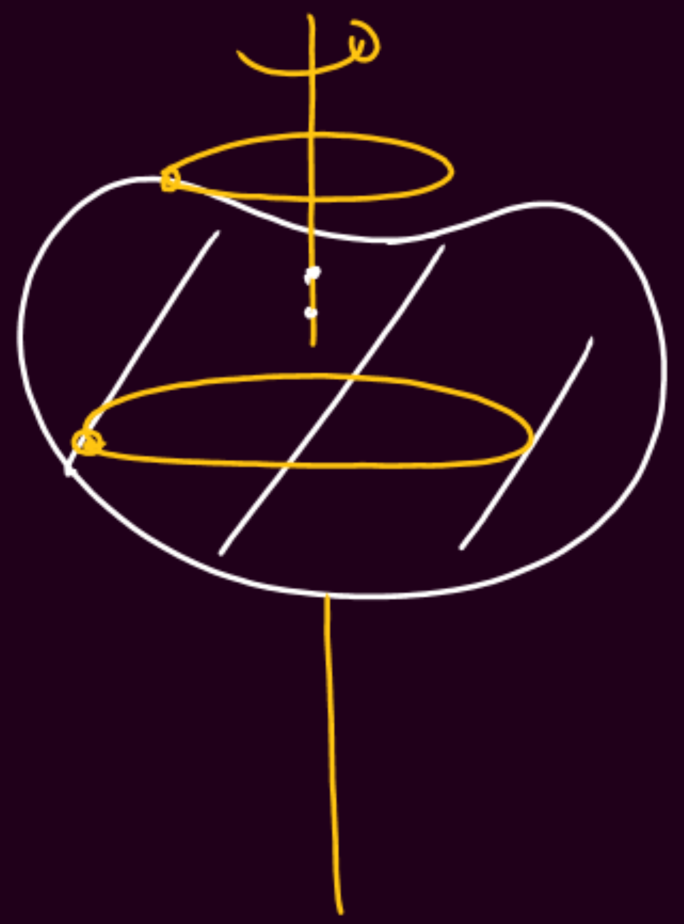
Rotational Motion

→ Rigid Body

→ Pure Rotation



→ Pt. on Axis

→ All the particles
same $\Delta\theta$ / ω / α

→ $v_i = \omega r_i$

→ $a_{c_i} = \omega^2 r_i$

→ $a_{t_i} = \alpha r_i$

Moment of Inertia (2)

→ Inertia of Rotation

Pt. Mass

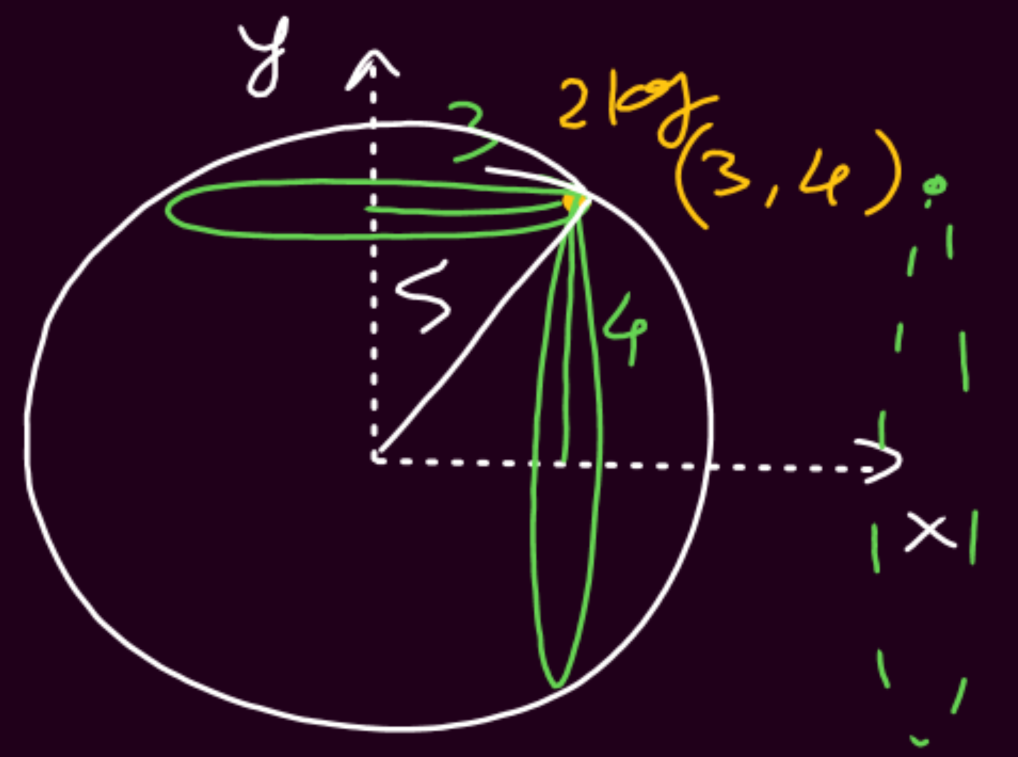
$$\boxed{I = mr^2}$$

unit kg-m^2

Tensor → MoI
→ Stress

Depends on → mass
→ Axis
→ Distribution

Q.



$I_x = 2(4)^2$

$I_y = 2(3)^2$

$I_z = 2(3^2+4^2)$

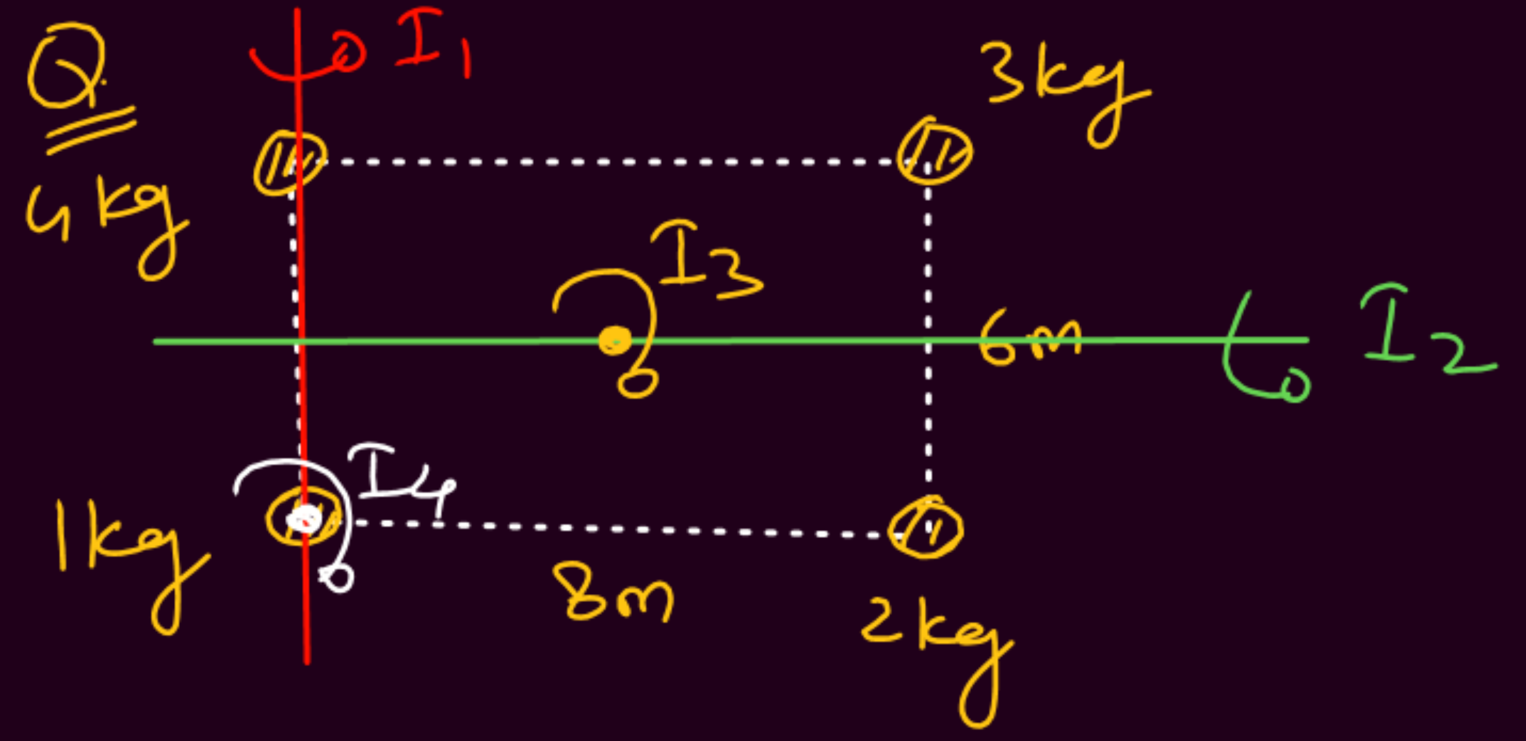
Pt. Mass

$I_x = m(y^2+z^2)$

$I_y = m(z^2+x^2)$

$I_z = m(x^2+y^2)$

Q.



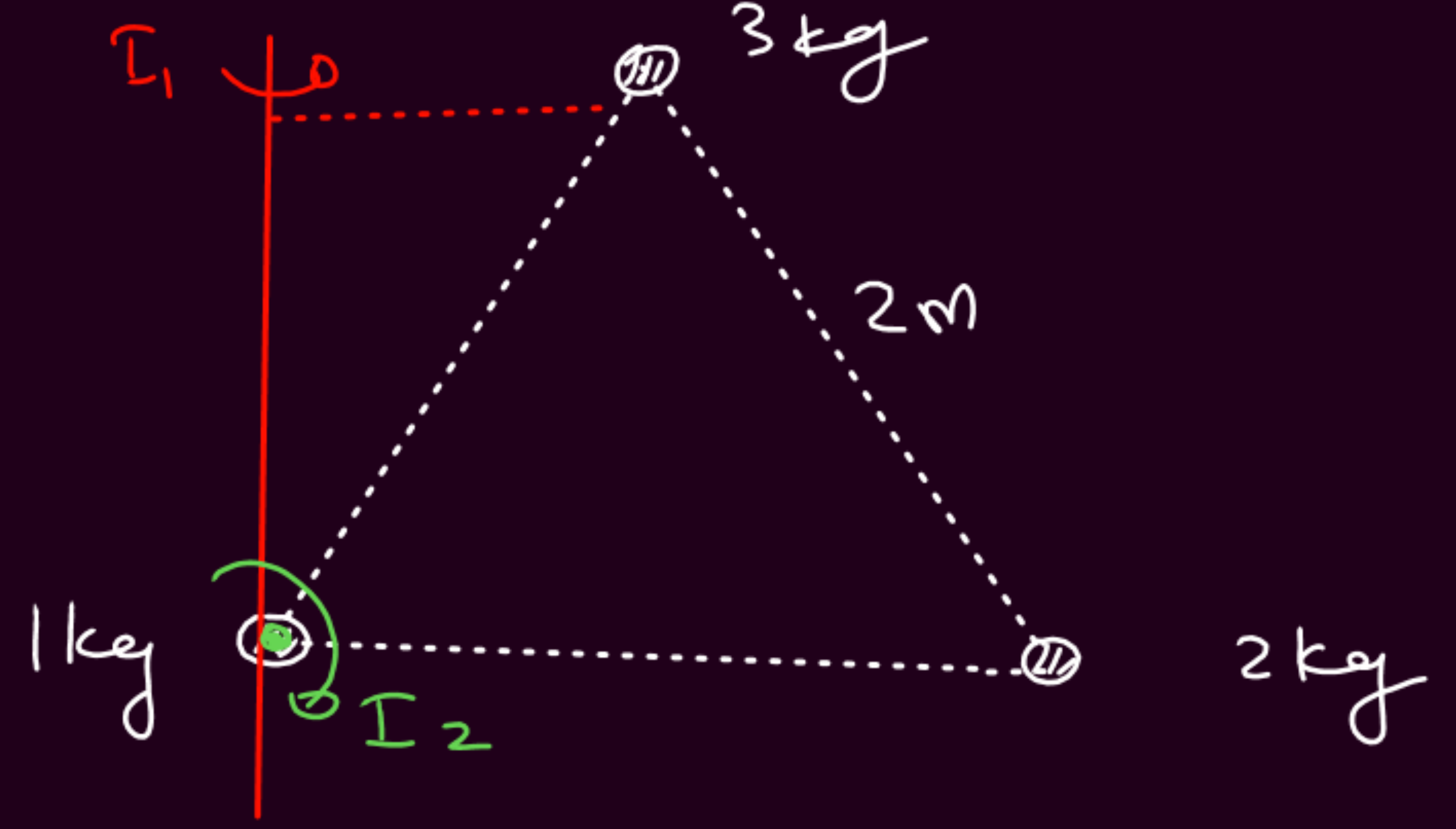
$$I_1 = 1(0)^2 + 2(8)^2 + 3(8)^2 + 4(0)^2$$

$$I_2 = 1(3)^2 + 2(3)^2 + 3(3)^2 + 4(3)^2$$

$$I_3 = 1(5)^2 + 2(5)^2 + 3(5)^2 + 4(5)^2$$

$$I_4 = 1(0)^2 + 2(8)^2 + 3(10)^2 + 4(6)^2$$

Q.



$$I_1 = 1(0)^2 + 2(2)^2 + 3(1)^2$$

$$I_2 = 1(0)^2 + 2(2)^2 + 3(2)^2$$

① HS

About its axis

$I = \frac{2}{3} mR^2$

② SS

About its axis

$I = \frac{2}{5} mR^2$

③ Triangular Plate

About its Edge

$I = \frac{1}{6} M b^2$

$\boxed{MR^2}$

Pt. Mass

Ring about its axis

HC about its Axis

Rod ‖ to Axis

$\boxed{\frac{1}{2} MR^2}$

Disc about its axis

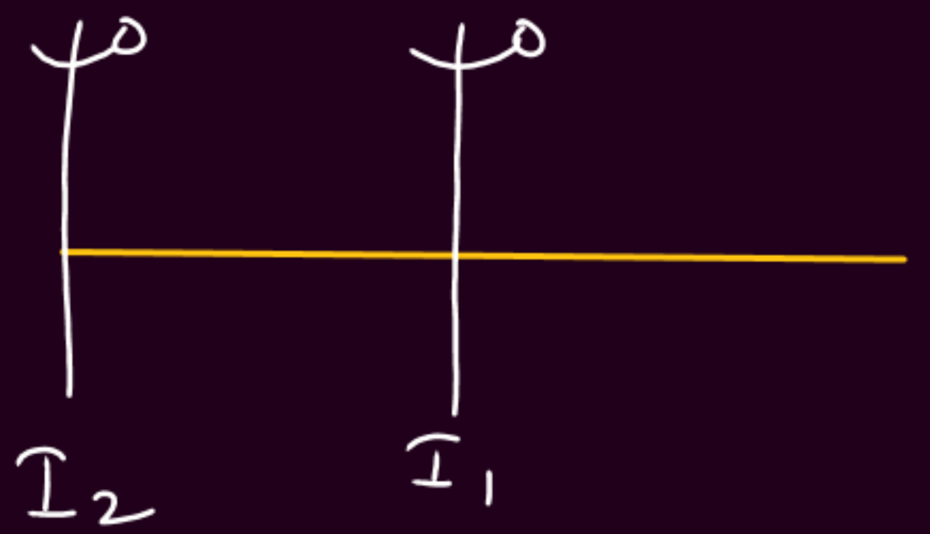
SC about its axis

Slice

$\frac{\frac{1}{2}(nM)R^2}{n}$

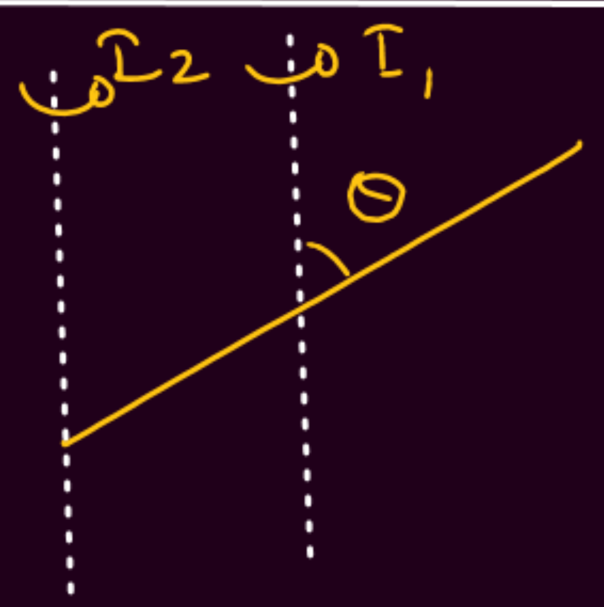
Pastry

Rod



$\perp$ Bisector $\quad I_1 = \frac{1}{12} m l^2$

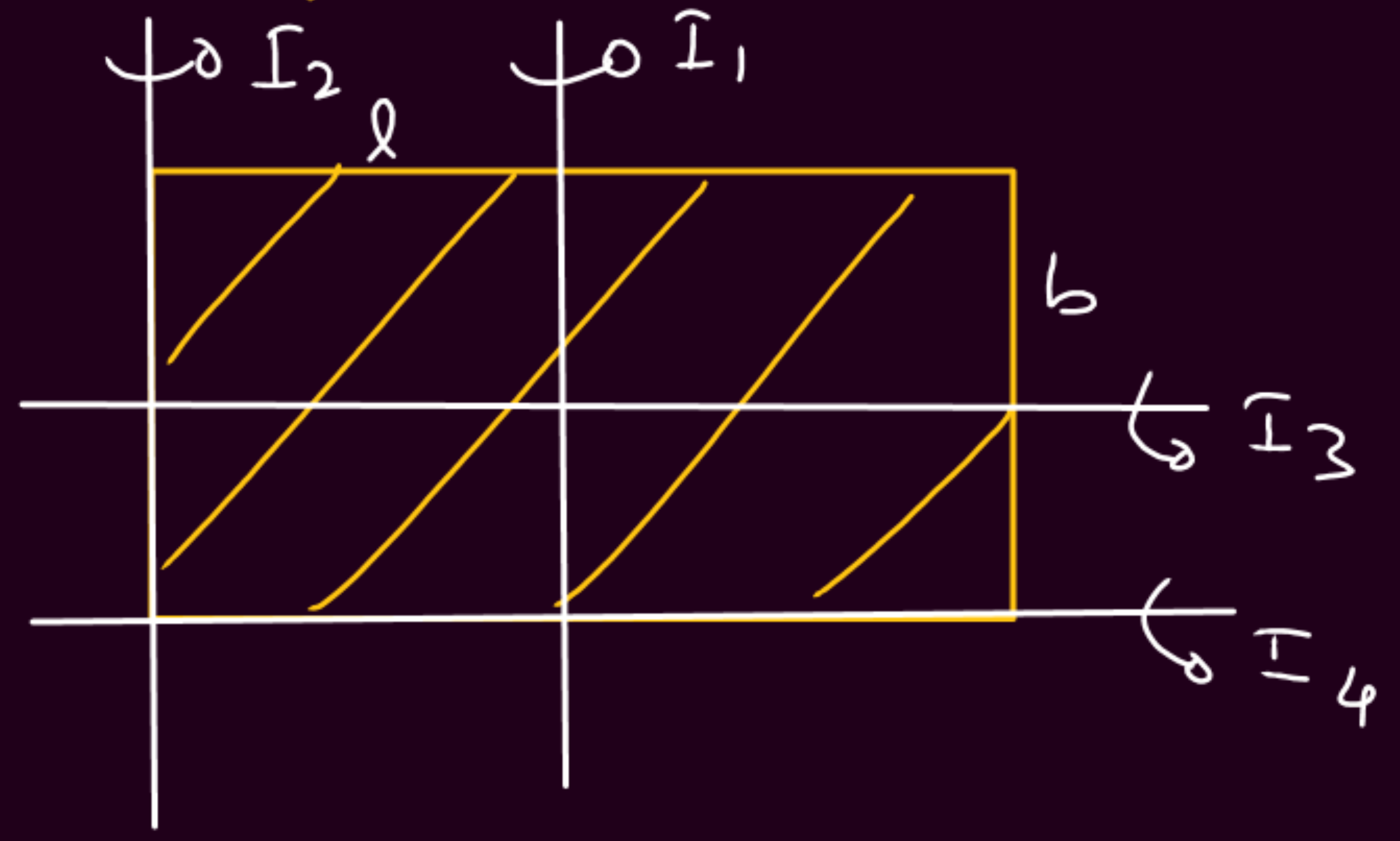
$\perp$ through End $\quad I_2 = \frac{1}{3} m l^2$



$$I_1 = \frac{1}{12} m l_1^2 = \frac{1}{12} m (l \sin\theta)^2$$

$$I_2 = \frac{1}{3} m l_1^2 = \frac{1}{3} m (l \sin\theta)^2$$

Rectangular Sheet



$$I_1 = \frac{1}{12} m l^2$$

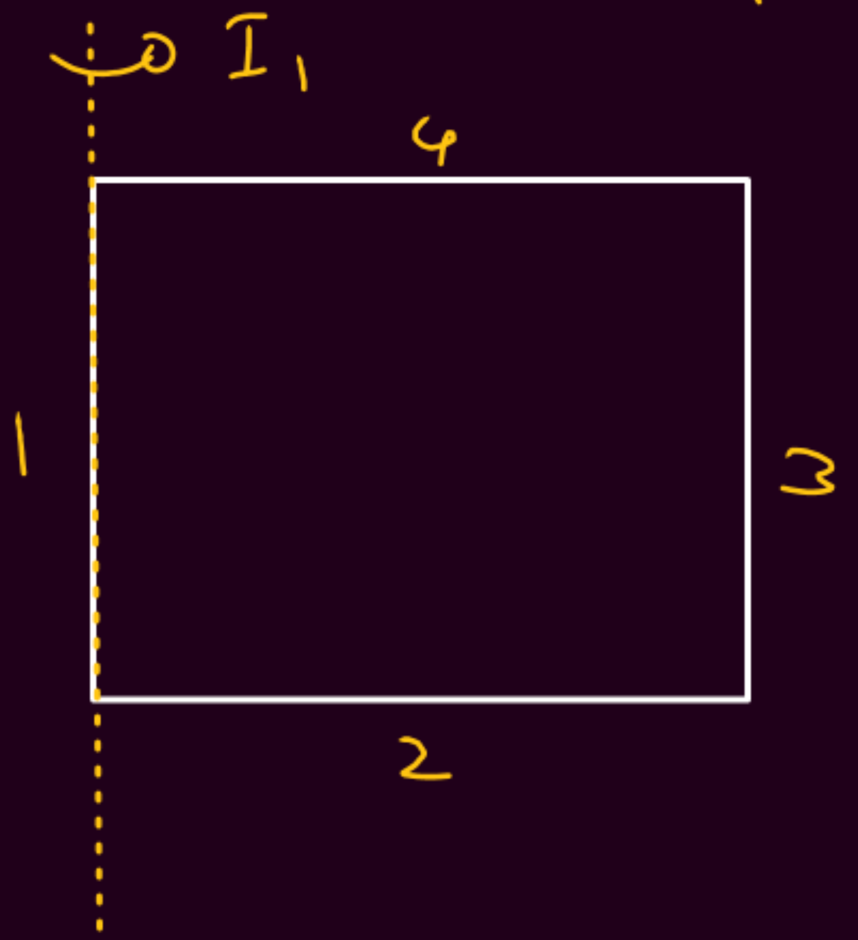
$$I_2 = \frac{1}{3} m l^2$$

$$I_3 = \frac{1}{12} m b^2$$

$$I_4 = \frac{1}{3} m b^2$$

Q Square frame

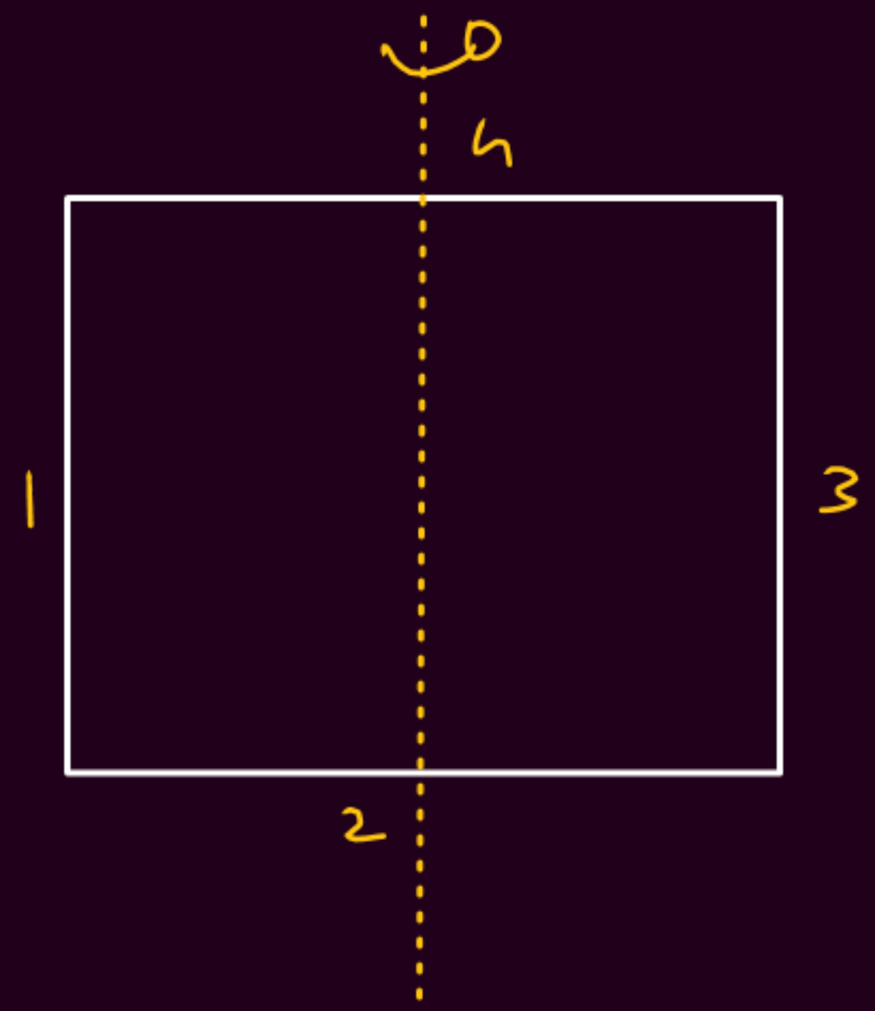
Each (m, ℓ)



$$I = 0 + \frac{1}{3}m\ell^2 + m\ell^2 + \frac{1}{3}m\ell^2$$

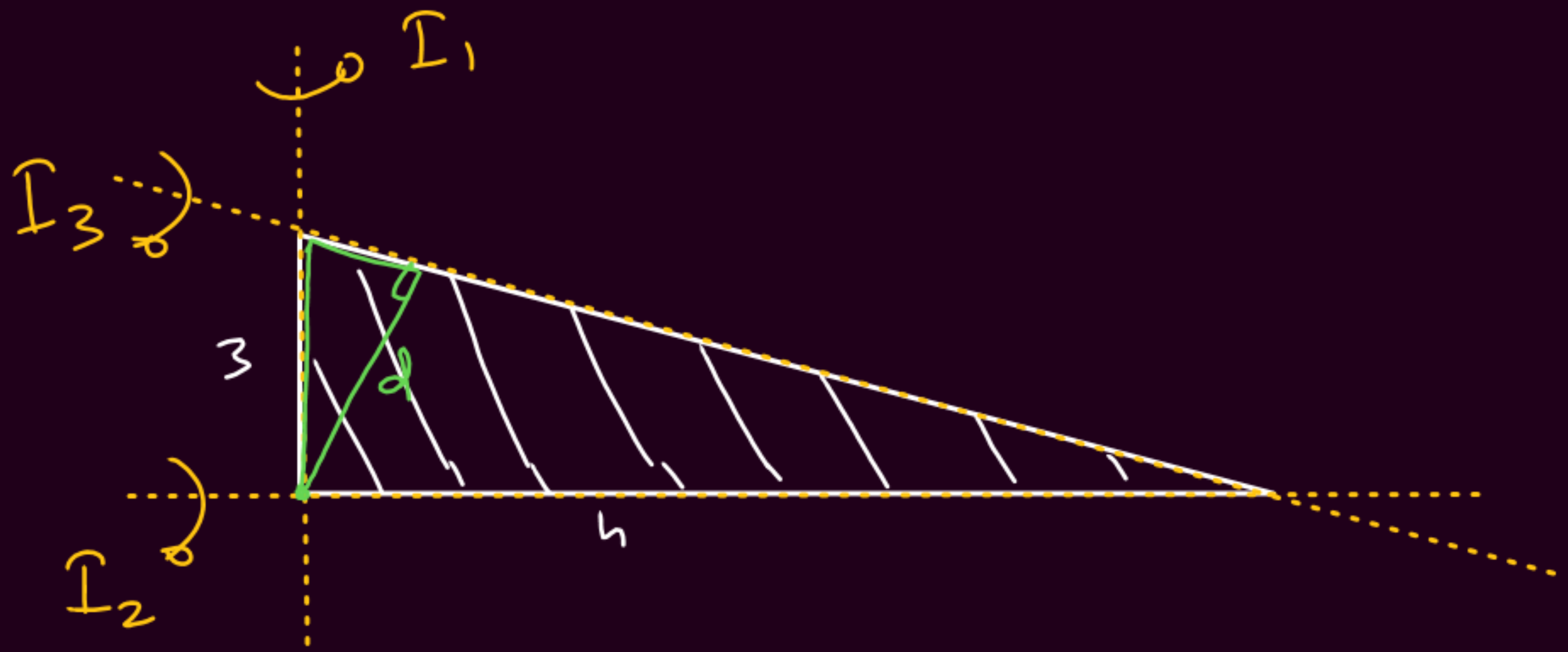
$$I = \frac{5}{3}m\ell^2$$

Q



$$I = m\left(\frac{\ell}{2}\right)^2 + \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 + \frac{1}{12}m\ell^2$$

$$I = \frac{2}{3}m\ell^2$$



Triangular plate

$I_1 = \frac{1}{6} m (4)^2$

$I_2 = \frac{1}{6} m (3)^2$

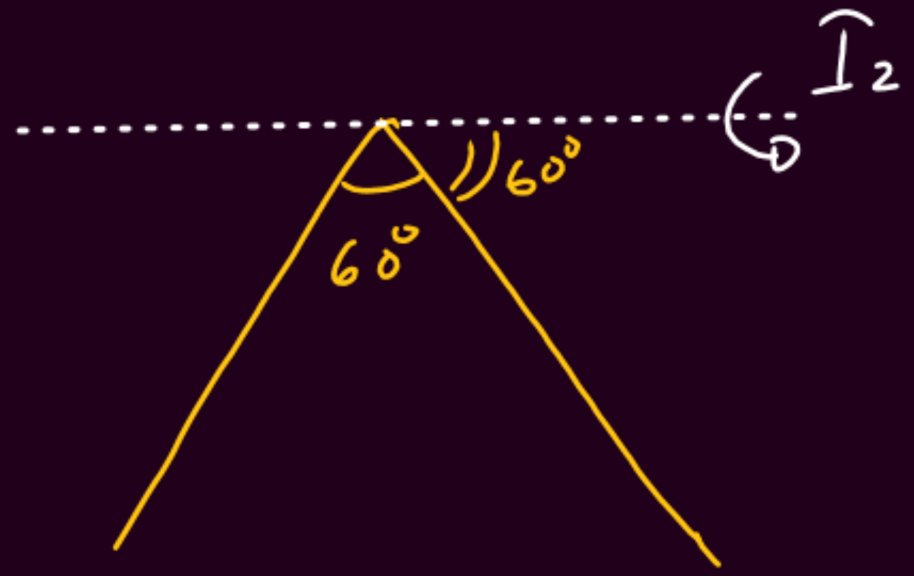
$I_3 = \frac{1}{6} m (d^2)$

$I_3 < I_2 < I_1$

Q

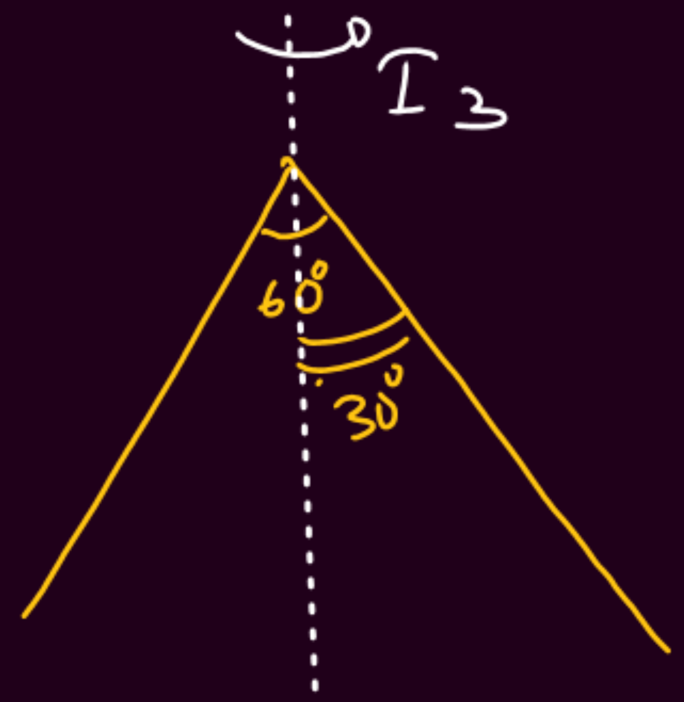
Each (m, ℓ)

$$I_1 = 2\left(\frac{1}{3} m\ell^2\right)$$



$$I_2 = 2\left(\frac{1}{3} m\,\ell_\perp^2\right)$$

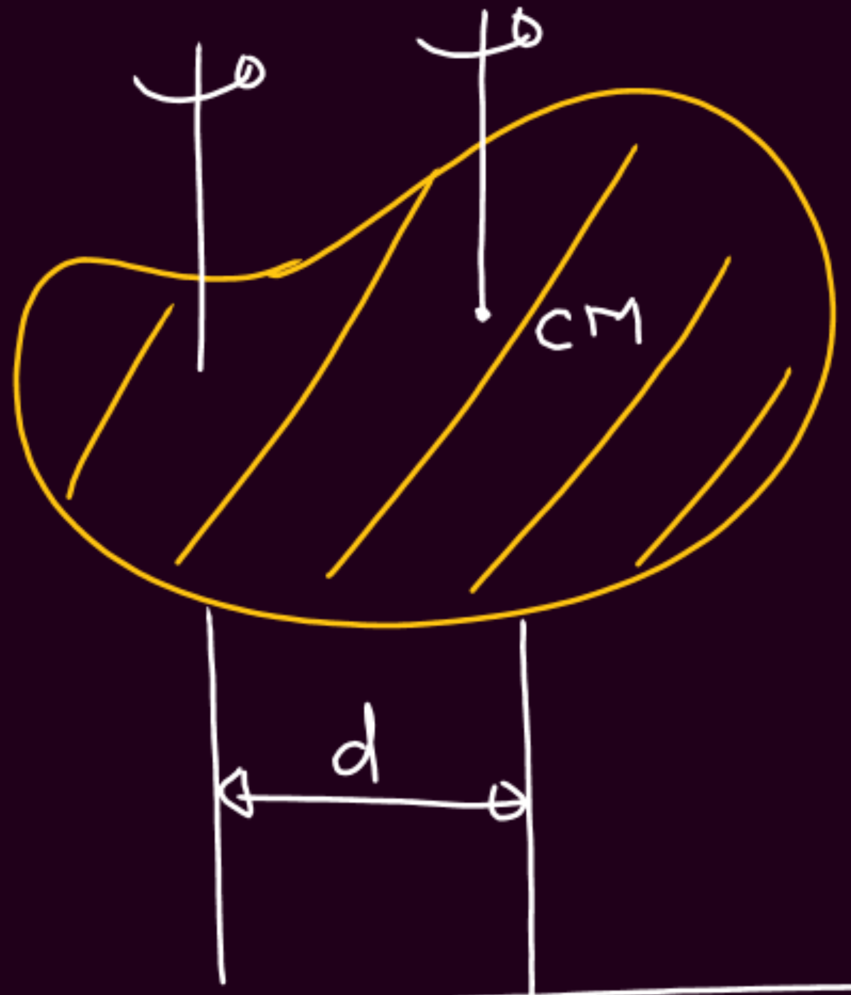
$$= \frac{2}{3} m\left(\ell \sin 60^\circ\right)^2$$



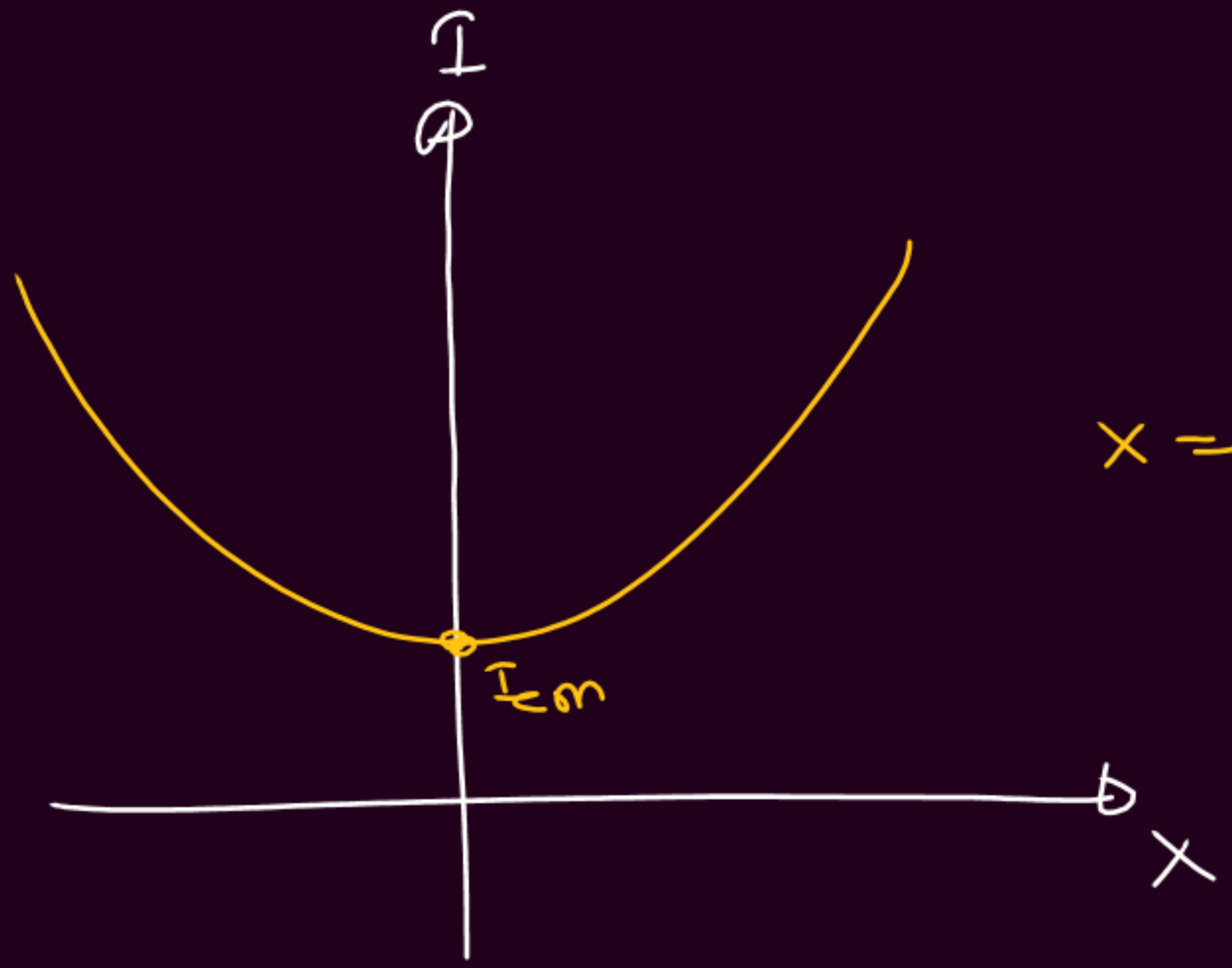
$$I_3 = 2\left(\frac{1}{3} m\,\ell_\perp^2\right)$$

$$= \frac{2}{3} m\left(\ell \sin 30^\circ\right)^2$$

Parallel Axis Theorem

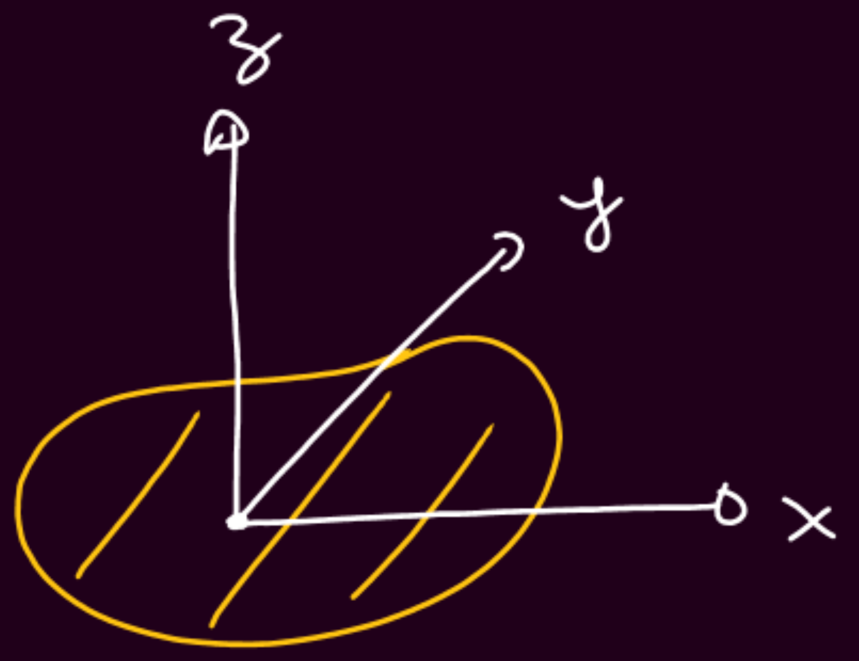


$$\boxed{I = I_{cm} + md^2}$$



x = dist. from CM

Perpendicular Axis Theorem



→ only for planner

$$\boxed{I_z = I_x + I_y}$$

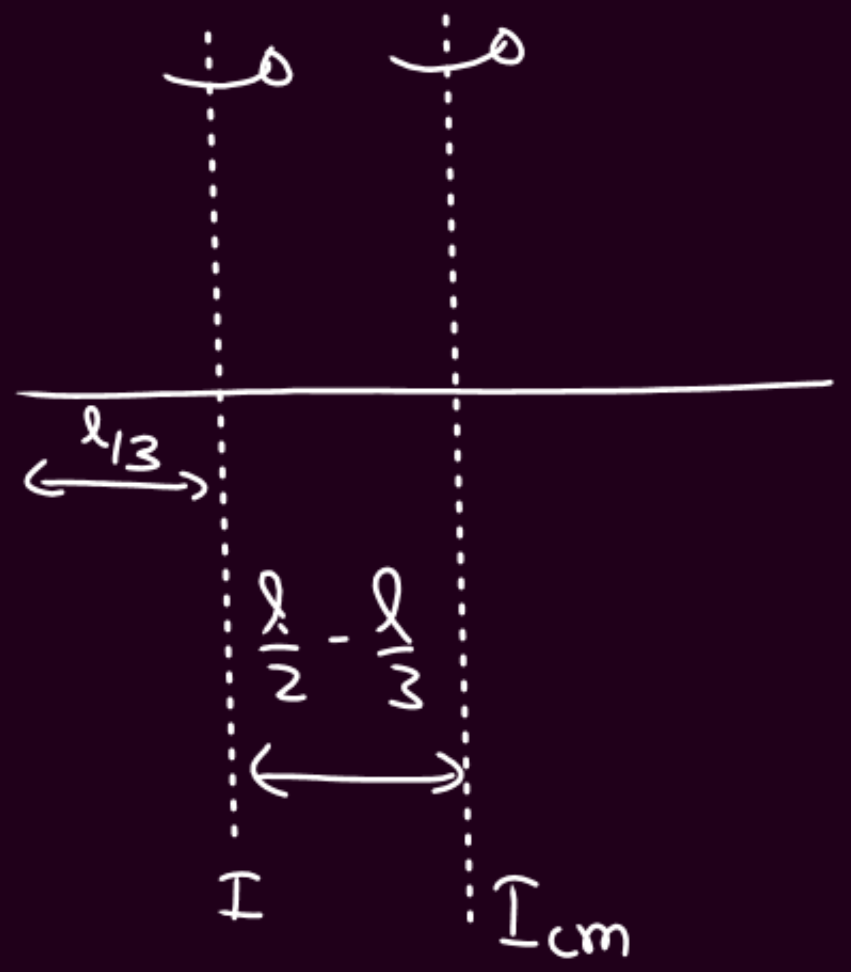
I_z: ⊥ to plane

$I_x + I_y$: In the plane

Q Uniform Rod

⊥ Axis

$\frac{l}{3}$ from End



$$I = \frac{1}{12}ml^2 + m\left(\frac{l}{6}\right)^2$$

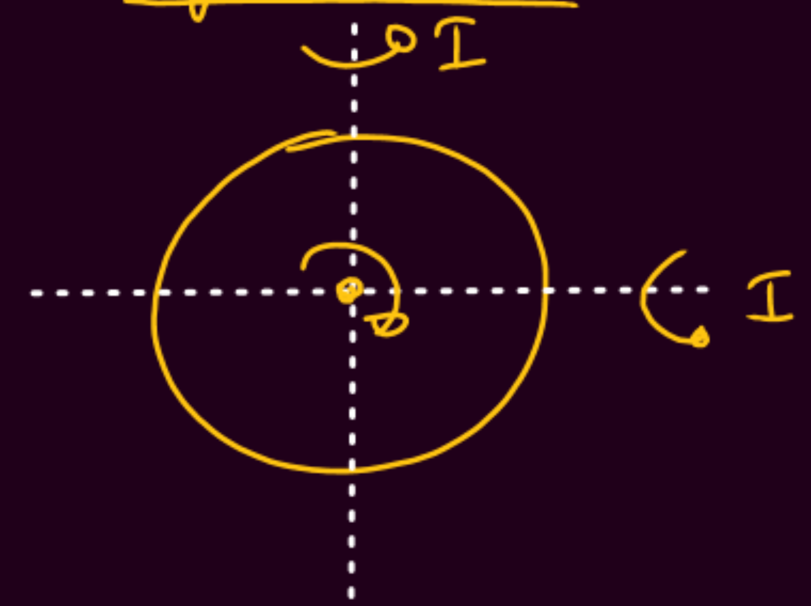
$$I = \frac{ml^2}{9}$$

Q. SS about Tangent

$$I = \frac{2}{5}mR^2 + mR^2$$

$$I = \frac{7}{5}mR^2$$

Q Ring about diameter

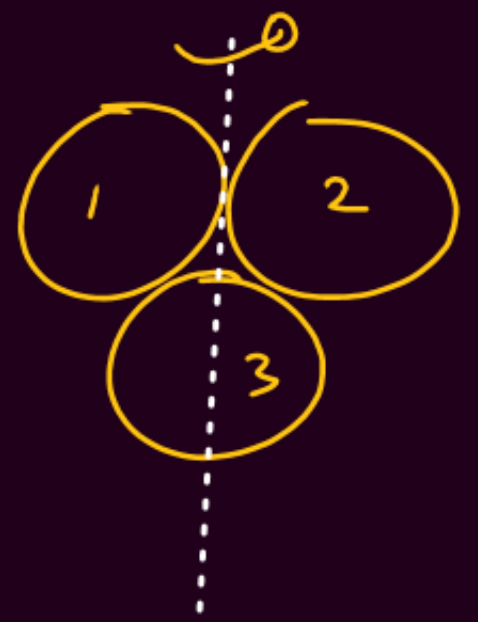


$$I_z = I_x + I_y$$

$$MR^2 = I + I$$

$$I = \frac{1}{2}MR^2$$

Q.

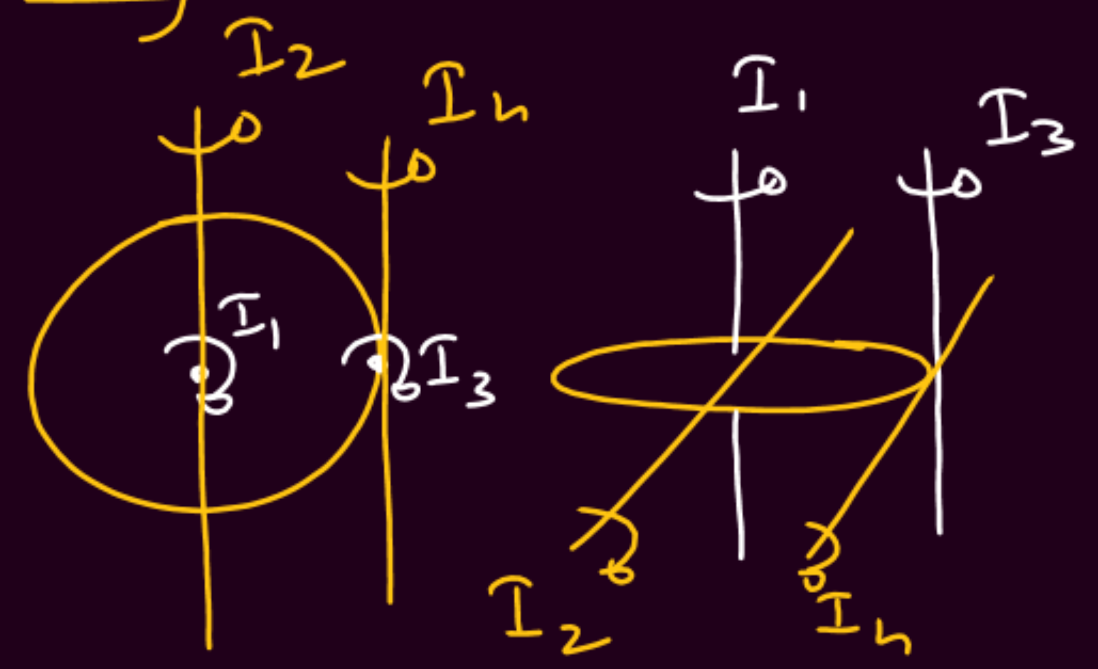


HS Each (m, R)

$$I = 2\left[\frac{2}{3}mR^2 + mR^2\right] + \frac{2}{3}mR^2$$

$$I = 4mR^2$$

Ring



① **Axis** $I_1 = mR^2$

⊥
② **Diameter** $I_2 = \frac{I_1}{2} = \frac{1}{2}mR^2$

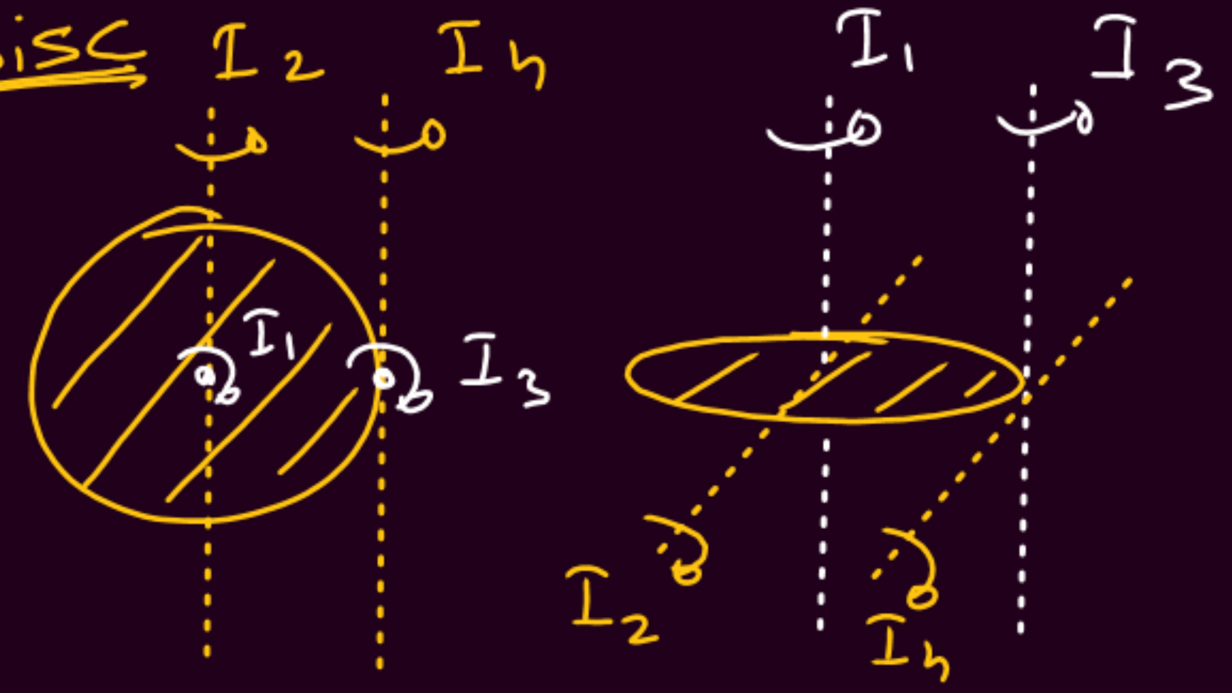
∥
③ Tangent ∥ to Axis
Tangent ⊥ to plane

$$\left.\begin{array}{l}\end{array}\right\} \quad I_3 = I_1 + mR^2, \quad I_3 = 2mR^2$$

∥
④ Tangent in the plane
— " — ∥ to diameter

$$\left.\begin{array}{l}\end{array}\right\} \quad I_4 = I_2 + mR^2 = \frac{3}{2}mR^2$$

Disc



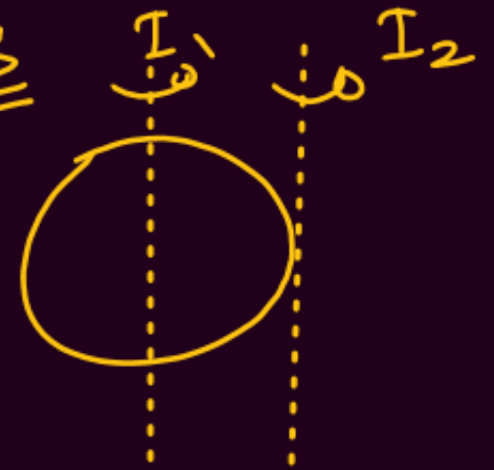
① Axis $I_1 = \frac{1}{2}mR^2$

② Dia $I_2 = \frac{I_1}{2} = \frac{1}{4}mR^2$

③ Tangent ∥ to Axis $I_3 = \left(\frac{1}{2}+1\right) = \frac{3}{2}mR^2$

④ Tangent ∥ to dia $I_4 = \left(\frac{1}{4}+1\right) = \frac{5}{4}mR^2$

HS

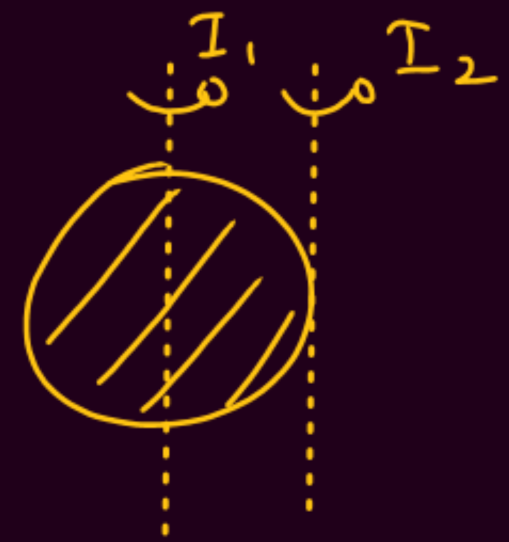


Axis $I_1 = \frac{2}{3}mR^2$

Tangent $I_2 = \left(\frac{2}{3}+1\right)mR^2$

$= \frac{5}{3}mR^2$

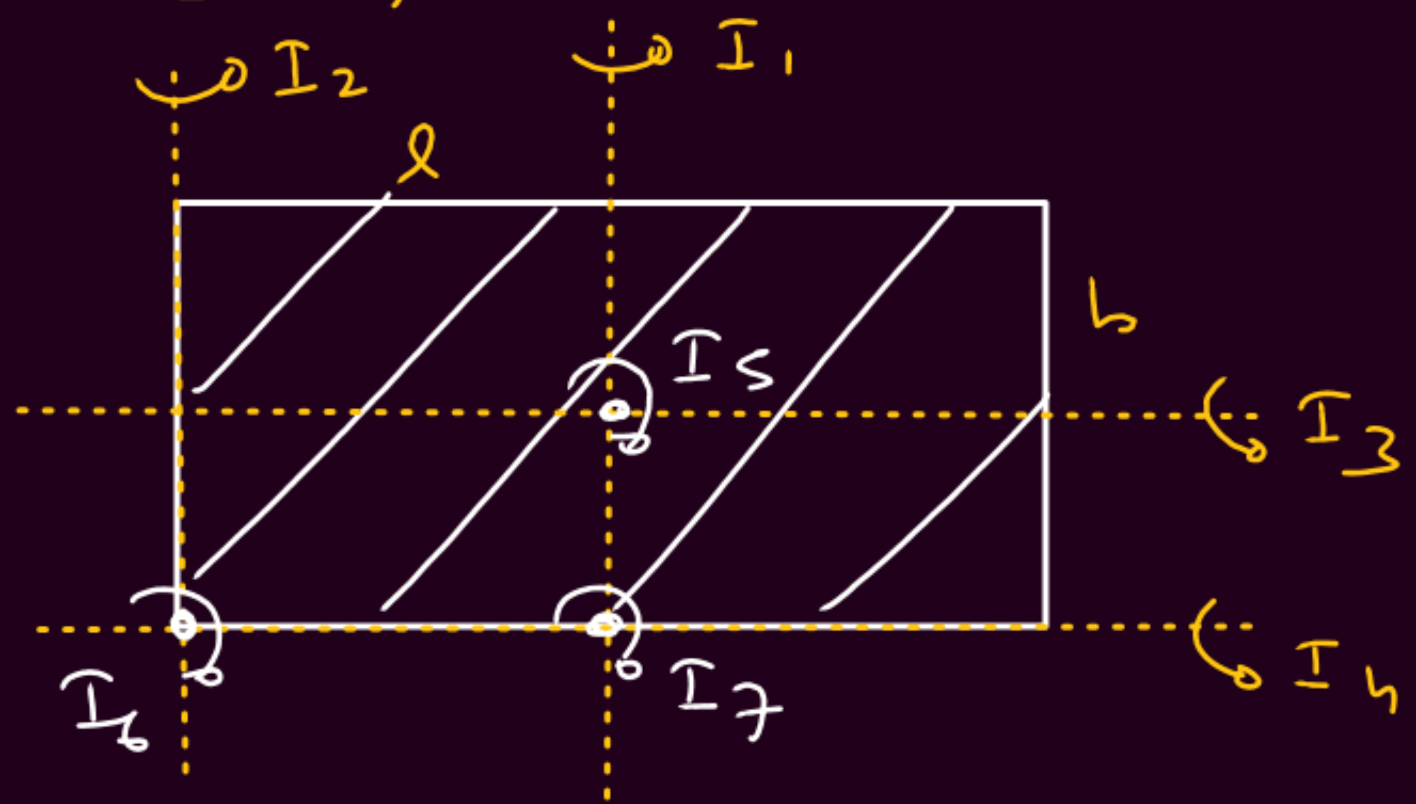
SS



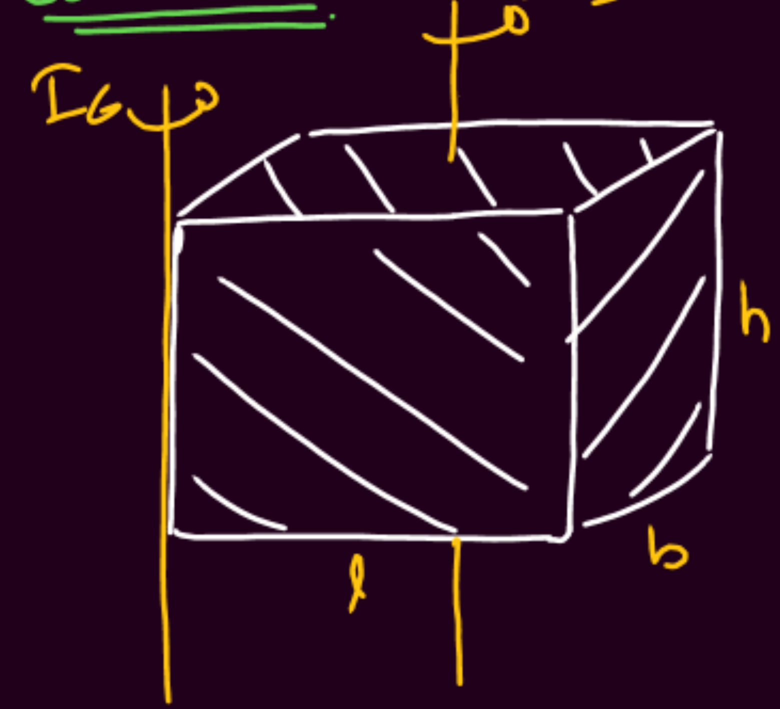
Axis $I_1 = \frac{2}{5}mR^2$

Tangent $I_2 = \left(\frac{2}{5}+1\right) = \frac{7}{5}mR^2$

Rectangular Sheet



Cuboid



$$I_5 = \frac{1}{12} m(l^2+b^2)$$

$$I_6 = \frac{1}{3} m(l^2+b^2)$$

$$I_1 = \frac{1}{12} ml^2$$

$$I_2 = \frac{1}{3} ml^2$$

$$I_3 = \frac{1}{12} mb^2$$

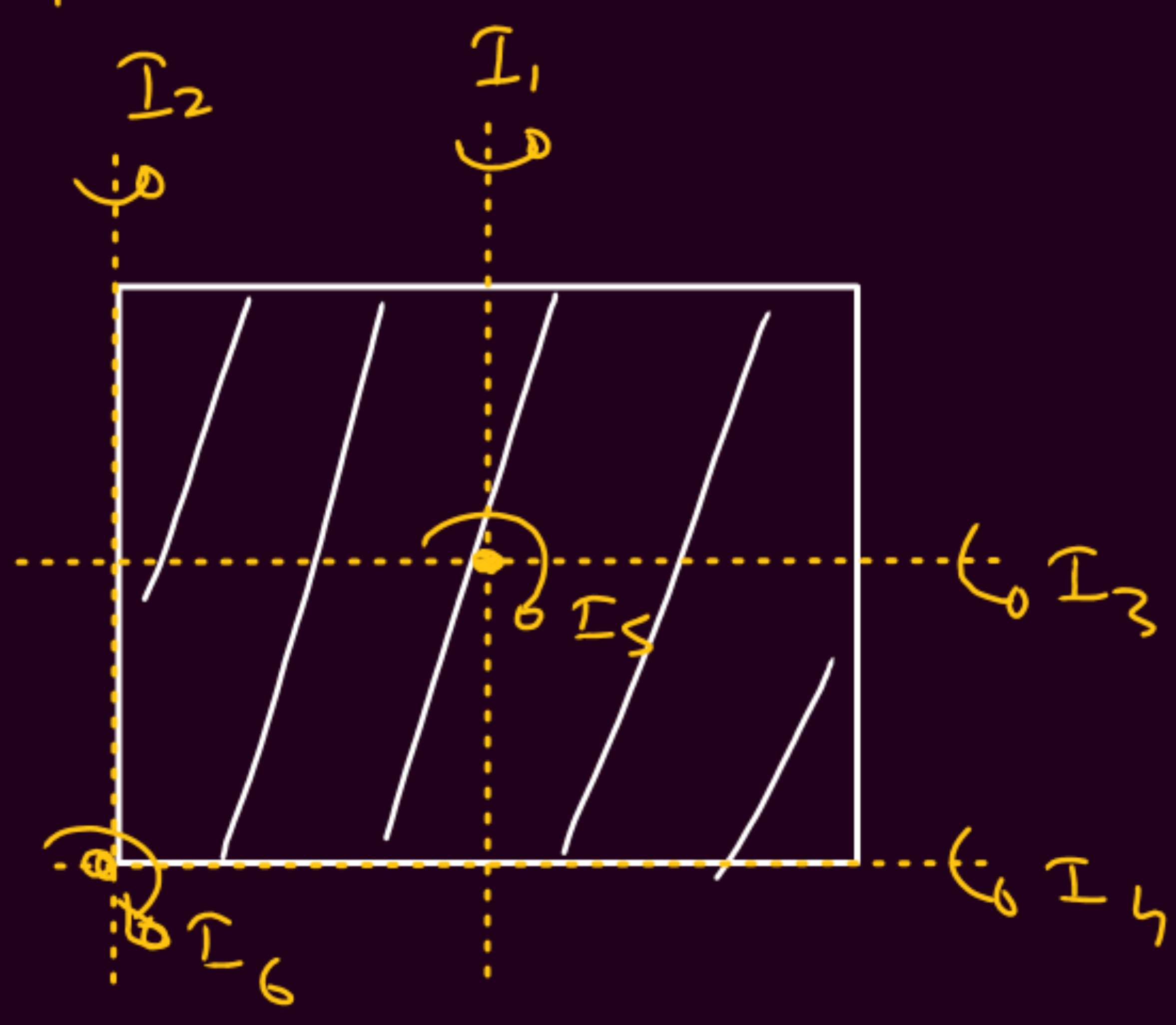
$$I_4 = \frac{1}{3} mb^2$$

$$I_5 = I_1 + I_3 = \frac{1}{12} m(l^2+b^2)$$

$$I_6 = I_2 + I_4 = \frac{1}{3} m(l^2+b^2)$$

$$I_7 = I_1 + I_4 = \frac{1}{12} ml^2 + \frac{1}{3} mb^2$$

Square Sheet

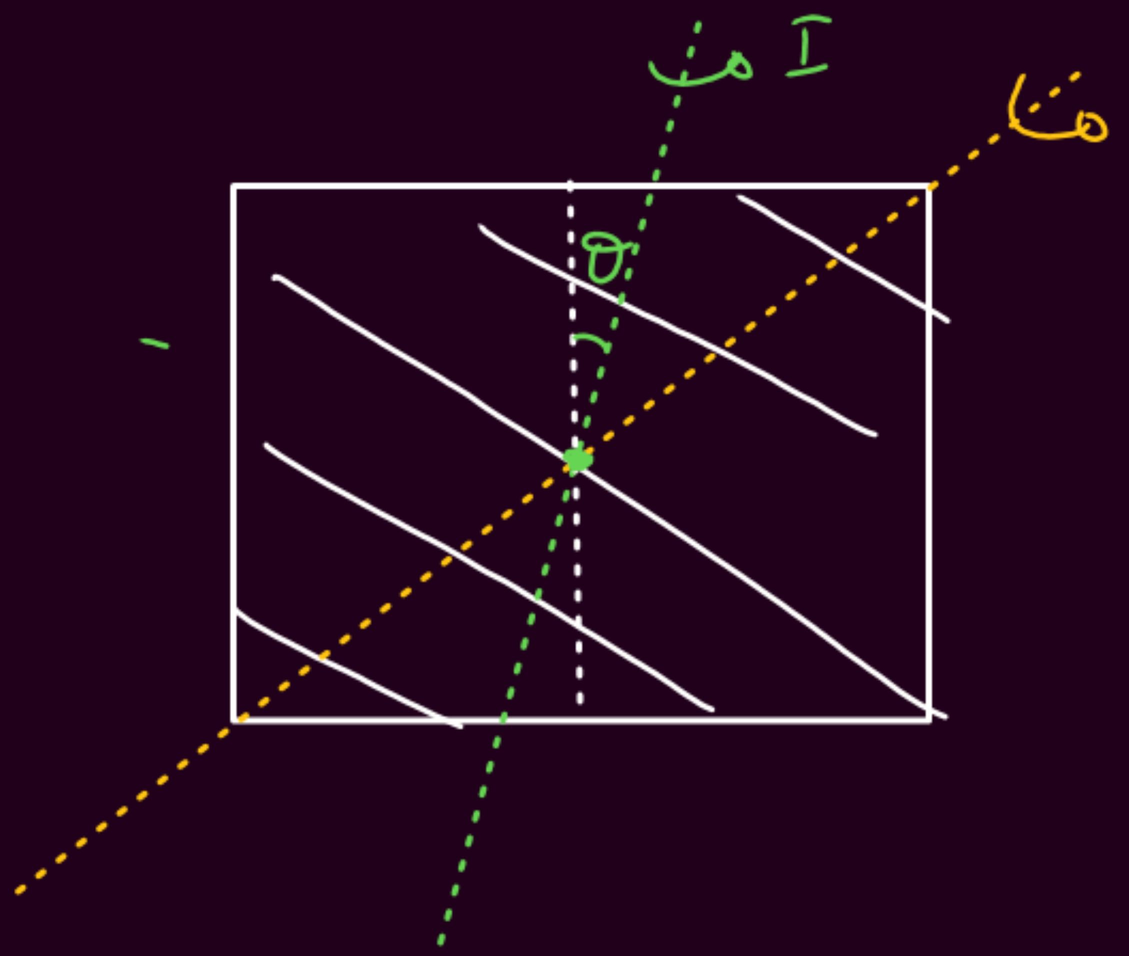


$$I_1 = I_3 = \frac{1}{12} ml^2$$

$$I_2 = I_4 = \frac{1}{3} ml^2$$

$$I_5 = \frac{1}{6} ml^2$$

$$I_6 = \frac{2}{3} ml^2$$



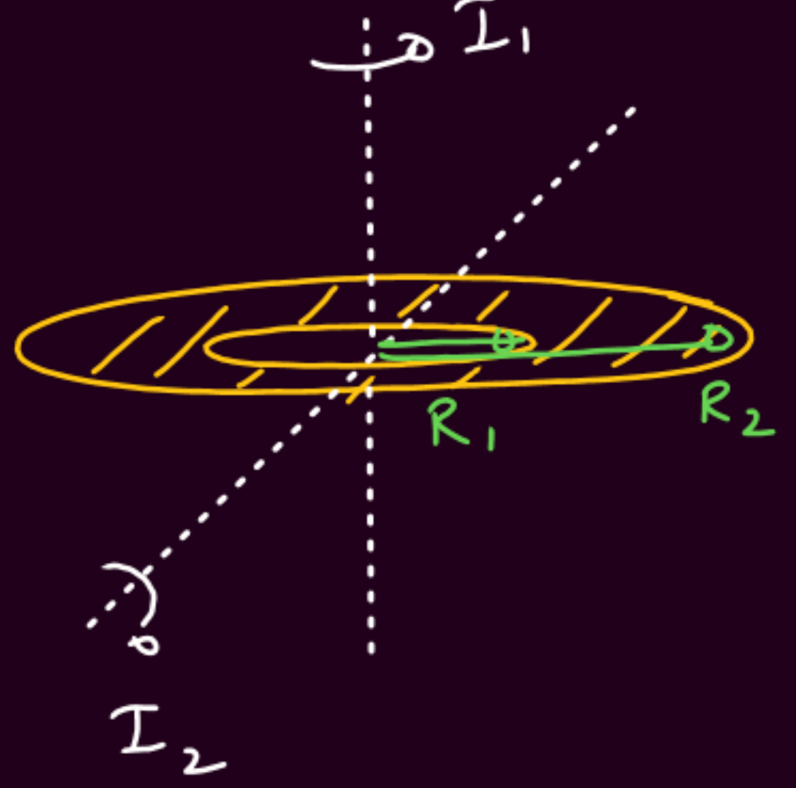
$$\text{Dia} + \text{Dia} = I_5 = I_1 + I_1$$

$$I_{Dia} = I_1 = \frac{1}{12} ml^2$$

$$\boxed{I = \frac{1}{12} ml^2}$$

Extra

Annular Disc



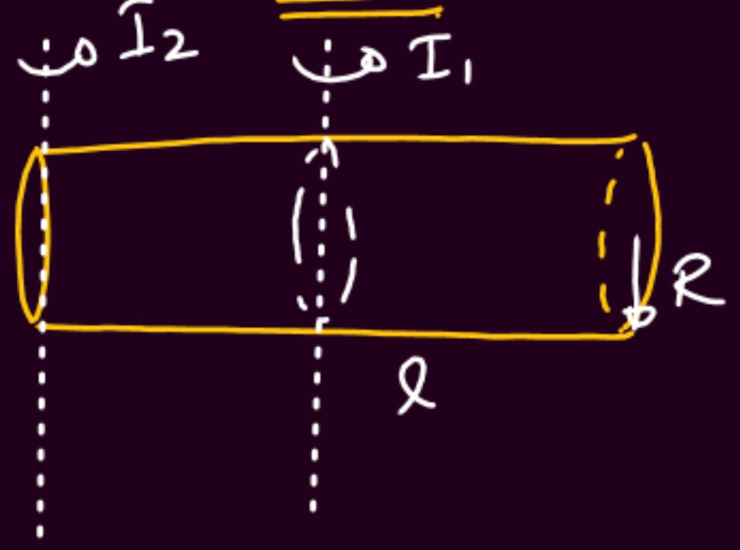
① **Axis**

$$I_1 = \frac{1}{2} m\left(R_1^2 + R_2^2\right)$$

② **Diameter**

$$I_2 = \frac{1}{4} m\left(R_1^2 + R_2^2\right)$$

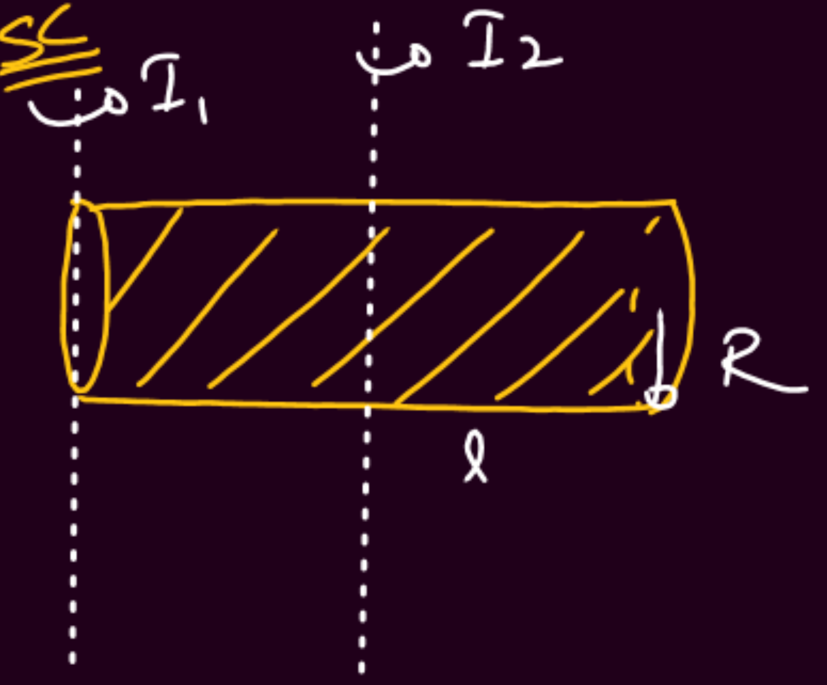
HC



$$I_1 = \frac{1}{2} MR^2 + \frac{1}{12} ml^2$$

$$I_2 = \frac{1}{2} mR^2 + \frac{1}{3} ml^2$$

SC



$$I_1 = \frac{1}{4} mR^2 + \frac{1}{12} ml^2$$

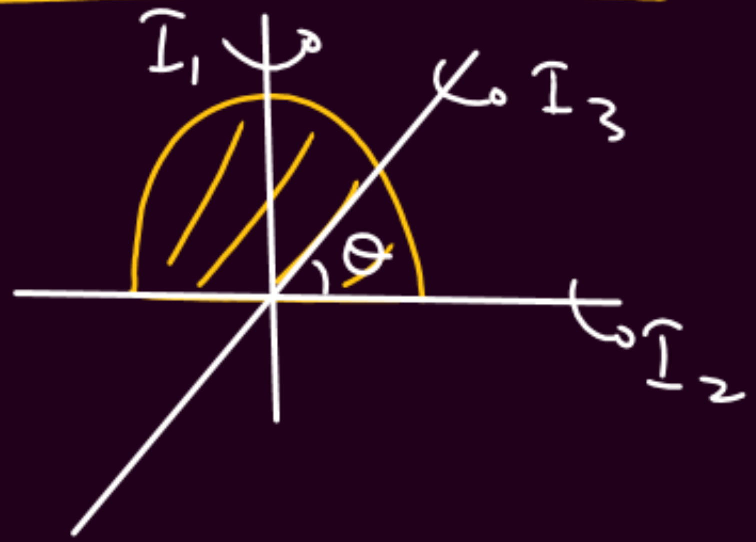
$$I_2 = \frac{1}{4} mR^2 + \frac{1}{3} ml^2$$

Symmetry

Disc

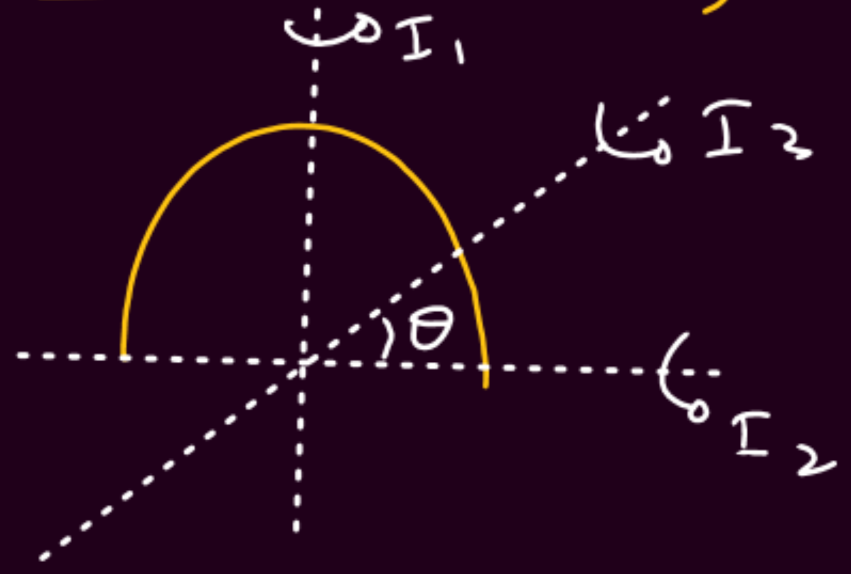
$$\boxed{\frac{1}{2}MR^2}$$

Semicircular Disc



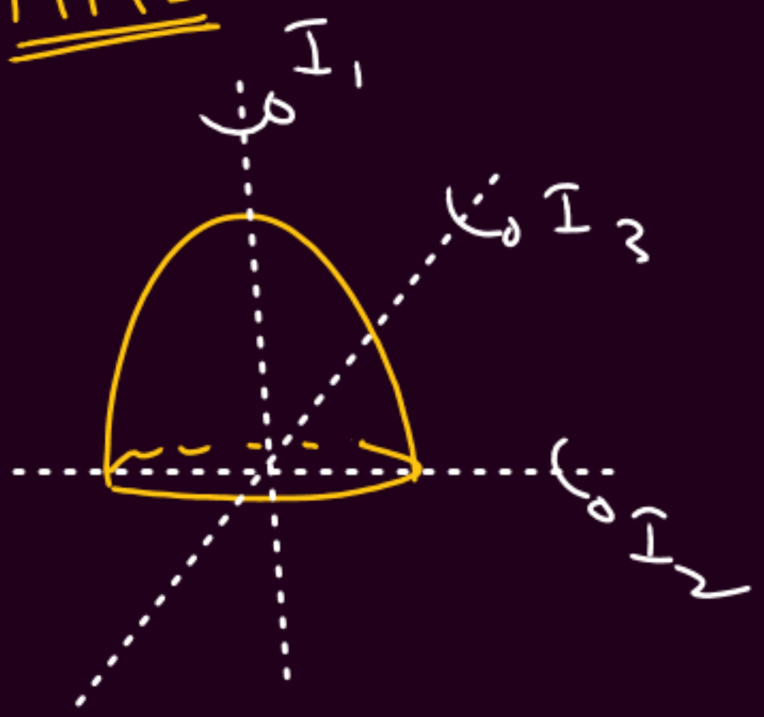
$$I_1 = I_2 = I_3 = \frac{1}{4}MR^2$$

Semicircular Ring



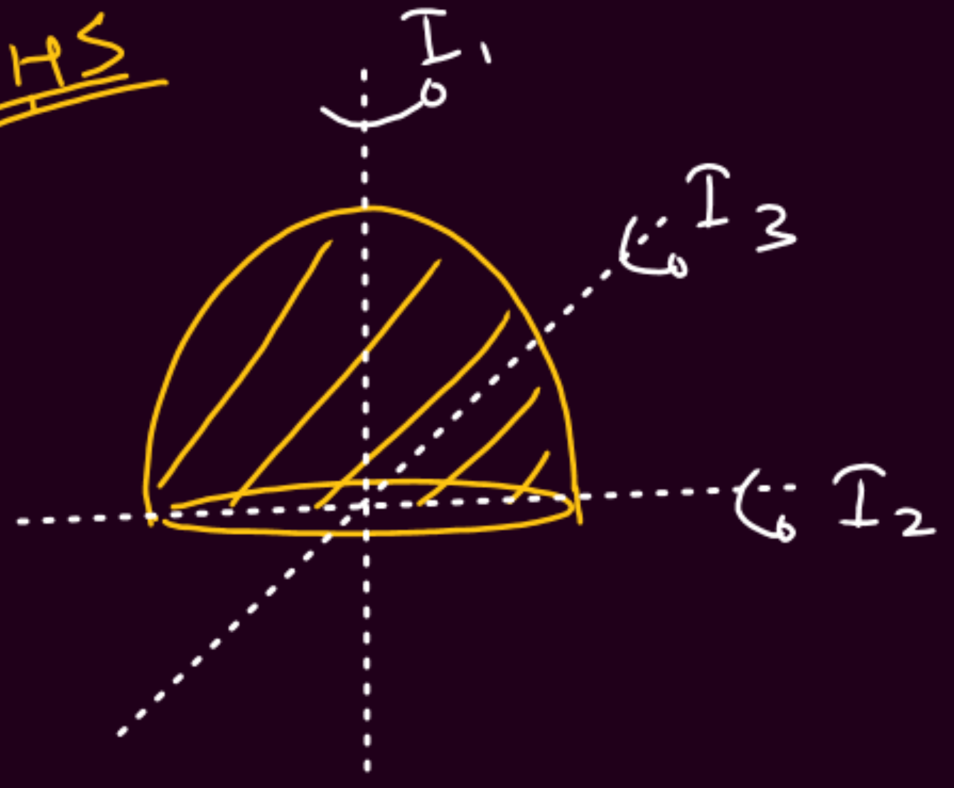
$$I_1 = I_2 = I_3 = \frac{1}{2}MR^2$$

HHS



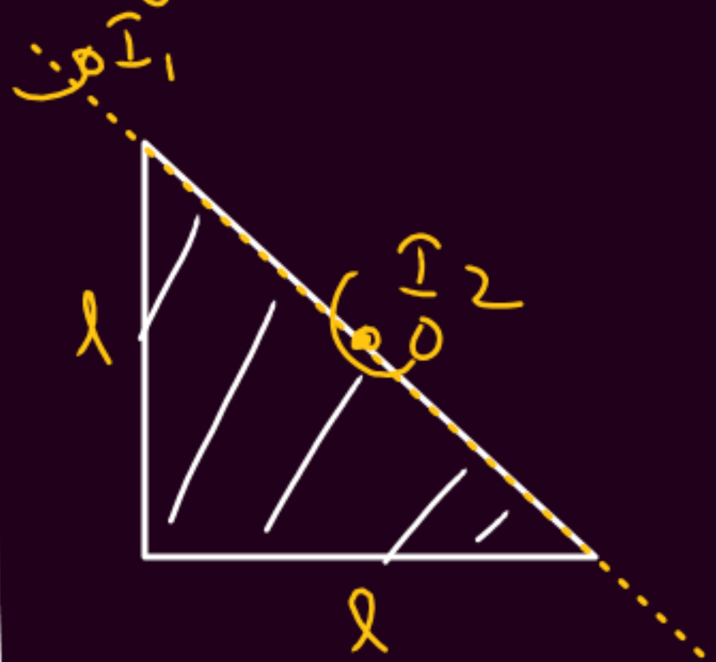
$$I_1 = I_2 = I_3 = \frac{2}{3}MR^2$$

SHS



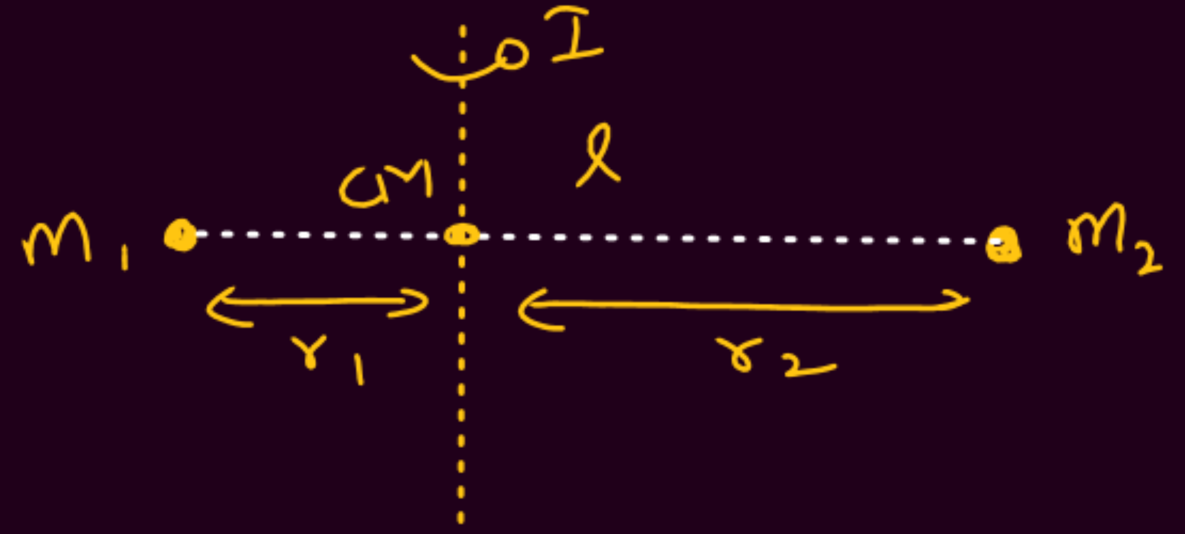
$$I_1 = I_2 = I_3 = \frac{2}{5}MR^2$$

Right Isosceles Triangular plate



$$I_1 = \frac{1}{12}ml^2$$

$$I_2 = \frac{1}{6}ml^2$$



$$\boxed{I_{cm} = \left(\frac{m_1 m_2}{m_1+m_2}\right) l^2}$$

$$I_{cm} = m_1 r_1^2 + m_2 r_2^2$$

$$r_1 = \frac{m_2 l}{m_1+m_2}$$

$$r_2 = \frac{m_1 l}{m_1+m_2}$$

Proportionality

Ring

$M \propto R$

$I \propto R^3$

Disc / HS

$M \propto R^2$

$I \propto R^4$

SS

$M \propto R^3$

$I \propto R^5$

Q Two disc

- Same material
- Same thickness

(2R, 3R)

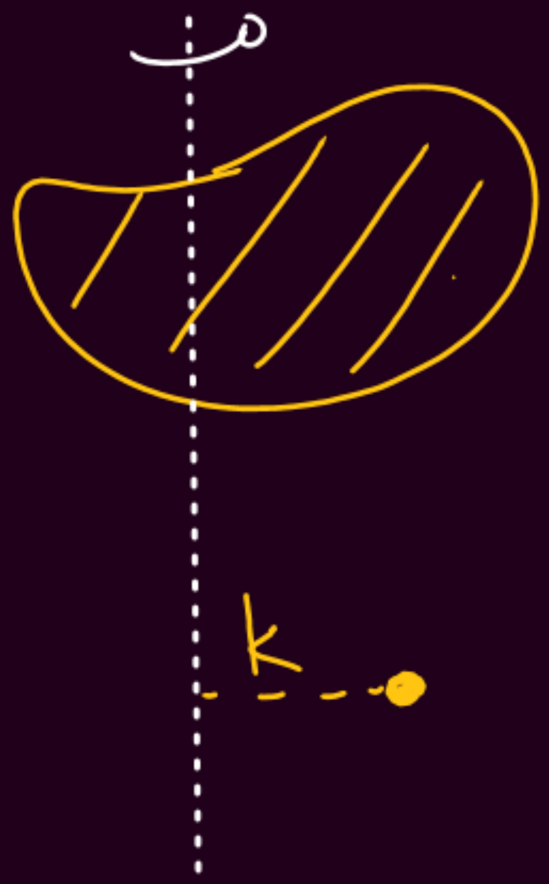
$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^4 = \left(\frac{2R}{3R}\right)^4 = \frac{16}{81}$$

Q Two spheres of same material

R & 2R

$$\frac{I_1}{I_2} = \left(\frac{R}{2R}\right)^5 = \frac{1}{32}$$

Radius of Gyration (k)



$$I = Mk^2$$

① Ring about dia

$$Mk^2 = \frac{1}{2}mR^2$$

$$k = \frac{R}{\sqrt{2}}$$

② Sphere about Dia

$$Mk^2 = \frac{2}{5}mR^2$$

$$k = \sqrt{\frac{2}{5}}\,R$$

③ HS about Tangent

$$Mk^2 = \frac{5}{3}mR^2$$

$$k = \sqrt{\frac{5}{3}}\,R$$

④ Disc about tangent in the plane

$$Mk^2 = \frac{5}{4}mR^2$$

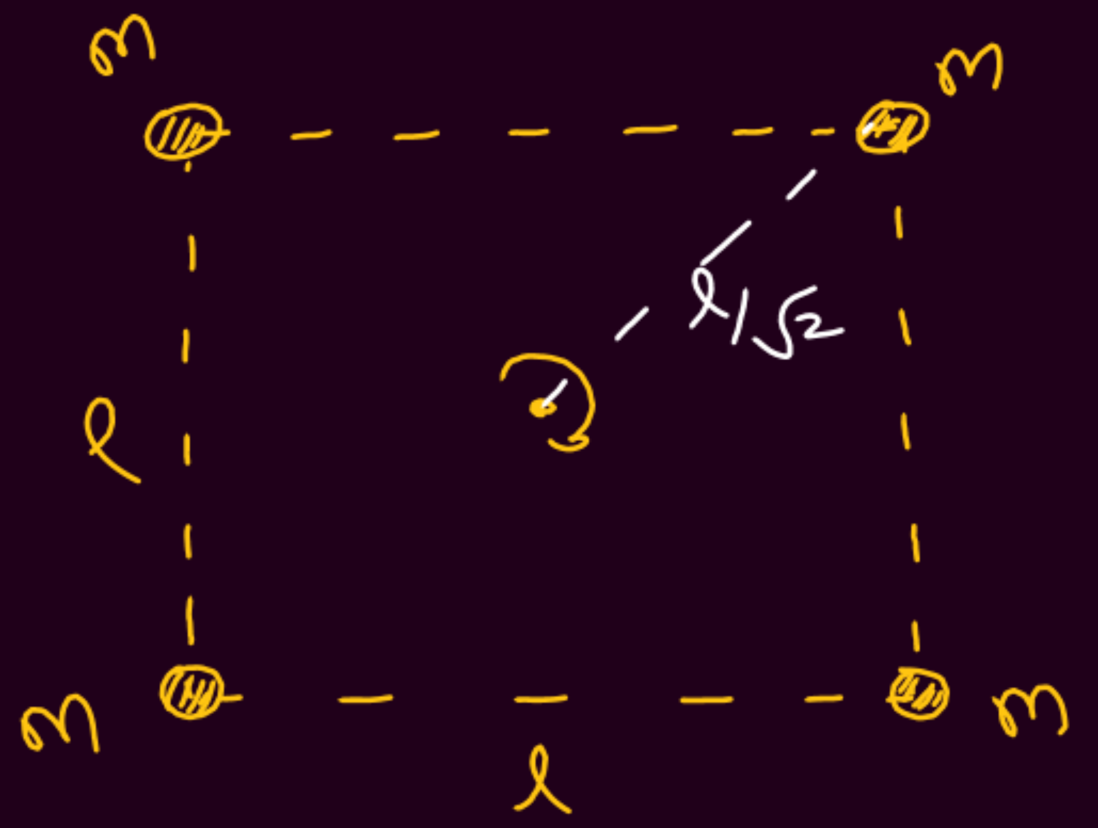
$$k = \frac{\sqrt{5}}{2}R$$

⑤ Square plate about Diagonal

$$Mk^2 = \frac{1}{12}ml^2$$

$$k = \frac{l}{2\sqrt{3}}$$

Q.

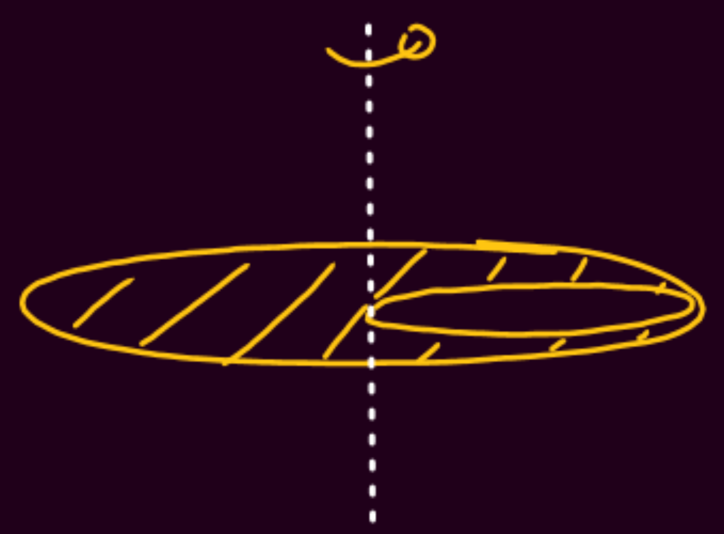


$$4mK^2 = 4\left[m\left(\frac{l}{\sqrt{2}}\right)^2\right]$$

$$K = \frac{l}{\sqrt{2}}$$

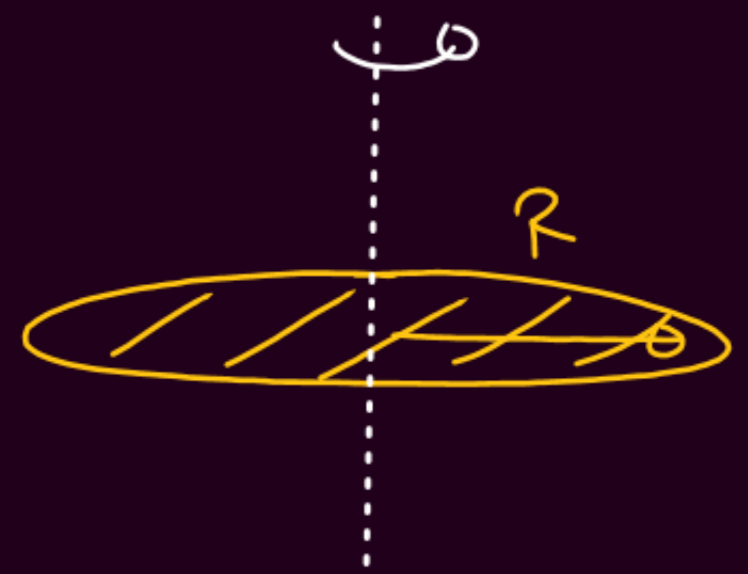
If Some part is Removed

Q.

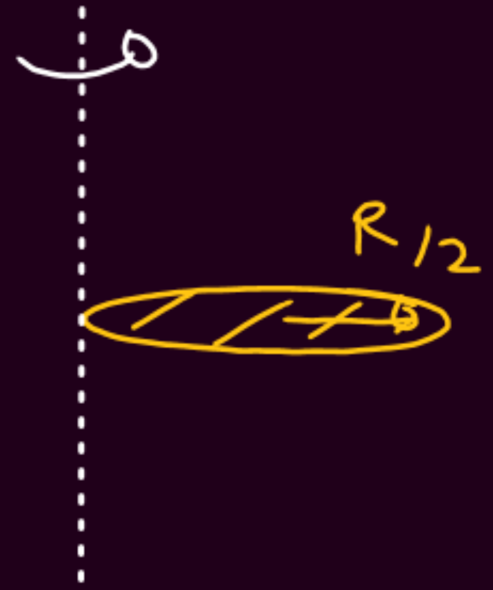


$=$

I_1



I_2



$\pi R^2 \to M$

$\pi\left(\frac{R}{2}\right)^2 \to \frac{M}{\pi R^2}\pi\left(\frac{R}{2}\right)^2$

$= \frac{M}{4}$

Extra

$$I = I_1 - I_2$$

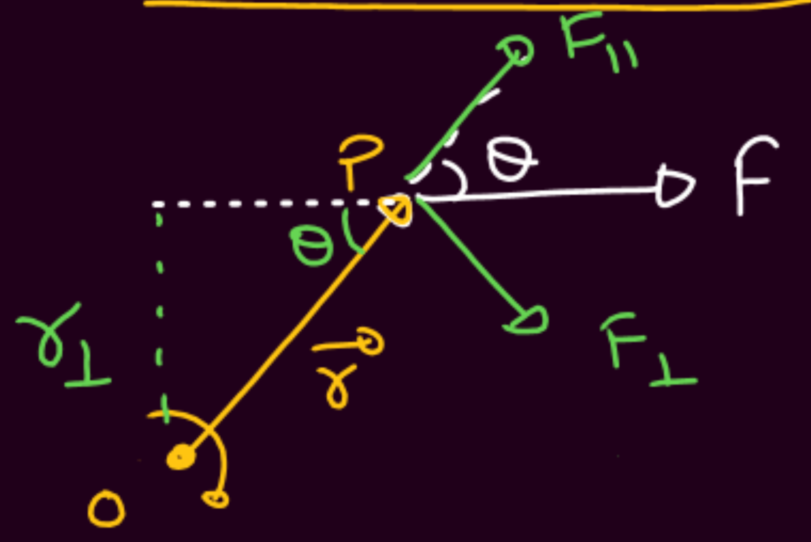
$$I = \frac{1}{2}MR^2 - \frac{3}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$I = \frac{1}{2}MR^2\left(1 - \frac{3}{16}\right)$$

$$I = \frac{13}{32}MR^2$$

Torque → Moment of force

① About a point



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \vec{OP} \times \vec{F}$$

$$\boxed{\tau = rF\sin\theta = r_{\perp}F = rF_{\perp}}$$

unit N-m

$\vec{\tau} \perp \vec{r}$ $\vec{\tau} \cdot \vec{r} = 0$

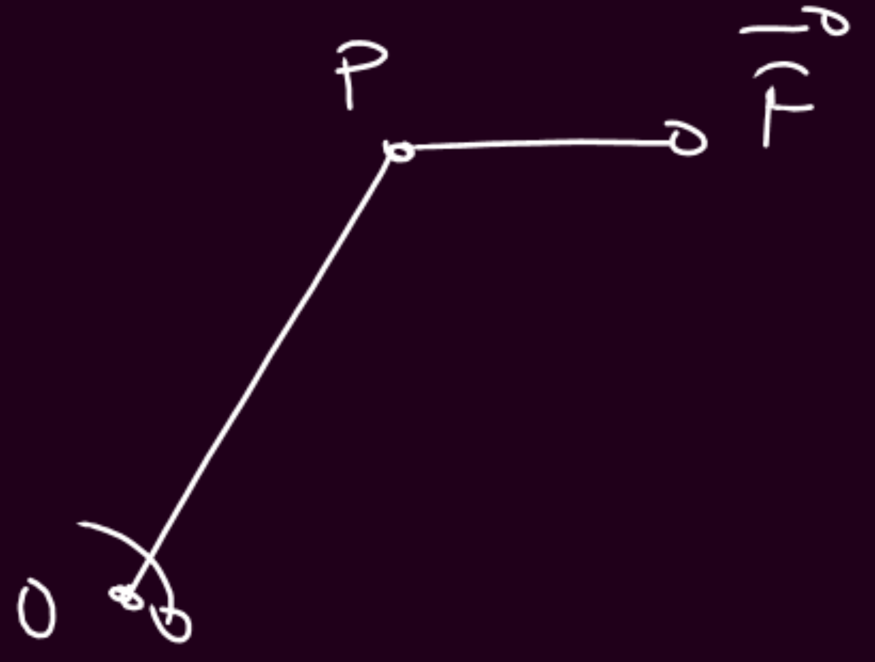
$\vec{\tau} \perp \vec{F}$ $\vec{\tau} \cdot \vec{F} = 0$

② Torque about an axis

Q. $\vec{F} = (2\hat{i} - \hat{j} + \hat{k})$ N

is applied at P $(1, 2, -1)$

Torque about O $(-1, 1, 2)$



$\vec{r} = \vec{OP} = \vec{P} - \vec{O}$

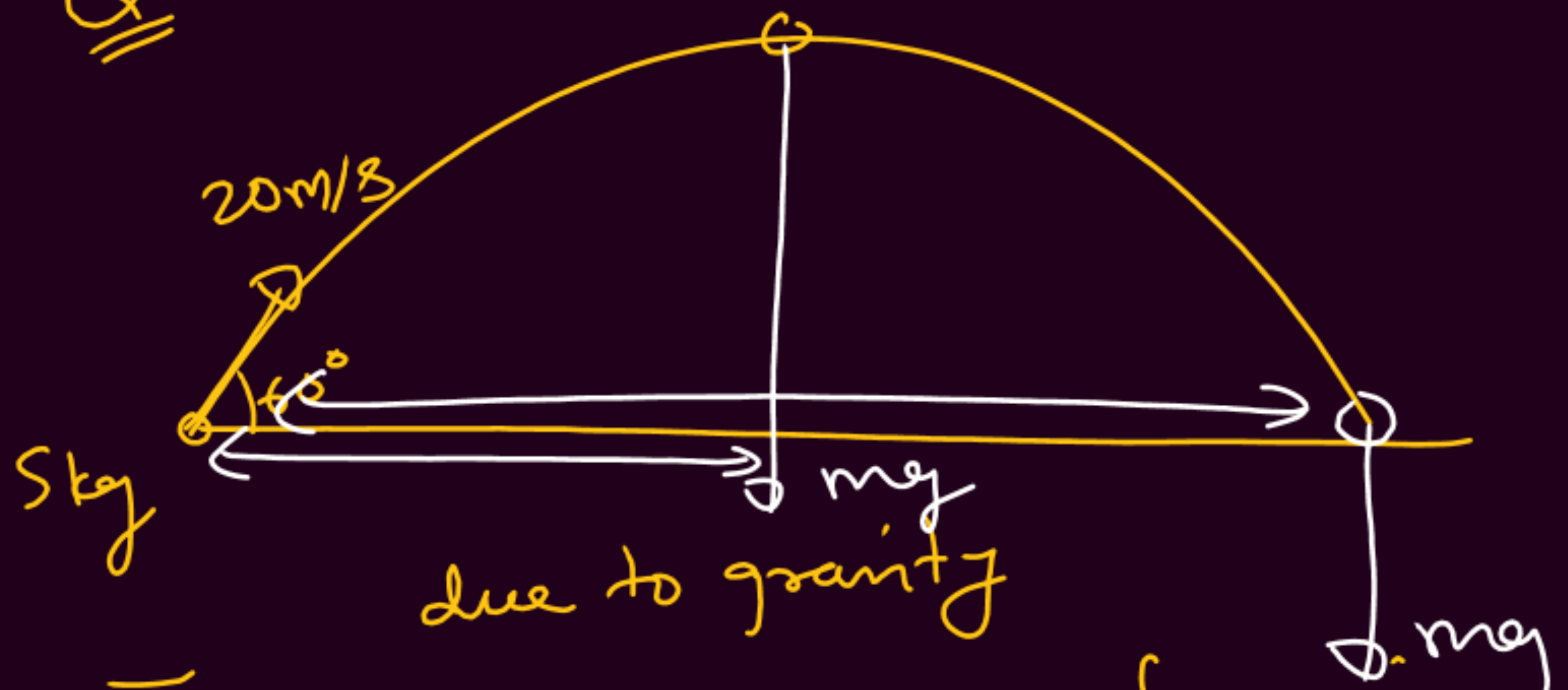
$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k})$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= (1 - 3)\hat{i} - (2 - (-6))\hat{j} + (-2 - 2)\hat{k} = -2\hat{i} - 8\hat{j} - 4\hat{k}$$

$$\boxed{\vec{\tau} = (-2\hat{i} - 8\hat{j} - 4\hat{k}) \text{ N-m}}$$

Q



Torque about pt. of proj.

when 1) Max height

2) Just before striking gr.

$$R = \frac{u^2 \sin(2\theta)}{g} = \frac{(20)(20)\sqrt{3}}{(10)\ 2} = \underline{\underline{20\sqrt{3}\,m}}$$

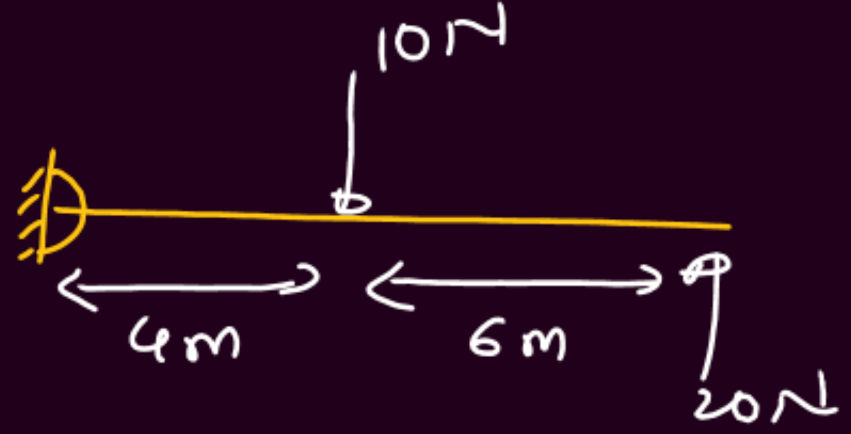
1) $\tau = r_\perp F = \left(\frac{R}{2}\right)mg$

$$\tau = \left(\frac{20\sqrt{3}}{2}\right)(50) = 500\sqrt{3}\ N\text{-}m$$

2) $\tau = r_\perp F = R(mg)$

$$\tau = (20\sqrt{3})(50) = 1000\sqrt{3}\ \underline{N\text{-}m}$$

Q

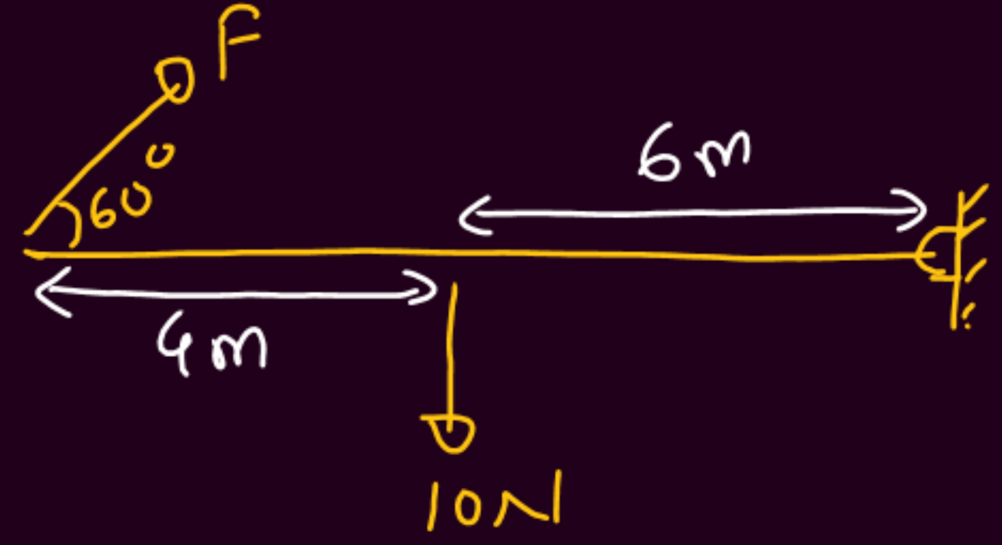


$$\tau_1 = 4(10) = 40 \text{ N-m}$$

$$\tau_2 = 10(20) = 200 \text{ N-m}$$

$$\tau_{net} = (200-40) = 160 \text{ N-m}$$

Q



Find F if $\tau = 0$

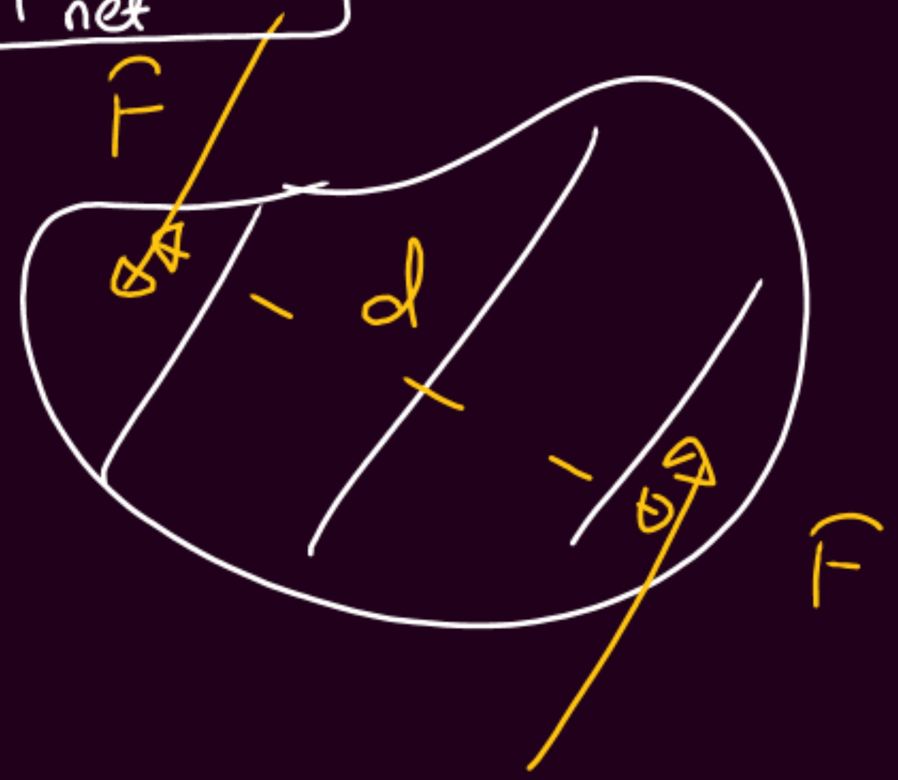
$$(6)(10) = (10)(F \sin 60^\circ)$$

$$6 = F\frac{\sqrt{3}}{2}$$

$$F = \frac{12}{\sqrt{3}} = 4\sqrt{3} \text{ N}$$

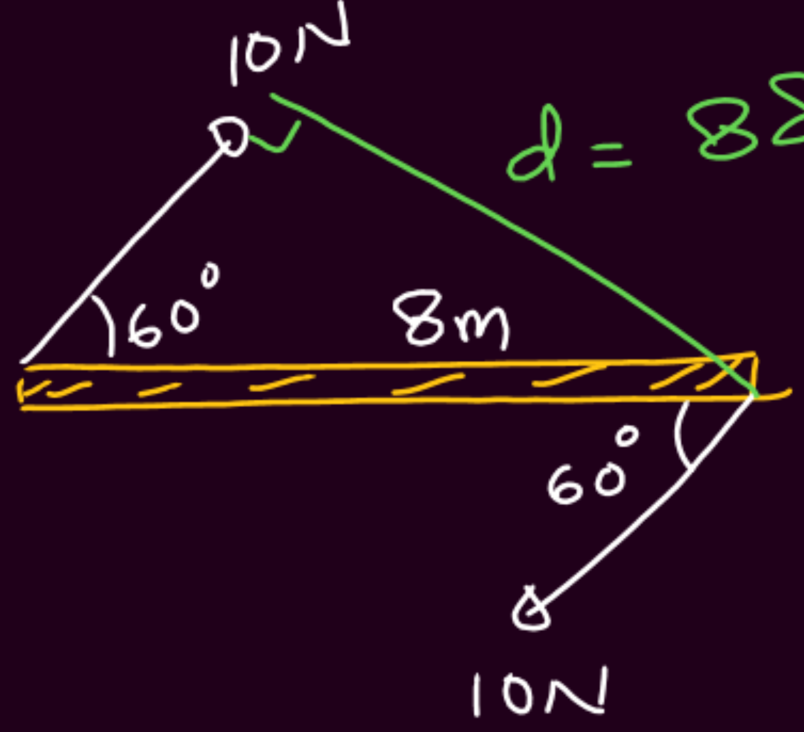
Torque due to Couple

- → Equal & opp forces
- → $\boxed{\vec{F}_{net} = 0}$



$$\boxed{\tau = Fd}$$

Q



$d = 8\sin 60°$

$$\text{Couple} = 10\,(8\sin 60°)$$

$$= 80\frac{\sqrt{3}}{2} = \underline{40\sqrt{3}\ \text{N-m}}$$

① Translational Eq.

$\vec{F}_{net} = 0$

$\vec{v}_{cm} = \text{const}$

② Rotational Eq.

$\vec{\tau}_{net} = 0$

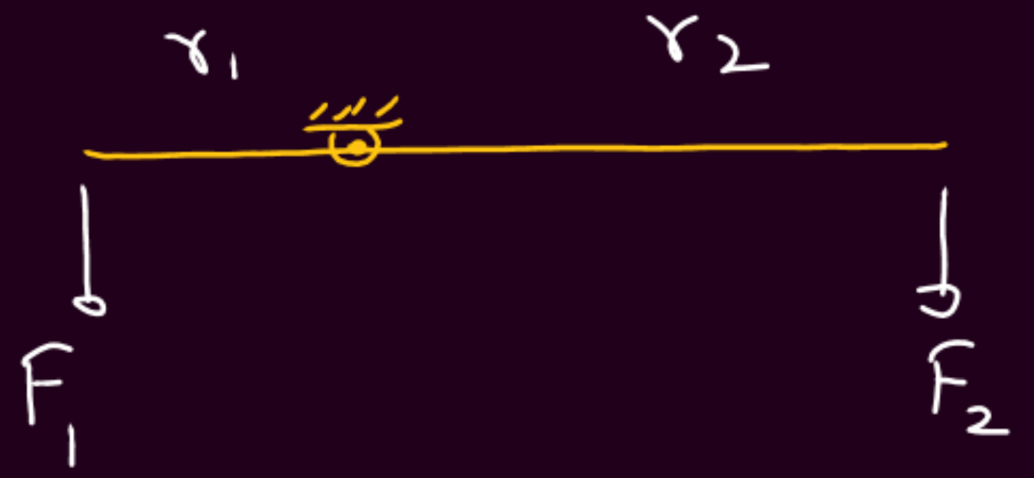
$\omega = \text{const}.$

③ Mech. Eq.

$\vec{F}_{net} = 0 \qquad \vec{v} = \text{const}$

$\vec{\tau}_{net} = 0 \qquad \omega = \text{const}.$

Principle of Moments $\boxed{\tau_{net} = 0}$



$$\boxed{r_1 F_1 = r_2 F_2}$$

Weighing Machine

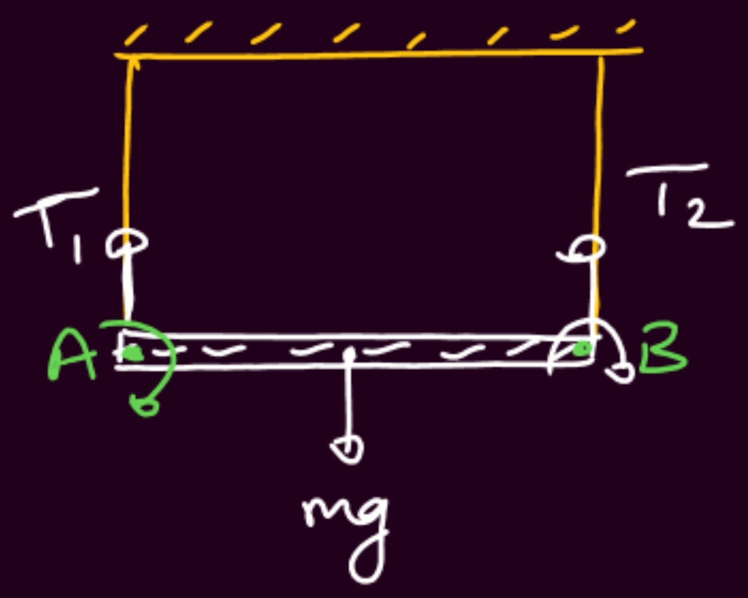
$mg = 10$

$m\left(g + g/2\right) = 3\left(\frac{10}{2}\right) = 15$

Physical Balance

$r(mg) = r(mg)$

Q.



$\Sigma F = 0$

① $T_1 + T_2 = mg$

$\Sigma \tau_A = 0$

② $T_2(l) = mg\left(\frac{l}{2}\right)$

$T_2 = \frac{mg}{2}$

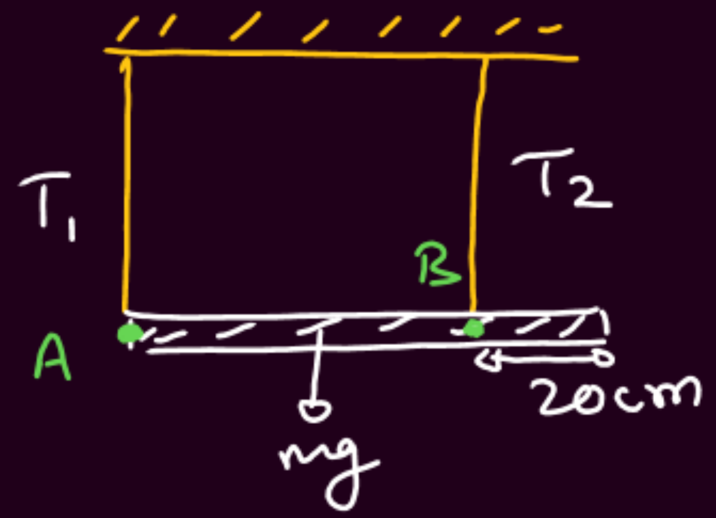
$\Sigma \tau_B = 0$

③ $T_1(l) = mg\left(\frac{l}{2}\right)$

$T_1 = \frac{mg}{2}$

Q

1m Rod



① $\boxed{\tau_B = 0}$

$T_1(80) = mg(30)$

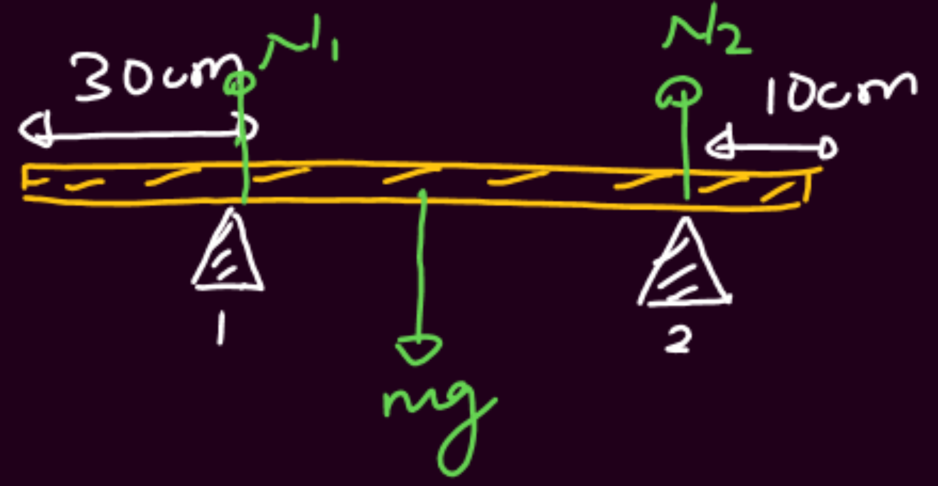
$T_1 = \frac{3}{8}mg$

② $\boxed{\tau_A = 0}$

$T_2(80) = mg(50)$

$T_2 = \frac{5}{8}mg$

Q



1m Rod

① $\boxed{\tau_2 = 0}$

$N_1(60) = mg(40)$

$N_1 = \frac{2}{3}mg$

② $\boxed{\tau_1 = 0}$

$N_2(60) = mg(20)$

$N_2 = \frac{mg}{3}$

Center of Gravity

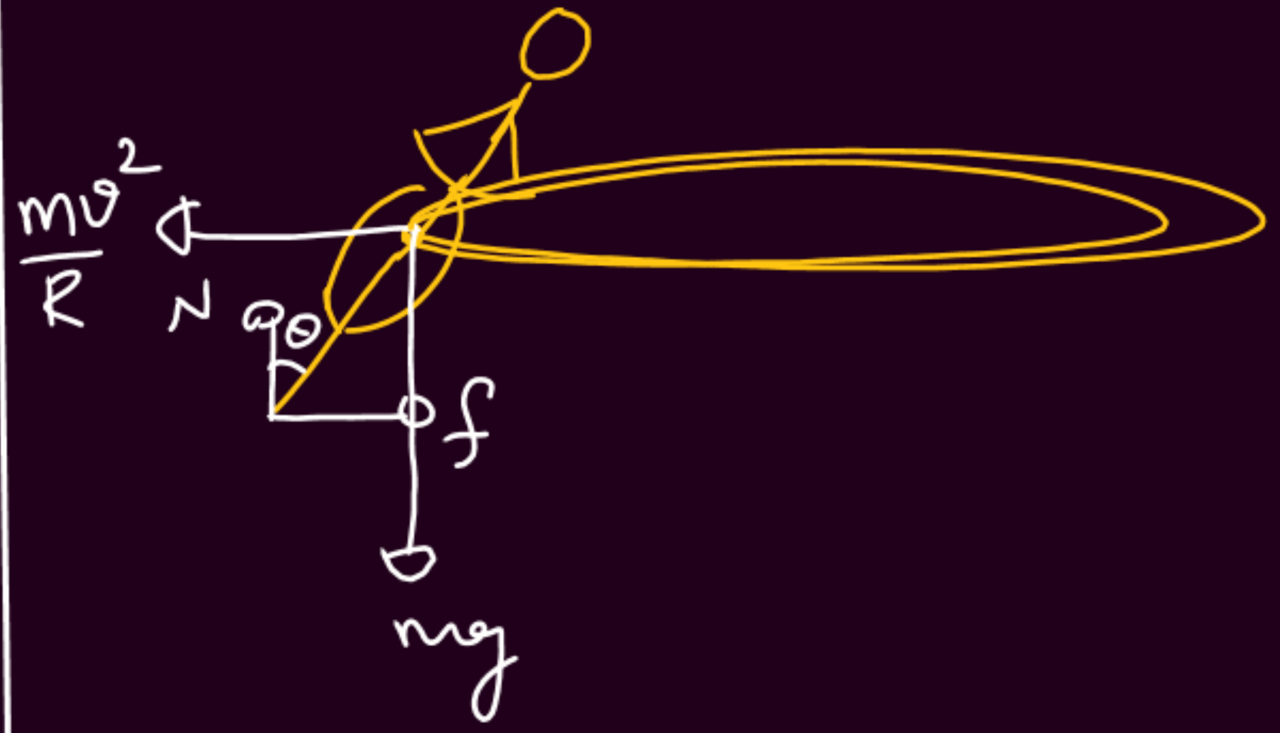
→ $\tau_g = 0$

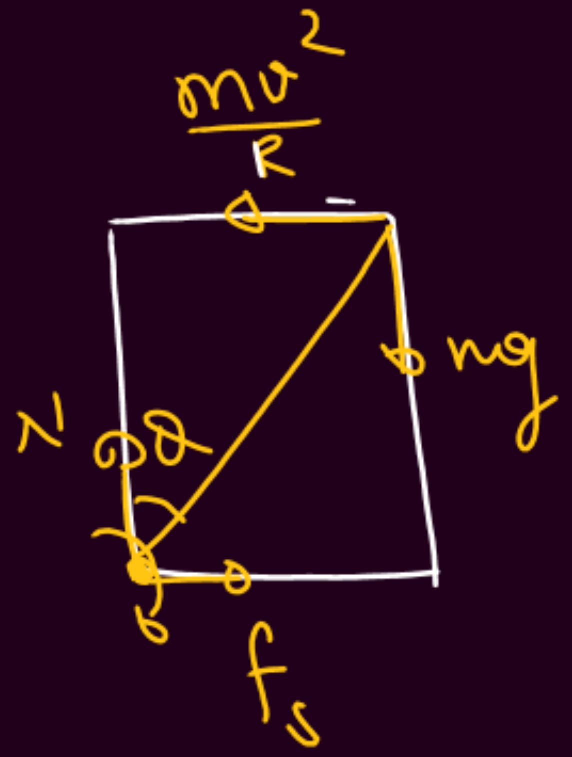
→ CM → mass

→ CG → mass & g

→ Small → same

Bending of Cyclist





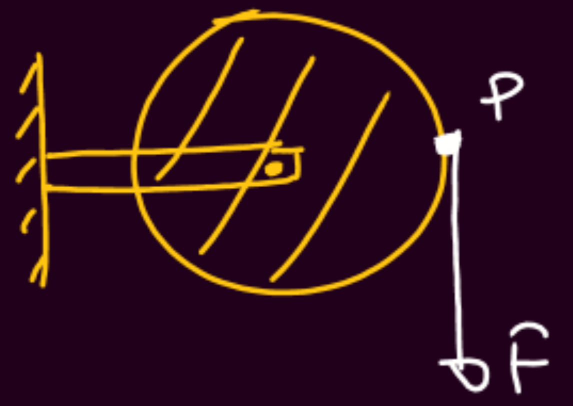
$$mg(P) = \frac{mv^2}{R}(B)$$

$$\boxed{\tan\theta = \frac{v^2}{gR}}$$

$$v = \sqrt{gR\tan\theta}$$

$$\boxed{\tau = I\alpha}$$

Wheel



① $\tau = FR$

② $\frac{1}{2}mR^2\alpha = FR$

$$\alpha = \frac{2F}{mR}$$

③ $a_t = \alpha R = \frac{2F}{M}$

Q. wheel (2kg, 50cm) $F = 10\,N$

① Torque $= FR = 10 \times \frac{50}{100} = 5$ N-m

② Ang. Acc $= \alpha = \frac{2F}{mR} = \frac{2 \times 10}{2 \times \left(\frac{50}{100}\right)} = 20$ rad/s²

③ Ang vel. after 2s

$$\omega = \omega_0 + \alpha t$$

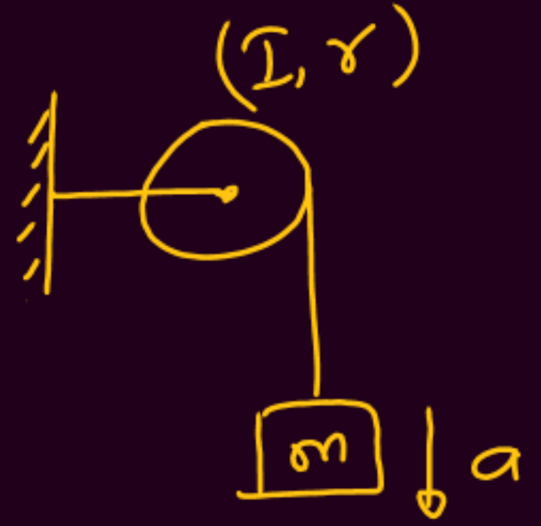
$$\omega = 0 + (20)(2) = 40 \text{ rad/s}$$

Q wheel (10kg, 2m), $\omega = 10$ rad/s $\longrightarrow$ Rest in 2s. by applying a const. tangential force. $F = ?$

$$\tau = I\alpha$$

$$FR = \left(\frac{1}{2}mR^2\right)\left(\frac{\omega}{t}\right)$$

$$F = \frac{mR\omega}{2t} = \frac{10(2)(10)}{2(2)} = \underline{\underline{50N}}$$



$$a = \frac{mg}{m + \frac{I}{r^2}}$$

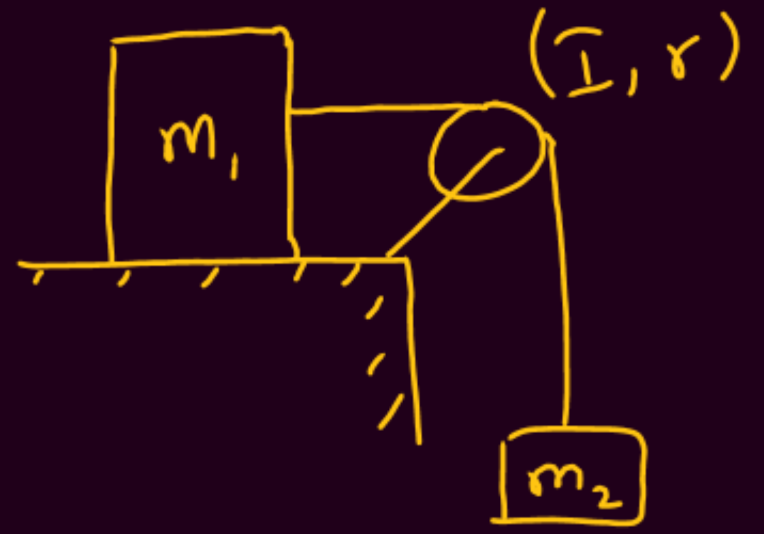
$$a = \frac{mg}{m + \frac{m_p}{2}}$$

$$I = \frac{1}{2} m_p r^2$$

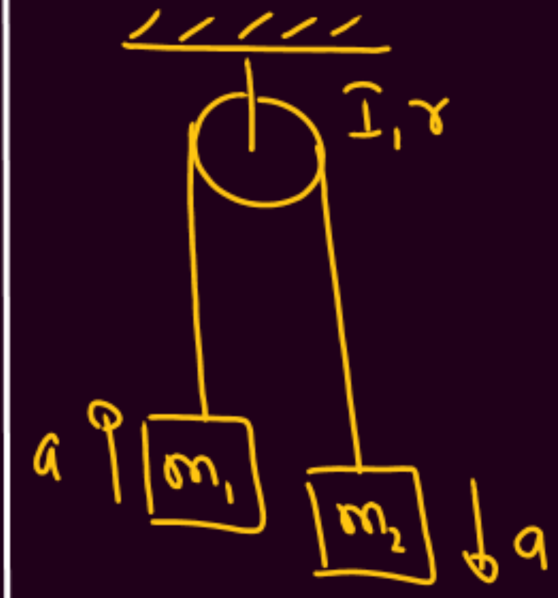
(1) $mg - T = ma$

(2) $Tr = I\alpha$

(3) $a = \alpha r$

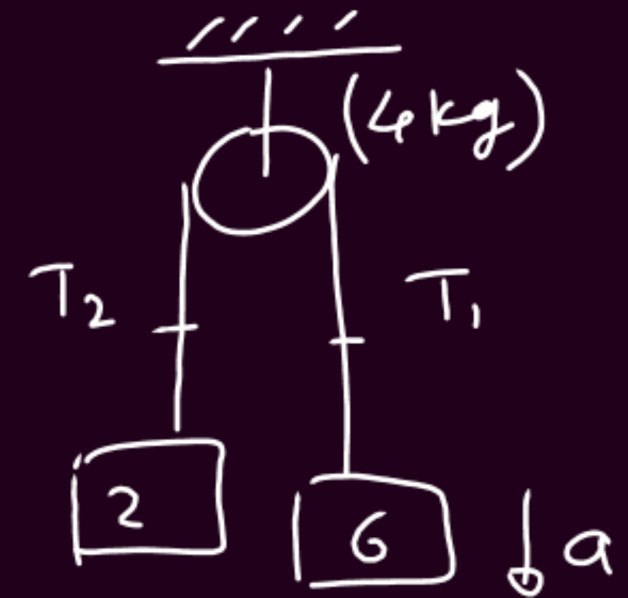


$$a = \frac{m_2 g}{m_1 + m_2 + \frac{I}{r^2}}$$



$$a = \frac{(m_2 - m_1) g}{m_1 + m_2 + \frac{I}{r^2}}$$

Q

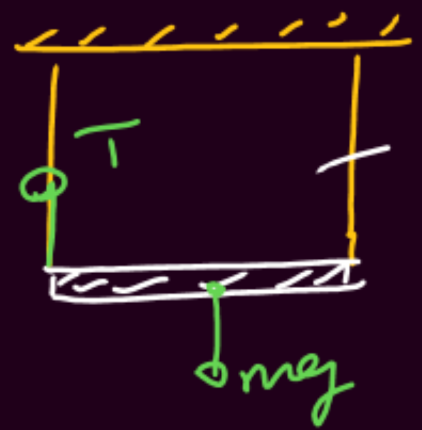


$$a = \frac{(6-2)g}{6+2+\frac{4}{2}} = 4 \text{ m/s}^2$$

$$T_1 = 6(g-4) = 36 N$$

$$T_2 = 2(g+4) = 28 N$$

Case

① 

② 

① $\tau = I\alpha$

$$mg\left(\frac{l}{2}\right) = \left(\frac{1}{3}ml^2\right)\alpha$$

$$\boxed{\alpha = \frac{3g}{2l}}$$

② $a_{cm} = \alpha\left(\frac{l}{2}\right)$

$$\boxed{a_{cm} = \frac{3g}{4}}$$

③ $mg - T = ma_{cm}$

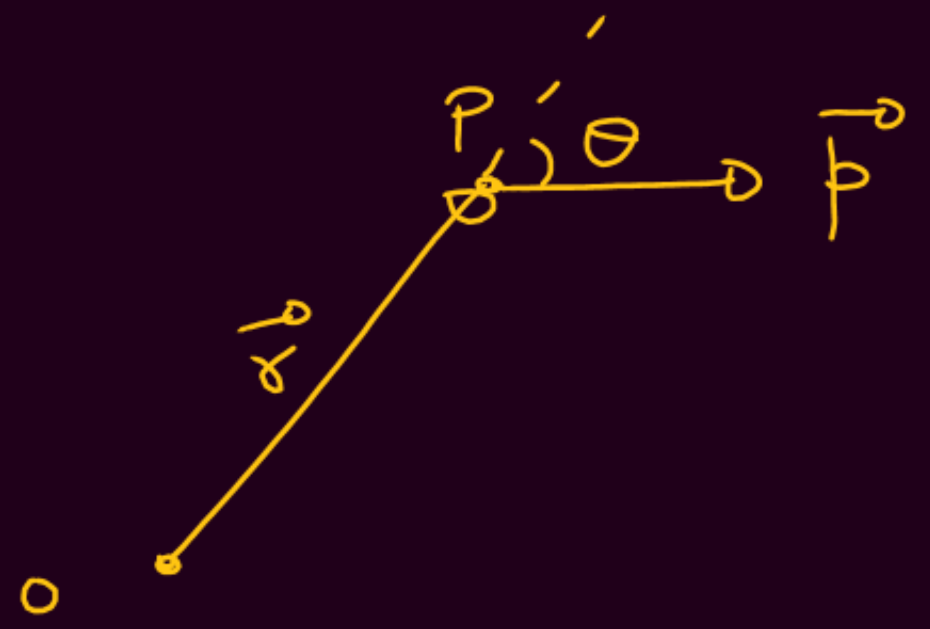
$$mg - T = m\left(\frac{3g}{4}\right)$$

$$\boxed{T = \frac{mg}{4}}$$

Angular Momentum ($\vec{L}$)

Moment of Momentum

1) Pt. mass



$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

$L = mvr\sin\theta = mvr_{\perp} = mv_{\perp}r$

Unit $kg\text{-}\frac{m^2}{s}$

Q. Find $\vec{L}$ of 2kg, $\vec{v} = (2\hat{i} - \hat{j} + 3\hat{k})$ m/s at point $\underset{P}{(2, 1, -1)}$ about $\underset{O}{(-1, 2, 1)}$

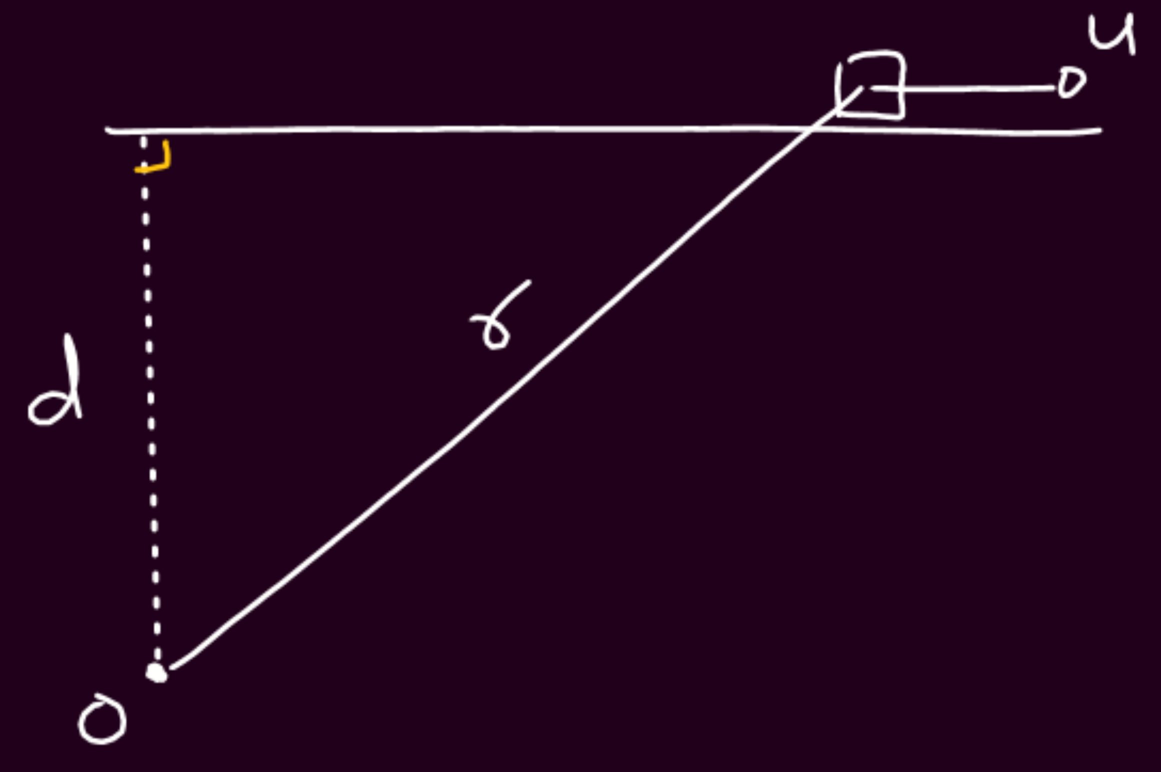
$$\vec{r} = \vec{OP} = \vec{P} - \vec{O} = (3\hat{i} - \hat{j} - 2\hat{k})\ m$$

$$\vec{L} = \vec{r} \times (m\vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -2 \\ 4 & -2 & 6 \end{vmatrix}$$

$$\vec{L} = (-6 - 4)\hat{i} - (18 - (-8))\hat{j} + (-6 - (-4))\hat{k}$$

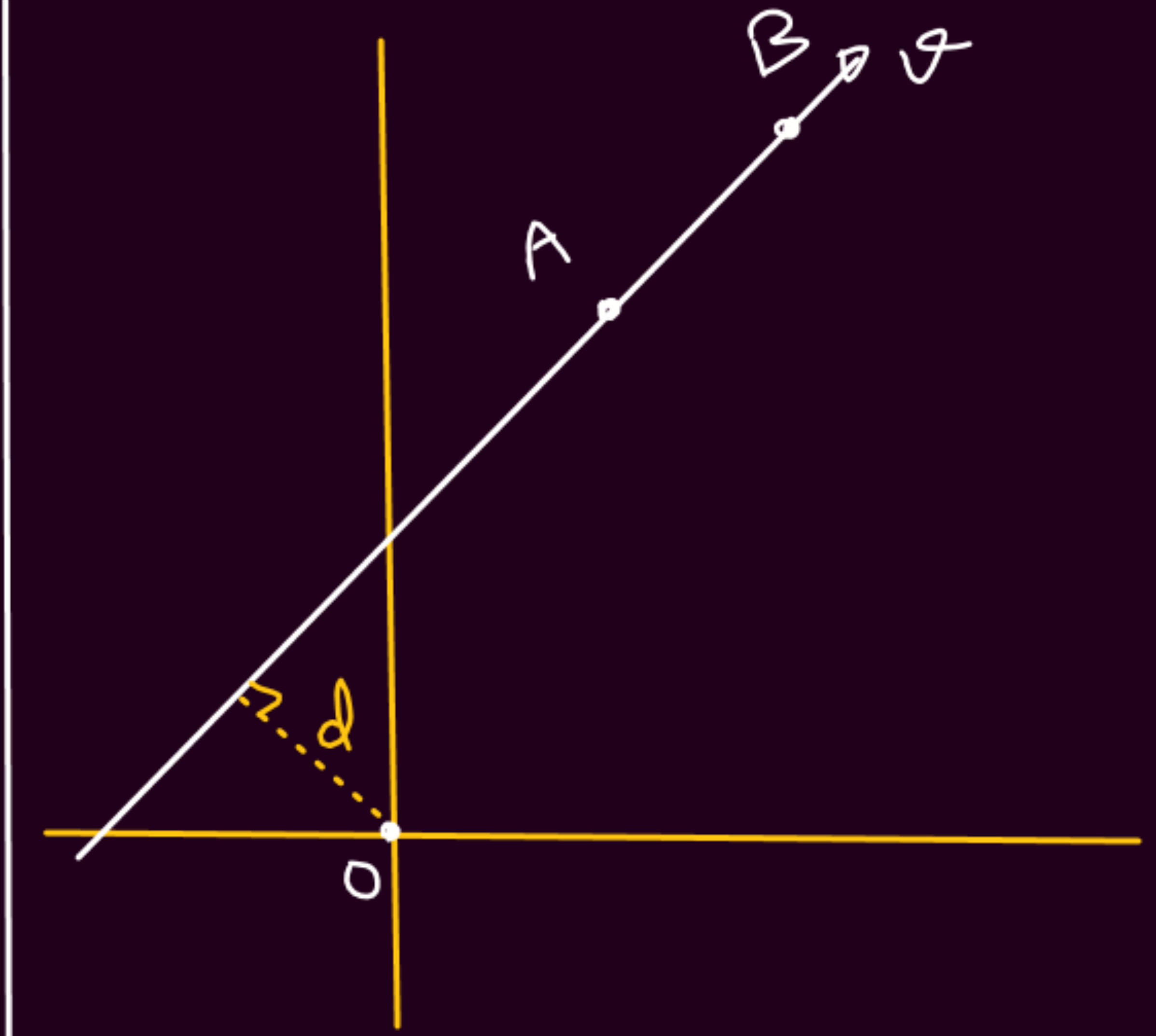
$$\vec{L} = (-10\hat{i} - 26\hat{j} - 2\hat{k})\ kg\text{-}\frac{m^2}{s}$$

2) Object in Linear Motion



$L = m v r_\perp$

$$\boxed{L = m u d}$$



$L = m u d$

const v

Angular momentum about origin when at A & B

1) $L_A > L_B$

2) $L_A < L_B$

3) $L_A = L_B$ ✓

3) L of Rigid Body about a fixed Axis

$$\boxed{L = I\omega}$$

Translational

m

x

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d^2x}{dt^2}$$

$$p = mv$$

$$F = ma \qquad F = \frac{dp}{dt}$$

$$\text{Impulse} = \Delta p = p_f - p_i = \int F\,dt$$

$$KE_{Trans} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Rotational.

I

θ

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta} = \frac{d^2\theta}{dt^2}$$

$$L = I\omega$$

$$\tau = I\alpha \qquad \tau = \frac{dL}{dt}$$

$$\text{Ang. Impulse } \Delta L = L_f - L_i = \int \tau\,dt$$

$$KE_{Rot} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

$$W = \int F\,dx$$

$$W = \Delta K = \frac{1}{2} m \left(v_2^2 - v_1^2\right)$$

$$P = \vec{F} \cdot \vec{v}$$

Conservation of Linear Momentum

If $F_{ext} = 0$

then $\vec{p}_f = \vec{p}_i$

$$W = \int \tau\,d\theta$$

$$W = \Delta K = \frac{1}{2} I \left(\omega_2^2 - \omega_1^2\right)$$

$$P = \vec{\tau} \cdot \vec{\omega}$$

Conservation of Angular Momentum

If $\tau_{ext} = 0$

then $\vec{L}_f = \vec{L}_i$

Q. Sphere (5kg, 2m)

about axis 10 rad/s

① $L = I\omega$

$L = \left(\frac{2}{5}mR^2\right)\omega$

$L = \frac{2}{5}(5)(2)^2(10)$

$L = 80 \ kg\text{-}\frac{m^2}{s}$

② W.D. in ↑ the speed to 15 rad/s

$W = \Delta K = \frac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$

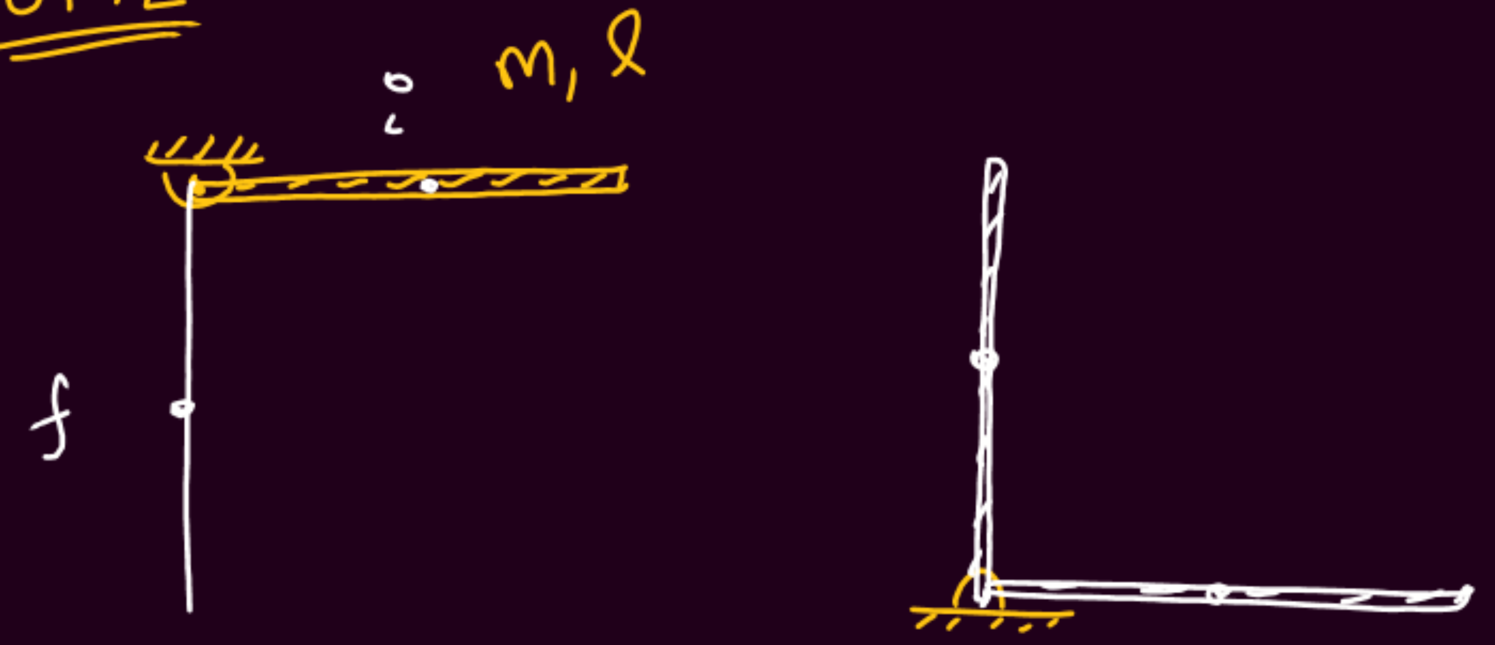
$W = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\omega_2^2 - \omega_1^2\right)$

$W = \frac{1}{2}\left(\frac{2}{5}\right)(5)(2)^2\left[15^2 - 10^2\right]$

$W = 4(225 - 100)$

$\underline{W = 500 \ J}$

COME



$$U_i + K_i = U_f + K_f$$

$$mg\left(\frac{l}{2}\right) + 0 = 0 + \frac{1}{2} I\omega^2$$

$$mg\frac{l}{2} = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{l}}$$

Conservation of $\vec{L}$

If $\tau_{ext} = 0$

then $\vec{L} = \text{Const}$

$$\vec{L}_f = \vec{L}_i$$

$$(L_1 + L_2 + \cdots)_f = (L_1 + L_2 + \cdots)_i$$

① Ice skating

$I\omega = \text{Const.}$

$I \downarrow \quad \omega \uparrow$

② Global warming / Sky Scrapper

$I \uparrow \quad \omega \downarrow \quad T \uparrow$

(Duration of Day) $\uparrow$

$$T_1 (365) = T_2 (N)$$

(No. of Days in a year) $\downarrow$

③

$I \downarrow$ min $\uparrow$

$\omega \uparrow$ max $\downarrow$

④

$I \uparrow$

$\omega \downarrow$

Q. Earth → Shrinks to Half

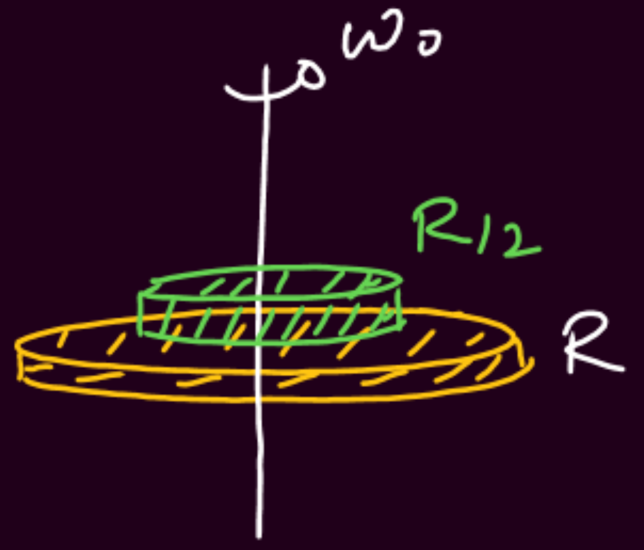
$$I_1 \omega_1 = I_2 \omega_2$$

$$\left(\frac{2}{5} m R^2\right)\left(\frac{2\pi}{T_1}\right) = \left[\frac{2}{5} m \left(\frac{R}{2}\right)^2\right]\left(\frac{2\pi}{T_2}\right)$$

$$\frac{R^2}{T_1} = \frac{(R/2)^2}{T_2}$$

$$T_2 = \frac{T_1}{4} = \frac{24}{4} = \underline{\underline{6 \text{ hrs}}}$$

Q.

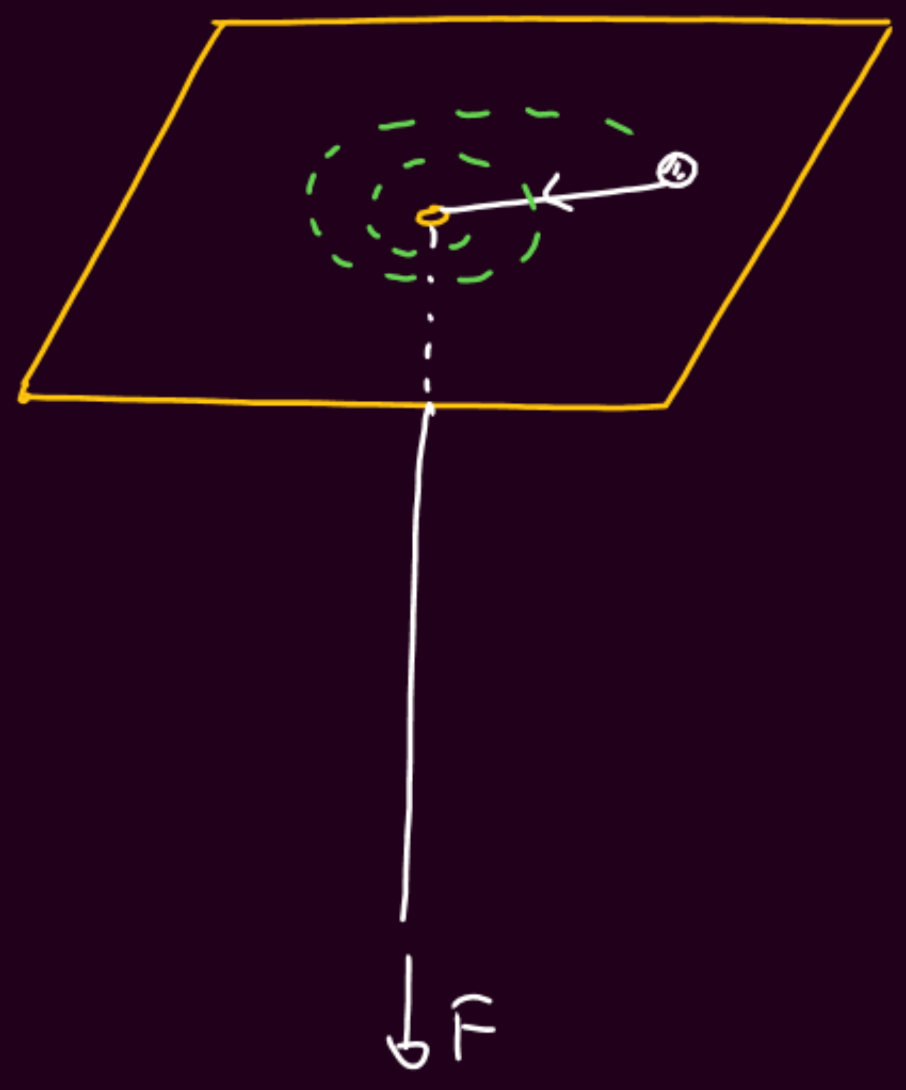


$$\left(\frac{1}{2} m R^2\right)\omega_0 + 0 = \left(\frac{1}{2} m R^2 + \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{R}{2}\right)^2\right)\omega$$

$$\omega_0 = \left(1 + \frac{1}{16}\right)\omega$$

$$\boxed{\omega = \frac{16}{17}\omega_0}$$

Case



$\tau = 0$

$I\omega = \text{const}$

$mvr = \text{const}.$

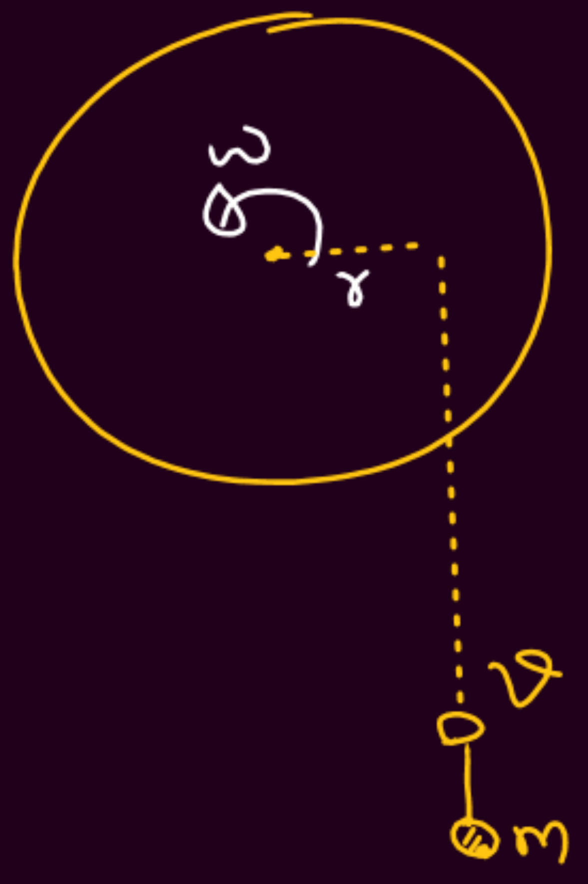
① $v_1 r_1 = v_2 r_2$

② $\omega_1 r_1^2 = \omega_2 r_2^2$

③ $\dfrac{K_2}{K_1} = \left(\dfrac{v_2}{v_1}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$

$r \downarrow \quad v \uparrow \quad \omega \uparrow \quad KE \uparrow$

Case Marry-go-Round



$$L_i = L_f$$

$$mvr = (I_p + mr^2)\omega$$

$$\boxed{\omega = \frac{mvr}{I_p + mr^2}}$$

If man starts walking on platform with speed v

$$0 = m(v - \omega r)r - I_p\omega$$

$$\boxed{\omega = \frac{mvr}{I_p + mr^2}}$$

Q.

(100kg, 2m)

Starts walking (10m/s)

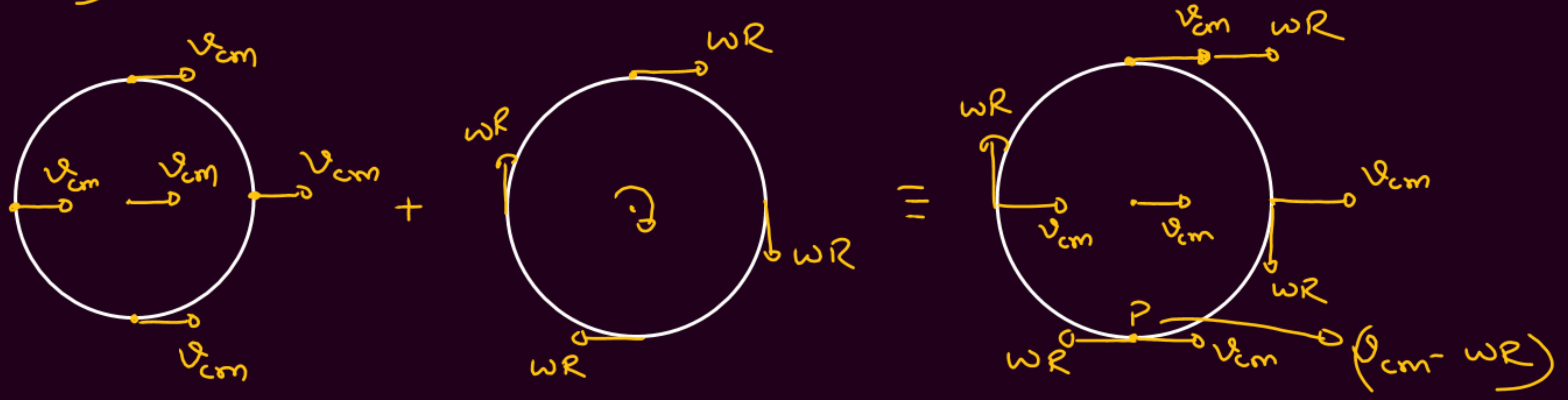
$\omega = ?$

$$\omega = \frac{mvr}{I_p + mr^2}$$

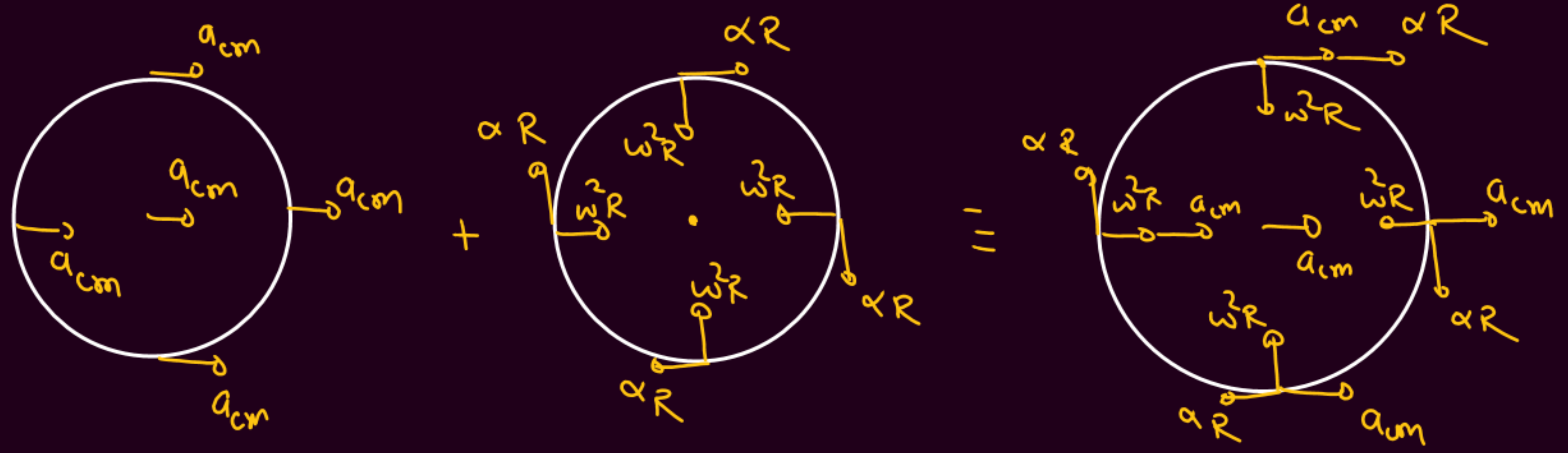
$$\omega = \frac{mvr}{\frac{1}{2}Mr^2 + mr^2}$$

$$\omega = \frac{(50)(10)}{\frac{1}{2}(100)(2) + 50(2)} = \frac{5}{2}\ \text{rad/s}$$

Rolling = Translation + Rotation



(Pure Translation) + (Pure Rotation) = Rolling



		Rolling on fixed surface $v_Q = 0$ 	In one rotation
Pure Rolling	$v_P = v_Q$	$v = \omega R$ $a = \alpha R$	$L = 2\pi R$
Forward Slipping (Fast Braking)	$v_P > v_Q$	$v > \omega R$	$L > 2\pi R$
Backward Slipping (Mud)	$v_P < v_Q$	$v < \omega R$	$L < 2\pi R$

Pure rolling on fixed Surface

→ Pt. of contact → Inst. Rest

→ $v = \omega R$

→ $a = \alpha R$

→ 

In one Rotation

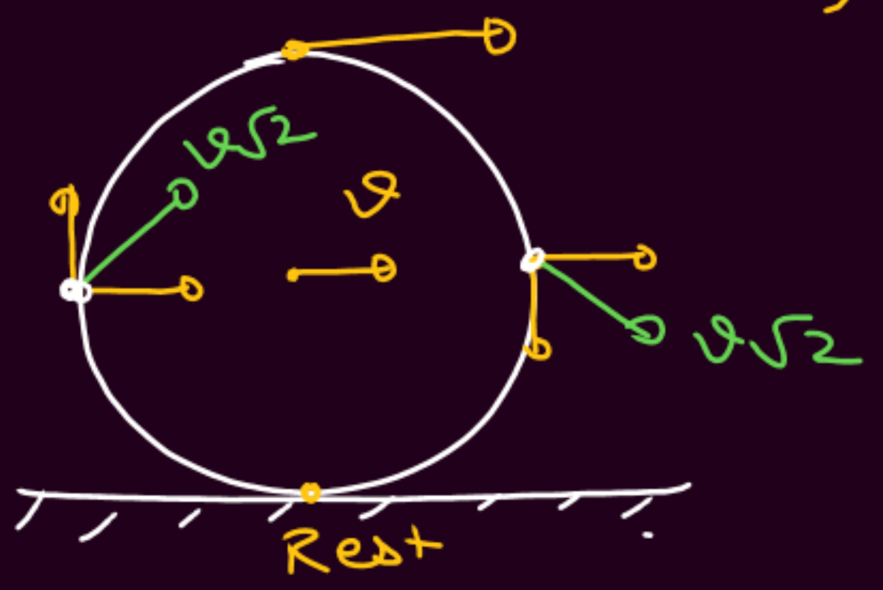
① Displacement $= 2\pi R$
② Distance $= 8R$

} for any point on circumference

→ friction → static

→ $W_f = 0$

→ Pure Rolling → fixed / Rough / Horizontal
→ Roll forever
$2v$ → $f = 0$



⇒ Bottom → Rest
⇒ Center → v
⇒ Top → $2v$
⇒ Extreme of H-Dia ⇒ $v\sqrt{2}$

→ Pt. of Contact is IAoR

→ 

$v_p = 2v\sin(\theta/2)$

Ring/HC	Disc/SC	HS	SS
			
$I = MR^2$	$I = \frac{1}{2}(mR^2)$	$I = \frac{2}{3}mR^2$	$I = \frac{2}{5}mR^2$
$\gamma = 1$	$\gamma = \frac{1}{2}$	$\gamma = \frac{2}{3}$	$\gamma = \frac{2}{5}$

$$I = \gamma mR^2 = Mk^2$$

$$\boxed{\gamma = \frac{k^2}{R^2}}$$

KE of Rolling

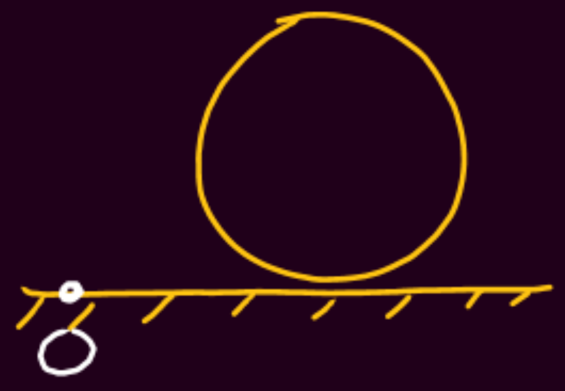
$$KE_{Trans} = \frac{1}{2} M v^2$$

$$KE_{Rot} = \frac{1}{2} I \omega^2$$

$$KE = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

Angular Momentum of Rolling

(About any point on Surface)



$$L = M v R + I_{cm} \omega$$

Pure Rolling

$$KE_{Trans} = \frac{1}{2} M v^2$$

$$KE_{Rot} = \gamma \left(\frac{1}{2} M v^2\right)$$

$$KE = (1+\gamma)\left(\frac{1}{2} M v^2\right)$$

Pure Rolling

$$L = (1+\gamma) M v R$$

Q HC → Pure Rolling
→ (2 kg, 1 m)
→ $v = 10\,m/s$

$$KE = (1+1)\frac{1}{2}(2)(10)^2$$
$$= 200\,J$$

Q Sphere (10 kg)
Rolling (2 m/s)

W.D. in Rest ?

$$W.D. = \Delta KE = \left(1+\frac{2}{5}\right)\frac{1}{2}(10)(2)^2$$
$$= 28\,J$$

Q. Sphere in pure Roll

$$\frac{KE_{Rot}}{KE_{Tot}} = \frac{\gamma\left(\frac{1}{2}mv^2\right)}{(1+\gamma)\left(\frac{1}{2}mv^2\right)}$$
$$= \frac{2/5}{1+2/5}$$
$$= \frac{2}{7}$$

Q. HS → Pure Rolling
% of KE → Translational

$$\frac{KE_{Trans}}{KE}\times 100$$
$$= \frac{\left(\frac{1}{2}mv^2\right)}{\left(1+\frac{2}{3}\right)\frac{1}{2}mv^2}\times 100$$
$$= \frac{3}{5}\times 100 = 60\%$$

Q What fraction of KE of a body in P.R. is translational

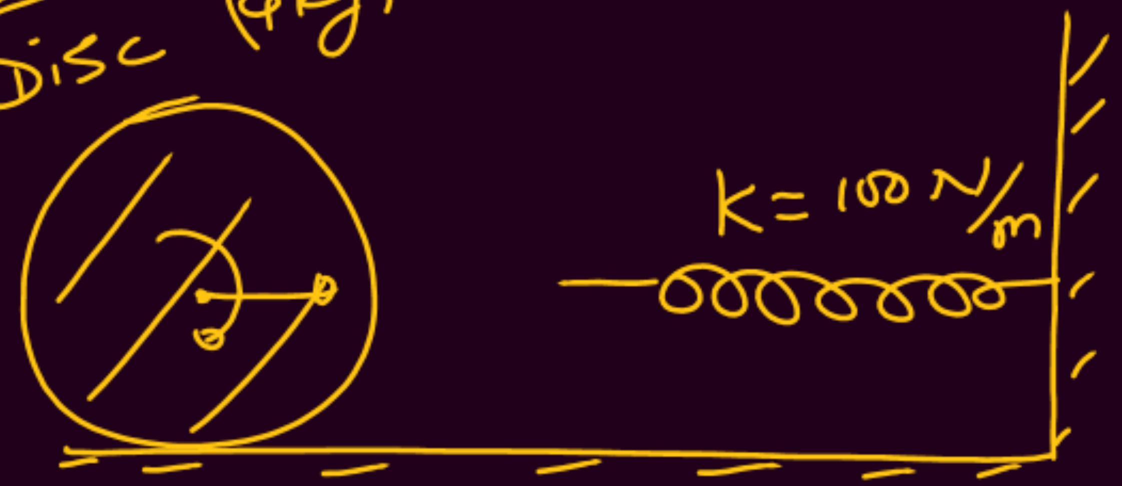
1) $\frac{K^2}{R^2}$ 2) $\frac{R^2}{K^2}$

3) $\frac{K^2}{R^2+K^2}$ ✓ 4) $\frac{R^2}{R^2+K^2}$

$$\frac{KE_{Trans}}{KE_{Total}} = \frac{\frac{1}{2}mv^2}{(1+\gamma)\frac{1}{2}mv^2}$$
$$= \frac{1}{1+\gamma}$$
$$= \frac{1}{1+\frac{K^2}{R^2}}$$
$$= \frac{R^2}{R^2+K^2}$$

Q

Disc $(4kg, 2m)$



P.R. $(10\,m/s)$

Max. compression?

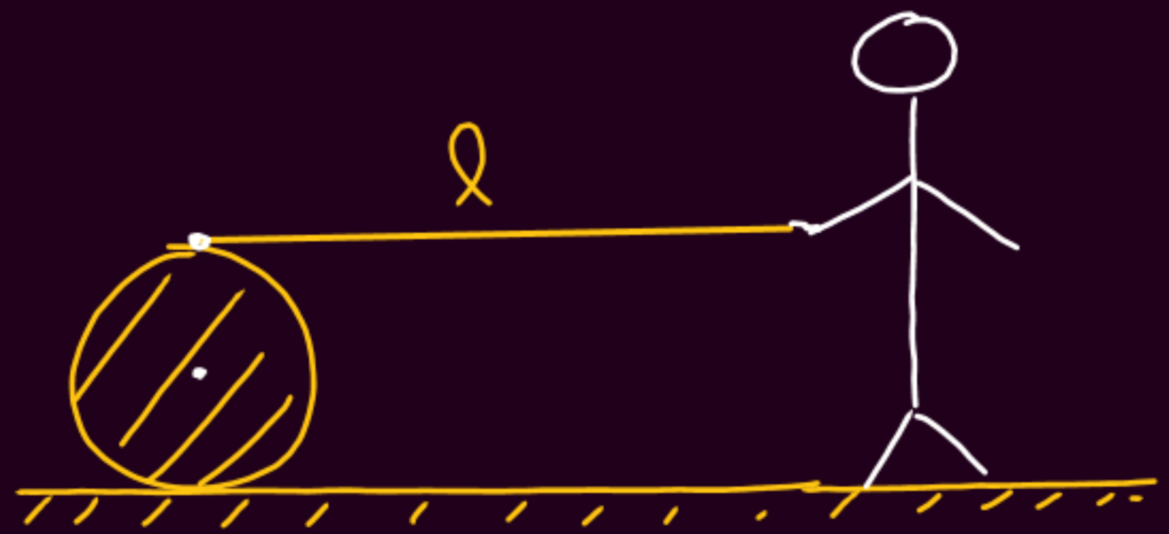
$$\frac{1}{2}kx^2 = (1+\gamma)\frac{1}{2}mv^2$$

$$x = v\sqrt{\frac{m}{k}(1+\gamma)}$$

$$x = 10\sqrt{\frac{4}{100}\left(1+\frac{1}{2}\right)}$$

$$x = \sqrt{6}\,m$$

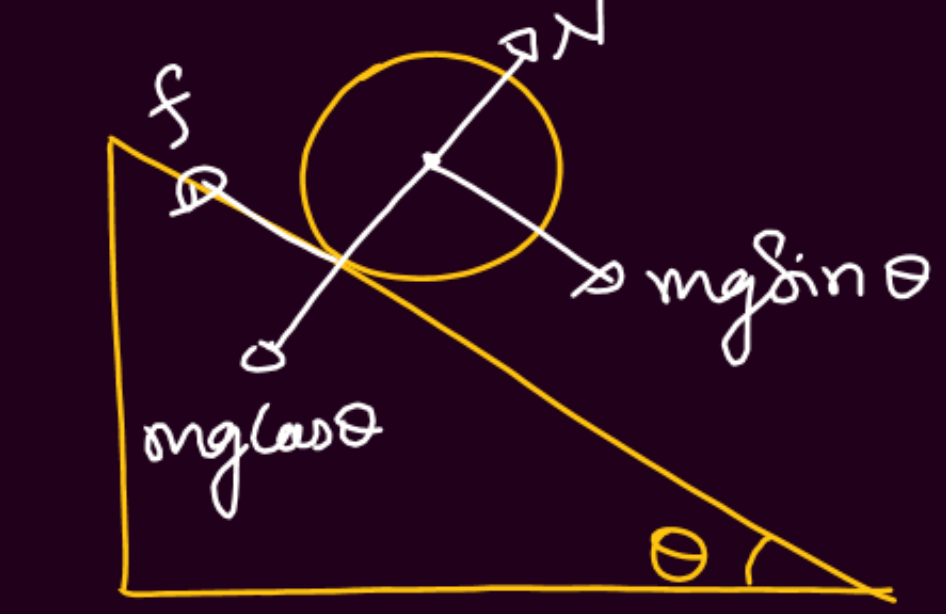
Q.



Length of String passed through hand till the spool reaches the man?

Ans $\Rightarrow \boxed{2l}$

Rolling on Inclined Plane



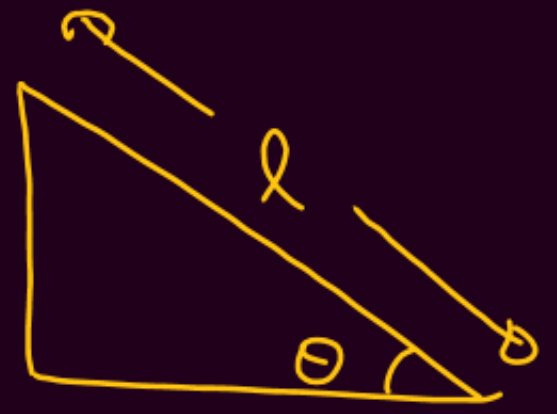
① $N = mg\cos\theta$

② $f_L = \mu mg\cos\theta$

③ $mg\sin\theta - f = ma$

④ $fR = I\alpha$

① Friction is not suff. to prevent Slipping



① $f_k = \mu mg\cos\theta$

② $a = g(\sin\theta - \mu\cos\theta)$

③ $v = \sqrt{2al}$

④ $t = \sqrt{\dfrac{2l}{a}}$

→ Same for all

→ independent of m, R

Forward Slipping $\Rightarrow \omega_f = \ominus$

⑤ $\alpha = \dfrac{\tau}{I} = \dfrac{(\mu mg\cos\theta)R}{\gamma mR^2} = \dfrac{\mu g\cos\theta}{\gamma R}$

② Pure Rolling on Inclined plane

① $a = \alpha R$

② $a = \dfrac{g\sin\theta}{1+\gamma}$

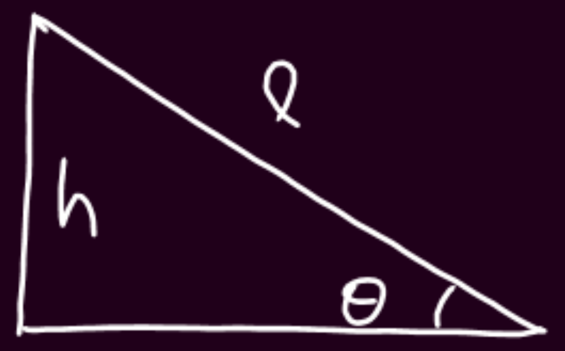
③ $t = \sqrt{\dfrac{2l}{a}} = \sqrt{\dfrac{2l(1+\gamma)}{g\sin\theta}}$

④ $v = \sqrt{2al} = \sqrt{\dfrac{2g\sin\theta\, l}{1+\gamma}}$

$$\boxed{v = \sqrt{\dfrac{2gh}{1+\gamma}}}$$

Extra ⑤ $f_s = \dfrac{\gamma\, mg\sin\theta}{1+\gamma}$

⑥ $W_f = 0$ $\therefore$ ME $\rightarrow$ conserved



⑦ $KE = (1+\gamma)\frac{1}{2}mv^2 = mgh$

Extra ⑧ Condition for pure rolling

$$\mu \geq \frac{\gamma\tan\theta}{1+\gamma}$$

⑨

Q  P.R.

HS & SS

(m, R) (m, R)

① $\frac{a_1}{a_2} = \frac{1+\frac{2}{5}}{1+\frac{2}{3}} = \frac{7/5}{5/3} = \frac{21}{25}$

② $\frac{KE_1}{KE_2} = \frac{mgh}{mgh} = \frac{1}{1}$

③ $\frac{v_1}{v_2} = \sqrt{\frac{1+2/5}{1+2/3}} = \sqrt{\frac{21}{25}}$

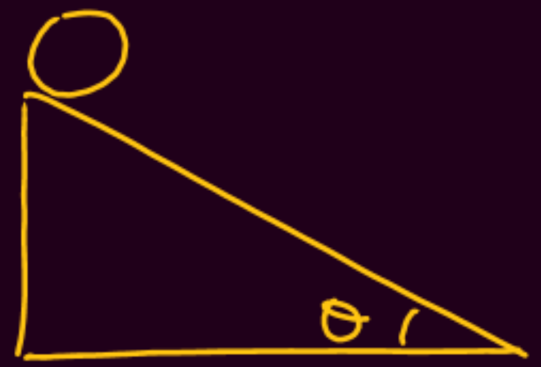
④ Which one reaches the bottom first?

$$t = \sqrt{\frac{2l(1+\gamma)}{g\sin\theta}}$$

$$\frac{t_{HS}}{t_{SS}} = \sqrt{\frac{1+\frac{2}{3}}{1+\frac{2}{5}}} = \sqrt{\frac{25}{21}}$$

SS reaches the bottom first

Q



Pure Rolling

SS — (M, R) 1

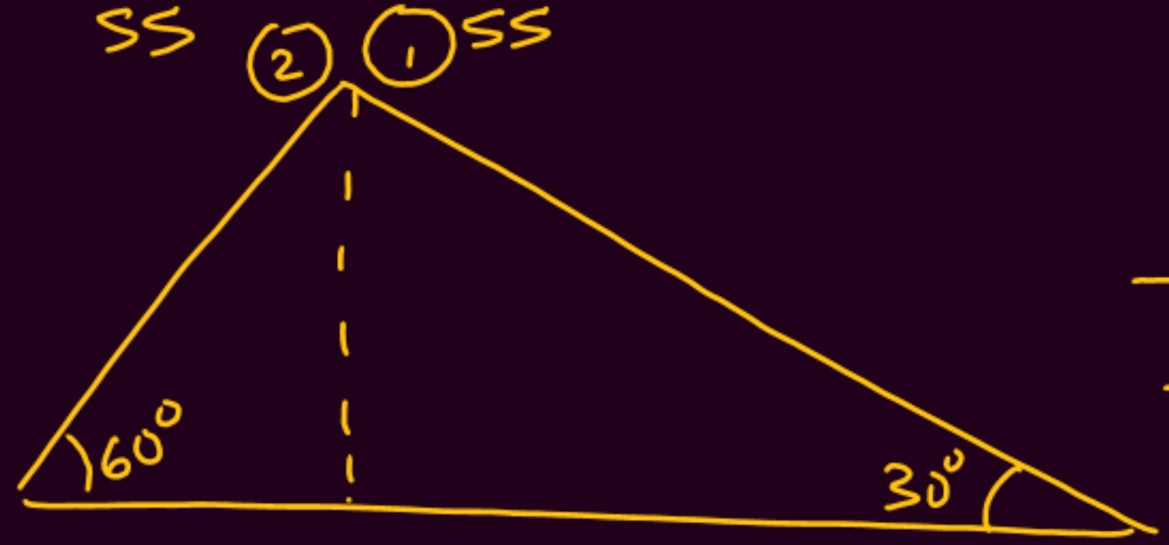
SS — $(2M, \frac{R}{2})$ 2

① $\frac{a_1}{a_2} = \frac{1}{1}$

② $\frac{v_1}{v_2} = \frac{1}{1}$

③ $\frac{KE_1}{KE_2} = \frac{Mgh}{2Mgh} = \frac{1}{2}$

Q



→ identical

→ P.R.

① $\frac{a_1}{a_2} = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}}$

② $\frac{v_1}{v_2} = \frac{1}{1}$

③ $\frac{KE_1}{KE_2} = \frac{1}{1}$

Q

Ball (2kg, 50cm)

Pure Rolling

① $a = \dfrac{(10)\sin 30^\circ}{1 + 2/5} = \dfrac{25}{7}\ m/s^2$

② $v = \sqrt{\dfrac{2(10)(20)}{1 + 2/5}} = 20\sqrt{\dfrac{5}{7}}\ m/s$

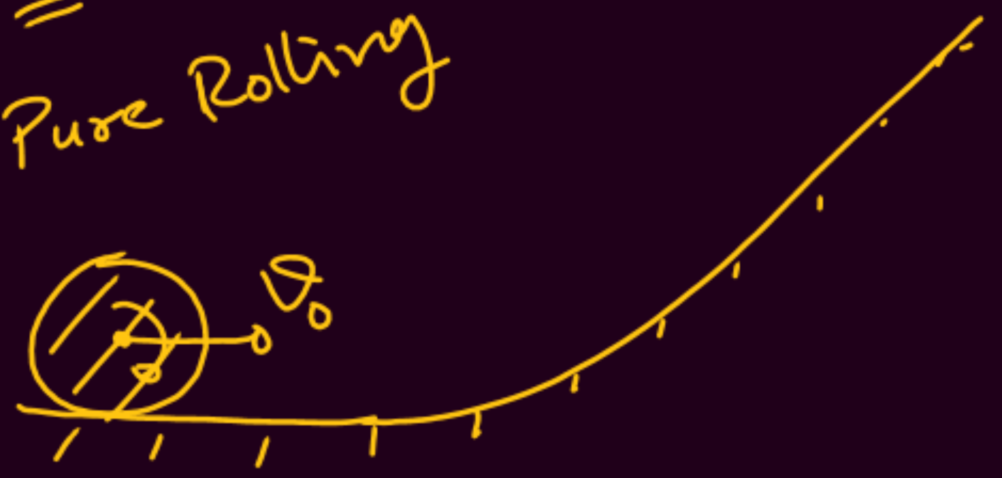
③ $t = \sqrt{\dfrac{2l}{a}} = \sqrt{\dfrac{2(40)\times 7}{25}} = \sqrt{\dfrac{2\times 8\times 7}{5}} = 4\sqrt{7/5}\ sec$

$l \sin 30^\circ = 20$

$l = 40$

Q.

Pure Rolling



if max height is $h = \frac{3v^2}{4g}$

then the object is

1) HC

2) SC

3) HS

4) SS

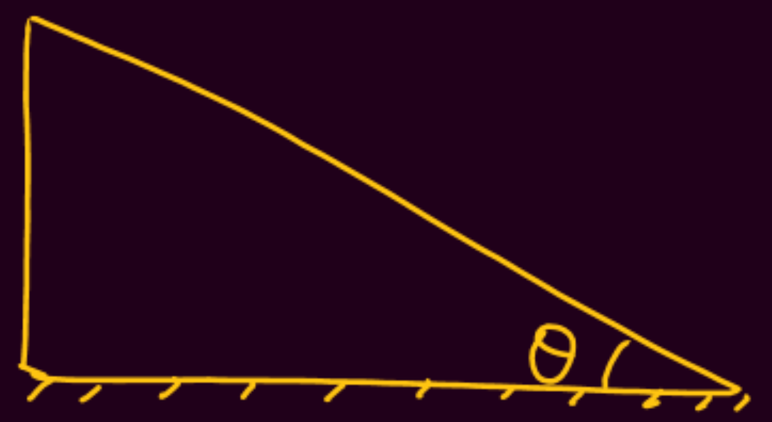
$$(1+\gamma)\frac{1}{2}mv^2 = mgh$$

$$(1+\gamma)\frac{1}{2}v^2 = g\,\frac{3v^2}{4g}$$

$$1+\gamma = \frac{3}{2}$$

$$\gamma = \frac{1}{2}$$

Q



Sphere

→ Slides Down → Smooth

→ Rolls Down → P.R.

① $\dfrac{a_1}{a_2} = \dfrac{g\sin\theta}{\left(\dfrac{g\sin\theta}{1+\gamma}\right)} = 1+\gamma = 1+\dfrac{2}{5} = \dfrac{7}{5}$

② $\dfrac{v_1}{v_2} = \dfrac{\sqrt{2gh}}{\sqrt{\dfrac{2gh}{1+\gamma}}} = \sqrt{1+\gamma} = \sqrt{\dfrac{7}{5}}$

③ $\dfrac{KE_1}{KE_2} = \dfrac{mgh}{mgh} = \dfrac{1}{1}$

④ $\dfrac{t_1}{t_2} = \sqrt{\dfrac{a_2}{a_1}} = \sqrt{\dfrac{5}{7}}$

$t = \sqrt{\dfrac{2l}{a}}$