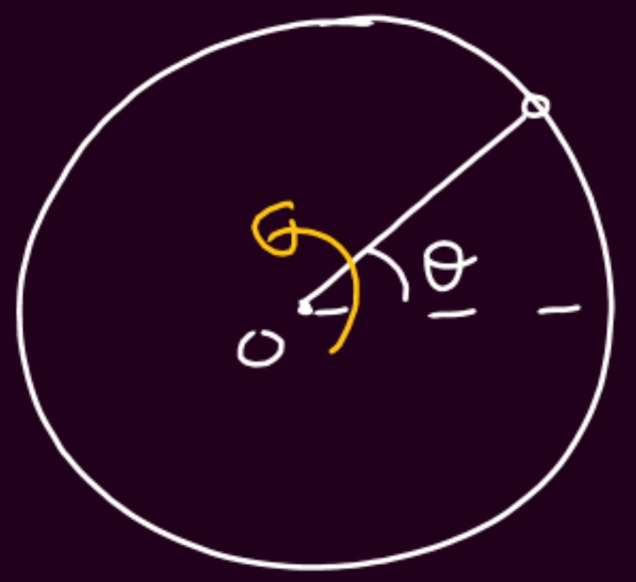
Circular Motion

Kinematics of CM



① Angular Position (θ)
② Angular Disp. $\Delta\theta$
} rad

③ Angular Velo (ω)

$$\omega_{inst} = \frac{d\theta}{dt}$$ rad/s

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

→ 1 round ⇒ Disp. = 0
⇒ Ang Disp. = 2π rad

→ Axial vectors

→ Ang. vel. Thumb outside ACW ⊙
— " — inside CW ⊗

④ Ang. Acc. (α)

$$\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta} = \frac{d^2\theta}{dt^2}$$

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

unit rad/s²

Dir → ① $\omega\uparrow$ then α in dir of ω
② $\omega\downarrow$ — " — opp dir of ω

If $\alpha = \text{const}$

① $\omega = \omega_0 + \alpha t$

② $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

③ $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

Q. $\omega_0 = 4$ rad/s

$\alpha = 2$ rad/s^2

in 4s

$\Delta\theta = 4(4) + \frac{1}{2}(2)(4)^2$

$= 32$ rad

Q. $\theta = (3t^2 - 4t + 5)$

① Initial ω

$\omega = 6t - 4$

$\omega_0 = -4$ rad/s

② Ang Acc.

$\alpha = \frac{d\omega}{dt}$

$\alpha = 6$ rad/s^2

Uniform CM

$\Rightarrow \omega = \text{const.}$

$\Rightarrow \alpha = 0$

$\Rightarrow$ Periodic

$$T = \frac{2\pi}{\omega}$$

$\Rightarrow \omega = 2\pi f$

$\Rightarrow$ rpm = rev/min

$$\boxed{\omega = \frac{2\pi}{60}(\text{rpm})}$$

Q: Fan 100 rpm $\longrightarrow$ Rest in 30s

no. of rev. = ? [assuming α = const.]

① $0 = (100)\frac{2\pi}{60} - \alpha(30)$

$$\alpha = \frac{200\pi}{60 \times 30} = \pi/9 \text{ rad/s}^2$$

② $0 = \left(\frac{10\pi}{3}\right)^2 - 2\left(\frac{\pi}{9}\right)\Delta\theta$

$$\frac{2\pi}{9}\Delta\theta = \frac{100\pi^2}{9}$$

$$\Delta\theta = 50\pi \text{ rad}$$

$$\text{rev.} = \frac{50\pi}{2\pi} = \underline{\underline{25 \text{ rev.}}}$$

Disp = (Avg. Vel.) (time)

$$\Delta\theta = \left(\frac{100+0}{2}\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1}{2}\text{ min}\right)$$

$$\Delta\theta = 25 \text{ rev}$$

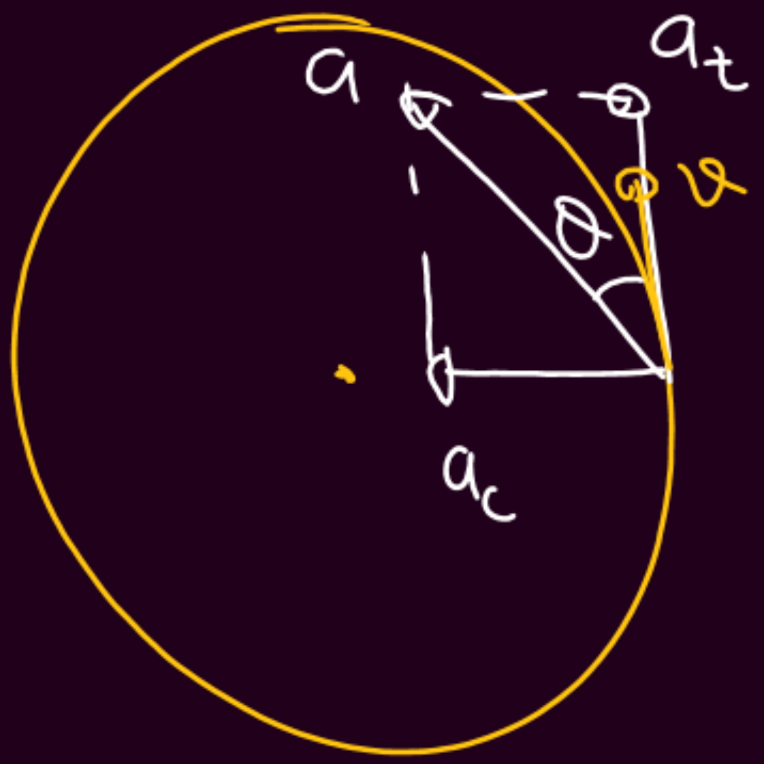
1. $v = \omega r$

2. <u>Centripetal Acc</u> (Dir)

$$a_c = \omega^2 r = \frac{v^2}{r}$$

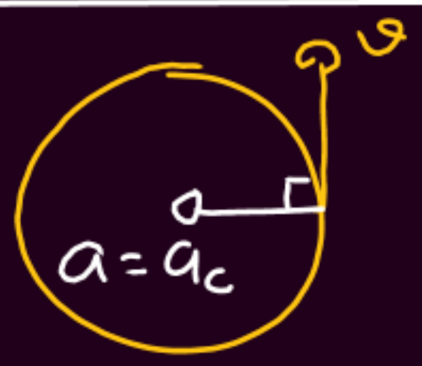
3. <u>Tangential Acc</u> (speed)

$$a_t = \alpha r = \frac{dv}{dt}$$

4. 

5. <u>Acc</u> (vel.) $\quad a = \sqrt{a_c^2 + a_t^2}$

6. $a_c \perp a_t$



$\theta = 90°$

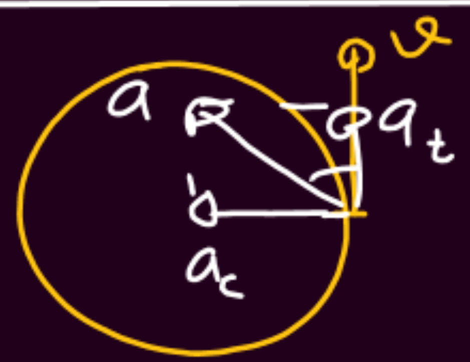
UCM

$a_t = 0$

$\vec{a} \perp \vec{v}$

[$W = 0 \quad P = 0$]

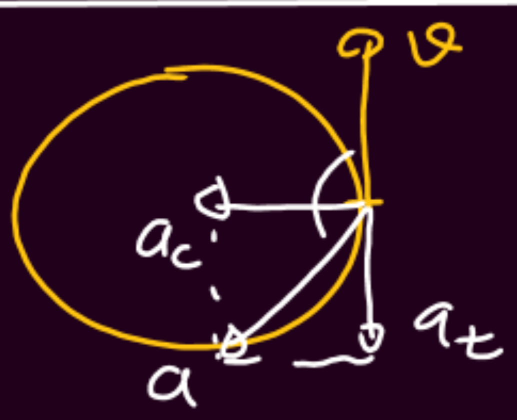
$K = \text{const.}$



$0° < \theta < 90°$

$v \uparrow$

$\omega \uparrow$



$90° < \theta < 180°$

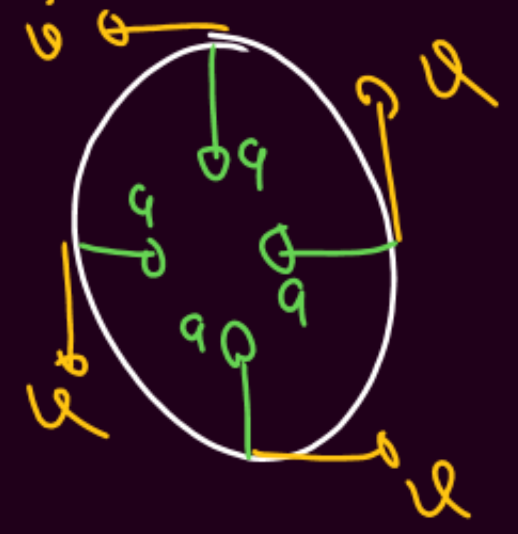
$v \downarrow$

$\omega \downarrow$

[W.D./Power by a_c is '0']

UCM

- $\omega / v = \text{const.}$
- $\alpha = 0$
- $a_t = 0$
- $a = a_c = \frac{v^2}{r} = \omega^2 r$
- Only dir of vel. changes
- Non-uniform motion (vel. is variable)
- Non-uniformly Acc. motion (dir of a_c is changing)



CM

- a_c is must
- a_t may / may not be '0'

If $a_t = \text{const}$

Scalar

① $v = u + a_t t$

② $\text{Dist.} = ut + \frac{1}{2} a_t t^2$

③ $v^2 = u^2 + 2 a_t (\text{Dist.})$

Q. Watch
second's hand (10 cm)
speed & acc. of Tip.

$T = 60\,s$

$\omega = \frac{2\pi}{60}$

① $v = \omega r = \frac{2\pi}{60}(10)$

$v = \frac{\pi}{3}$ cm/s

② $a_c = \omega^2 r = \frac{\pi}{30}\frac{\pi}{30}(10)$

$a_c = \frac{\pi^2}{90}$ cm/s²

Q. CM from rest $\alpha = 2$ rad/s²
$R = 4m$, At $t = 2s$

① $\omega = \alpha t = (2)(2) = 4$ rad/s

② $v = \omega r = (4)(4) = 16$ m/s

③ $a = \sqrt{a_c^2 + a_t^2} = \sqrt{64^2 + 8^2} = 8\sqrt{8^2+1} \approx 64$ m/s²

$a_c = \omega^2 r = (4)^2(4) = 64$ m/s²

$a_t = \alpha r = (2)(4) = 8$ m/s²

④ $\Delta\theta = \frac{1}{2}\alpha t^2$

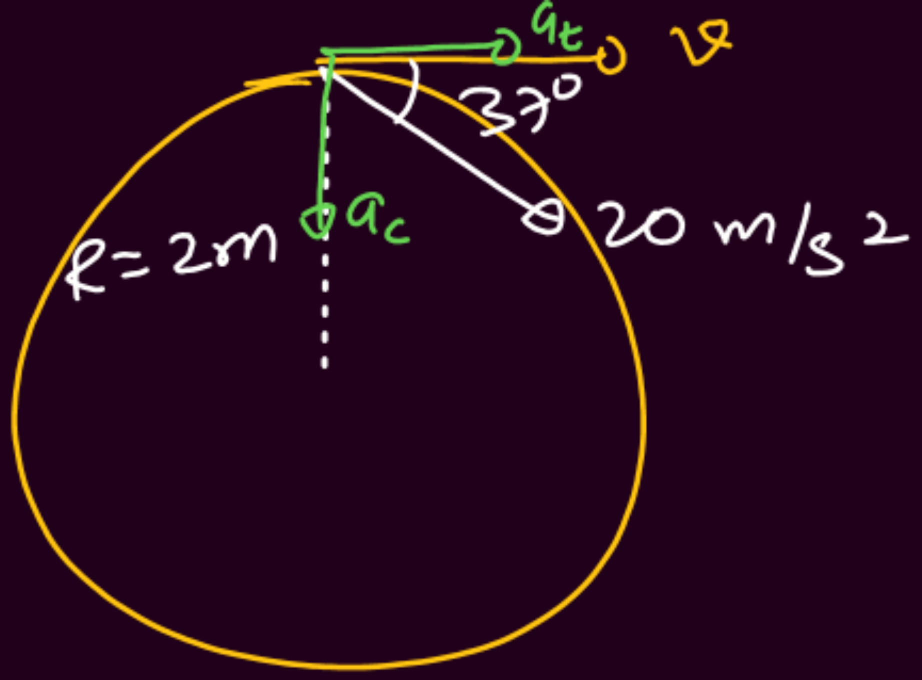
$2\pi = \frac{1}{2}(2)t^2$

$t = \sqrt{2\pi}$ sec

Dist $= \frac{1}{2}a_t t^2$

$2\pi(4) = \frac{1}{2}(8)t^2$

Q.



$v = ?$

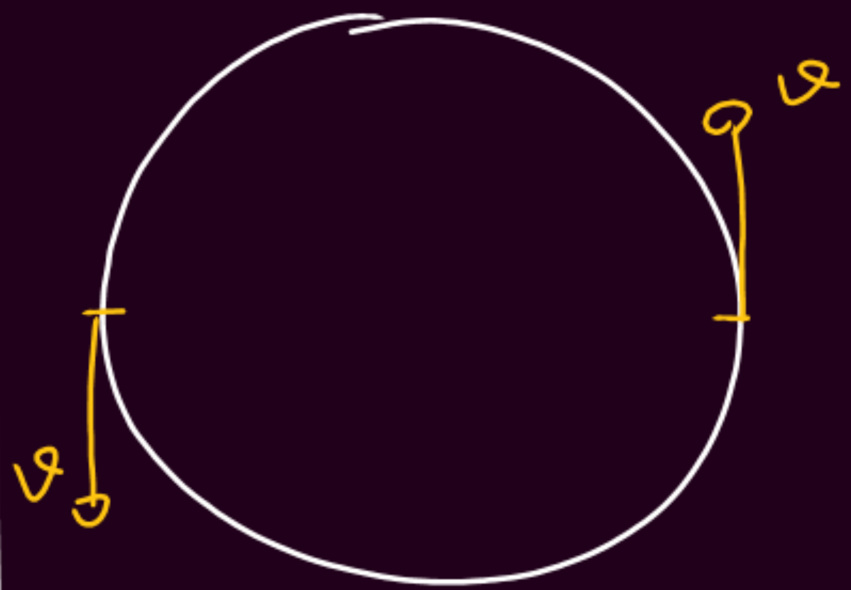
$a_c = 20 \sin 37^0 = 20 \times \frac{3}{5} = 12\ m/s^2$

$\frac{v^2}{2} = 12$

$v^2 = 24$

$v = 2\sqrt{6}\ m/s$

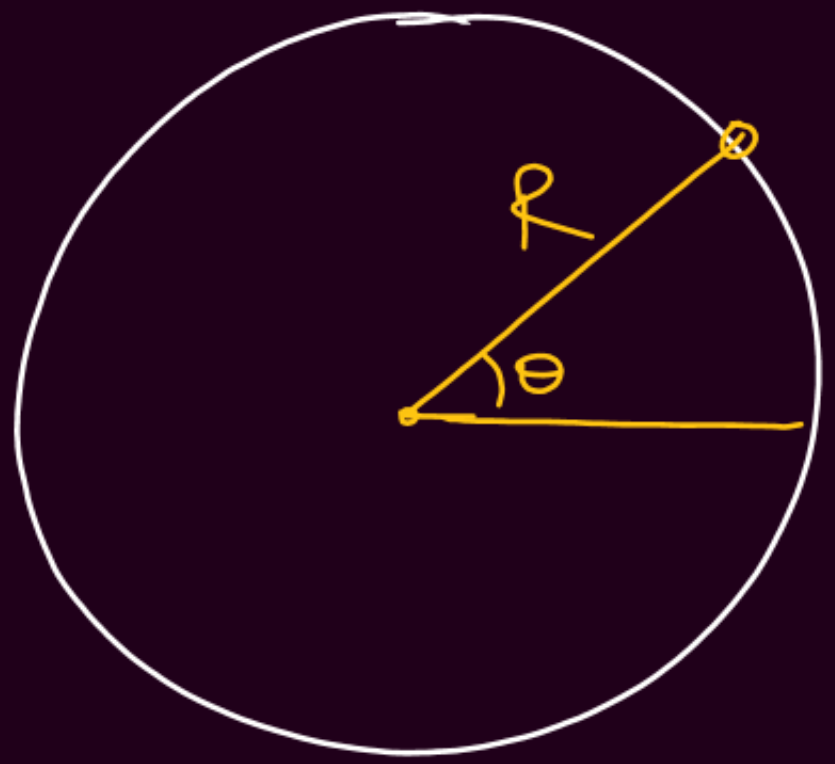
Q. UCM $\frac{a_{av}}{a_c}$ in Half Rev



$a_{av} = \frac{\Delta v}{\Delta t} = \frac{2v}{\left(\frac{\pi R}{v}\right)} = \frac{2v^2}{\pi R} = \frac{2}{\pi} a_c$

$\frac{a_{av}}{a_c} = \frac{2}{\pi}$

Q. $\vec{r} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$



① Path → Circular

② Radius → 4

③ speed $= \omega r = (2\pi)(4) = 8\pi$

④ Acc. $= a_c = \omega^2 r$

$= (2\pi)^2 (4)$

$= 16\pi^2$

Q. Speed $v = 4t$, Radius $= 2m$

① $a_t = \frac{dv}{dt} = 4$

② time when $a_c = a_t$

$$\frac{v^2}{R} = a_t$$

$$\frac{(4t)^2}{2} = 4$$

$$4t = 2\sqrt{2}$$

$$\boxed{t = \frac{1}{\sqrt{2}} \text{ sec}}$$

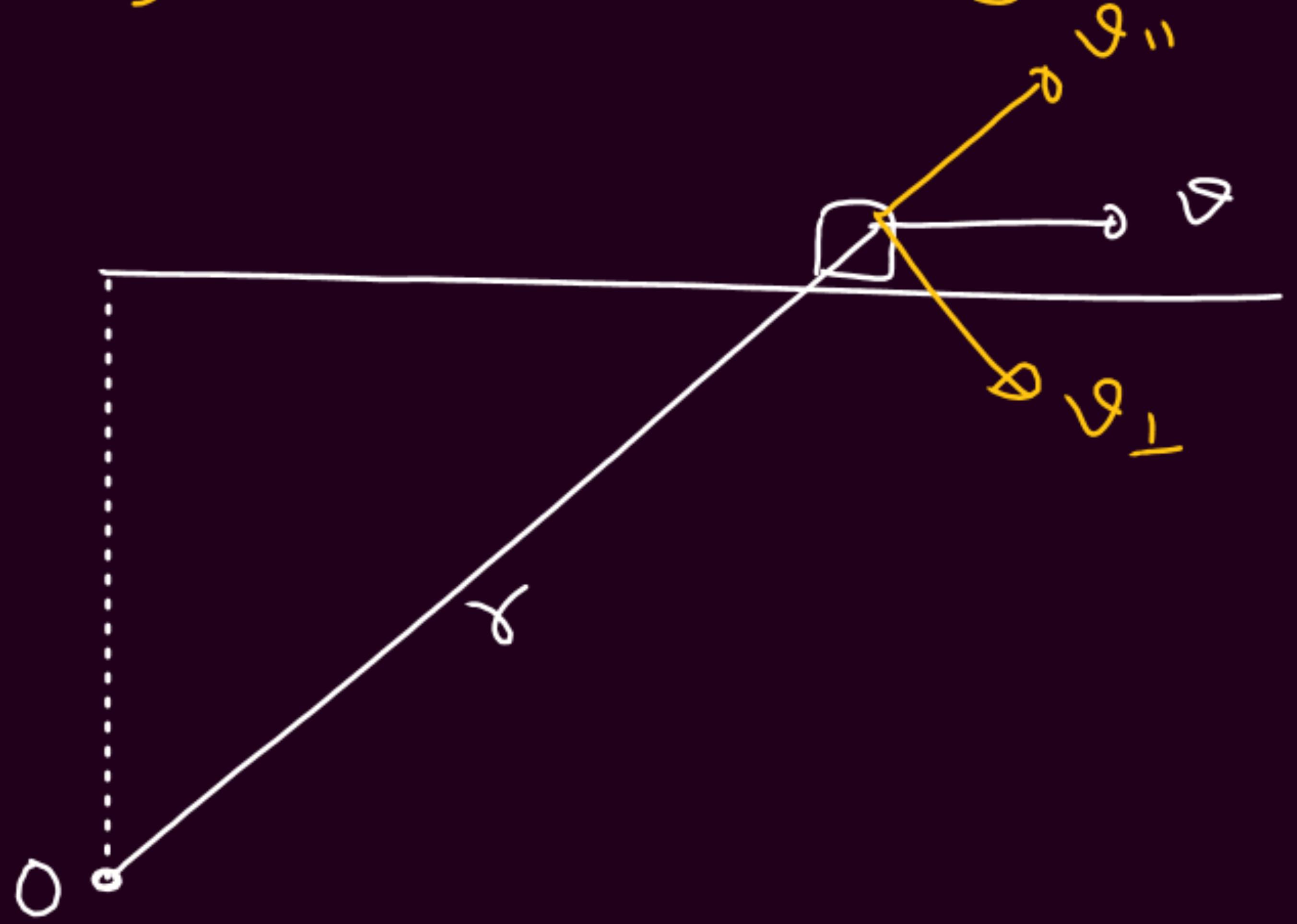
$v = \omega(2) = 4t$

$\omega = 2t$

$\alpha = \frac{d\omega}{dt} = 2$

$a_t = \alpha r = 4$

Ang. Vel. when obj → linear Motion (Extra)



$$\boxed{\omega = \frac{v_{\perp}}{r}}$$

Dynamics of CM

$F_c = m\frac{v^2}{r} = m\omega^2 r$

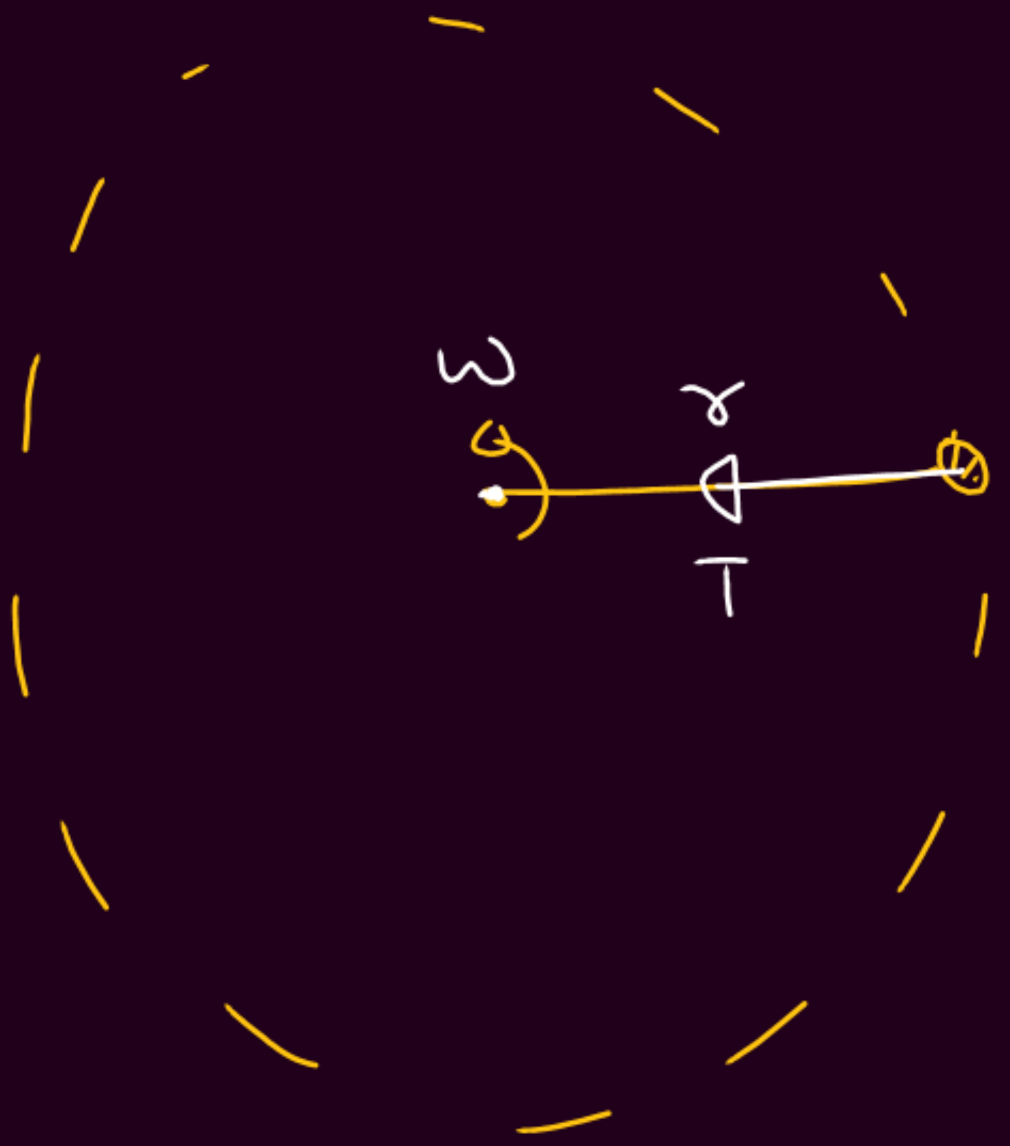
$F_t = m a_t = m\alpha r$

Centrifugal force → (Pseudo force)

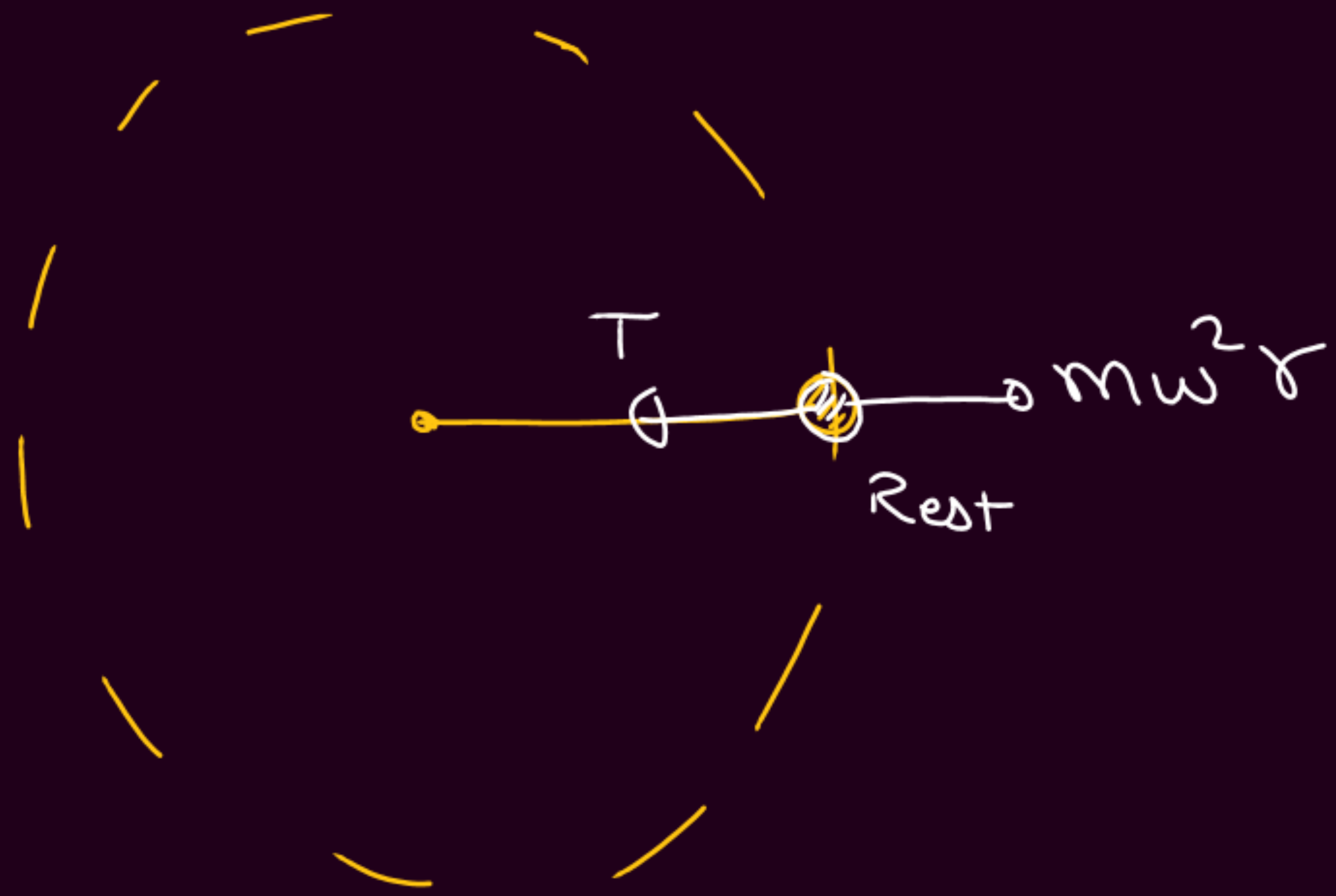
→ NIF → (frame Circular Motion)

→ Radially Outward

$T = m\omega^2 r$



NIF



Q 2 kg whirled

$l = 2\ m$

$T_{max} = 100\ N$

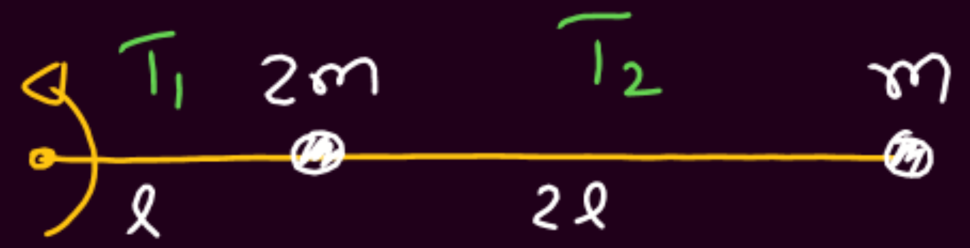
$$T = \frac{mv^2}{R}$$

$$100 = \frac{2v^2}{(2)}$$

$$v^2 = 100$$

$$v = 10\ m/s$$

Q.



(1)

$m\omega^2(3l)$

$$\boxed{T_2 = 3m\omega^2 l}$$

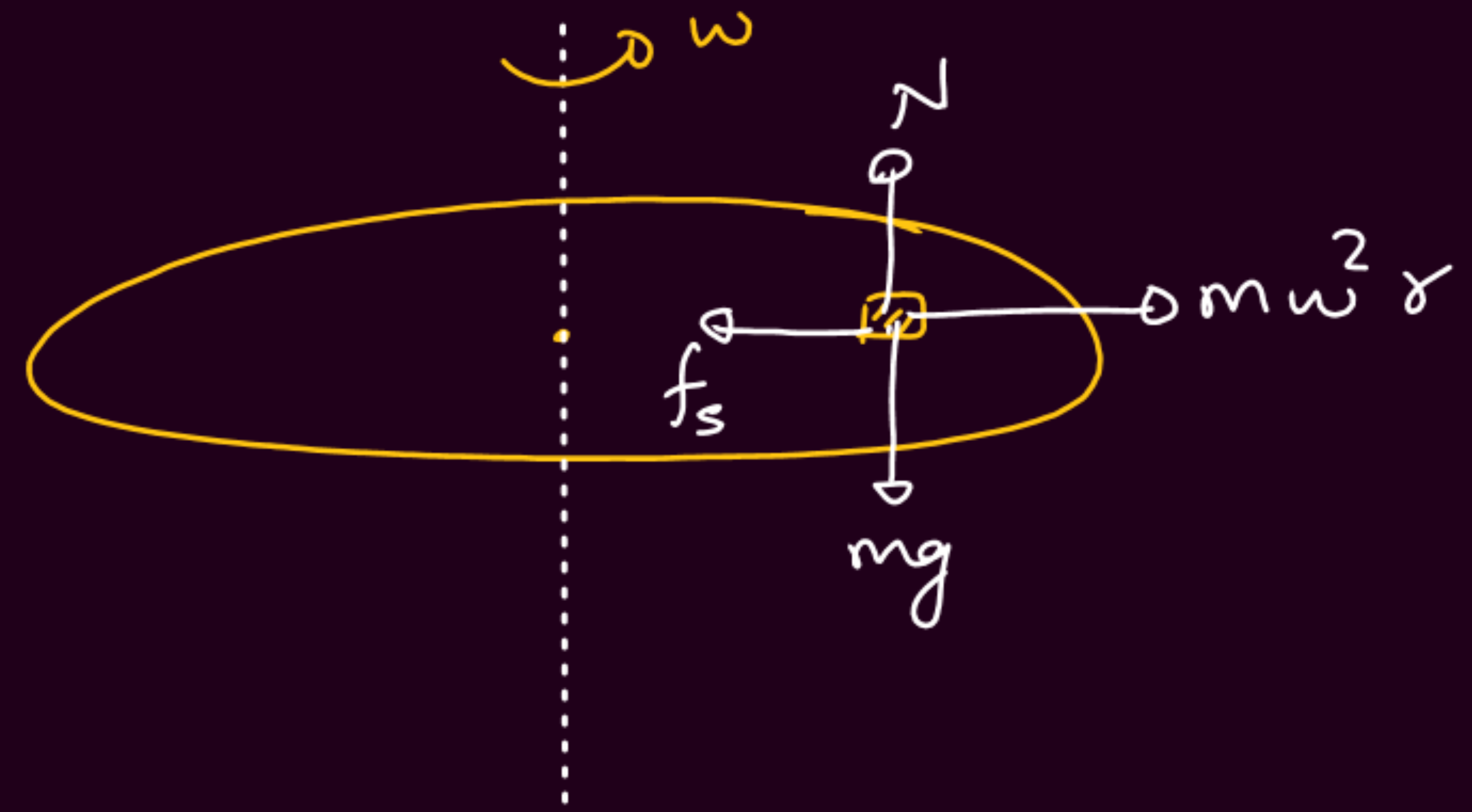
(2)

$(2m)\omega^2 l$

$$T_1 = T_2 + 2m\omega^2 l$$

$$\boxed{T_1 = 5m\omega^2 l}$$

Coin & plate



① $N = mg$

② $f_L = \mu mg$

③ $f_s = m\omega^2 r$

④ $m\omega^2 r \le \mu mg$

$$\boxed{\omega \le \sqrt{\frac{\mu g}{r}}}$$

Q Coin 100g rotated with 10 rad/s
$r = 4\,cm$ if $\mu = 0.5$ Find $f = ?$

① $f_L = \mu mg = 0.5\,(0.1)(10) = 0.5\,N$

② $f = m\omega^2 r = 0.1\,(10)^2\left(\frac{4}{100}\right) = 0.4\,N$

$$\boxed{\therefore\ f_s = 0.4\,N}$$

Ⓘ ω_{max}

$\omega^2 r = \mu g$

$\omega = \sqrt{\frac{0.5(10)}{0.04}}$

$\omega = 5\sqrt{5}$ rad/s

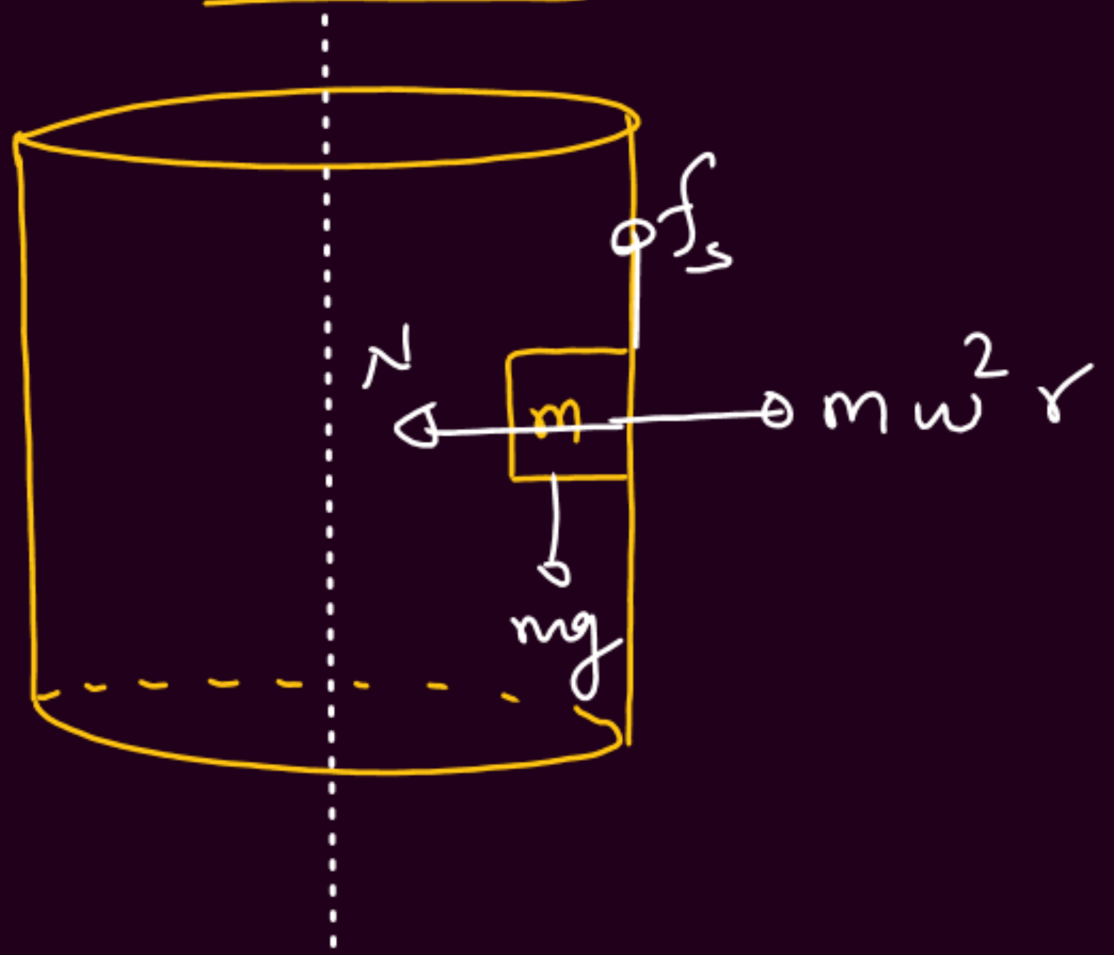
Ⓘ r_{max}

$(10)^2 r = 0.5(10)$

$r = \frac{5}{100}\,m$

$r = 5\,cm$

Death well / Rotor



① $N = m\omega^2 r$

② $f_L = \mu m\omega^2 r$

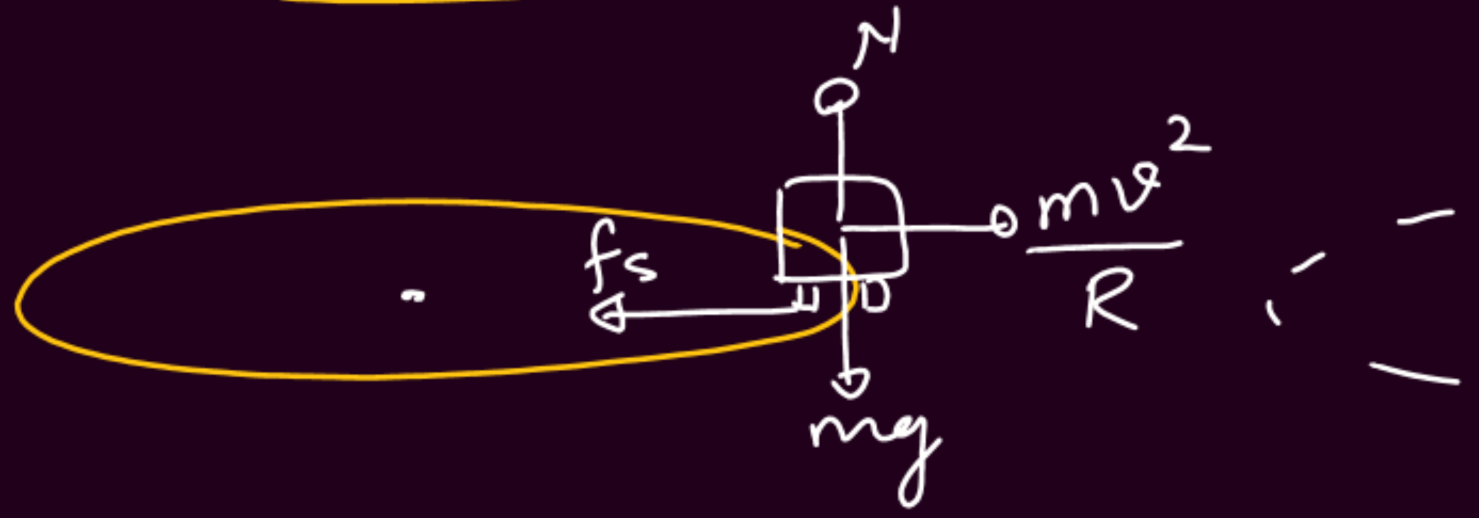
③ $f_s = mg$

$mg \le \mu m\omega^2 r$

$\omega \ge \sqrt{\frac{g}{\mu r}}$

$\boxed{v \ge \sqrt{\frac{gr}{\mu}}}$

Horizontal Turning of Road

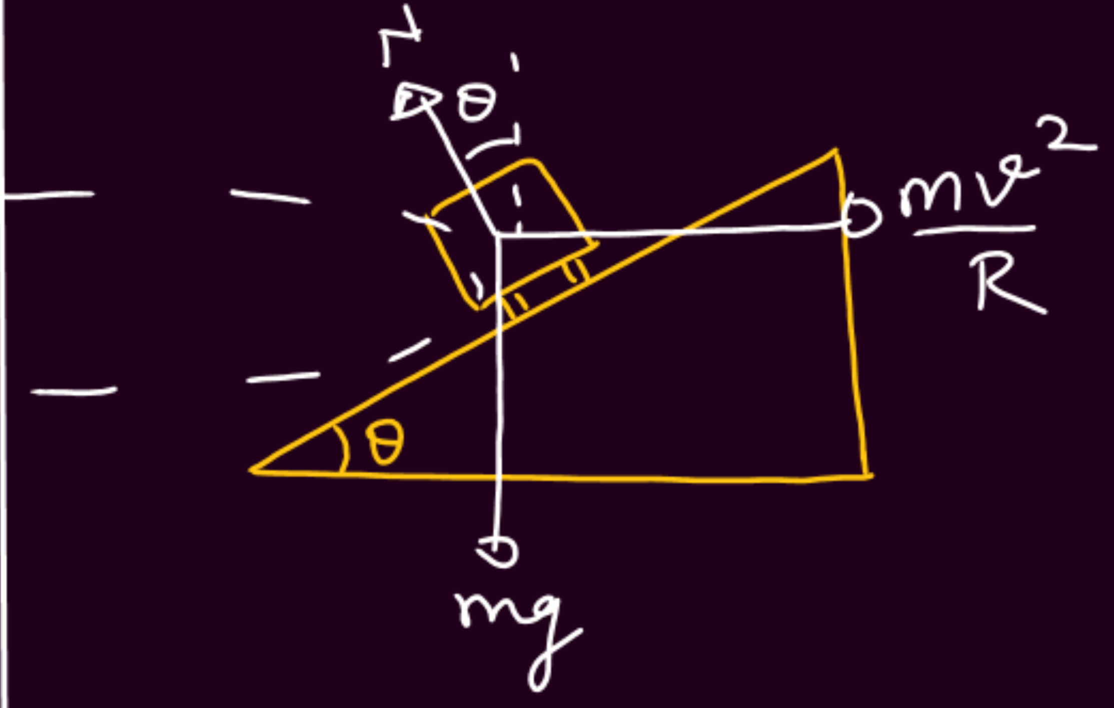


① $f_L = \mu mg$

② $f_s = \frac{mv^2}{R}$

③ $\boxed{v_{max} = \sqrt{\mu g R}}$

Banking of Road



$\tan\theta = \frac{v^2}{Rg}$

① $v = \sqrt{gR\tan\theta}$ [friction is absent or not acting]

② $\sqrt{gR\left(\frac{\tan\theta - \mu}{1+\mu\tan\theta}\right)} \le v \le \sqrt{gR\left(\frac{\tan\theta + \mu}{1-\mu\tan\theta}\right)}$

min max

Q Car $R = 50m$

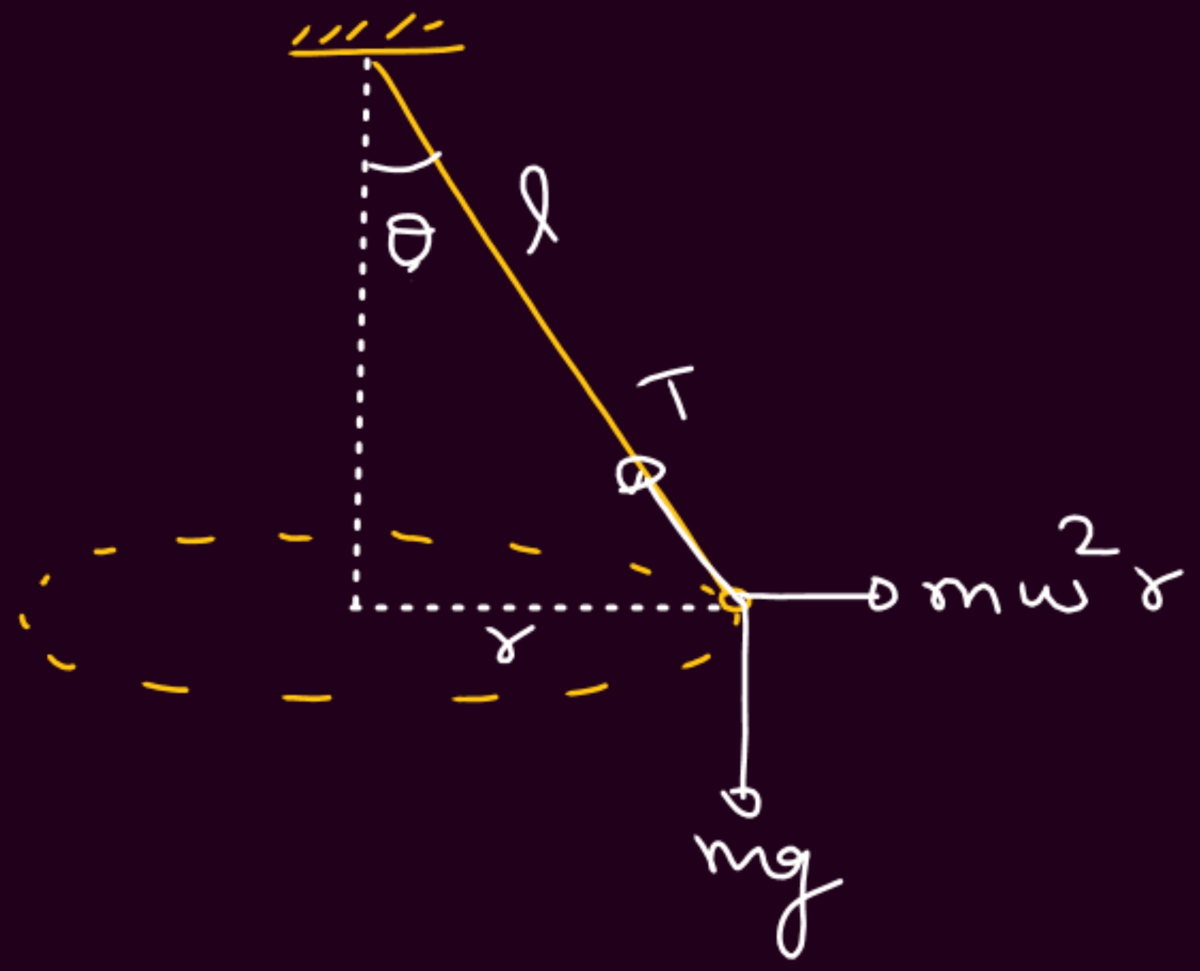
$\mu = 0.2$

$v_{max} = ?$

$$v_{max} = \sqrt{(0.2)(10)(50)}$$

$$v_{max} = 10\ m/s$$

Conical Pendulum



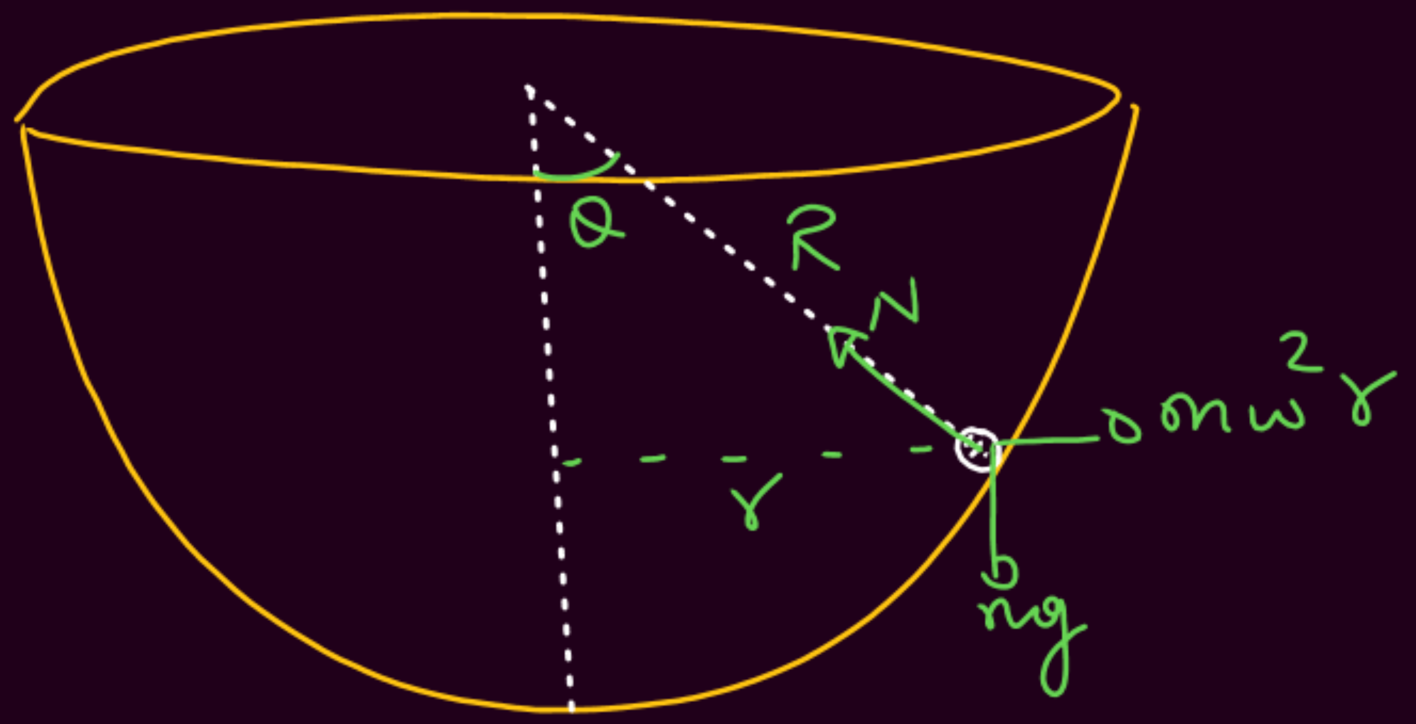
① $r = l\sin\theta$

② $\tan\theta = \dfrac{\omega^2 r}{g}$

③ $T = \dfrac{mg}{\cos\theta}$ (Tension)

④ Time Period $= 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

Marbal in - Hemispherical Bowl



Q. $m = 2\ kg$

HCM

String $l = 20\ cm$

60°

① Tension?

$$T = \frac{mg}{\cos 60^\circ}$$

$$T = \frac{20}{1/2} = \underline{40N}$$

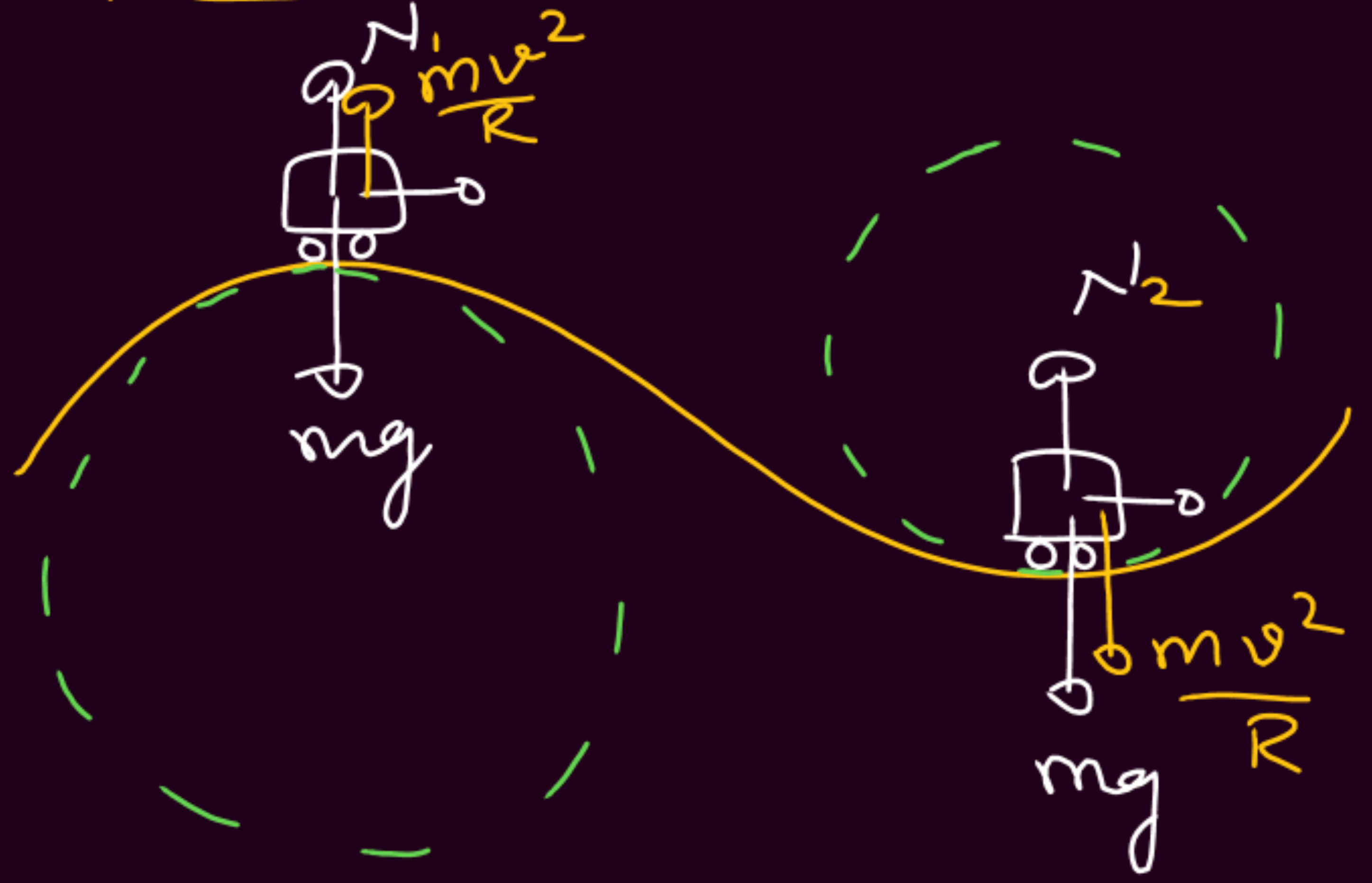
② Time Period

$$T = 2\pi\sqrt{\frac{(0.2)\left(\frac{1}{2}\right)}{10}}$$

$$T = 2\pi(0.1)$$

$$\underline{T = \frac{\pi}{5}\ sec}$$

Vertical Circular Motion



$$N_1 + \frac{mv^2}{R} = mg$$

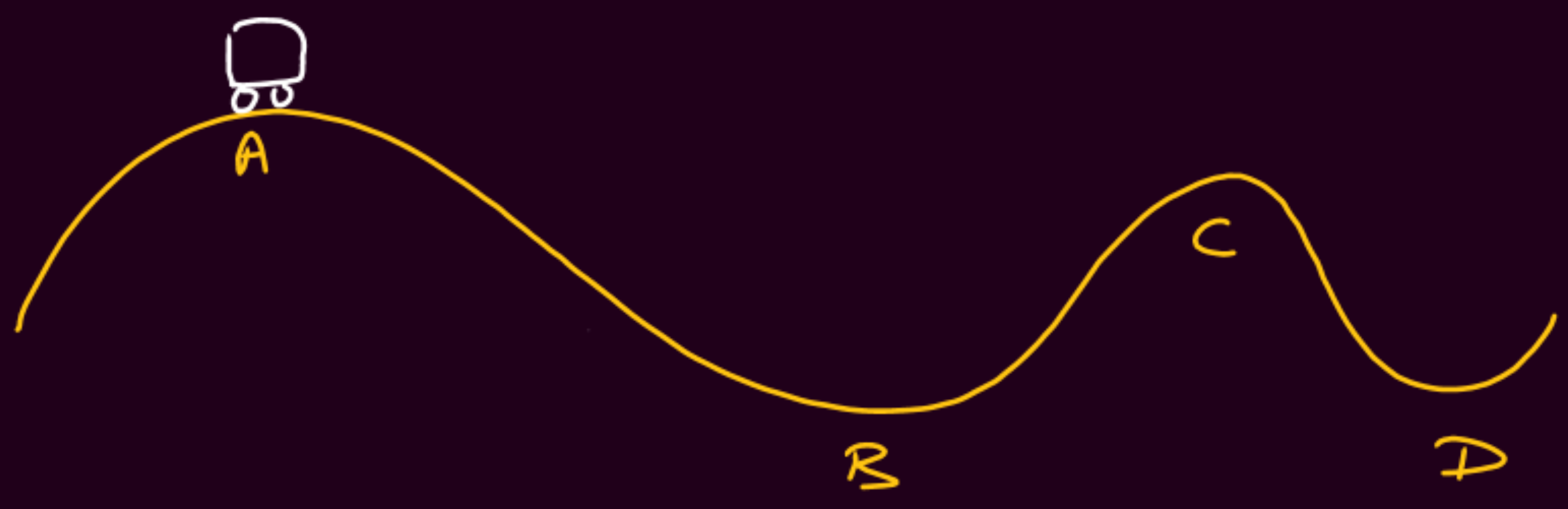
$$N_1 = mg - \frac{mv^2}{R}$$

$$\boxed{A/C}$$

$$N_2 = mg + \frac{mv^2}{R}$$

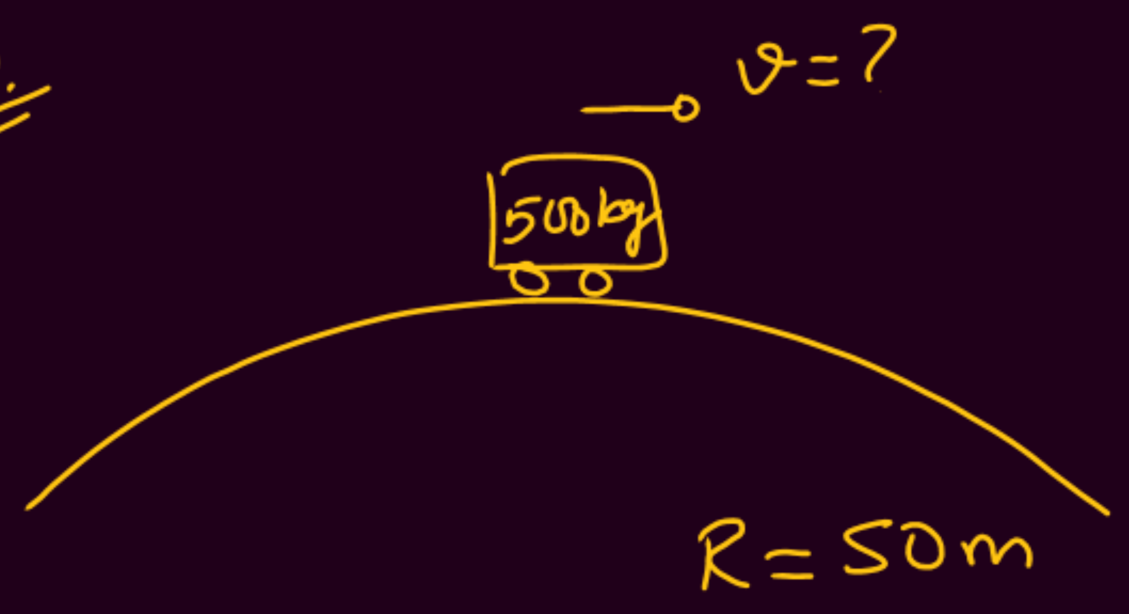
$$\boxed{B/D}$$

Const. Speed



$$N_C < N_A < N_B < N_D$$

Q.



$N = 3000\ N$

(1) $N + \frac{mv^2}{R} = mg$

$3000 + \frac{500v^2}{50} = 5000$

$10v^2 = 2000$

$v = 10\sqrt{2}\ m/s$

(2) Find v_{min} for which it loses contact?

$N = 0$

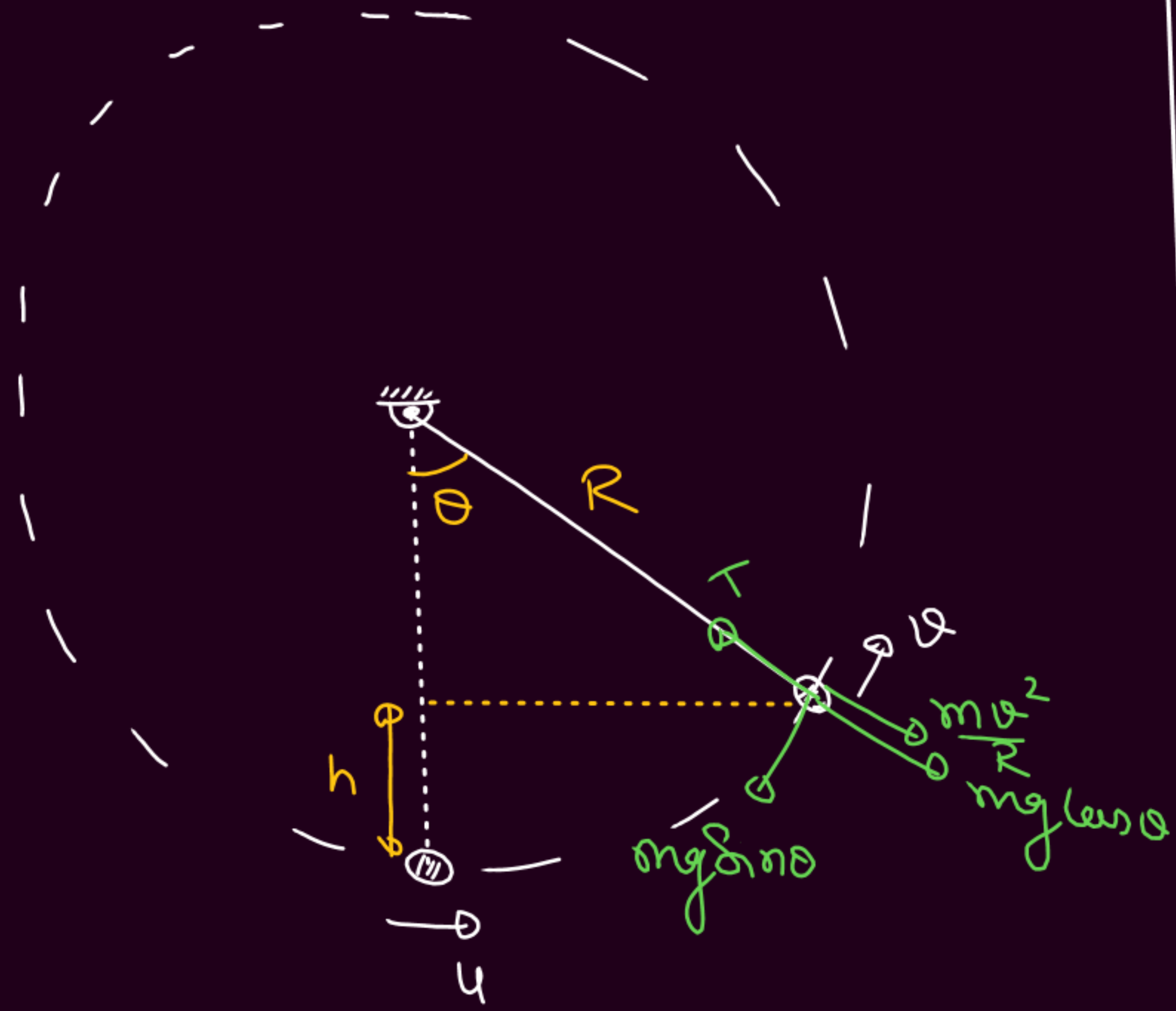
$\frac{mv^2}{R} = mg$

$v = \sqrt{gR}$

$v = \sqrt{10 \times 50}$

$v = 10\sqrt{5}\ m/s$

String



① $h = R(1 - \cos\theta)$

② $v^2 = u^2 - 2gh$

③ $a_c = \frac{v^2}{R}$

④ $a_t = g\sin\theta$

⑤ $a = \sqrt{a_c^2 + a_t^2}$

⑥ $T = mg\cos\theta + \frac{mv^2}{R}$

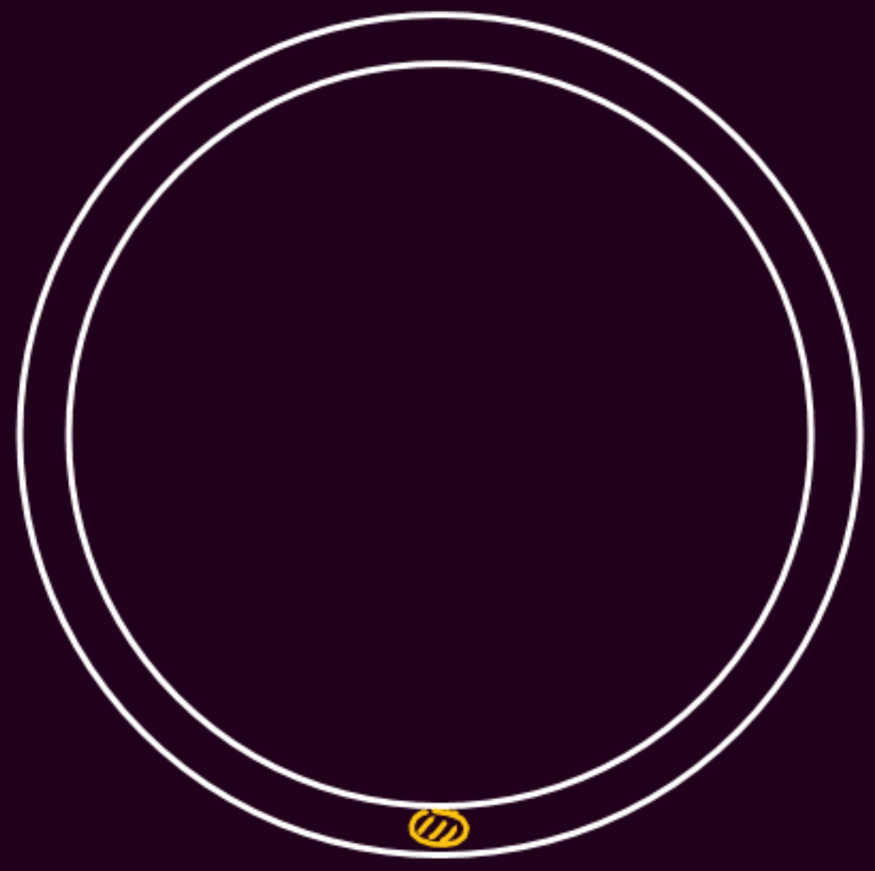
⑦ $T_{max} - T_{min} = 6mg$

$T_{Bottom} - T_{Top} = 6mg$

String / Track

① $u \le \sqrt{2gR}$	oscillation	
② $\sqrt{2gR} < u < \sqrt{5gR}$	leaves the circle	
③ $u \geq \sqrt{5gR}$	Completes the loop	

Tube



① $u < \sqrt{4gR}$ oscillates

② $u \geq \sqrt{4gR}$ completes the loop

String → Just completing the loop

① Topmost Point

$$\frac{mv^2}{R} = mg + T$$

① $\boxed{T=0}$

② $v_{min} = \sqrt{gR}$

③ $\theta = 180^\circ$

④ $h = 2R$

⑤ $a_t = 0$

⑥ $a = a_c = \frac{v^2}{R} = g$

② Bottommost Point

① $\theta = 0^\circ$

② $h = 0$

③ $u = \sqrt{5gR}$

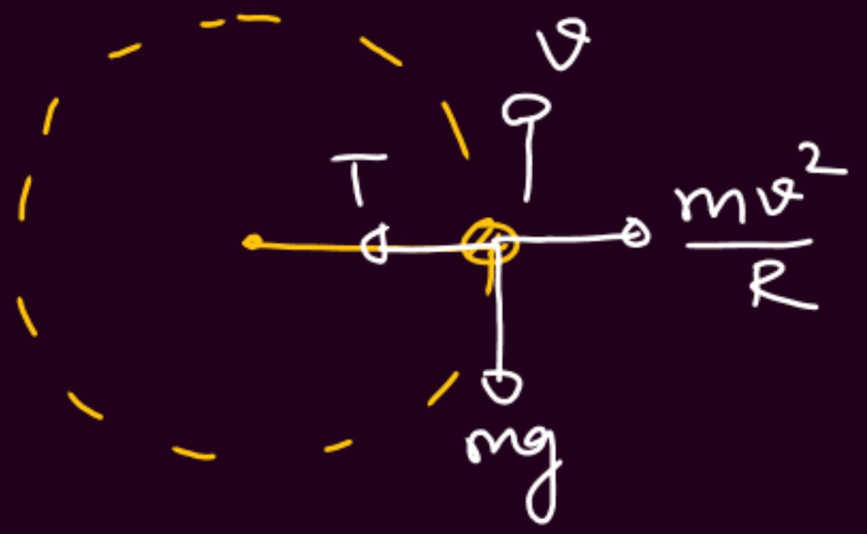
④ $a_t = 0$

⑤ $a = a_c = \frac{u^2}{R} = 5g$

⑥ $T = mg + \frac{mu^2}{R} = 6mg$

$$u^2 = gR + 2g(2R)$$

③ String is horizontal (velocity is verticle)



① $\theta = 90^\circ$ ② $h = R$

③ $v = \sqrt{5gR - 2gR}$

$v = \sqrt{3gR}$

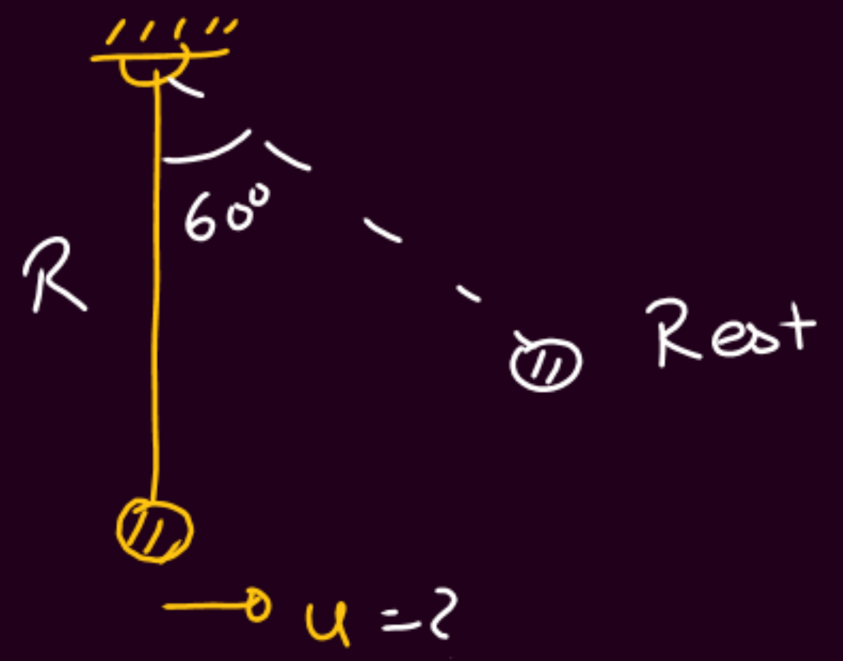
④ $a_t = g$

⑤ $a_c = \frac{v^2}{R} = 3g$

⑥ $a = g\sqrt{10}$

⑦ $T = 3mg$

Q.



① $h = R(1 - \cos 60°)$

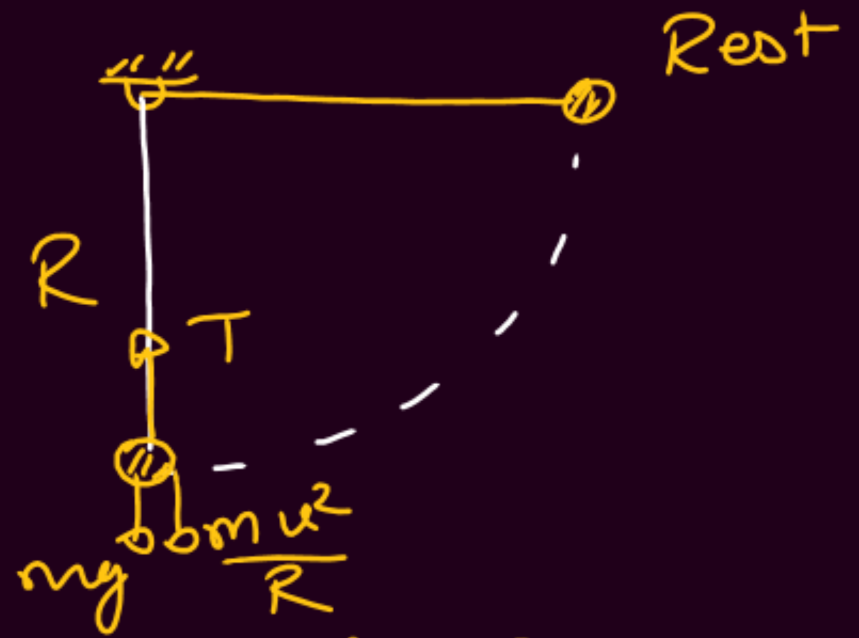
$= R\left(1 - \frac{1}{2}\right)$

$h = \frac{R}{2}$

② $0^2 = u^2 - 2g(R/2)$

$u = \sqrt{gR}$

Q



① Speed at Bottom

$u = \sqrt{2gR}$

② $T = mg + \frac{mu^2}{R}$

$T = mg + \frac{m(2gR)}{R}$

$T = 3mg$

Q. $u = \sqrt{6gR}$

① $T_{Bottom} = mg + \frac{mu^2}{R}$

$= mg + \frac{m(6gR)}{R}$

$= 7mg$

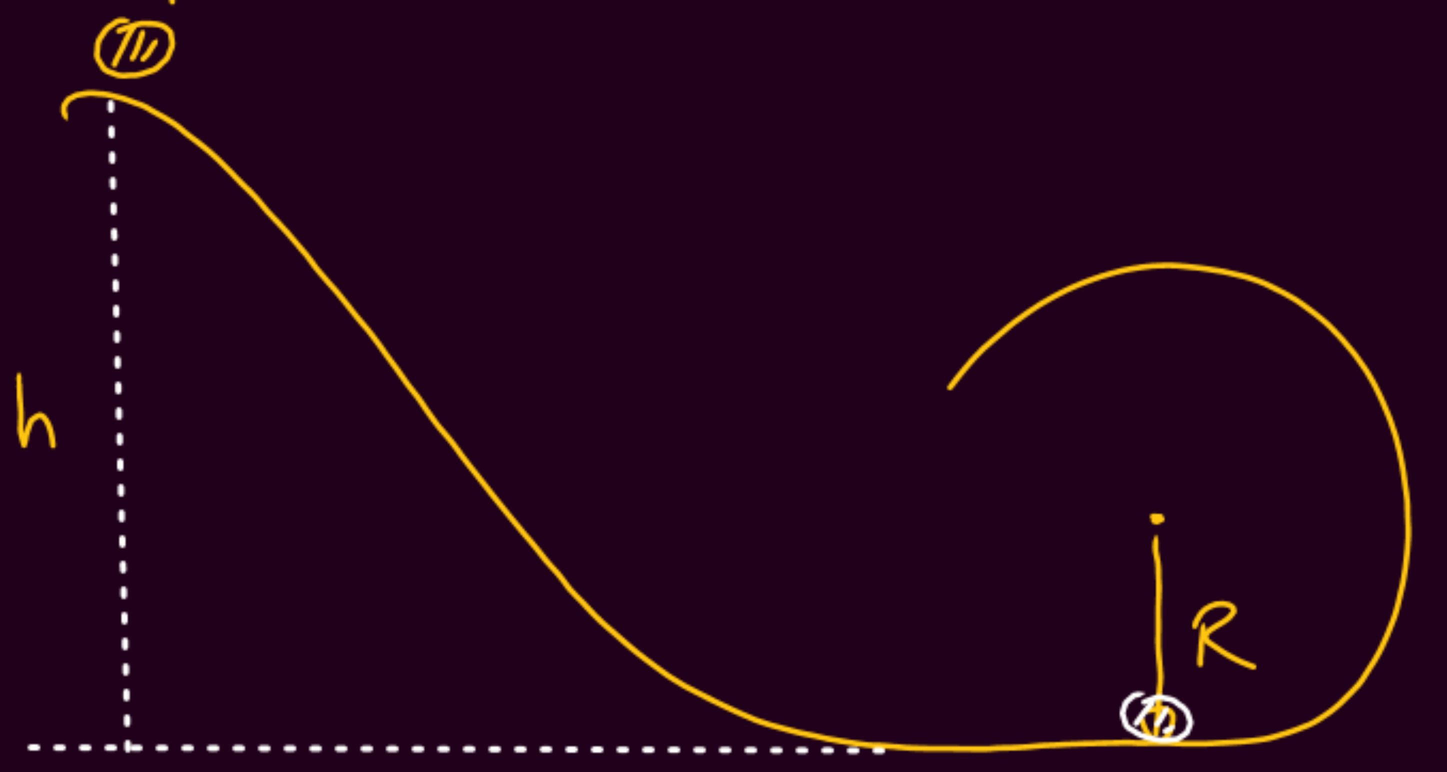
② $T_{Top} = ?$

$Bot - Top = 6mg$

$7mg - T_{Top} = 6mg$

$T_{Top} = mg$

Q.

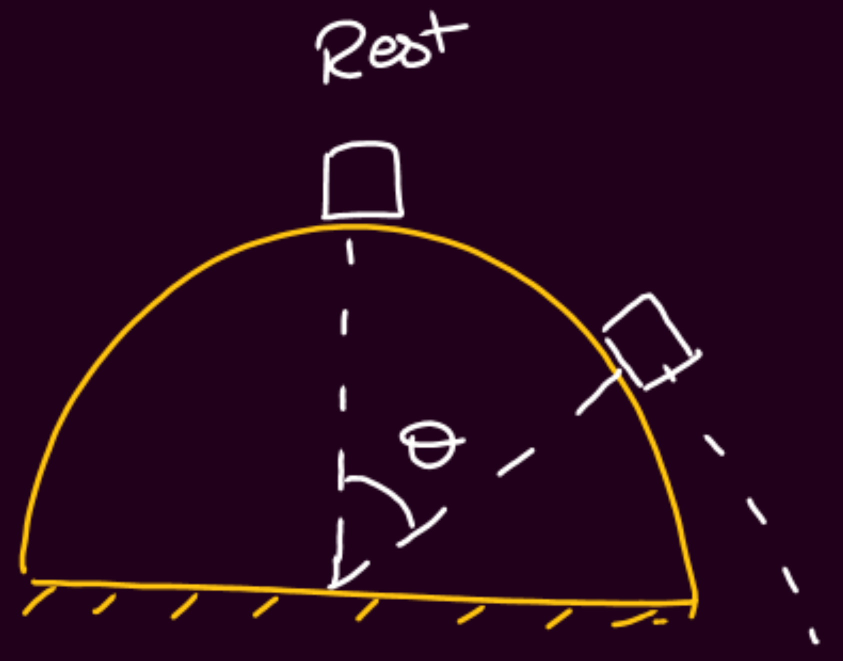


$\sqrt{2gh} = \sqrt{5gR}$

min $h = ?$

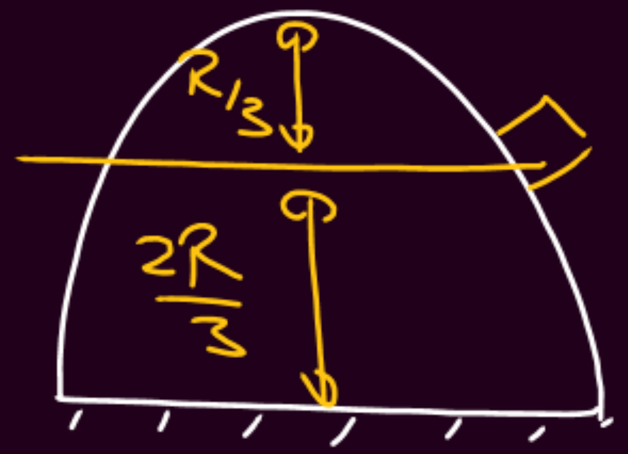
for loop

$$\boxed{h = \frac{5R}{2}}$$

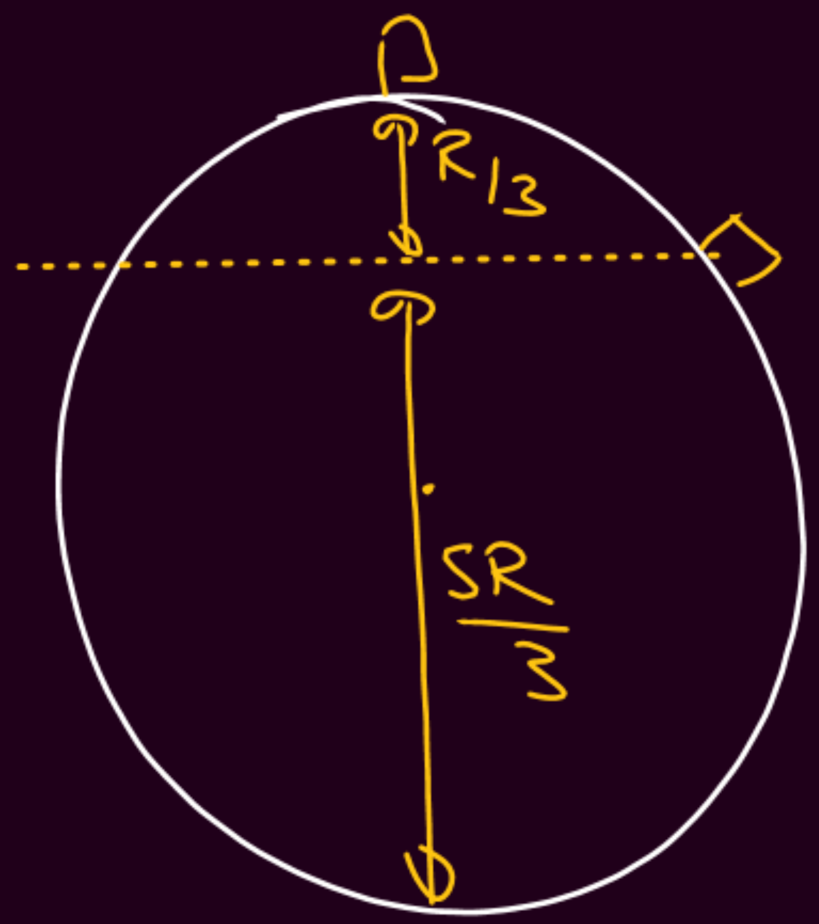


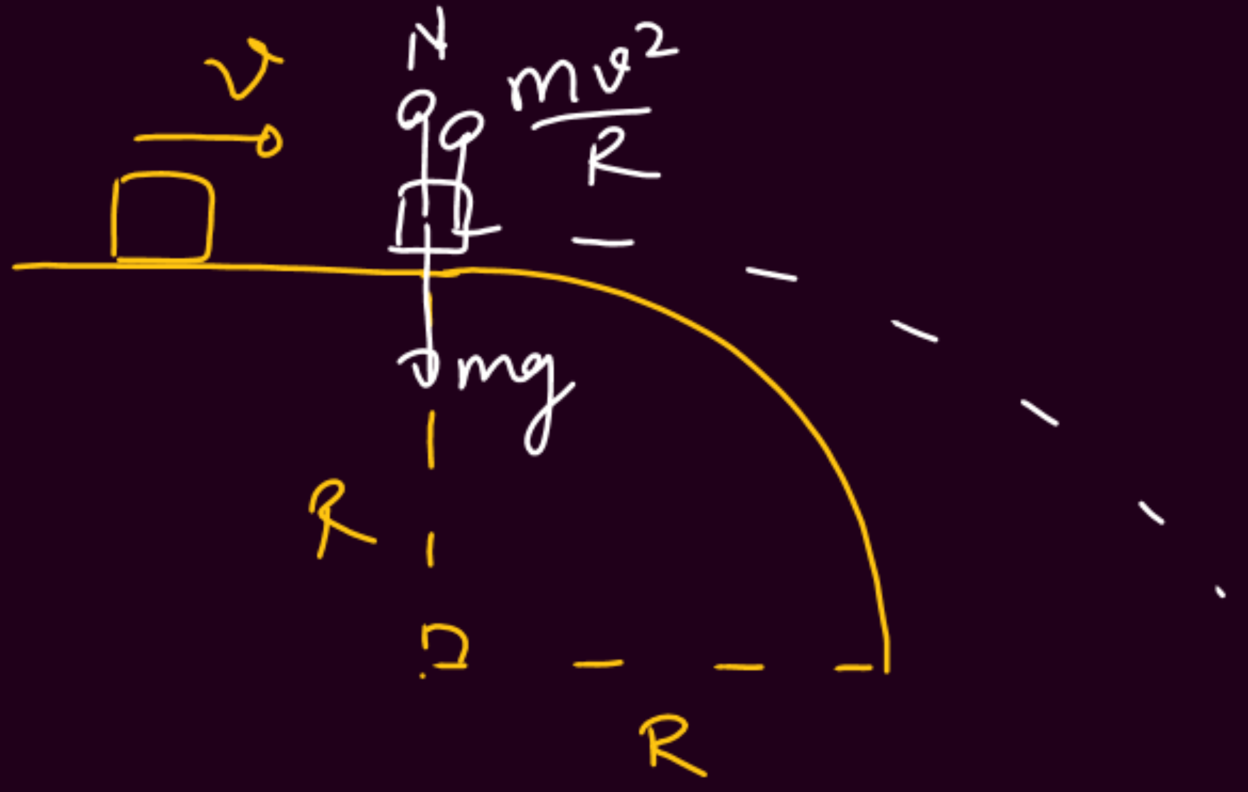
① $\cos\theta = \frac{2}{3}$

Hemisphere



Sphere





min speed

doesn't slide on track

$$N = 0$$

$$\frac{mv^2}{R} = mg$$

$$\boxed{v = \sqrt{gR}}$$