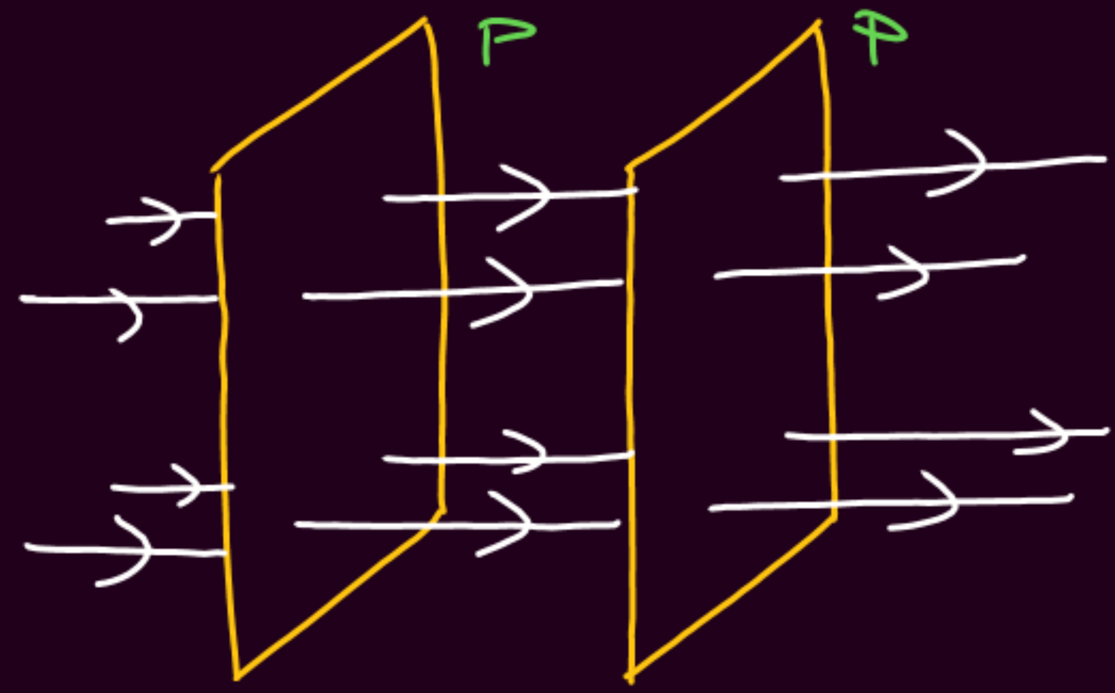
Cylindrical

$$I = \frac{P}{2\pi r l}$$

$$\frac{I_2}{I_1} = \frac{r_1}{r_2}$$

$$\frac{A_2}{A_1} = \sqrt{\frac{r_1}{r_2}}$$

Infinitely large Source (or Source at ∞)



parallel planes

$$I = \frac{P}{A}$$

$$\boxed{I = \text{const.}}$$

$$\boxed{A = \text{const}}$$

$$y_1 = 10\sin\left(\pi t - \frac{\pi}{2}x\right)$$

$$I = \frac{1}{2}\rho\omega^2 A^2 v$$

$$y_2 = 5\sin\left(2\pi t - \pi x\right)$$

① $\dfrac{A_1}{A_2} = \dfrac{10}{5} = \dfrac{2}{1}$

② $\dfrac{v_1}{v_2} = \dfrac{\left(\dfrac{\pi}{\pi/2}\right)}{\left(\dfrac{2\pi}{\pi}\right)} = \dfrac{2}{2} = \dfrac{1}{1}$

③ $\dfrac{I_1}{I_2} = \dfrac{(\omega_1 A_1)^2}{(\omega_2 A_2)^2} = \left[\dfrac{\pi\,(10)}{2\pi\,(5)}\right]^2 = \dfrac{1}{1}$

Wave Speed on a Stretched String

$$\boxed{v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}}$$

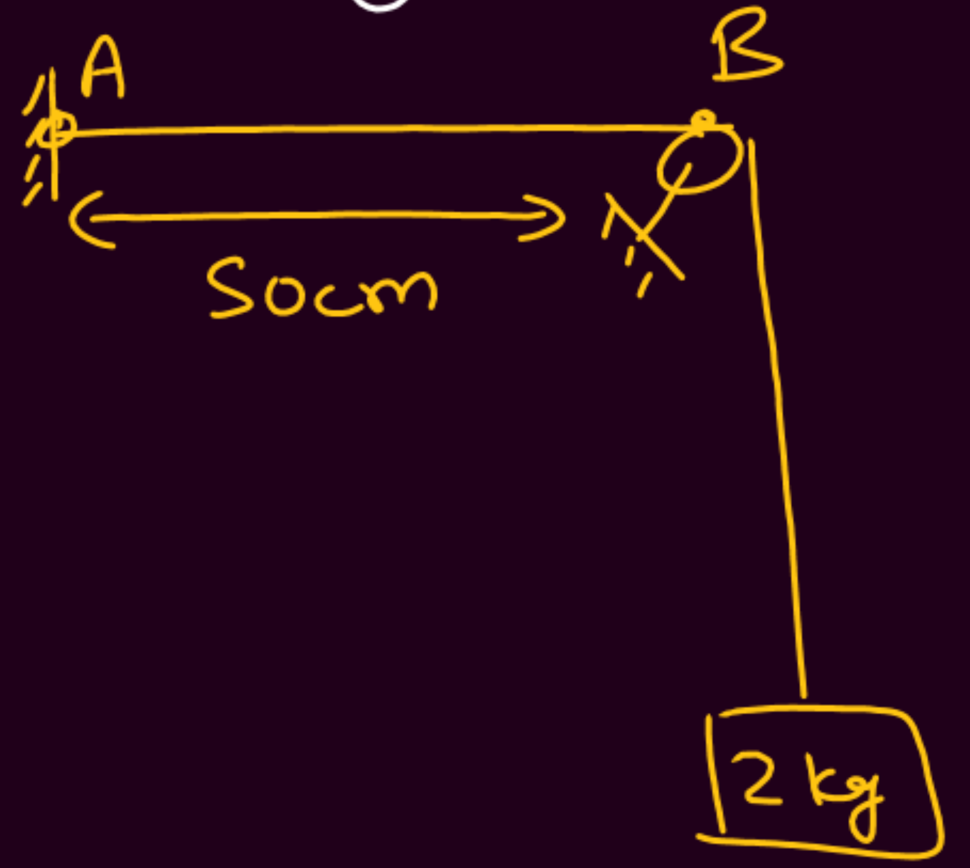
$T =$ Tension

$\mu = \frac{m}{l}$

$\rho = \frac{m}{Al}$

$\rho A = \mu$

$$\boxed{A = \pi r^2}$$

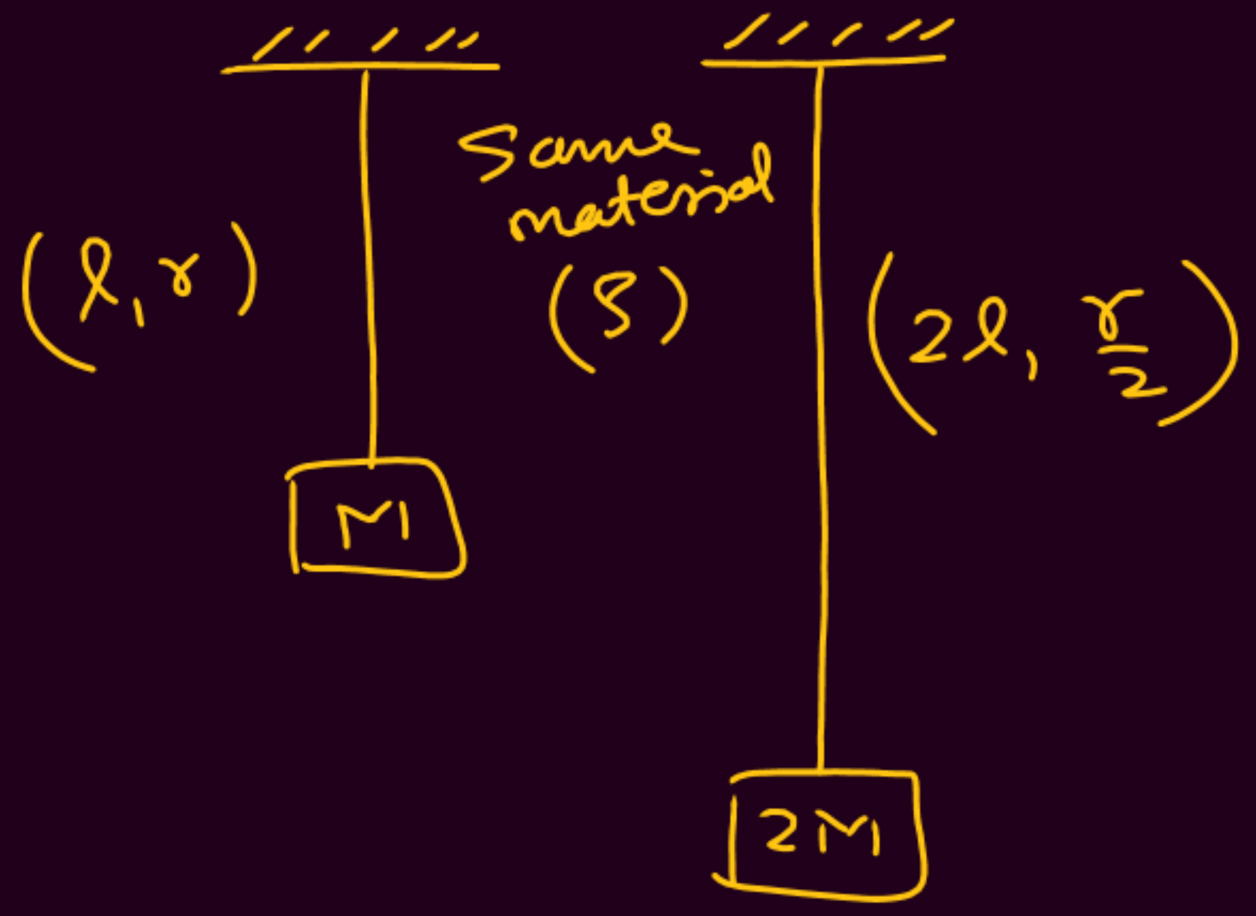


Rope $\Rightarrow$ (50g , 2m)

$t_{A \to B} = ?$

$$t = \frac{d}{v} = \frac{0.5}{20\sqrt{2}} = \frac{1}{40\sqrt{2}} \text{ sec}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{\left(\frac{0.05}{2}\right)}} = \sqrt{\frac{\cancel{20}^{4} \times 2 \times 100}{\cancel{5}}} = \underline{20\sqrt{2} \text{ m/s}}$$



$$v = \sqrt{\frac{T}{\rho A}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}\,\frac{r_2^2}{r_1^2}} = \sqrt{\frac{mg}{2mg}\,\frac{(r/2)^2}{r^2}} = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

Speed of wave in diff med

1. Stretched String $\quad v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{\rho A}}$ $\quad$ Transverse

2. Rod

$$v = \sqrt{\frac{Y}{\rho}}$$ longitudinal $\quad Y =$ Young's Modulus

$$v = \sqrt{\frac{\eta}{\rho}}$$ Transverse

$\eta =$ modulus of rigidity (Shear Modulus)

3. Bulk Solid $\rightarrow$ Transverse & longitudinal (Earthquake)

④ Bulk Liq → $v = \sqrt{\frac{B}{S}}$ longitudinal

B = Bulk Modulus

⑤ Liquid Surface → Transverse + longitudinal

(Surface Tension)

water ripples → elliptical

⑥ Gases → $v = \sqrt{\frac{B}{S}}$ (longitudinal)

Sound Wave

- → Mech. [req. med.]
- → Longitudinal
- → No sound on moon / in vacuum
- → "Pressure"

$$\boxed{\Delta P_0 = BAk}$$

$$k = \frac{2\pi}{\lambda}$$

20 Hz — 20 kHz

Infrasonic

Ultrasonic

Pitch → freq

Loudness → Amp.

Quality → Waveform / Timbre

Speed Sound in a medium

$v = \sqrt{\frac{B}{\rho}}$

$B \propto \frac{1}{\text{Compressibility}}$

① $\rho_s > \rho_l >> \rho_{gas}$

② $B_s > B_l >> B_{gas}$

$\boxed{v_s > v_l > v_{gas}}$

4000 m/s 1500 m/s 340 m/s

Speed of Sound in gas

Newton → isothermal

$B_T = P$

$v = \sqrt{\frac{P}{\rho}} \approx 280 \text{ m/s}$

$$\boxed{v = \sqrt{\frac{\gamma R T}{M}}}$$

$v_{Dry} < v_{Humid}$

Laplace Correction → Adiabatic

$B_a = \gamma P$

$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma R T}{M}}$

$$\boxed{P = \frac{\rho R T}{M}}$$

γ = Adiabatic Exponent

$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

Q. $T \longrightarrow 2T$

$$v \longrightarrow \sqrt{2}\,v = 1.414\,v$$

41.4 %

Q. $27^\circ C \longrightarrow v_0$

$227^\circ C \longrightarrow v =$

$$\frac{v_2}{v_0} = \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{500}{300}}$$

$$\boxed{v_2 = \sqrt{5/3}\,v_0}$$

Q. $v_{He} = v_{O_2}$

$$\sqrt{\frac{\gamma_{He}\,\cancel{R}\,T}{4}} = \sqrt{\frac{\gamma_{O_2}\,\cancel{R}\,(600)}{32}}$$

$$\frac{\left(\frac{5}{3}\right)T}{4} = \frac{\left(\frac{7}{5}\right)600}{32}$$

$$T = \frac{7 \times 600 \times 4 \times 3}{5 \times 32 \times 5} = 63\,K$$

Q. O_2 at 300K

$$v = \sqrt{\frac{\gamma R T}{M}}$$

$$v = \sqrt{\frac{\left(\frac{7}{5}\right) 25 \times 300}{3 \times 32 \times 10^{-3}}}$$

$$\frac{v}{340} = \sqrt{\frac{310}{300}}$$

Q. $27^\circ C \longrightarrow 340$ m/s

$37^\circ C \longrightarrow v = ?$

$$v = \sqrt{\frac{\gamma R T}{M}}$$

$$\frac{\Delta v}{v} = \frac{1}{2}\left(\frac{\Delta T}{T}\right)$$

$$\frac{\Delta v}{34\cancel{0}} = \frac{1}{2}\left(\frac{1\cancel{0}}{3\cancel{00}}\right)$$

$$\Delta v = \frac{17}{3} = 5.67 \text{ m/s}$$

$$\boxed{v = 345.67 \text{ m/s}}$$

Loudness

$$\boxed{L = 10 \log_{10}\left(\frac{I}{I_0}\right)}$$ dB (decibel)

I = Intensity

I_0 = Threshold of sound

$= 10^{-12}\ W/m^2$

	Intensity (W/m^2)	Loudness (dB)
Threshold	10^{-12} ✓	1
whisper	10^{-11}	10
Normal	10^{-6}	60
Loud	10^{-5}	70
Tolerance	1	120

logarithm

$$\boxed{\begin{gathered} y = 10^{x} \\ \log_{10} y = x \end{gathered}}$$

$$\log_{10} 100 = 2$$

$$\boxed{e = 2.72}$$

$$\log_{10}(1) = 0$$

$$\log_{10}(10) = 1$$

$$\log_{10}(2) = 0.3$$

$$\log_{10}(3) = 0.4771$$

$$\log_{e}(1) = 0$$

$$\log_{e}(2) = 0.693 \approx \underline{0.7}$$

$$\boxed{\log_{e}(x) = 2.303 \log_{10}(x)}$$

$$\log(2^{4}) = 4 \log(2)$$

$$\log(2 \times 3) = \log(2) + \log(3)$$

$$\log\left(\frac{2}{3}\right) = \log(2) - \log(3)$$

$$\log\left(\frac{1}{2}\right) = -\log(2)$$

$$\log(x) = \log_{10} x$$

$$\ln(x) = \log_{e} x$$

① $I = 10^{-12}$

$$L = 10 \log\left(\frac{10^{-12}}{10^{-12}}\right) = 10 \log(1) = 0$$

② $I = 10^{-11}$

$$L = 10 \log\left(\frac{10^{-11}}{10^{-12}}\right) = 10 \log(10) = \underline{10 \text{ dB}}$$

③ $I = 10^{-5}$

$$L = 10 \log\left(\frac{10^{-5}}{10^{-12}}\right) = 10 \log(10^{7}) = 70 \log(10) = \underline{70 \text{ dB}}$$

$$\Delta L = 10\log\left(\frac{I_2}{I_1}\right)$$

$$I = \frac{P}{4\pi r^2}$$

$$I \propto \frac{1}{r^2}$$

Q. $$L_2 - L_1 = 10\log\left(\frac{I_2}{I_1}\right) = 10\log\left(\frac{r_1}{r_2}\right)^2$$

$$60 - 40 = 10\log\left(\frac{r}{2}\right)^2$$

$$2 = \log_{10}\left(\frac{r}{2}\right)^2$$

$$2 = 2\log_{10}\left(\frac{r}{2}\right)$$

$$1 = \log_{10}\left(\frac{r}{2}\right)$$

$$\boxed{\frac{r}{2} = 10}$$

(1) $$10^2 = \left(\frac{r}{2}\right)^2$$

$$10 = \frac{r}{2}$$

$$r = 20\,\text{m}$$

Echo

Persistance of Sound = 0.1 s

$$(330)(0.1) = 2d$$

Min dist. for echo $\boxed{d = 16.5\,m}$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$= I_0 + 4I_0 + 2\sqrt{I_0(4I_0)}\cos 120^\circ$$

$$= 5I_0 + 4I_0\left(-\frac{1}{2}\right)$$

$$= 5I_0 - 2I_0$$

$$= \underline{\underline{3I_0}}$$

① $I_0 = I_1 + I_1 + 2\sqrt{I_1 I_1}$

$I_0 = 4 I_1$

$$\boxed{I_1 = \frac{I_0}{4}}$$

② $I = I_1 + I_1 + 2\sqrt{I_1 I_1} \cos(\pi/2)$

$I = 2 I_1$

$I = 2\left(\frac{I_0}{4}\right) = \frac{I_0}{2}$

① $I_0 = 4 I_1$

$$\boxed{I_1 = \frac{I_0}{4}}$$

② $I = (I_0) \cos^2\left(\frac{\pi/2}{2}\right)$

$I = I_0 \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{2}$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$A = \sqrt{A^2 + A^2 + 2A^2\cos\phi}$$

$$A^2 = 2A^2 + 2A^2\cos\phi$$

$$-\cancel{A^2} = 2\cancel{A^2}\cos\phi$$

$$\cos\phi = -1/2$$

$$\phi = 120^\circ = \frac{2\pi}{3}$$

$$A = 2A\cos(\phi/2)$$

$$\cos(\phi/2) = \frac{1}{2}$$

$$\phi/2 = 60^\circ$$

$$\phi = 120^\circ$$

$$(\cancel{A_1} + A_2) - (\cancel{A_1} - A_2) = 2A_2$$

$$(A_1 + \cancel{A_2}) - (\cancel{A_2} - A_1) = 2A_1$$

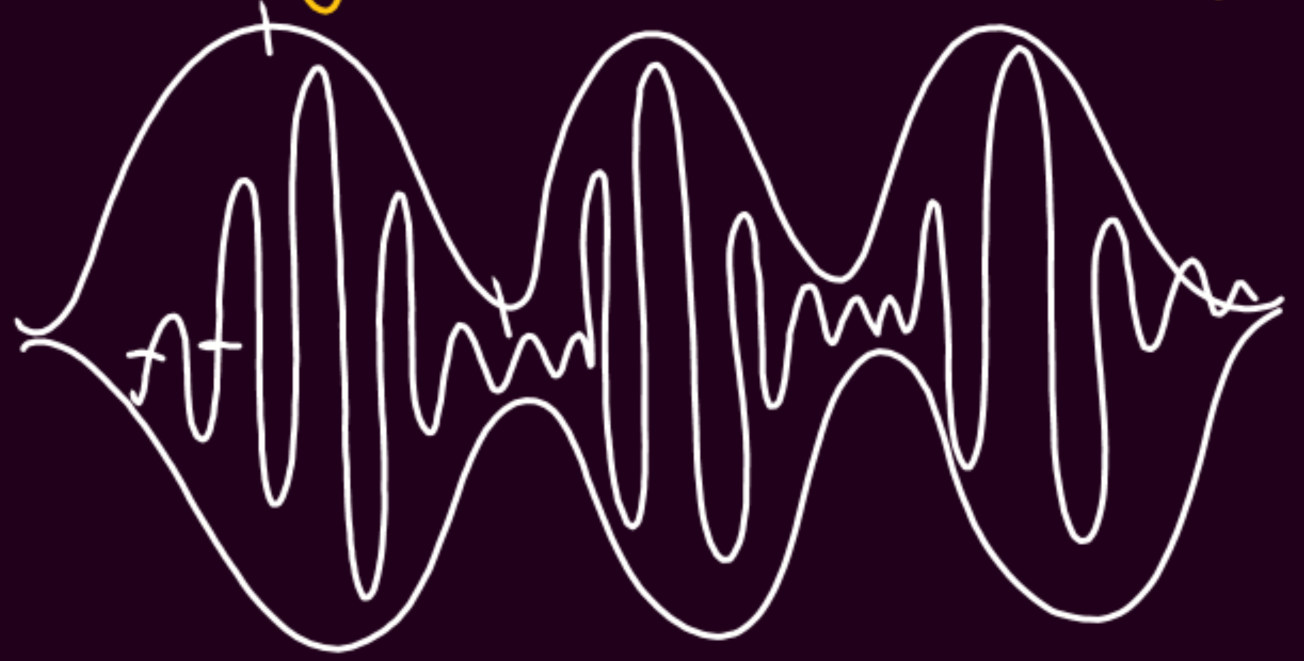
Beats

$$y_1 = A\cos(\omega_1 t)$$

$$y_2 = A\cos(\omega_2 t)$$

$$y_1 + y_2 = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\cos\left(\frac{\omega_1+\omega_2}{2}t\right)$$

$$y_1 + y_2 = \left[2A\cos\left(\boxed{\frac{\Delta\omega}{2}}t\right)\right]\cos(\omega_{av}t)$$



Beats → max/min per sec

→ $\Delta f \le 10$ Hz

Beat freq → $\Delta f = \dfrac{\Delta\omega}{2\pi}$

$$y_1 = 10\sin(408\pi t)$$

$$y_2 = 5\sin(400\pi t)$$

① Beat freq. $= \dfrac{8\pi}{2\pi} = \underline{4\text{ Hz}}$

② $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{10+5}{10-5}\right)^2 = \left(\dfrac{15}{5}\right)^2 = 9$

Tuning Fork

Stem ↓ longitudinal

Prongs ↓ Transverse [out of phase - $\Delta\phi = \pi$]

① Loading / weight (waxing) freq ↓

② Filing (wanning) freq ↑

Unison

$$f_2 = f_1$$

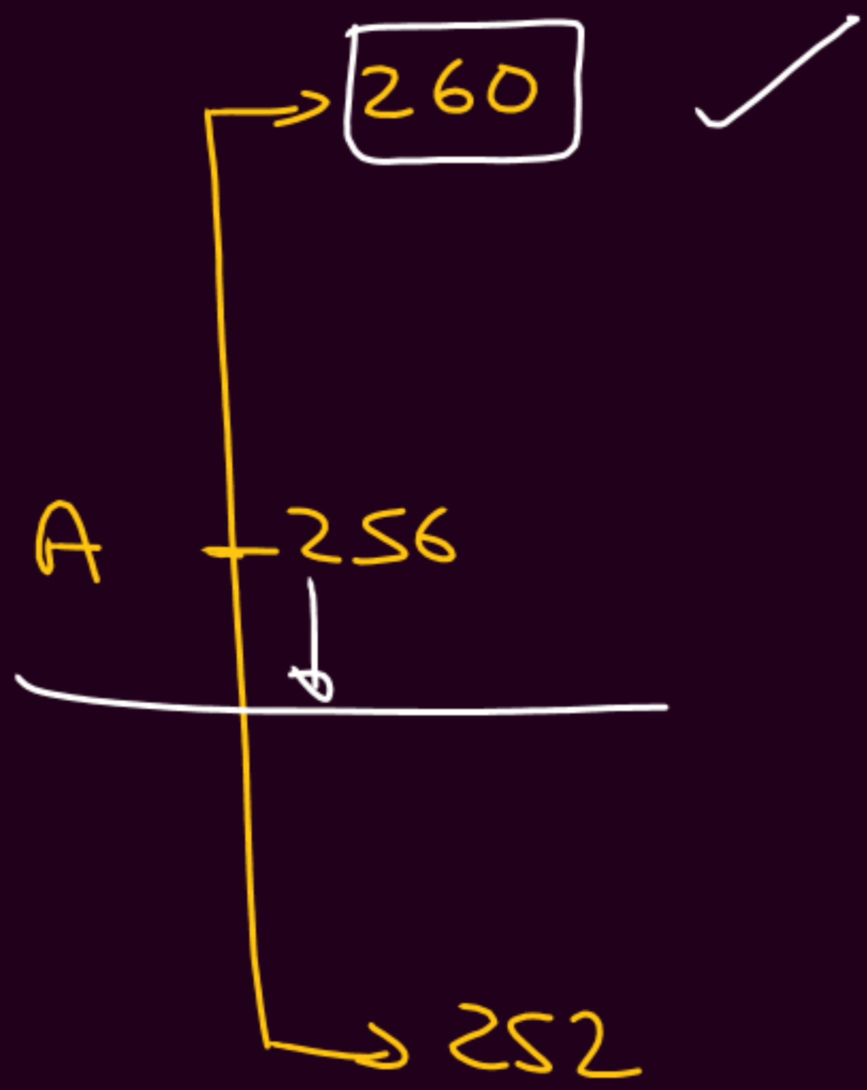
Octave

$$\frac{f_2}{f_1} = 2$$

$A = 256\ Hz$

$B \rightarrow$ 4 Beats

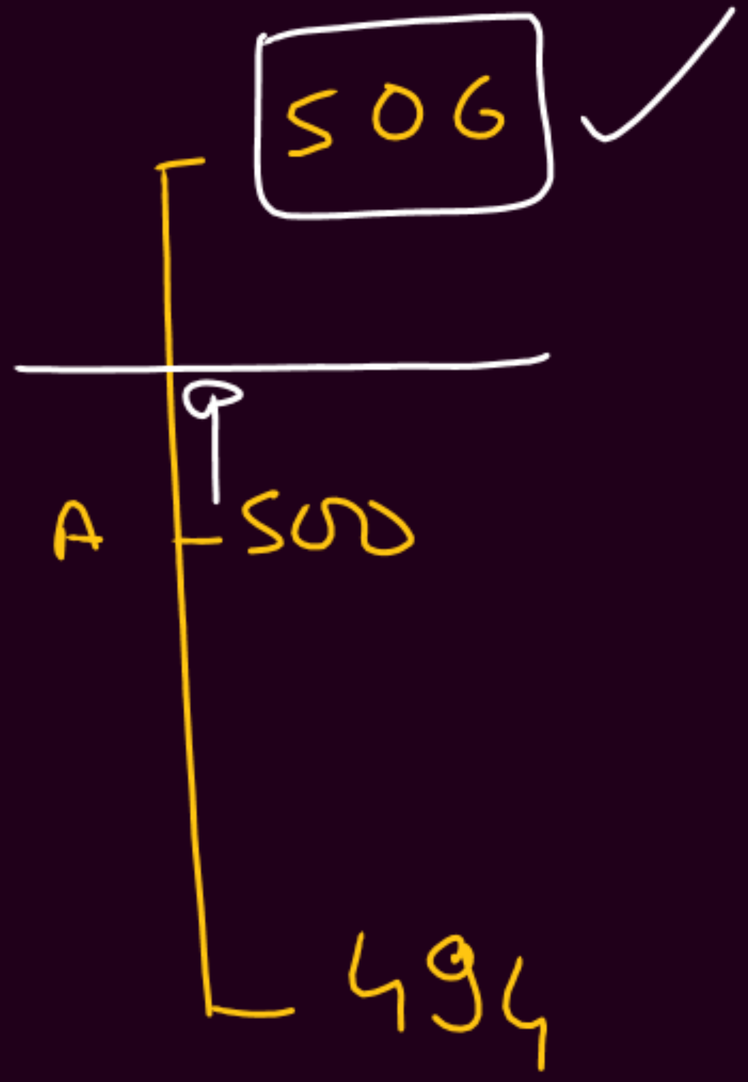
{ if A is loaded then 6 Beats



$A = 500\ Hz$

$B \rightarrow$ 6 Beats

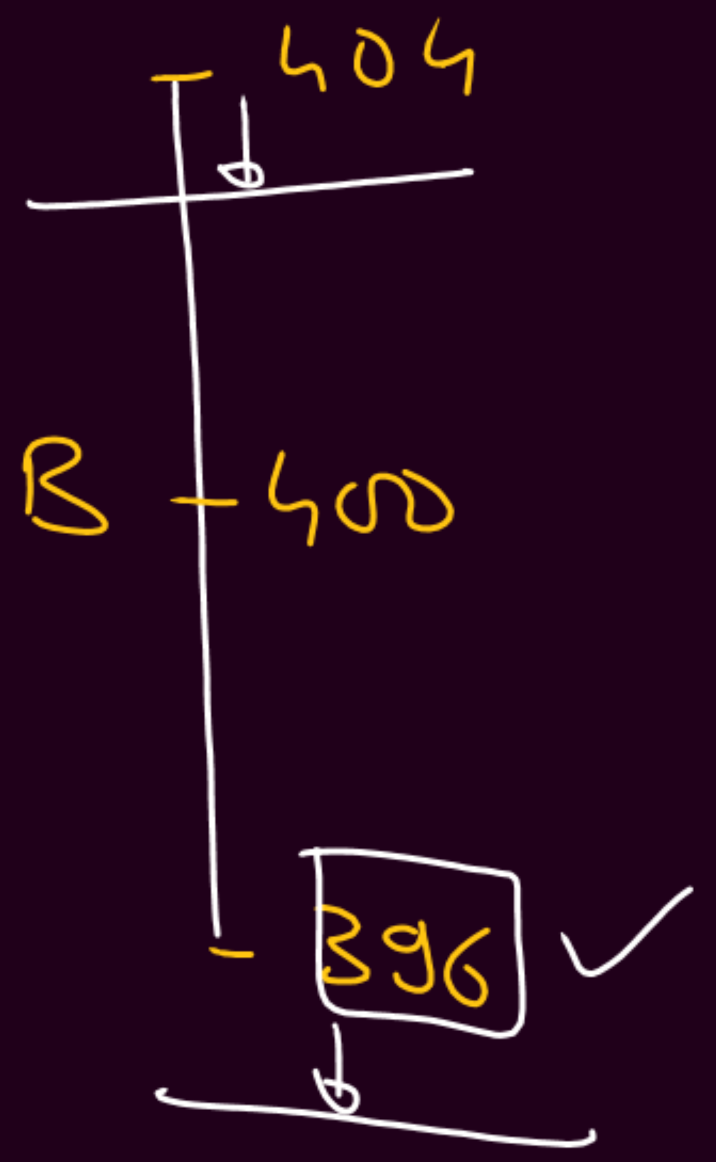
$A \rightarrow$ filing $\Rightarrow$ 4 Beats



$B = 400\ Hz$

$A \rightarrow 4$ Beats

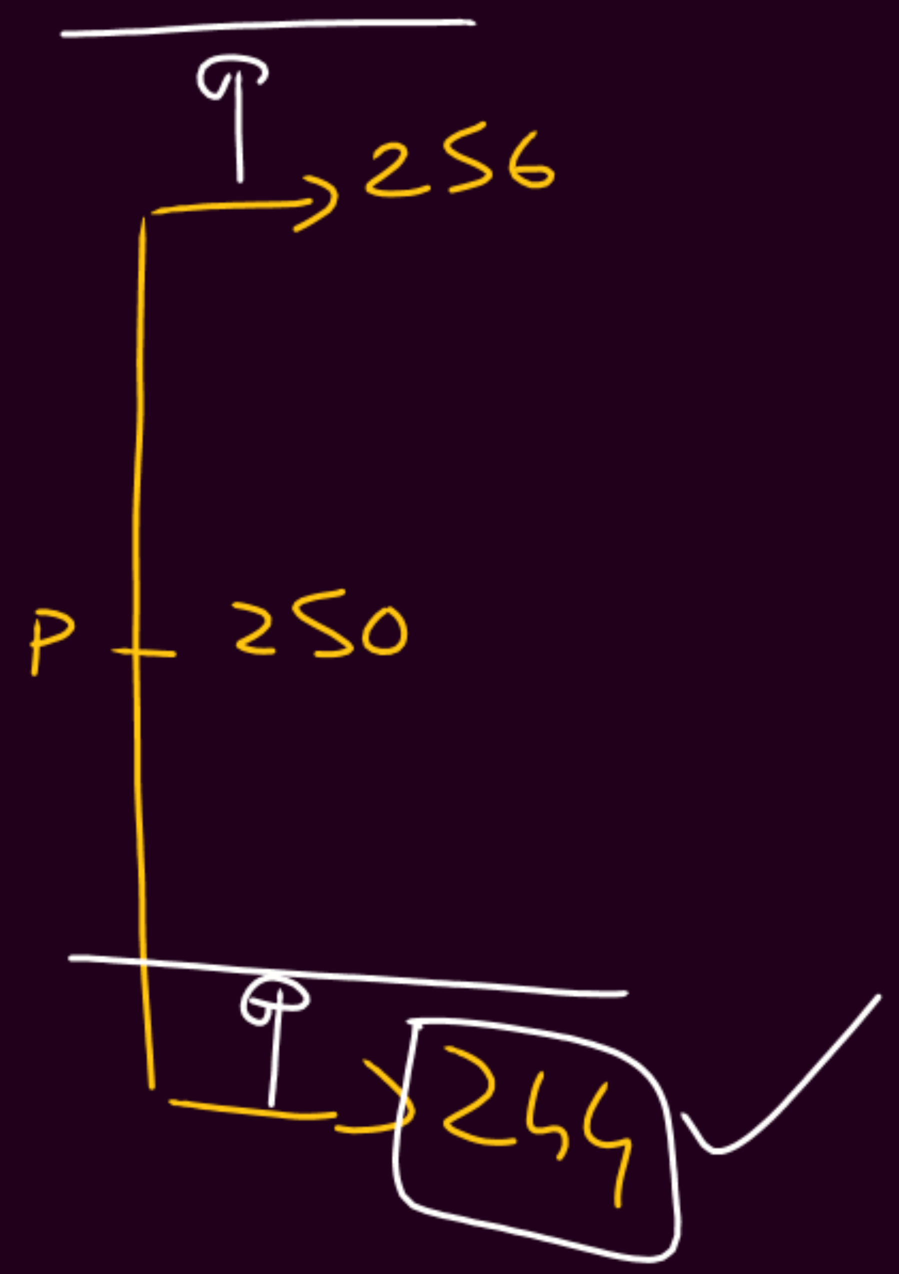
$A \rightarrow$ load $\Rightarrow 6$ Beats



$P = 250\ Hz$

$Q \rightarrow 6$ Beats

$Q \rightarrow$ filing $\rightarrow$ 4 Beats



3TF

300 302 304

(2) 300/302 $\frac{1}{2}$ $\frac{2}{2}$

(2) 302/304 $\frac{1}{2}$ $\frac{2}{2}$

(4) 300/304 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

4 Beats

BTF

	400	401	403
400/401	$\frac{1}{1}$		
401/403	$\frac{1}{2}$	$\frac{2}{2}$	
400/403	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

4 Beats

	500	502	505		
500/502	$\frac{1}{2}$	$\frac{2}{2}$			
502/505	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$		
500/505	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$

8 beats

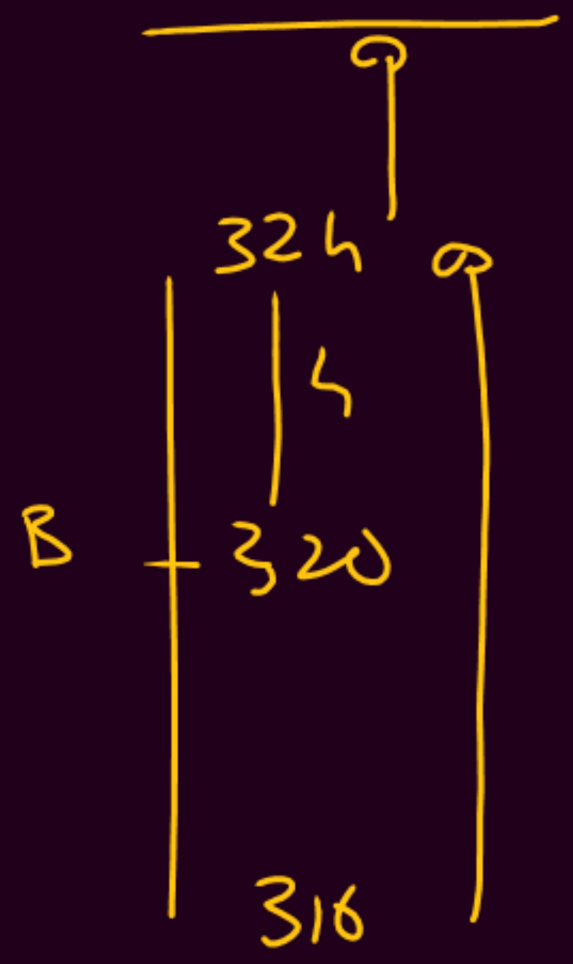
$$a + (n-1)d$$

$$\underline{f} \quad f+4 \quad f+8 \quad - - - - - - \quad \underline{f + 9(4)}$$

$$f + 9(4) = 2f$$

$$f + 36 = 2f$$

$$\boxed{f = 36\ \text{Hz}}$$



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33, 35

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16, 18

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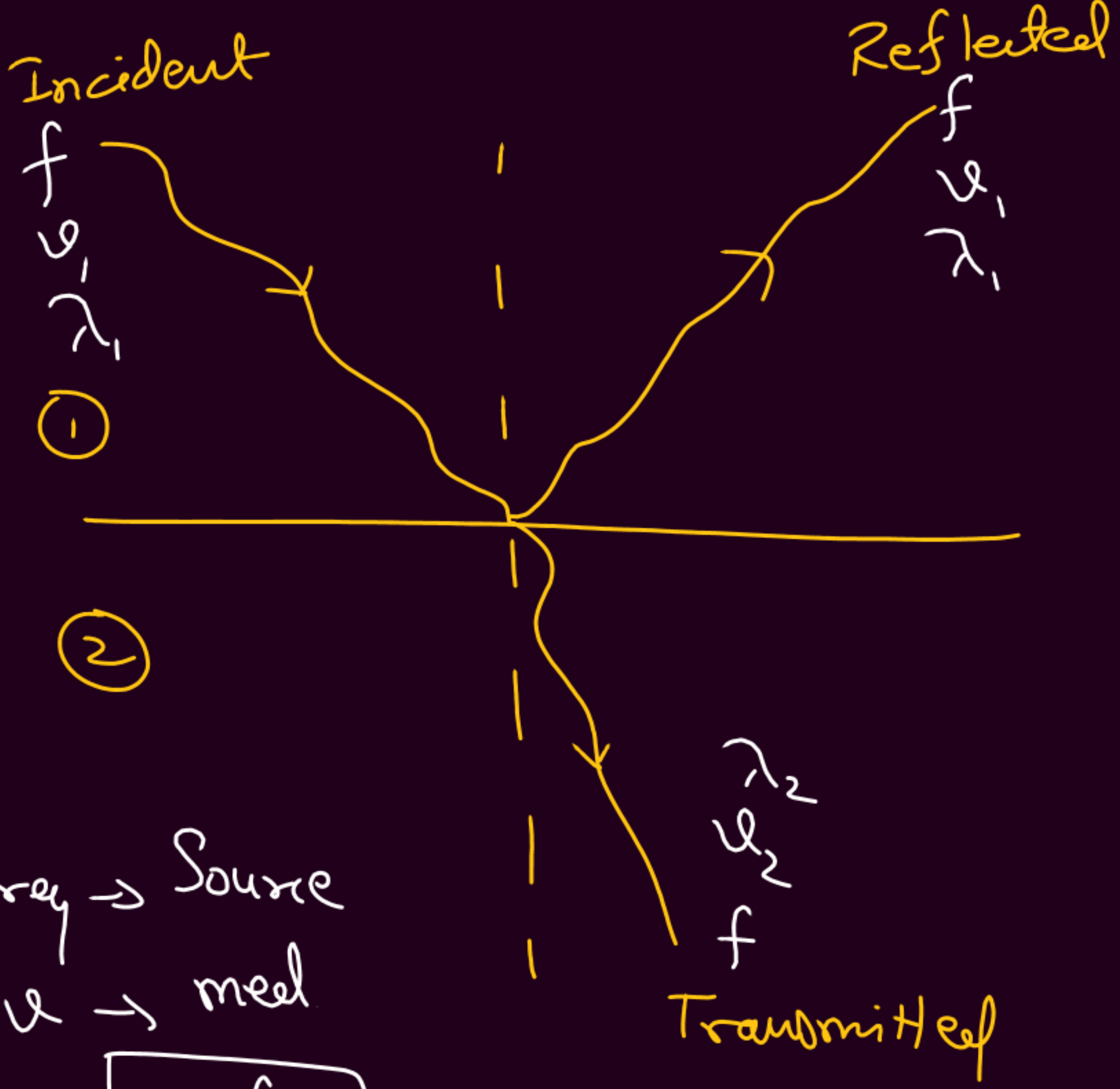
6, 18,

Reflection & Transmission of Wave



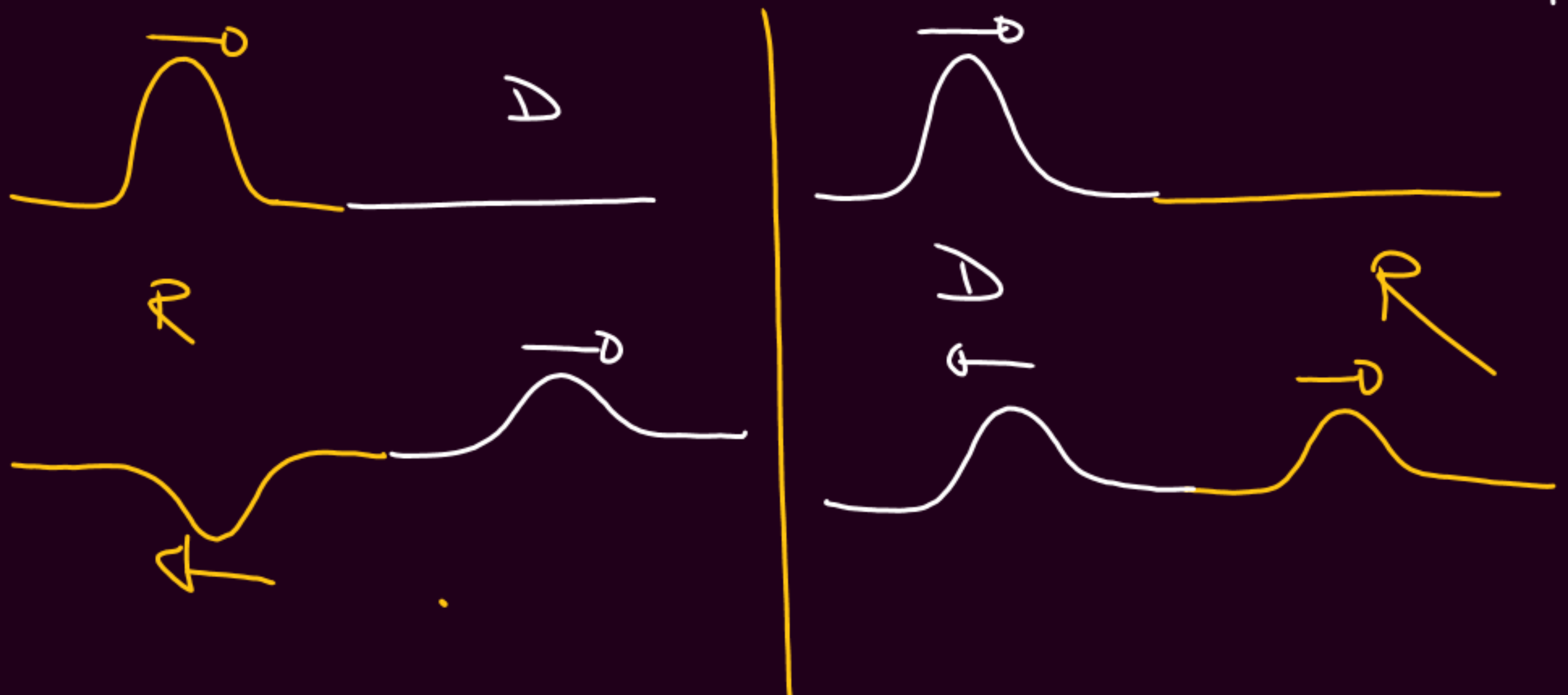
freq → Source

v → med

$$\boxed{v = \underline{f}\lambda}$$

Rarer and Denser

	Light	Sound
Air	3×10^8 m/s (R)	330 m/s (D)
Water	2.25×10^8 m/s (D)	1500 m/s (R)



Standing Wave

$y_1 = A \sin(kx - wt)$

$y_2 = A \sin(kx + wt)$

$$y_1 + y_2 = [2A \sin(kx)] \cos(wt)$$

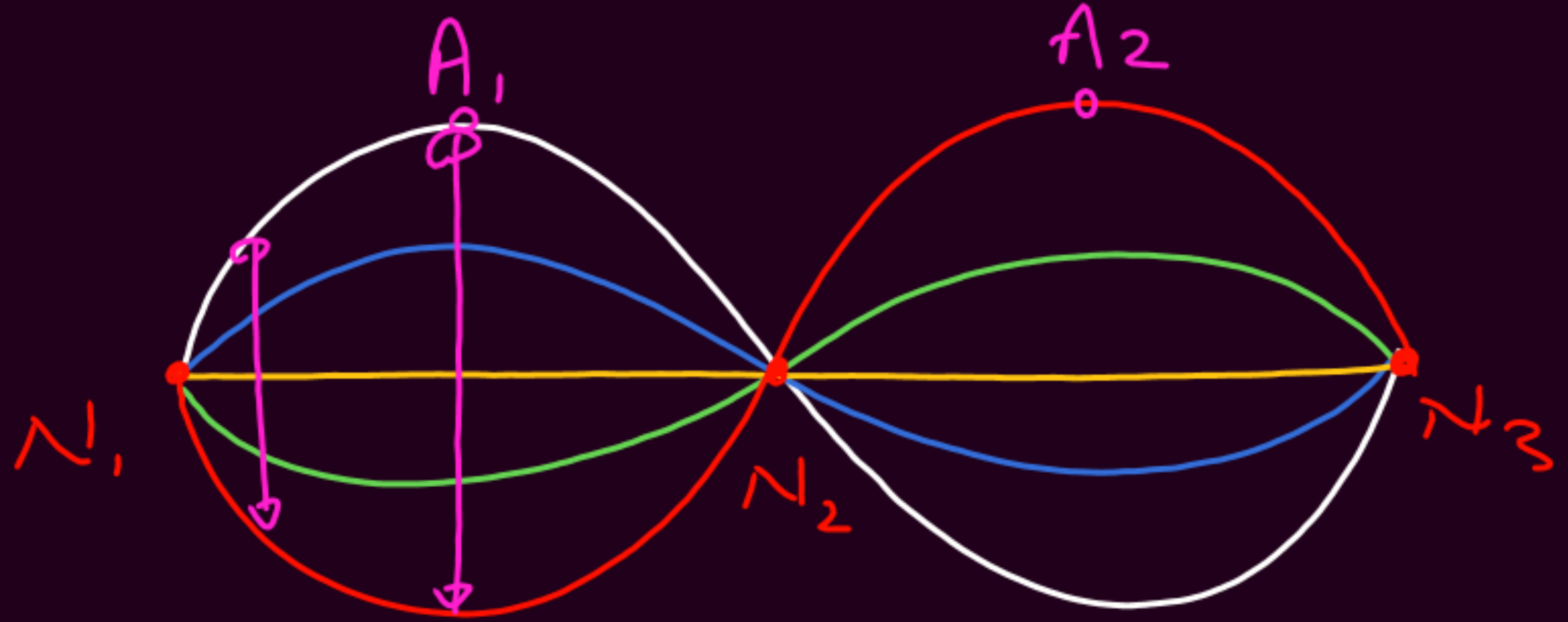
$wt = 0$ $\quad \cos(wt) = 1$

$wt = \frac{\pi}{4}$ $\quad \cos(wt) = \frac{1}{\sqrt{2}}$

$wt = \frac{\pi}{2}$ $\quad \cos(wt) = 0$

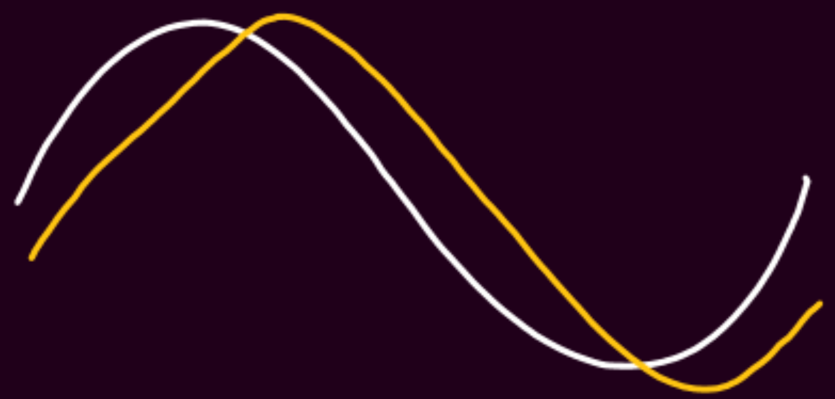
$wt = \frac{3\pi}{4}$ $\quad \cos(wt) = -\frac{1}{\sqrt{2}}$

$wt = \pi$ $\quad \cos(wt) = -1$



P

S



String fixed at Both Ends (Sonometer)

$L = \frac{\lambda}{2}$	f_0	1 H	fundamental
$L = 2\left(\frac{\lambda}{2}\right)$	$2f_0$	2 H	1 OT
$L = 3\left(\frac{\lambda}{2}\right)$	$3f_0$	3 H	2 OT
$L = 4\left(\frac{\lambda}{2}\right)$	$4f_0$	4 H	3 OT

$$\boxed{f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}}$$

n^{th} Harmonic

$n \Rightarrow A$

$n+1 \Rightarrow N$

$$L = n\left(\frac{\lambda}{2}\right)$$

H = Harmonic

OT = overtone

$$\boxed{f = n\left(\frac{v}{2L}\right) = n f_0}$$

→ Resonant freq

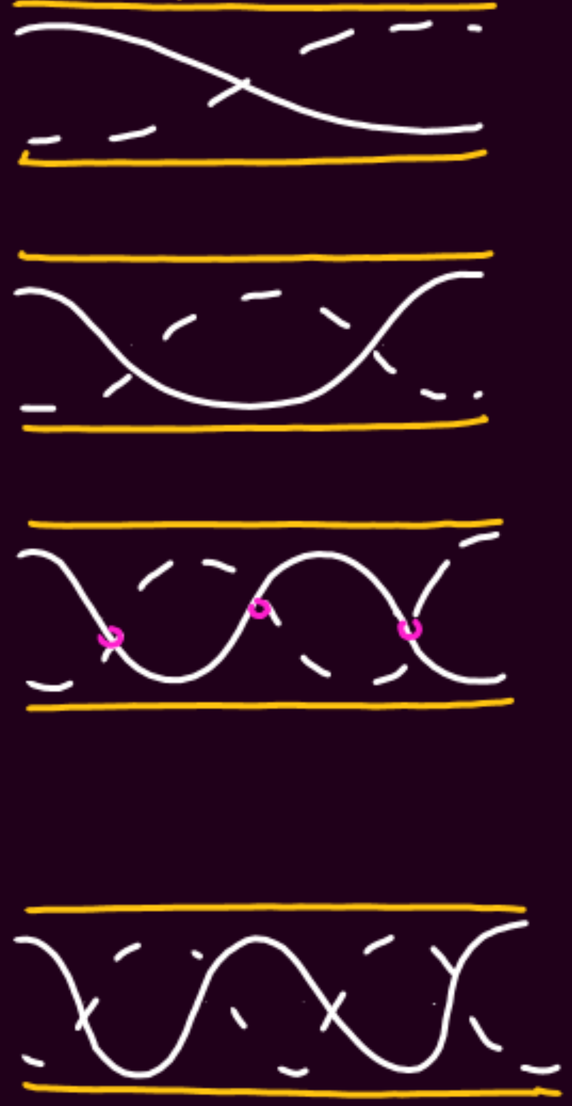
→ natural freq

→ normal modes of Vibration

$$f_0 = \frac{v}{2L} = \text{fundamental frq}$$

Open organ pipe (OOP)

Displacement



Pressure

$n \Rightarrow N$

$n+1 \Rightarrow A$

$$\boxed{f = \frac{n}{2L}\sqrt{\frac{\gamma RT}{M}}}$$

String fixed at one End (only odd harmonics)

$L=\left(\frac{\lambda}{4}\right)$ f_0 1H fundamental

$L=3\left(\frac{\lambda}{4}\right)$ $3f_0$ 3H 1OT

$L=5\left(\frac{\lambda}{4}\right)$ $5f_0$ 5H 2OT

$L=7\left(\frac{\lambda}{4}\right)$ $7f_0$ 7H 3OT

$L=(2n+1)\frac{\lambda}{4}$

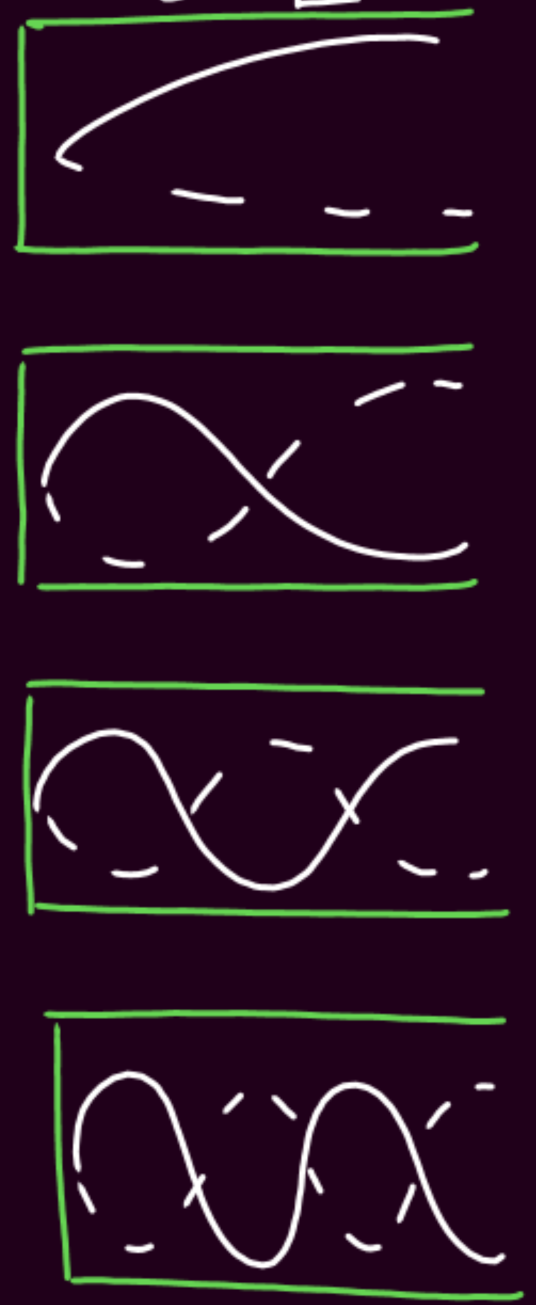
$$\boxed{f=(2n+1)\frac{v}{4L}=(2n+1)f_0}$$

$$\boxed{f_0=\frac{v}{4L}}$$

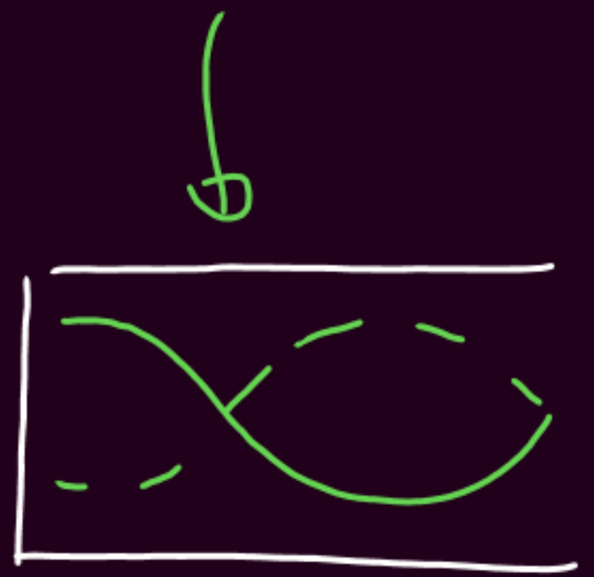
$\boxed{N=A}$

Closed organ pipe (COP)

Disp



Pressure



$\boxed{N=A}$

$$y = 10 \sin(\pi x)\cos\left(\frac{\pi}{2} t\right)$$

$$\begin{bmatrix} y & \text{in} & \text{cm} \\ x & \text{in} & \text{m} \\ t & \text{in} & \text{s} \end{bmatrix}$$

① Wave Speed $= 0$

② $2A = 10$

$A = 5\,\text{cm}$

③ N & A $= \dfrac{\lambda}{4} = \dfrac{2}{4} = \dfrac{1}{2}\,\text{m}$

$\pi = \dfrac{2\pi}{\lambda}$

$\lambda = 2$

④ $\boxed{\begin{matrix} x = 0.2\,\text{m} \\ x = 0.7\,\text{m} \end{matrix}}$

$x = 0$ N

$\boxed{\Delta\phi = 0}$

$$y = 10 \sin(\pi x - 2\pi t)$$

① $$\frac{E_2}{E_1} = \left(\frac{A_2}{A_1}\right)^2$$

$$0.64 = \left(\frac{A_2}{10}\right)^2$$

$$0.8 = \frac{A_2}{10}$$

$$\boxed{A_2 = 8}$$

② $$y = 8 \sin(-\pi x - 2\pi t + \pi)$$

$$y = 8 \sin[\pi - (\pi x + 2\pi t)]$$

$$\boxed{y = 8 \sin(\pi x + 2\pi t)}$$

OOP $4H = 600\ Hz$

$f_0 = ?$

$4 f_0 = 600$

$f_0 = 150\ Hz$

OOP $3OT = 500\ Hz$

1	f_0
2	1OT
3	2
$\boxed{4}$	$\boxed{3}$

$4 f_0 = 500$

$f_0 = 125\ Hz$

OOP $5OT = 1200\ Hz$

$3OT = ?$

1	f_0
2	1OT
3	2
$\boxed{4}$	$\boxed{3}$
5	4
$\boxed{6}$	$\boxed{5}$

$6 f_0 = 1200$

$f_0 = 200\ Hz$

$3OT = 4 f_0 = 4 \times 200 = 800\ Hz$

COP $5H = 1200\ Hz$

$3H = ?$

$5f_0 = 1200$

$f_0 = 240\ Hz$

$3H = 3f_0 = 3(240)$

$= 720\ Hz$

COP $20T = 1200\ Hz$

$40T = ?$

1	f
3	1OT
$\boxed{5}$	$\boxed{2}$
7	3
$\boxed{9}$	$\boxed{4}$

$20T = 5f_0 = 1200$

$f_0 = 240\ Hz$

$40T = 9f_0 = 9(240) = \underline{2160\ Hz}$

COP $f_0 = 100\ Hz$

$7H = 7f_0 = 7 \times 100 = 700\ Hz$

$10T = 3f_0 = 300\ Hz$

1	f
$\boxed{3}$	$\boxed{1OT}$
5	2
7	3

COP $30T = 600\ Hz$

$5H = ?$

1	f
3	1OT
5	2
$\boxed{7}$	$\boxed{3}$

$30T = 7f_0 = 600$

$f_0 = \frac{600}{7}$

$5f_0 = \frac{5 \times 600}{7} = \frac{3000}{7}\ Hz$

Q.

$$450 : 600 : 750$$

$$3 : 4 : 5 \Rightarrow \underline{\underline{OOP}}$$

$$3f_0 = 450$$

$$\underline{f_0 = 150\,Hz}$$

Q.

$$150 : 250 : 350$$

$$3 : 5 : 7 \quad \underline{\underline{COP}}$$

$$3f_0 = 150\,Hz$$

$$\underline{f_0 = 50\,Hz}$$

Q.

OOP COP

$$f_o = f_o$$

$$\frac{v}{2L_o} = \frac{v}{4L_c}$$

$$\frac{4}{2} = \frac{L_o}{L_c}$$

$$\boxed{\frac{L_o}{L_c} = \frac{2}{1}}$$

Q.

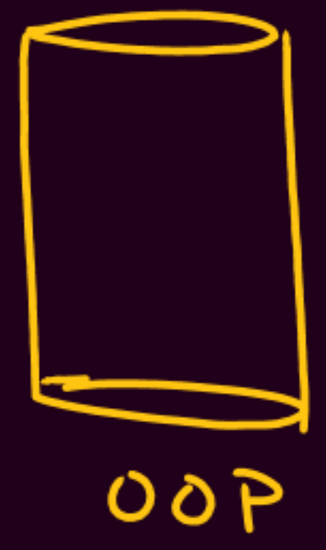
OOP COP

1OT 1OT

$$2\left(\frac{v}{2L_o}\right) = 3\left(\frac{v}{4L_c}\right)$$

$$\boxed{\frac{4}{3} = \frac{L_o}{L_c}}$$

Q.



$$f_o = \frac{v}{2L}$$

COP

$$f = \frac{v}{4\left(\frac{L}{2}\right)} = \frac{v}{2L} = f_o$$

Q. Sonometer

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$L \to 2L$

$T \to T/2$

$$f = \frac{f_0}{2}\sqrt{\frac{1}{2}} = \frac{f_0}{2\sqrt{2}}$$

Q. $$\frac{f}{f-4} = \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{1.21} = 1.1 = \frac{11}{10}$$

$$\frac{f}{f-4} = \frac{11}{10}$$

$$\boxed{f = 44\text{ Hz}}$$

$$f \begin{cases} \to f+4 \\ \to f-4 \end{cases}$$

$$T_2 = T_1\left(1 + \frac{21}{100}\right) = 1.21\,T_1$$

Q. $100 \text{ kgf} = 100g = 1000 \text{ N}$

169 kgf

$$\frac{f+6}{f-6} = \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{169}{100}} = \frac{13}{10}$$

$$f \rightarrow \begin{cases} f+6 \\ f-6 \end{cases}$$

$$\frac{f+6}{f-6} = \frac{13}{10}$$

$$\boxed{f = 46 \text{ Hz}}$$

Sonometer

3H

N = 4

A = 3

COP 5H

N = 3

A = 3

OOP 4OT

N = 5

A = 6

COP 3OT

N = 4

A = 4

Q

COP

$\boxed{9H}$

Q

$$f = \frac{v}{\lambda} = \frac{100}{1} = 100$$

$$2f_0 = 100$$

$$f_0 = 50\,Hz$$

Q $y = a\cos(kx - \omega t)$ $\boxed{x = 0}$

1) $y = a\sin(kx + \omega t)$

3) $y = -a\cos(kx + \omega t)$

2) $y = -a\sin(kx - \omega t)$ ✗

4) $y = -a\cos(kx - \omega t)$ ✗

$\boxed{x = 0}$ $y = a\cos(-\omega t) = \underline{a\cos(\omega t)}$

1) $y = a\sin(\omega t)$

3) $y = -a\cos(\omega t)$

Q. $y = a\sin(kx - \omega t)$

1) $y = a\sin(kx + \omega t)$ ✓

2) $y = -a\sin(kx + \omega t)$

× 3) $y = a\sin(kx - \omega t)$

× 4) $y = -a\sin(kx - \omega t)$

$x = 0$ $\quad y = a\sin(-\omega t) = -a\sin(\omega t)$

$y = a\sin(\omega t)$

$y = -a\sin(\omega t)$

$$\frac{v}{4L_c} = \frac{1}{2}\left[3\left(\frac{v}{2L_o}\right)\right]$$

$L_o = 3L_c = 30\text{cm}$

End Correction

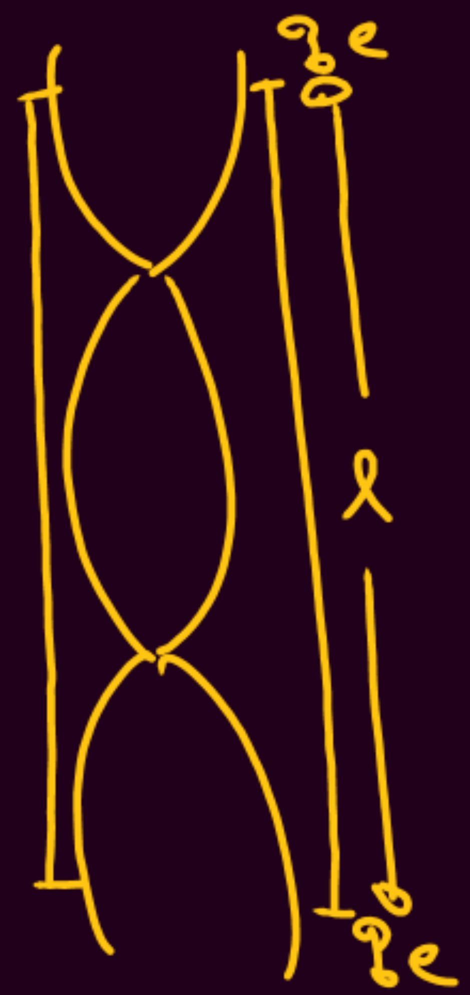
$\boxed{e = 0.6r}$

COP

$f = (2n+1)\frac{v}{4L}$

$$\boxed{L = l + e}$$

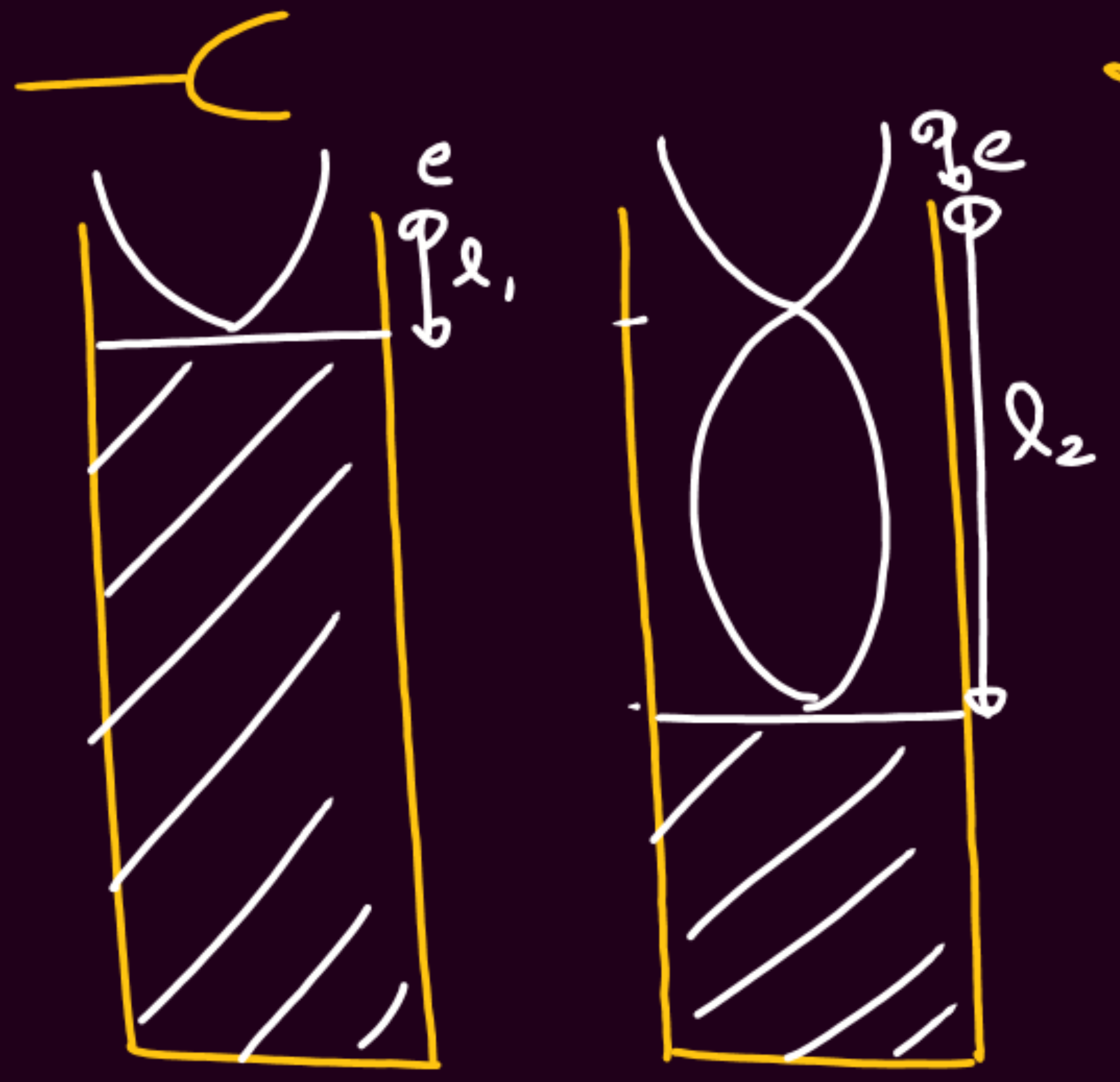
OOP



$f = n\left(\frac{v}{2L}\right)$

$$\boxed{L = l + 2e}$$

Resonance Tube



first Reso 2^{nd} reso

l_1/l_2

↳ length of Air coloumn

① $(l_2 - l_1) = \frac{\lambda}{2}$

② $\boxed{v = f\lambda = 2f(l_2 - l_1)}$

Extra

③ $\boxed{e = \frac{l_2 - 3l_1}{2}}$

$l_1 + e = \frac{\lambda}{4}$

$l_2 + e = 3\left(\frac{\lambda}{4}\right) = 3(l_1 + e)$

Q. $l_1 = 20\,cm$

$l_2 = 65\,cm$

(1) $v = 2f(l_2 - l_1)$

$= 2 \times 350 \dfrac{(65-20)}{100}$

$= 315\ m/s$

(2) $e = \dfrac{l_2 - 3l_1}{2} = \dfrac{65 - 3\times 20}{2} = \dfrac{5}{2} = 2.5\,cm$

$r = ?$

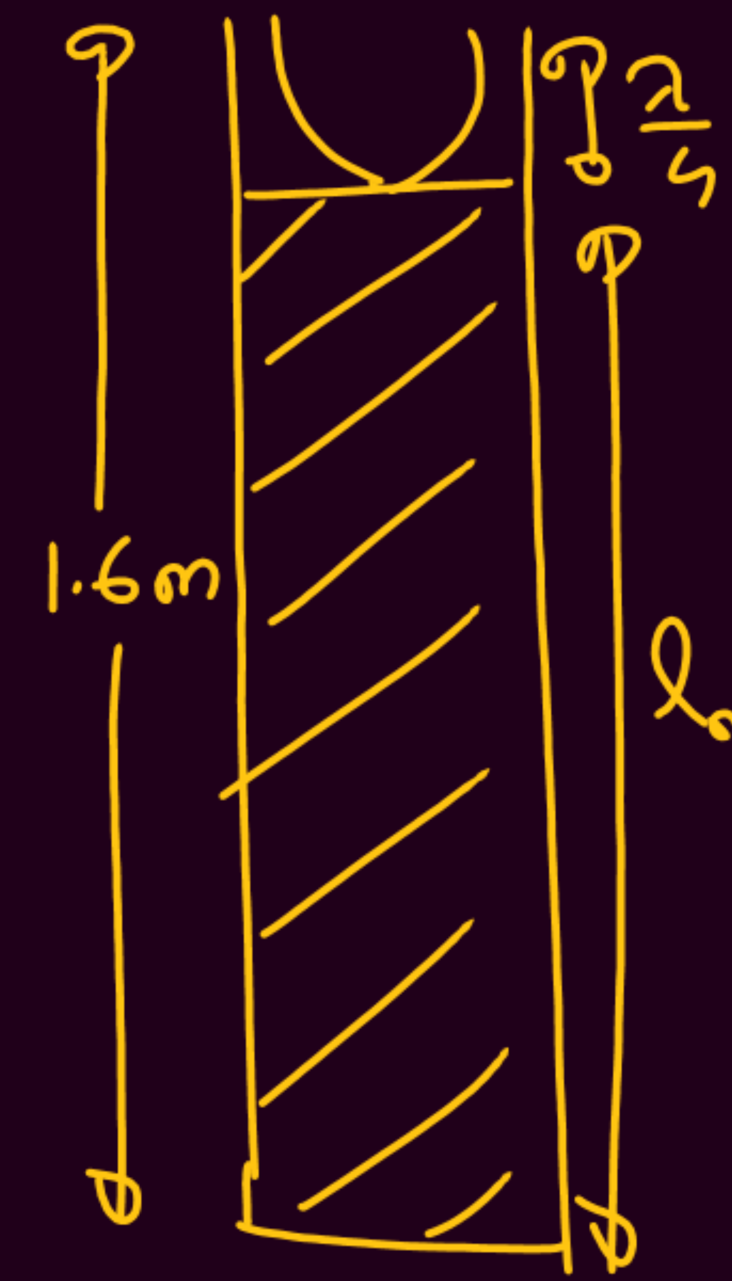
$0.6r = 2.5$

$\boxed{r = \dfrac{25}{6}\,cm}$

Q Extra

$v = 330\ m/s$

$f = 660\ Hz$



$$\lambda = \frac{v}{f} = \frac{330}{660} = \frac{1}{2} m = 50\ cm$$

$$l_{max} = 160 - \frac{50}{4}$$
$$= 147.5\ cm$$

$$l_{min} = 160 - 137.5$$
$$= 22.5\ cm$$

Q:

No Sound

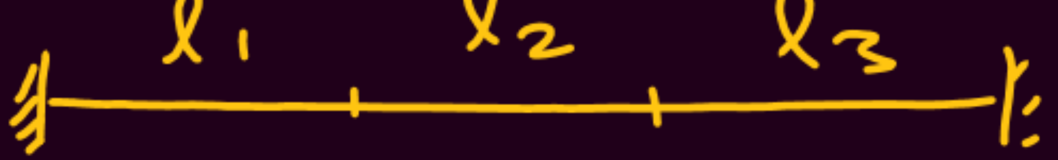
Destructive Interference

$$\Delta x_{min} = \frac{\lambda}{2} = \frac{32}{2} = 16\,cm$$

$$(\pi r - 2r) = 16$$

$$\left(\frac{22}{7} - 2\right) r = 16$$

$$\boxed{r = 14\,cm}$$



$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$l = l_1 + l_2 + l_3$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

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