Elasticity

Rigid →

Elastic → Restoring force

→ Quartz

"Steel & Rubber"

↳ more elastic

"Elastomer" → 'Tissue of Aorta'

Plastic → "Clay"

Stress = $\frac{\text{R.F.}}{\text{Area}}$ $\left(\frac{N}{m^2} = Pa\right)$

$GPa = 10^9\ Pa$

$\boxed{\text{Tensor}}$

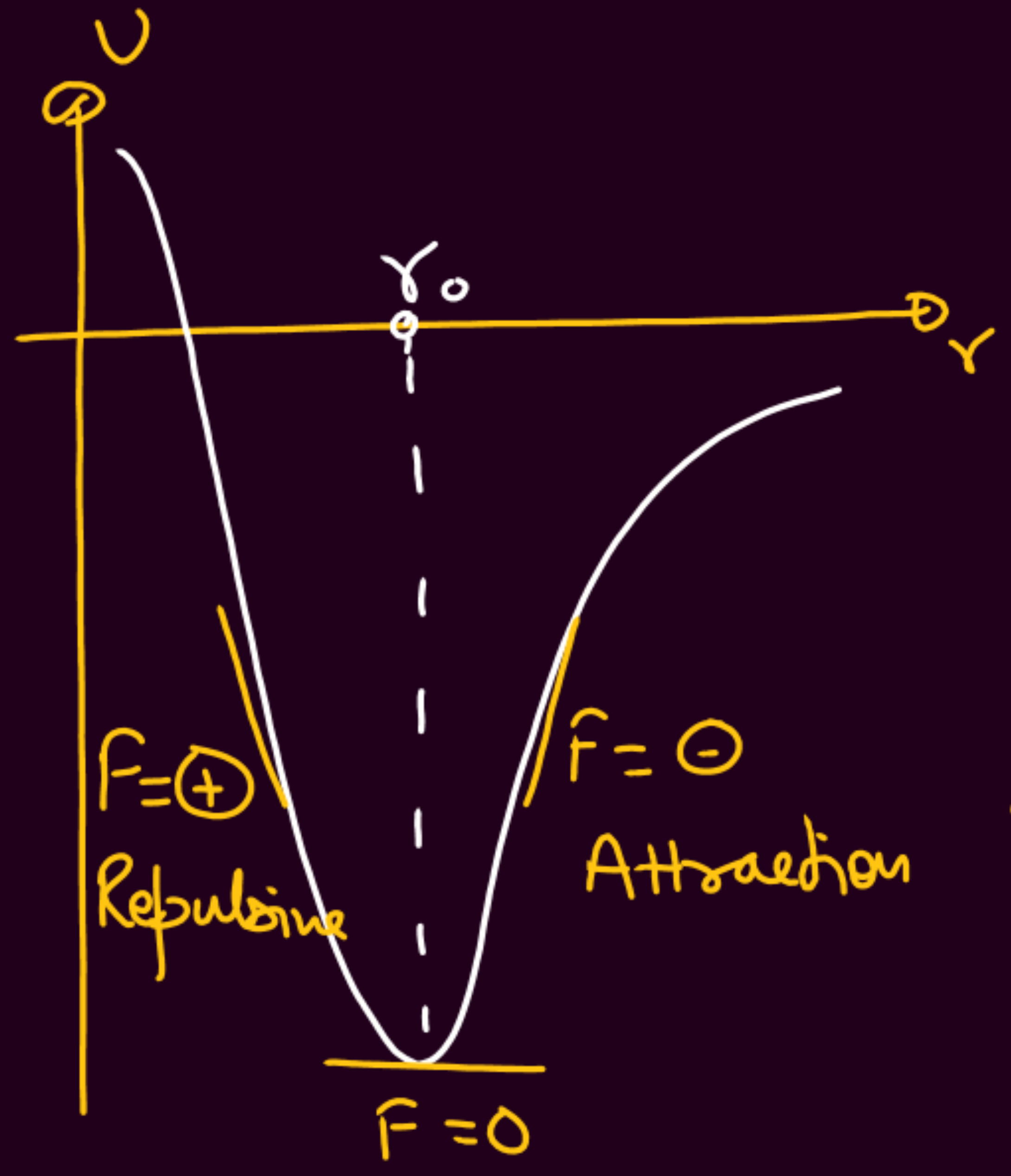
Strain = $\frac{\text{Change}}{\text{Original}}$

Hooke's Law → for small deformations

$(\text{Stress}) \propto (\text{strain})$

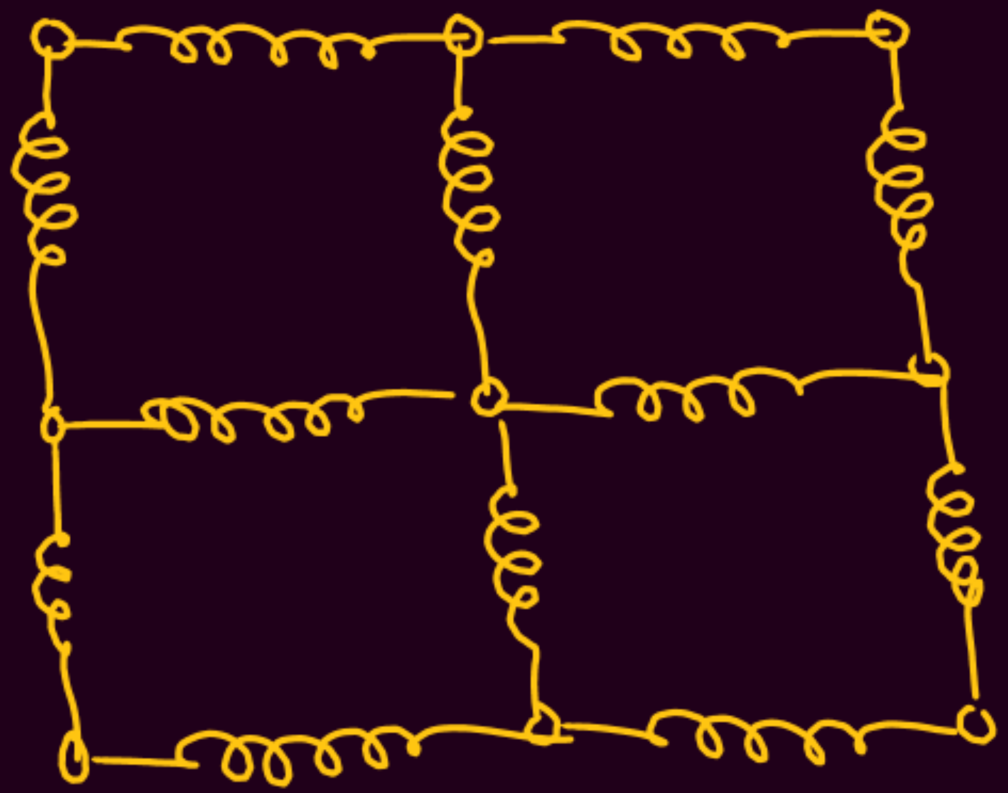
$\text{Stress} = E(\text{Strain})$

E = Modulus of Elasticity $\left(N/m^2 = Pa\right)$

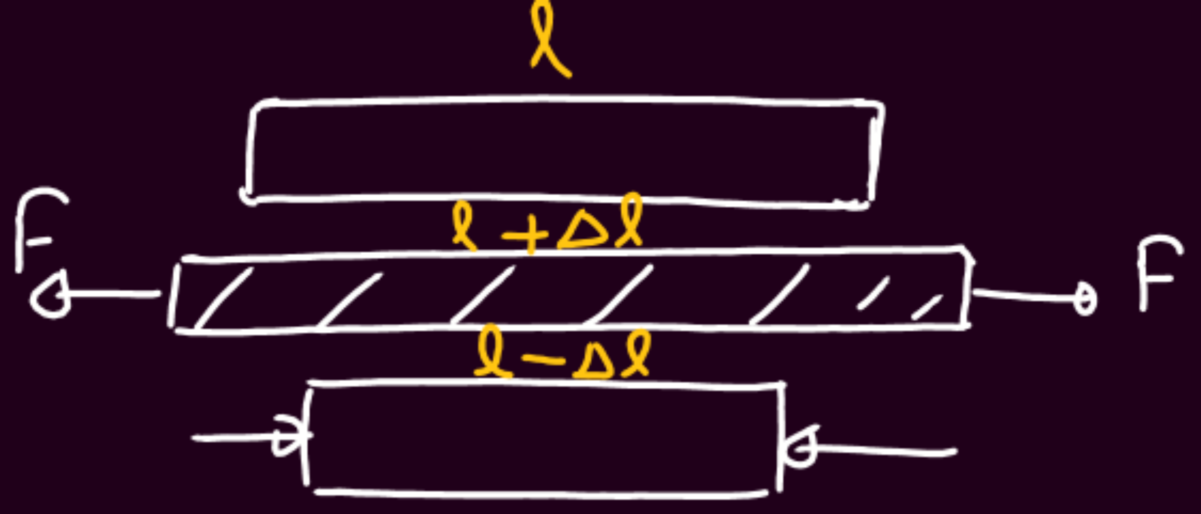


$$\boxed{F = -\left(\frac{dU}{dr}\right)}$$

Spring - Ball



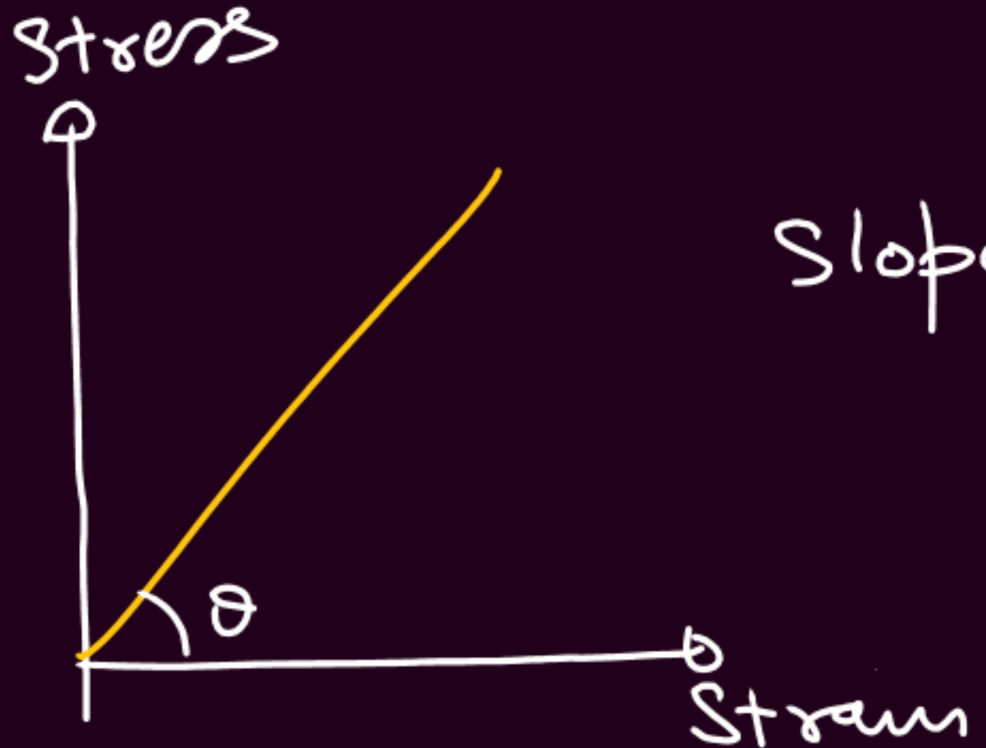
Longitudinal Stress $= \frac{F_\perp}{A}$



(Tensile)

(Compressive)

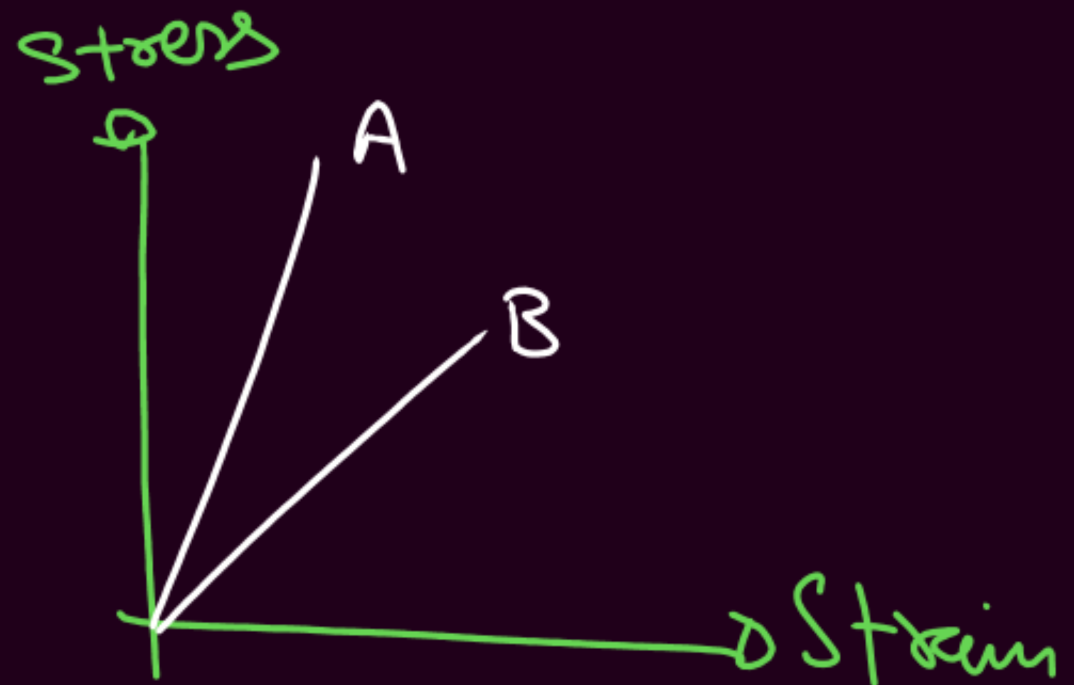
Longitudinal Strain $= \frac{\Delta l}{l}$



slope = $\tan\theta$ = Modulus of Elasticity

$$\frac{F_\perp}{A} = Y\left(\frac{\Delta l}{l}\right)$$

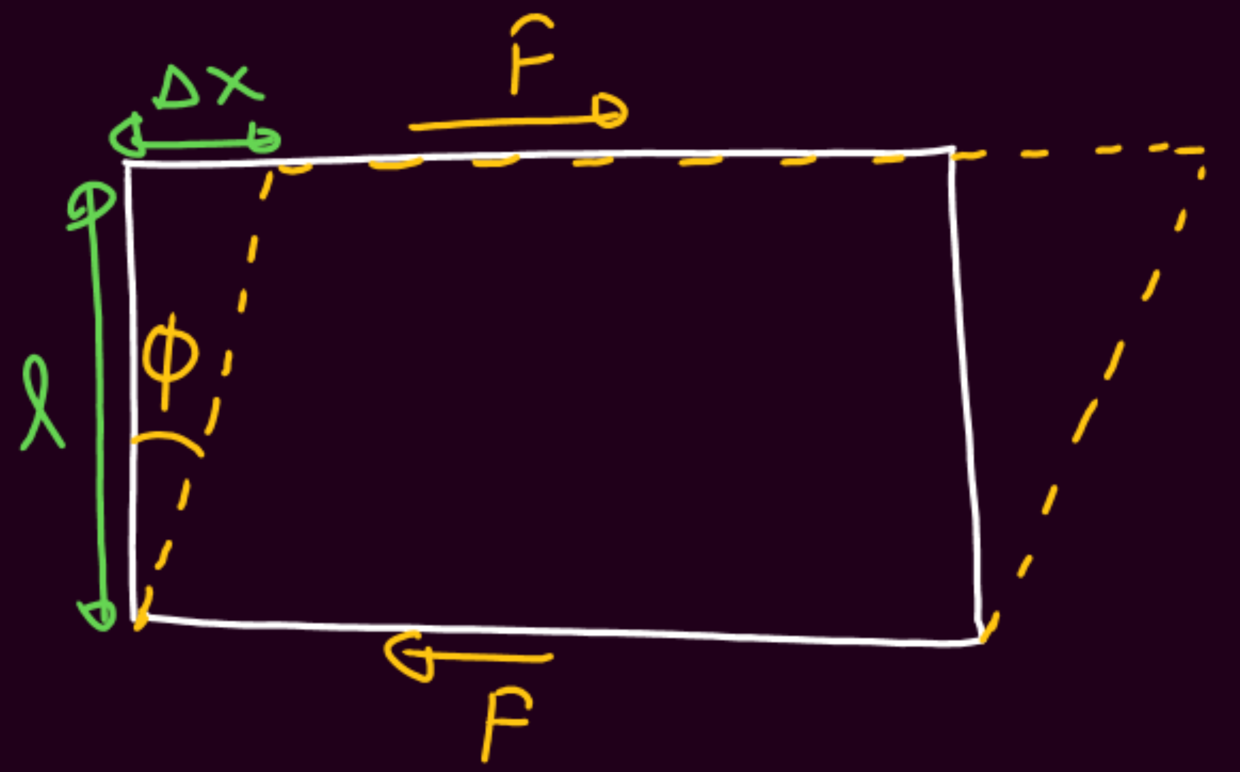
[Young's Modulus]



$Y_A > Y_B$

Shear Stress $= \frac{F_{\parallel}}{A}$

Shear Strain $\phi = \frac{\Delta x}{l}$



$$\boxed{\frac{F_{\parallel}}{A} = \eta \, \phi}$$

$\eta \rightarrow$ modulus of rigidity

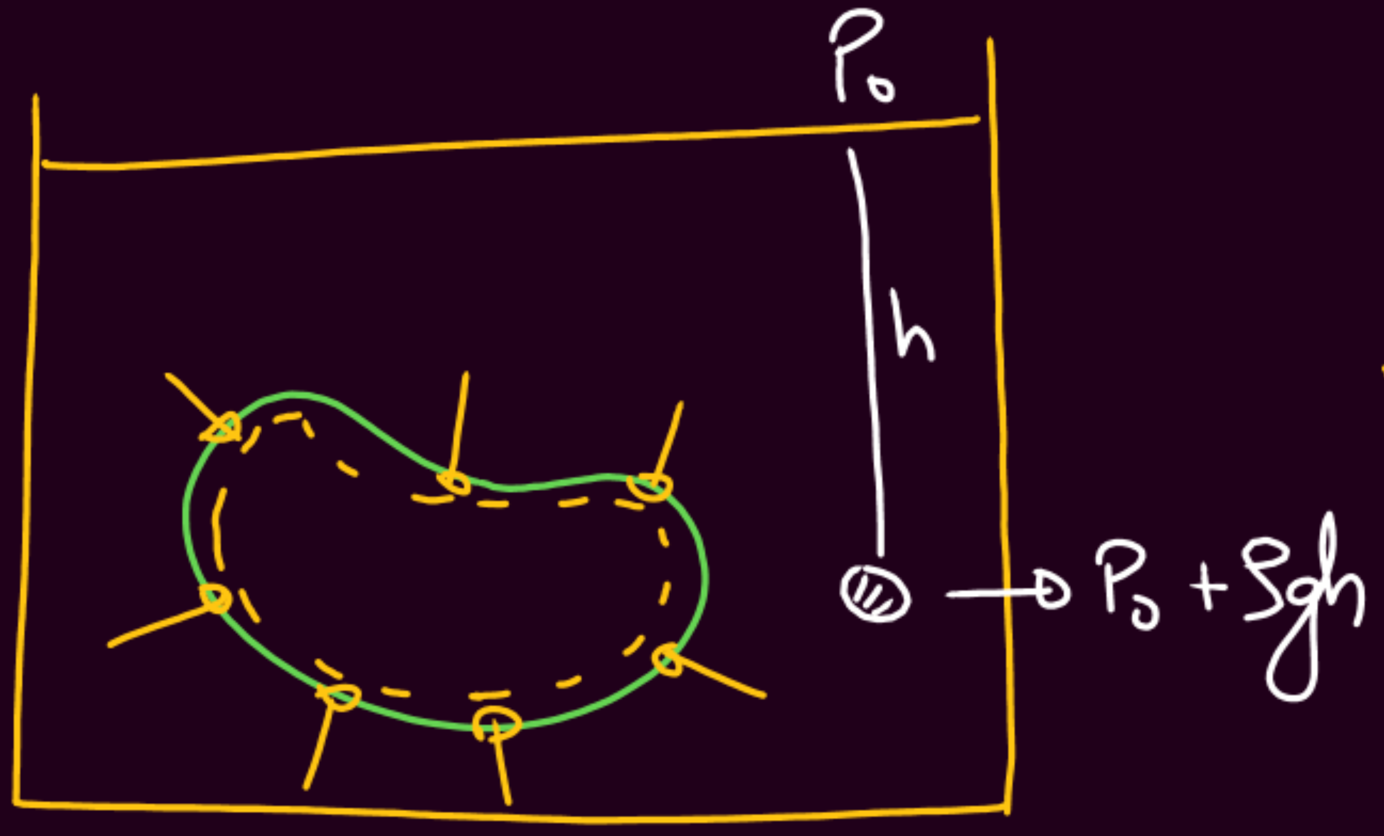
$\boxed{\text{Shape}}$

'Twist'

$\boxed{\text{Spring}}$ to

'Punching'

Bulk Stress = ΔP



Bulk Strain = $\frac{\Delta V}{V}$

Bulk Modulus $B = -\frac{\Delta P}{(\Delta V / V)}$

$\Delta P = \rho g h$

$\rho_w = 1000 \ kg/m^3$
$= 1 \ g/cc$

~~Sphere~~

$V = \frac{4}{3}\pi r^3$

$$\boxed{\frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)}$$

Compressibility $C = \frac{1}{B} = -\frac{\Delta V}{V \Delta P}$

$C_{gas} > C_{liq} > C_{solid}$

$B_{gas} < B_{liq} < B_{solid}$

Q.

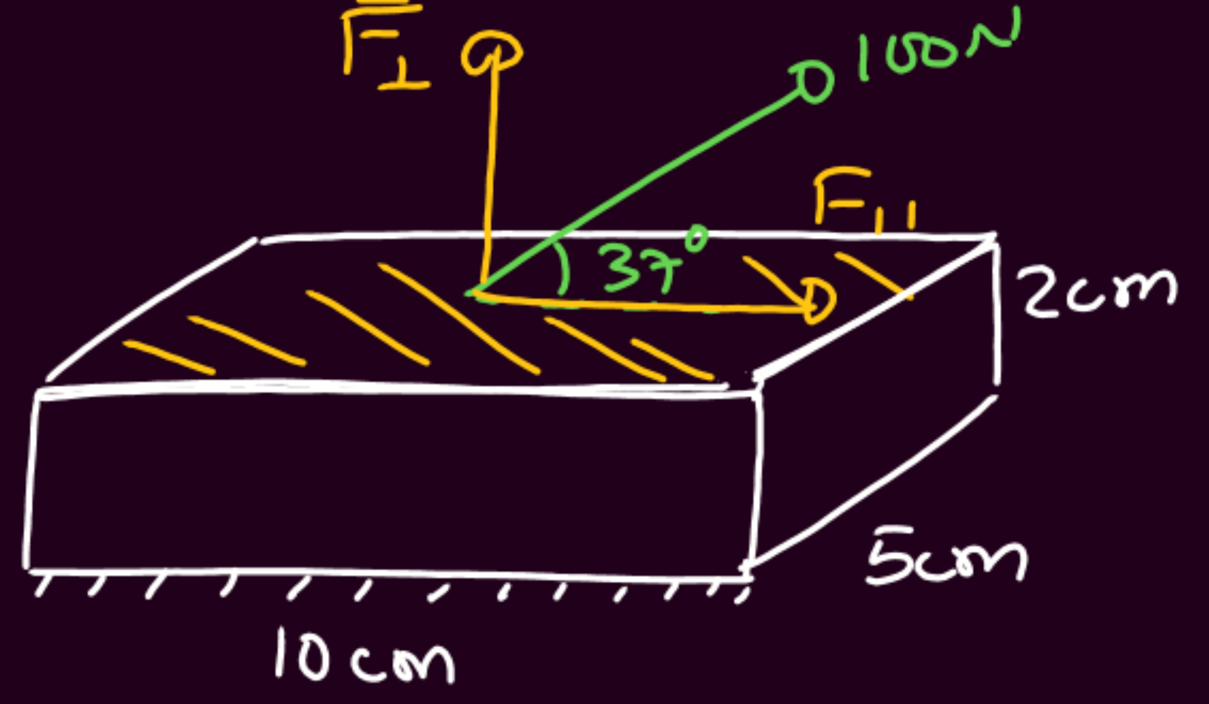
$$B = \frac{-\Delta P}{\frac{\Delta V}{V}}$$

$$B = \frac{-\rho g h}{3\left(\frac{\Delta r}{r}\right)}$$

$$B = \frac{-1000\,(10)\,(10)}{3\left(\frac{-0.1}{100}\right)}$$

$$B = \frac{1}{3} \times 10^{8} \ \frac{N}{m^2}$$

Q.



$$\text{long. stress} = \frac{F_\perp}{A} = \frac{100 \sin 37^\circ}{50 \times 10^{-4}} = \frac{100 \times \frac{3}{5}}{50 \times 10^{-4}}$$

$$= \frac{60}{50} \times 10^{4} = 1.2 \times 10^{4} \ N/m^2$$

$$\text{Shear stress} = \frac{F_{\parallel}}{A} = \frac{100 \cos 37^\circ}{50 \times 10^{-4}} = \frac{100\left(4/5\right)}{50 \times 10^{-4}}$$

$$= \frac{80}{50} \times 10^{4} = 1.6 \times 10^{4} \ N/m^2$$

Rigid Body $\rightarrow$ $[E \rightarrow \infty]$

Liquid & Gas $\rightarrow$ $\boxed{\begin{matrix} Y = 0 \\ \eta = 0 \end{matrix}}$

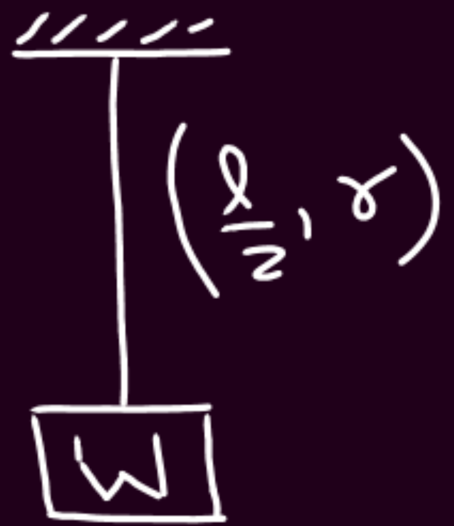
Temp. $\uparrow$ $Y \downarrow$

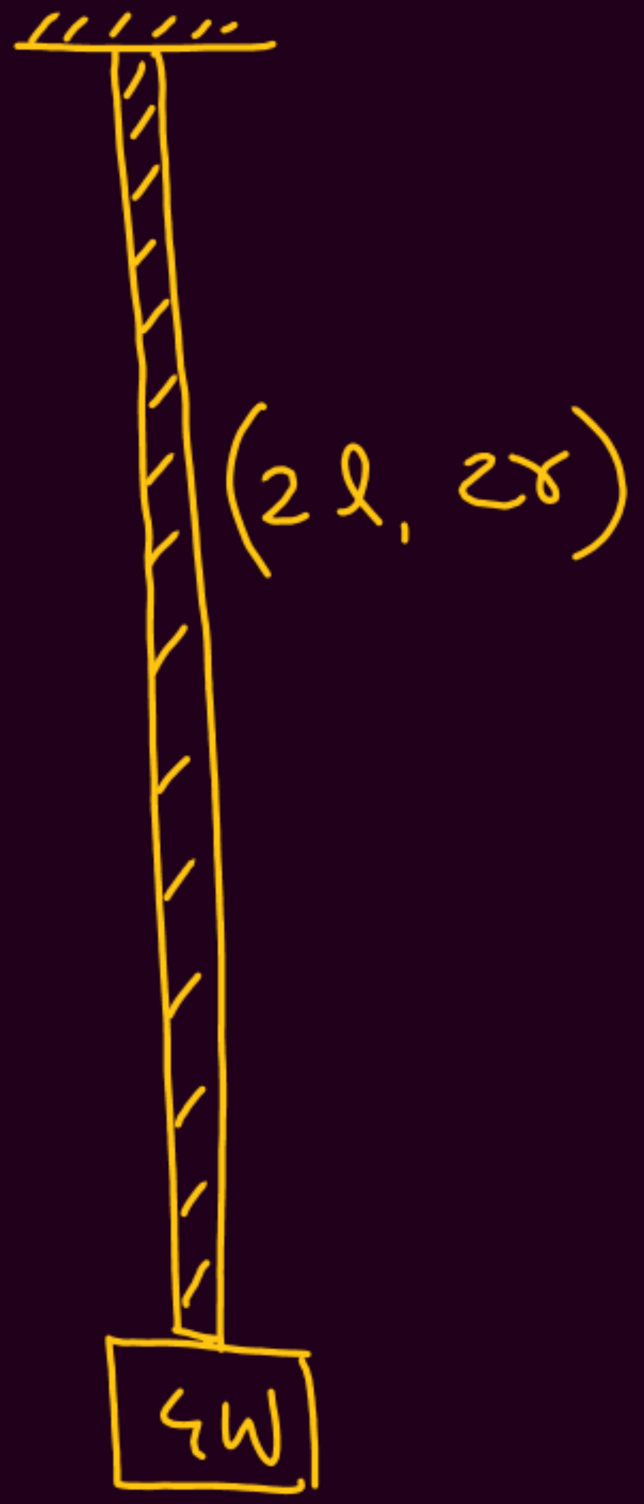
$\boxed{\text{INVAR}}$ $\rightarrow$

Impurities $\rightarrow$ $Y \uparrow$

Breaking Stress / Strength

↳ material property





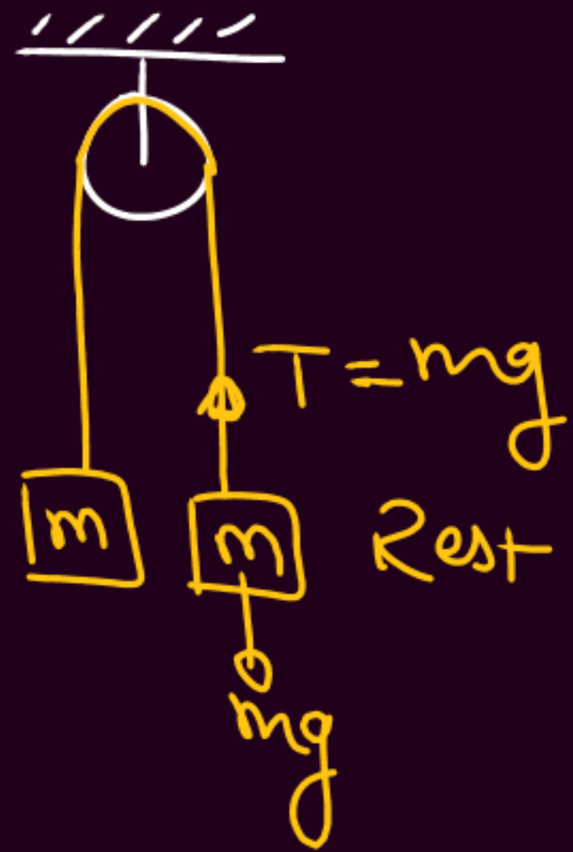
Max load = W

$$\boxed{B.S. = \frac{W}{\pi r^2} = \frac{(4W)}{\pi (2r)^2}}$$

$$\boxed{\Delta l = \frac{Fl}{AY}}$$

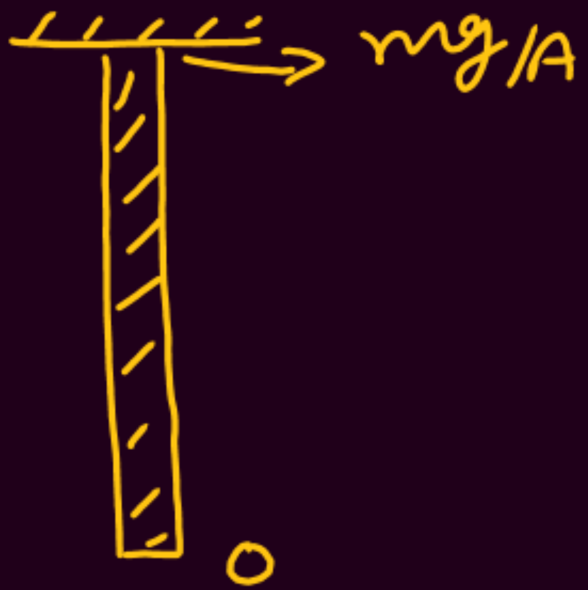
$$\boxed{\Delta l = \frac{Fl}{2AY}}$$

$$\boxed{\Delta l = \frac{mgl}{AY}}$$



Light thread

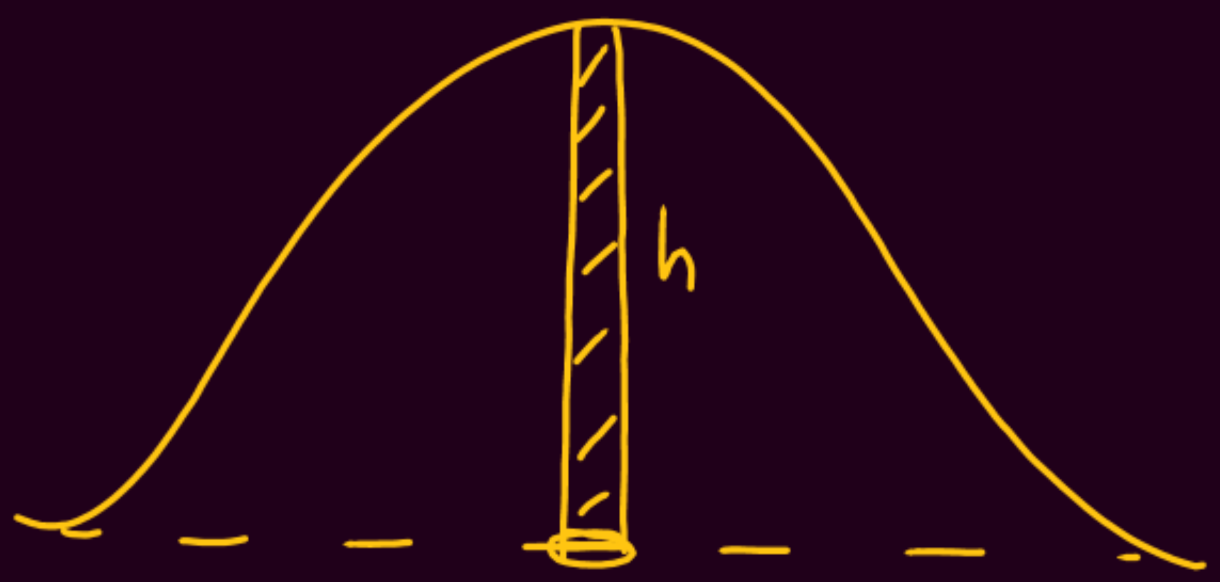
$$\Delta l = \frac{mgl}{AY}$$



Heavy Rope

$$\Delta l = \frac{mgl}{2AY}$$

Max height of Mountain



$$B.S. = \frac{\rho (Ah) g}{A}$$

$$\boxed{B.S. = \rho g h_{max}}$$

Q. Two Wires

Same material

Same Stress

$l = 1 : 2$

$$(\text{Stress}) = Y(\text{Strain})$$

$$\frac{F}{A} = Y\left(\frac{\Delta l}{l}\right)$$

$$(\text{Elongation}) = (\text{Strain})\, l$$

① $$\frac{(\text{Strain})_1}{(\text{Strain})_2} = \frac{1}{1}$$

② $$\frac{(\text{Elongation})_1}{(\text{Elongation})_2} = \frac{(\text{Strain})_1}{(\text{Strain})_2}\,\frac{l_1}{l_2} = \frac{1}{2}$$

Q. Two Wires

Same Stress

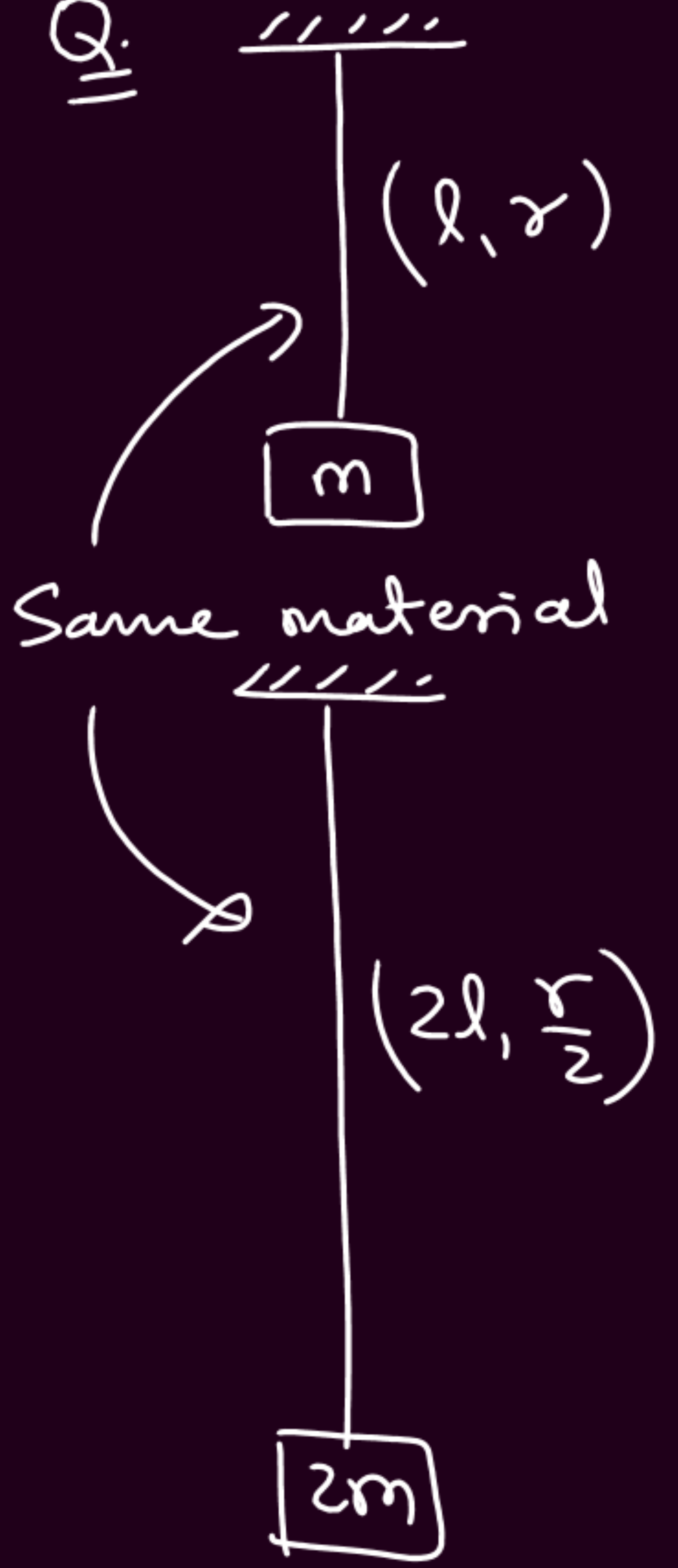
$Y \quad (2 : 1)$

$l \quad (1 : 2)$

① $$\frac{(\text{Strain})_1}{(\text{--}\,''\,\text{--})_2} = \frac{\cancel{\text{Stress}}}{Y_1} \frac{Y_2}{\cancel{\text{Stress}}} = \frac{1}{2}$$

② $$\frac{(\text{Elongation})_1}{(\text{--}\,''\,\text{--})_2} = \frac{(\text{Strain})_1}{(\text{Strain})_2} \frac{l_1}{l_2} = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

Q.



① $\dfrac{(\text{Stress})_1}{(\text{Stress})_2} = \dfrac{mg}{\pi r^2} \; \dfrac{\pi (r/2)^2}{2mg} = \dfrac{1}{8}$

② $\dfrac{(\text{Strain})_1}{(\text{Strain})_2} = \dfrac{1}{8}$

③ $\dfrac{(\text{Elongation})_1}{(\text{———''———})_2} = \dfrac{(\text{Strain})_1}{(\text{Strain})_2} \; \dfrac{l_1}{l_2} = \dfrac{1}{8} \times \dfrac{1}{2} = \dfrac{1}{16}$

$$F = \left(\frac{YA}{l}\right)\Delta l$$

Spring

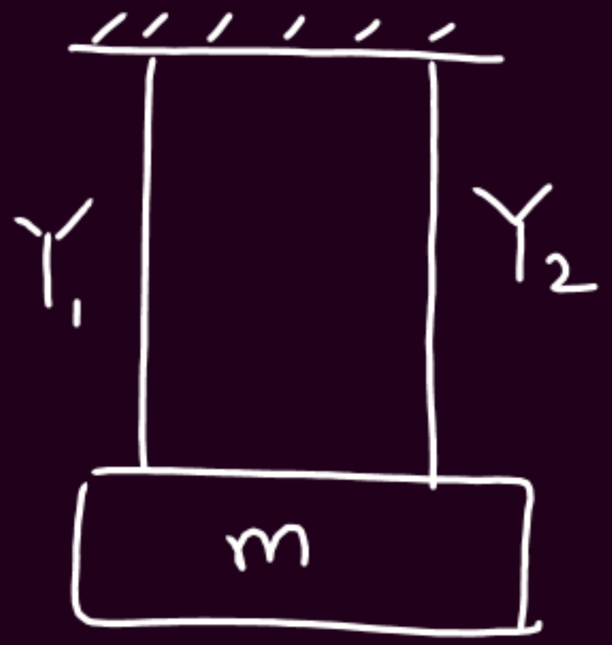
$$F = kx$$

$$\boxed{k = \frac{YA}{l}}$$

SHM

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{ml}{YA}}$$



$$K_{eq} = K_1 + K_2$$

$$K_{eq} = \frac{Y_1 A_1}{l} + \frac{Y_2 A_2}{l}$$

Energy

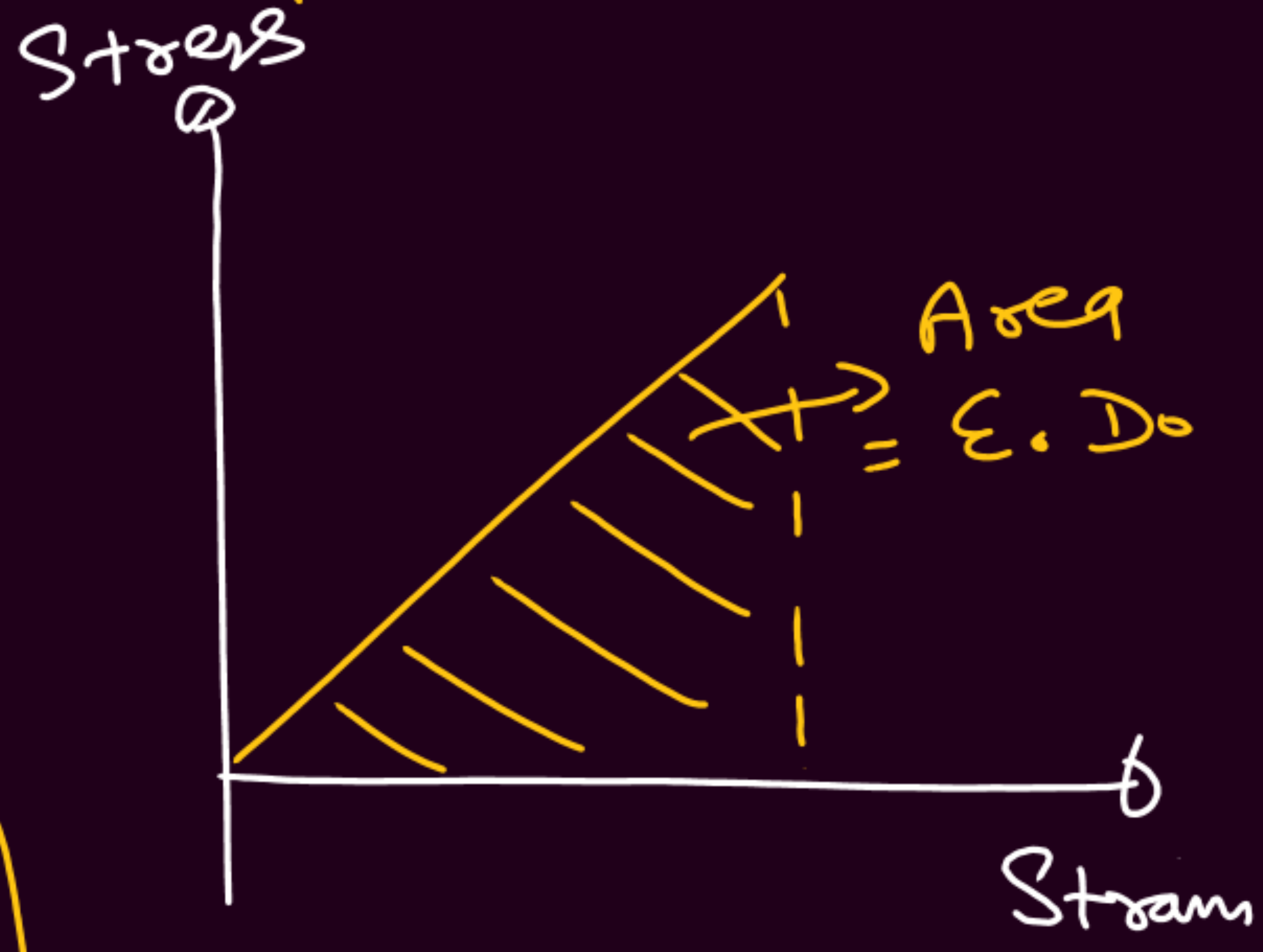
$F = kx$

$\text{Stress} = Y(\text{Strain})$

Spring $\quad E = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Wire $\quad \boxed{E = \frac{1}{2}\left(\frac{YA}{l}\right)\Delta l^2 = \frac{1}{2}F\,\Delta l}$

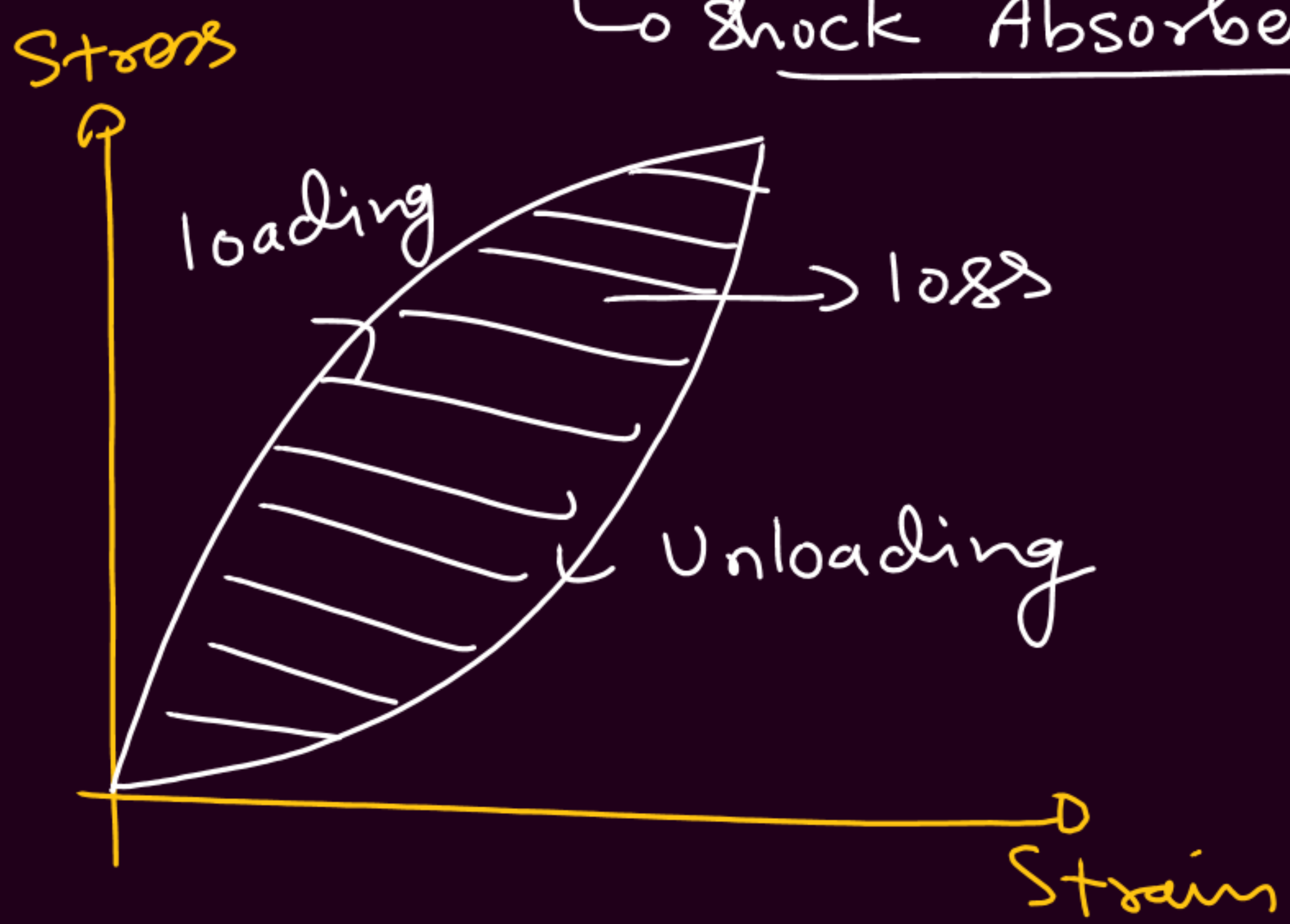
$$\boxed{\begin{aligned} \text{Energy} &= \frac{1}{2}(\text{load})(\text{Extension}) \\ \text{Energy} &= \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Volume}) \end{aligned}}$$



$$\frac{\text{Energy}}{\text{Volume}} = \boxed{\begin{aligned} \text{Energy Density} &= \frac{1}{2}(\text{Stress})(\text{Strain}) \\ &= \frac{1}{2}Y(\text{Strain})^2 \\ &= \frac{1}{2}\frac{(\text{Stress})^2}{Y} \end{aligned}}$$

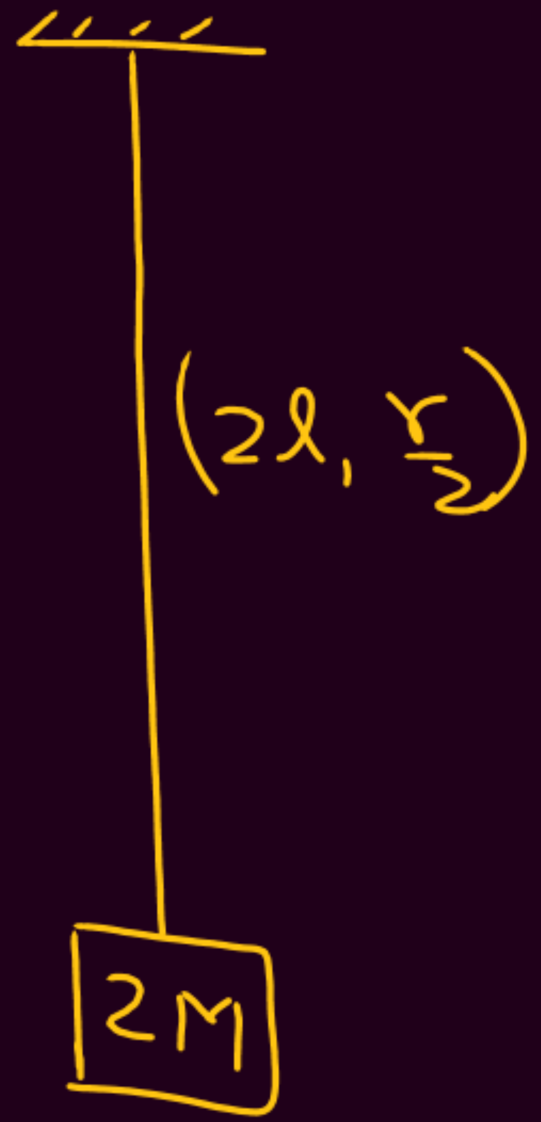
Elastic Hysteresis

↳ Shock Absorber →



Q.

Same material



$$\frac{(\text{Energy})_1}{(\text{Energy})_2} = \frac{F_1^2 \, l_1}{r_1^2} \; \frac{r_2^2}{F_2^2 \, l_2}$$

$$= \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) \left(\frac{1/2}{1}\right)^2$$

$$= \frac{1}{32}$$

$$\boxed{\text{Energy} = \frac{F^2 l}{2AY}}$$

$$\text{Energy} = \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Vol})$$

$$\frac{E_1}{E_2} = \left(\frac{1}{8}\right)\left(\frac{1}{8}\right) \frac{l \, \pi r^2}{2l \, \pi \left(\frac{r}{2}\right)^2}$$

Y 1 : 2

l 1 : 2

r 2 : 1

① $$\frac{(\text{Stress})_1}{(\text{Stress})_2} = \frac{F_1}{A_1}\frac{A_2}{F_2} = \frac{4mg}{r_1^2}\frac{r_2^2}{3mg} = \frac{4}{3}\left(\frac{1}{2}\right)^2 = \frac{1}{3}$$

② $$\frac{(\text{Elongation})_1}{(\text{— ,, —})_2} = \frac{F_1 l_1}{r_1^2 Y_1}\frac{r_2^2 Y_2}{F_2 l_2} = \frac{1}{3}\left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = \frac{1}{3}$$

$$\boxed{\Delta l = \frac{Fl}{AY}}$$

F l r Y

③ $$\frac{(\text{Energy})_1}{(\text{— ,, —})_2} = \frac{F_1^2 l_1}{r_1^2 Y_1}\frac{r_2^2 Y_2}{F_2^2 l_2} = \left(\frac{4}{3}\right)^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2\left(\frac{2}{1}\right) = \frac{4}{9}$$

$$\boxed{E = \frac{F^2 l}{2AY}}$$

Q Two wire

Same material

Same length

$\boxed{\text{Same stress}}$

r (1 : 2)

$$\Delta l = \frac{Fl}{AY}$$

$$\text{Energy} = \frac{F\,F\,l}{2\,A\,Y} = \frac{1}{2}F(\text{Ext.})$$

$$(\text{Stress})_1 = (\text{Stress})_2$$

$$\frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

$$\frac{F_1}{F_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

① $$\frac{(\text{Elongation})_1}{(\text{—''—})_2} = \frac{1}{1}$$

② $$\frac{(\text{Energy})_1}{(\text{Energy})_2} = \frac{F_1}{F_2} = \frac{1}{4}$$

Q. Two wires

Same material

Same load

Same volume

$l \quad (1:2)$

Stress $= \frac{F}{A}$

$V_1 = V_2$

$l_1 A_1 = l_2 A_2$

$\frac{A_1}{A_2} = \frac{l_2}{l_1} = \frac{2}{1}$

$\Delta l = \frac{F l}{A Y}$

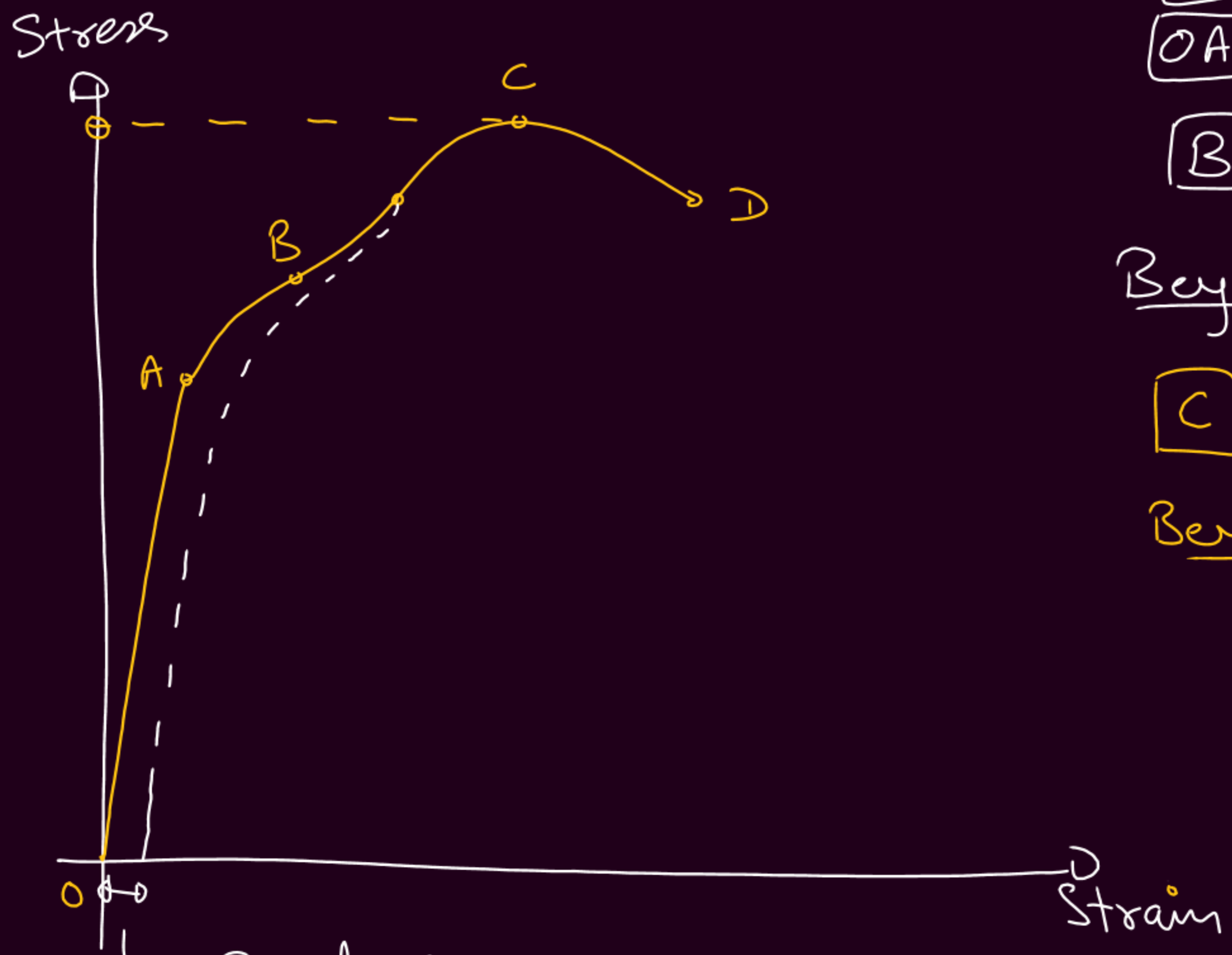
$E.D. = \frac{1}{2} \frac{(\text{Stress})^2}{Y}$

① $\frac{(\text{Stress})_1}{(\text{Stress})_2} = \frac{A_2}{A_1} = \frac{1}{2}$

② $\frac{(\text{Elongation})_1}{(\text{Elongation})_2} = \frac{l_1}{l_2} \frac{A_2}{A_1} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

③ $\frac{(\text{Energy Density})_1}{(\text{ —— " —— })_2} = \frac{(\text{Stress})_1^2}{(\text{Stress})_2^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Stress - Strain Curve



Residual strain

$\boxed{A}$

$\boxed{OA}$ → Hooke's law

$\boxed{B}$ → Yield point

Beyond B →

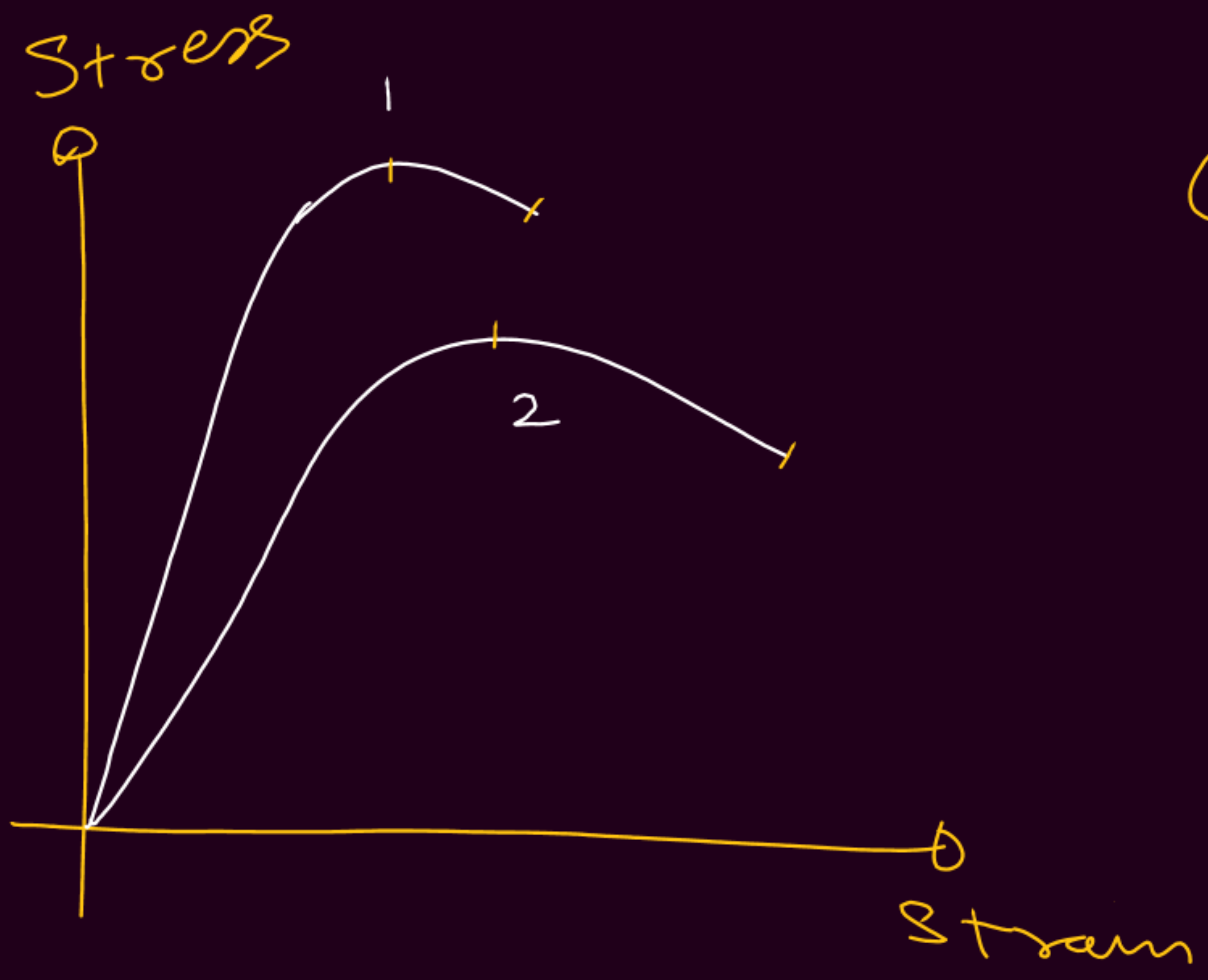
$\boxed{C}$ →

Beyond C → Necking start

$\boxed{CD}$ → large → Ductile

→ small → Brittle

$\boxed{D}$ → Fracture/Rupture



(2) → Ductile

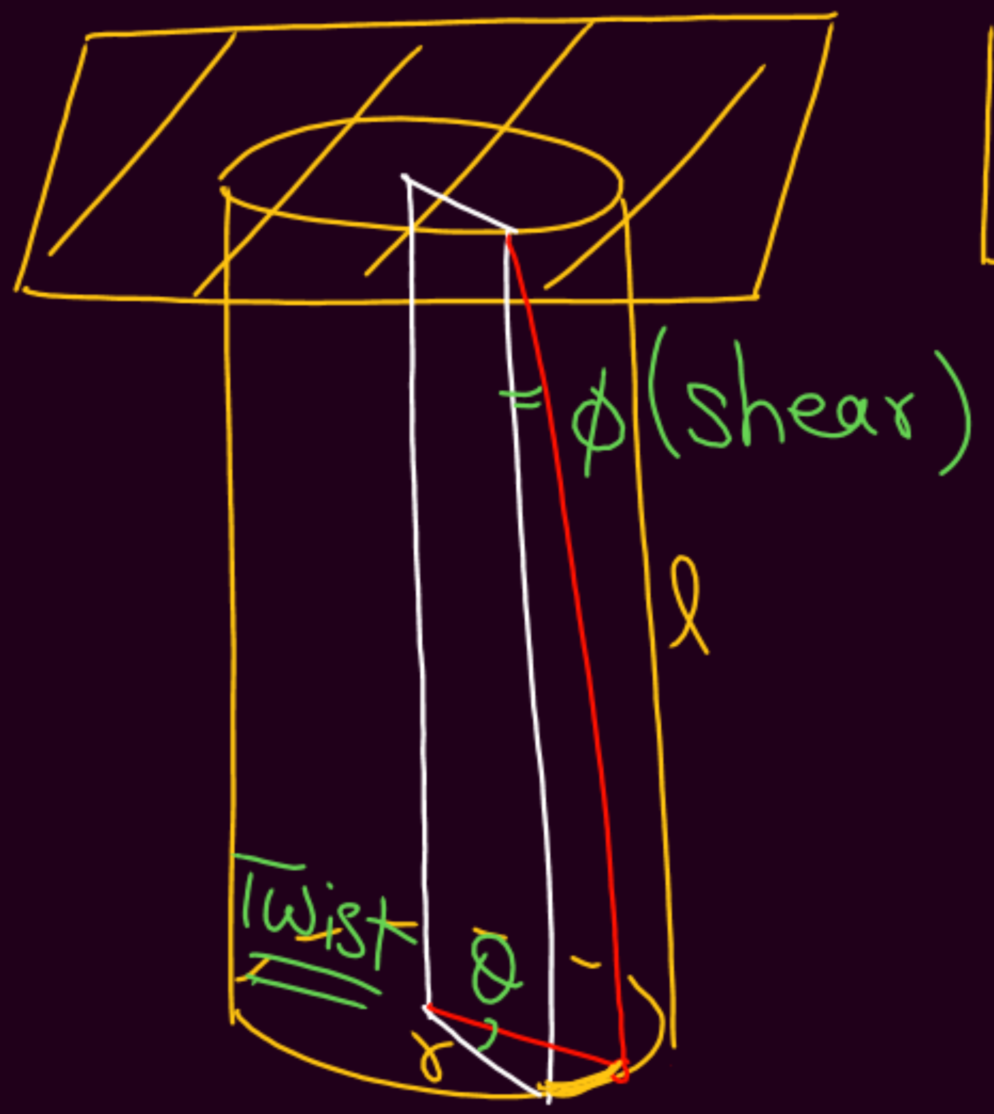
(1) → Brittle

Extra

Interatomic force const.

$$\boxed{k = Y r_0}$$

Relation Twist / Shear



$$\boxed{l\phi = r\theta}$$

Poisson's Ratio (σ)

$$\sigma = -\frac{\text{lateral strain}}{\text{longitudinal Strain}} = -\frac{\Delta r/r}{\Delta l/l}$$

1. $\sigma = \frac{1}{2}$ [if volume = const]
2. $-1 < \sigma \leq \frac{1}{2}$
3. $\sigma = \ominus$ [Auxatic materials]

① $Y = 2\eta(1+\sigma)$

② $Y = 3B(1-2\sigma)$

③ $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{B}$

Viscosity

Fluid → liq → shape
→ gas → Volume

Viscosity → 'Newton'

Viscous Force $\boxed{F = -\eta A\left(\frac{dv}{dx}\right)}$

η = Coeff of Viscosity

$\frac{N\text{-}s}{m^2}$ = 1 Poiseuilli

$\frac{dv}{dx}$ = Vel. gradient

A = Area

$1\ \frac{N\text{-}S}{m^2} = 10$ poise

$\boxed{SI}$ $\boxed{CGS}$

Effect of Temp

liq → T↑ η↓

gas → T↑ η↑

Effect of Pressure

liq →

gas →

Dimension

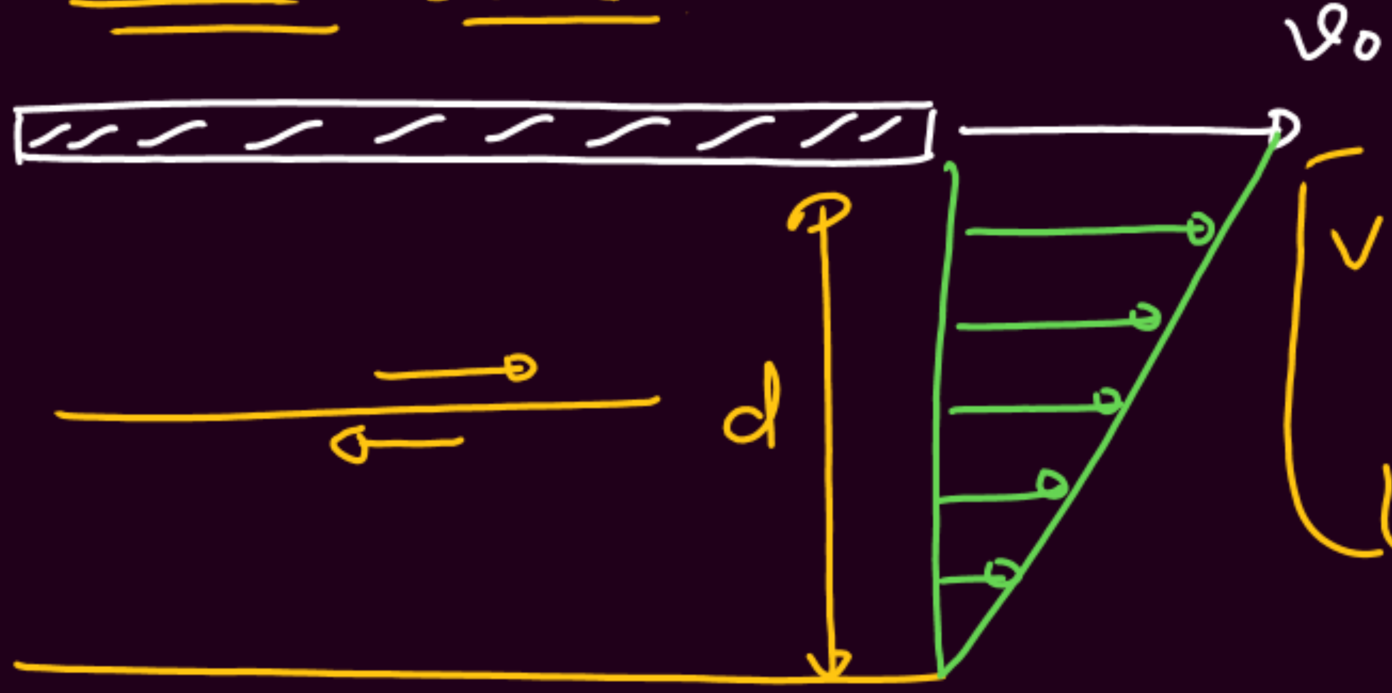
$F = MLT^{-2}$

$E = ML^2T^{-2}$

$[\eta] = \frac{MLT^{-2}\ T}{L^2}$

$[\eta] = [ML^{-1}T^{-1}]$

River flow



vel. profile

↓

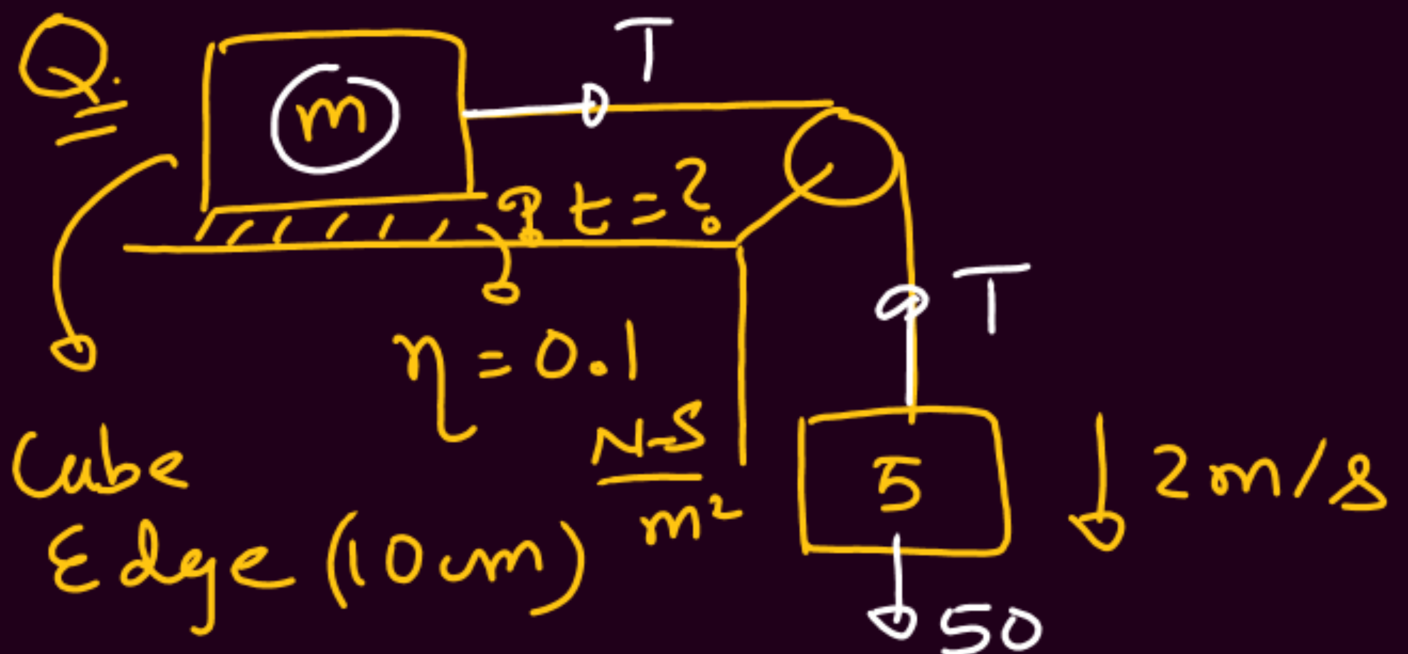
Linear

$$\boxed{F = \eta A\left(\frac{v_0}{d}\right)}$$

Pipe flow

parabolic

Q.



Const. v = 0 $\quad F_{net} = 0$

$$50 = T = \eta A\left(\frac{v}{t}\right)$$

$$50 = (0.1)(0.01)\left(\frac{2}{t}\right)$$

$$t = \frac{10^{-3} \times 2}{50} = 0.04 \times 10^{-3} \text{ m}$$

$$= 0.04 \text{ mm}$$

Q.

$$5g\sin 30° = \eta A\left(\frac{v}{t}\right)$$

$$50 \times \frac{1}{2} = (0.01)(0.01)\frac{\cancel{5}}{t}$$

$$t = 2\times10^{-5} \text{ m}$$

$$= 2\times10^{-2} \text{ mm}$$

$$t = 0.02 \text{ mm}$$

Stoke's law

— (only for spherical objects)

$\rho \rightarrow$ object

$\sigma \rightarrow$ fluid

$$\boxed{F = 6\pi\eta r v}$$

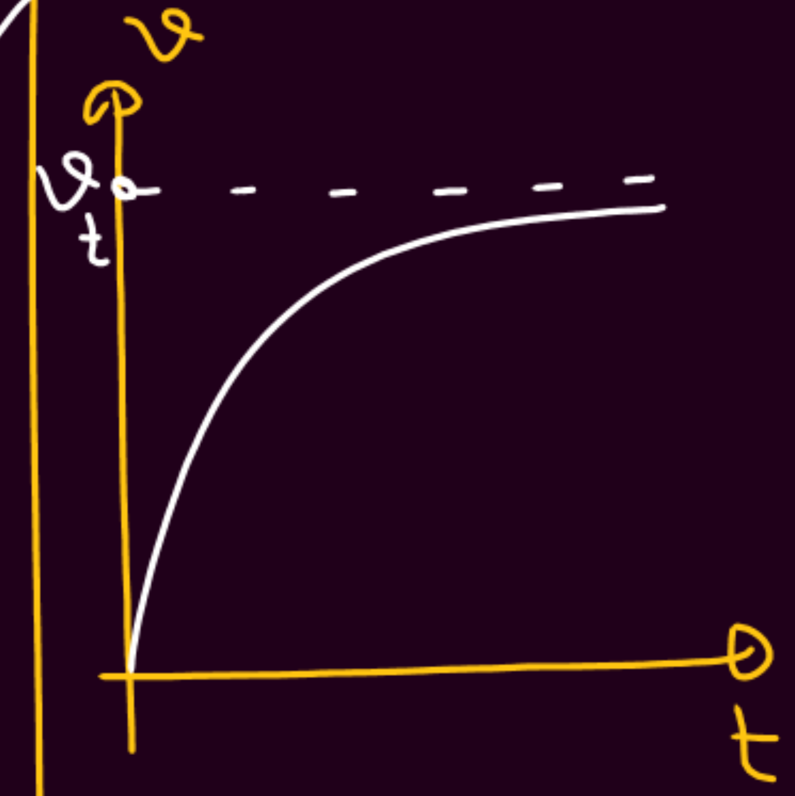
Steady State / Terminal Velocity

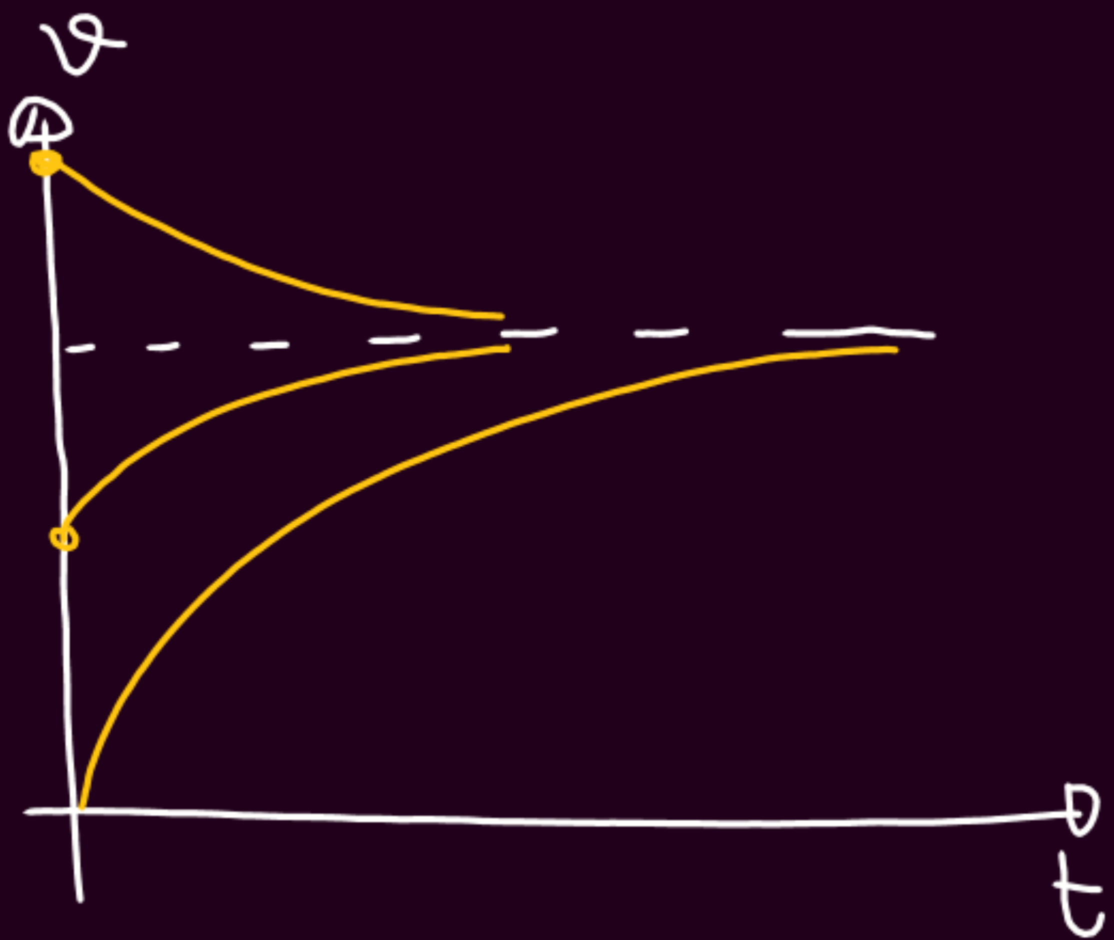
$$\boxed{v_t = \frac{2(\rho-\sigma) g r^2}{9\eta}}$$

Concept

$$F + B = mg$$

$$6\pi\eta r v_t + \sigma\left(\frac{4}{3}\pi r^3\right)g = \rho\left(\frac{4}{3}\pi r^3\right)g$$





Two drops

r_1 $\qquad$ r_2

$$\boxed{\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2}$$

n-identical drops coalesce

r $\Rightarrow$

v_0 $\qquad$ v

$$\boxed{R = n^{1/3} r}$$

$$\frac{v}{v_0} = \left(\frac{R}{r}\right)^2 = \left(n^{1/3}\right)^2$$

$$\boxed{v = n^{2/3} v_0}$$

$$n \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

Q. 64 drops $\boxed{2\ cm/s}$

merge $\rightarrow$ $v =$

$$v = (64)^{2/3}(2)$$

$$v = (4)^2(2) = \underline{32\ cm/s}$$

Q. Two drops 4 m/s

merge $v = ?$

$$v = (2)^{2/3}(4)$$

$$v = 4^{1/3}(4) = 4^{4/3}$$

Extra

Poiseuilli's Eqn

→ flow through narrow Tube

$$\boxed{\frac{dV}{dt} = \frac{\pi r^4 (P_2 - P_1)}{8 \eta L}}$$

Volume flow rate

Reynold's No.

$$R_e = \frac{\rho v D}{\eta}$$

D = diameter

v_c = Critical velocity

$R_e < 1000$ Streamline / Laminar

$R_e > 2000$ Turbulent

Turbulent → Mixer

→

Q. Rain drop $r = 1\,mm$

$$\eta_{air} = 0.1 \ \frac{N\text{-}s}{m^2}$$

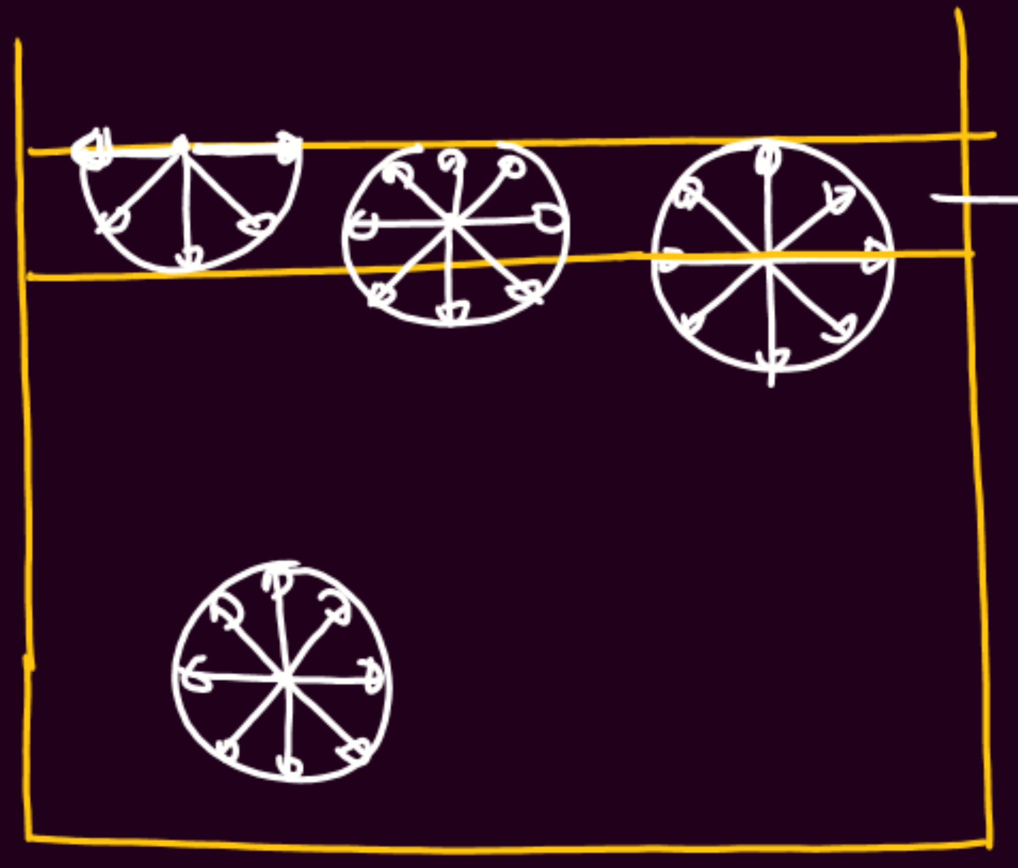
$$v_t = ?$$

$$v_t = \frac{2(\overset{\text{water}}{\rho} - \overset{\text{Air}}{\sigma})\, g r^2}{9\eta}$$

$$= \frac{2(\cancel{1000} -)\,10\,(10^{-3})^{\cancel{2}}}{9 \times 0.1}$$

$$\boxed{v_t = \frac{0.2}{9}\ m/s}$$

Surface Tension



→ Surface

→ 10 – 15 molecule

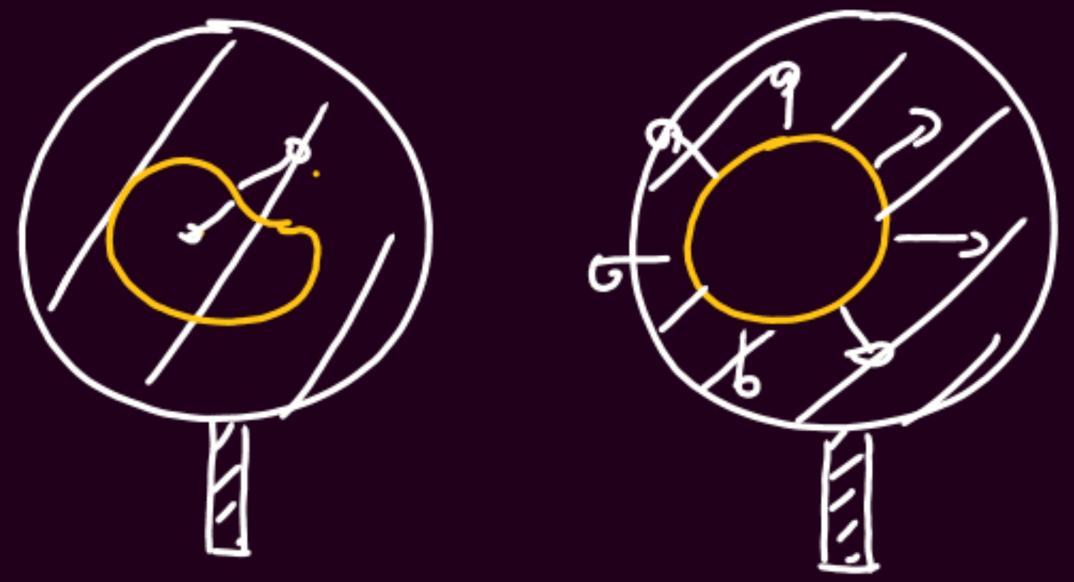
→ Higher Energy state

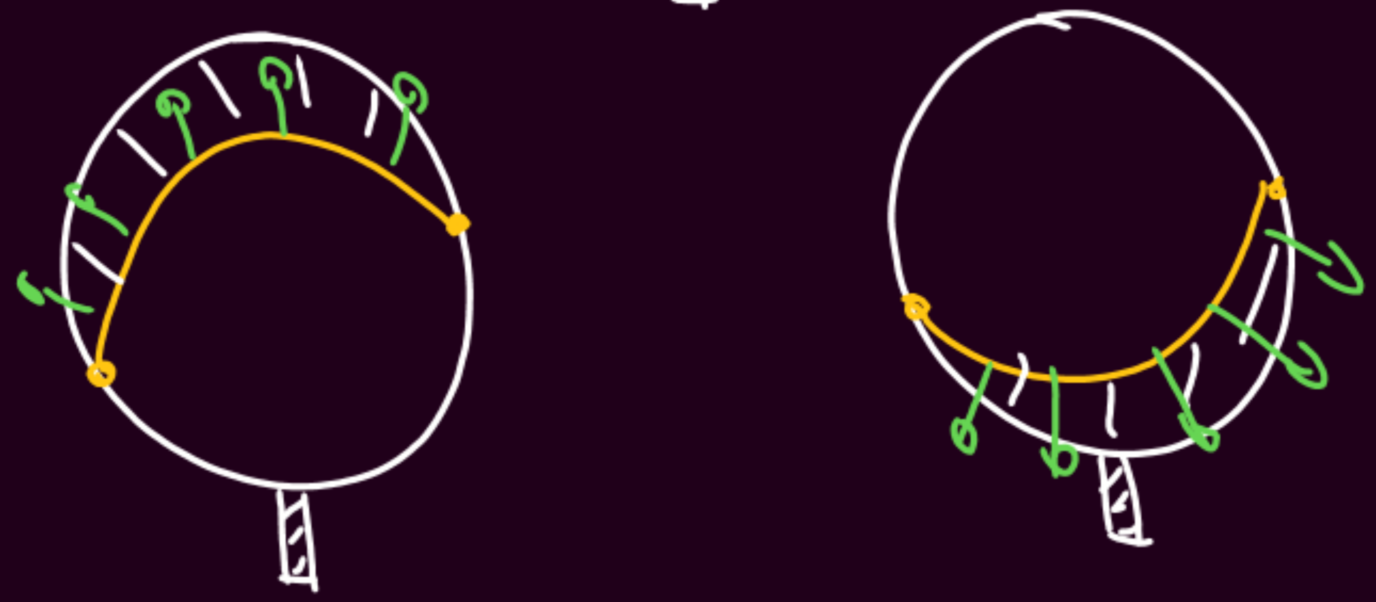
→ Minimise S. A.

→ Stretched membrane

①

②





Surface Tension (T)

$$\boxed{T = \frac{F}{\ell}}$$

$$\left[\frac{N}{m} = \frac{J}{m^2}\right]$$

Drop  Single Surface

Soap Bubble  Two Surfaces

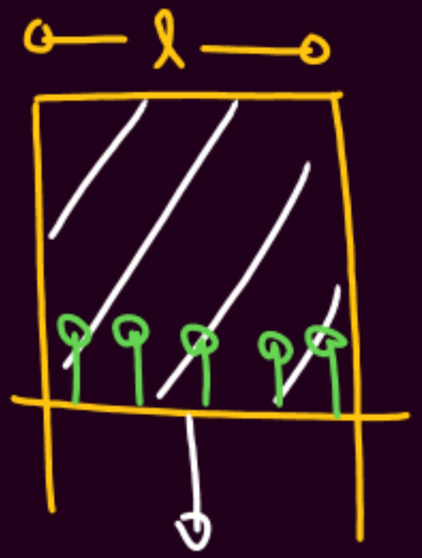
Excess force

$$f_{ex} = F - mg$$

$$\boxed{F_{ex} = 2T\ell}$$

Film

Film → Two surface



$$mg = 2(T\ell)$$

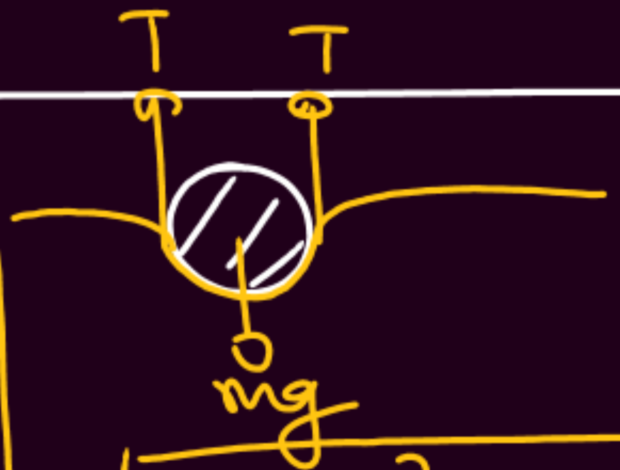
Needle

$$mg = (2T\cos\theta)\ell$$

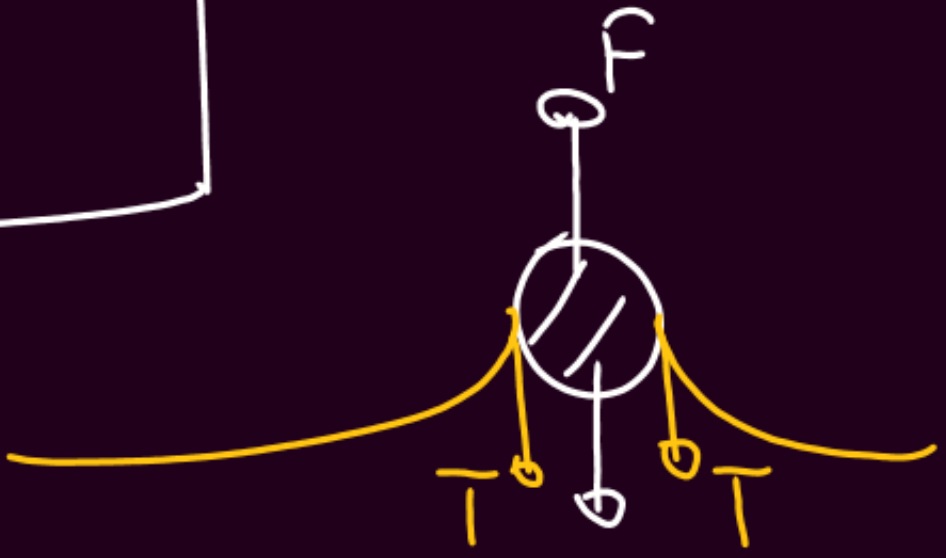
$$\rho(\pi r^2 \ell)g = (2T\cos\theta)\ell$$

$$\rho\pi r^2 g = 2T\cos\theta$$

max. Radius



$$\boxed{\rho\pi r_{max}^2 g = 2T}$$

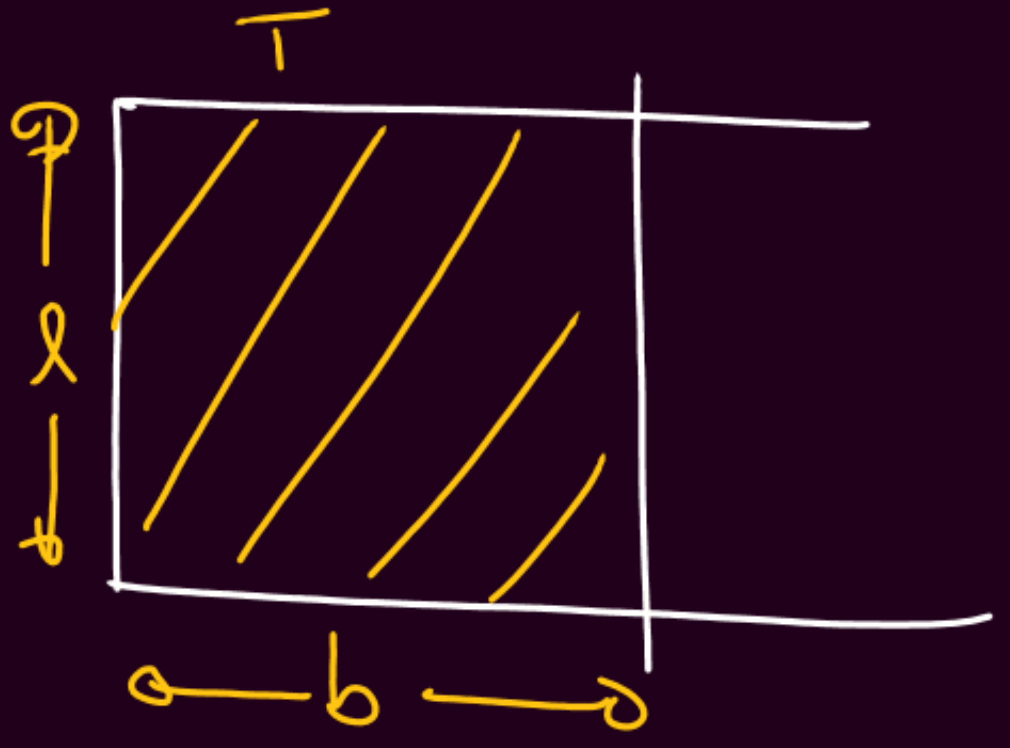


		Just float/Sink	Exess Force
① Needle		$mg = 2Tl$	$F_{ex} = 2Tl$
② Disc		$mg = T(2\pi R)$	$F_{ex} = T(2\pi R)$
③ Annular Disc		$mg = T[2\pi(R_1+R_2)]$	$F_{ex} = T[2\pi(R_1+R_2)]$
④ Thin Ring		$mg = T(2\pi R)(2)$	$F_{ex} = T(2\pi R)(2)$

$$\text{Surface Energy} = \left(\text{Surface Tension}\right) \times \left(\text{Surface Area}\right)$$

$$S.T. = \frac{S.E.}{S.A.} \quad \left(\frac{J}{m^2}\right)$$

Soap Film



$$S.E. = 2(T l b)$$

① W.D./S.E. of a drop $= T\left(4\pi R^2\right)$

② —— " ———— Soap Bubble $= 2T\left(4\pi R^2\right)$

③ W.D. / ΔSE in increasing the radius of soap bubble from R_1 to R_2 $= 2T\left[4\pi\left(R_2^2 - R_1^2\right)\right]$

$$\boxed{\frac{SE_n}{SE_1} = n^{1/3}}$$

$$\frac{n\cancel{T}(\cancel{4\pi} r^2)}{\cancel{T}\cancel{4\pi}R^2} = \frac{n}{n^{2/3}} = n^{1/3}$$

$$R = n^{1/3} r$$

① $V' = V\left(1 + \frac{700}{100}\right) = 8V$

$$\cancel{\frac{4}{3}\pi} R'^3 = 8\left(\cancel{\frac{4}{3}\pi} R^3\right)$$

$$R' = 2R$$

$$W.D. = 2T\left[4\pi\left((2R)^2 - R^2\right)\right]$$

$$= 24\pi R^2 T$$

$$\frac{SE_n}{SE_1} = (1000)^{1/3} = 10$$

$SE_1 = 10$

$$SE_n = 100 J$$

$$\Delta SE = 100 - 10$$

$$= 90 J$$

Q. $1000_{\,2} \longrightarrow 1$

$$\frac{SE_n}{SE_1} = (1000)^{1/3} = 10 = \frac{2000}{SE_1}$$

$$\text{loss} = \text{Initial} - \text{Final}$$

$$= 1000(2) - 200$$

$$= \underline{\underline{1800\,J}}$$

Q. 64 drops $\longrightarrow$

4 1

% Loss

$$\frac{SE_n}{SE_1} = (64)^{1/3} = \frac{4}{1}$$

$$\%\ \text{loss} = \frac{I-f}{I} \times 100$$

$$= \frac{4-1}{4} \times 100 = \underline{75\%}$$

2

$$\Delta SE = n\,T(4\pi r^2) - T(4\pi R^2)$$

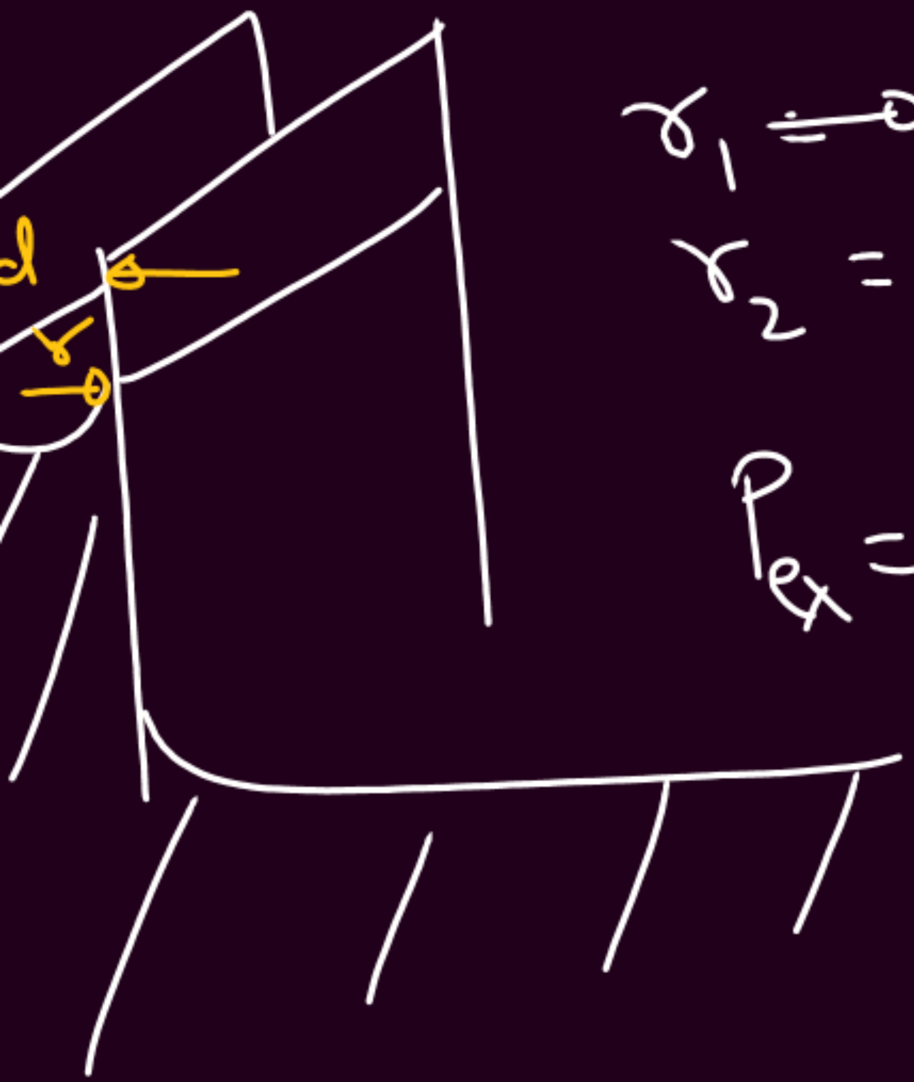
$$= VT\left[\frac{n\,4\pi r^2}{V} - \frac{4\pi R^2}{V}\right]$$

$$= VT\left[\frac{n(4\pi r^2)}{n\left(\frac{4}{3}\pi r^3\right)} - \frac{4\pi R^2}{\frac{4}{3}\pi R^3}\right]$$

$$= VT\left[\frac{3}{r} - \frac{3}{R}\right]$$

$$\boxed{\Delta SE = 3VT\left(\frac{1}{r} - \frac{1}{R}\right)}$$

② Touch $r = \dfrac{(3)(4)}{4-3} = 12\,cm$



$r_1 \to \infty$

$r_2 = r$

$P_{ex} = \dfrac{T}{r} = \dfrac{2T}{d}$

Q. $P_1 = 1.02$ atm

$P_2 = 1.03$ atm

$P_{ex_1} = 1.02 - 1 = 0.02$ atm $= \dfrac{4T}{r_1}$

$P_{ex_2} = 1.03 - 1 = 0.03$ atm $= \dfrac{4T}{r_2}$

$\dfrac{V_1}{V_2} = \left(\dfrac{r_1}{r_2}\right)^3 = \left(\dfrac{3}{2}\right)^3 = \dfrac{27}{8}$

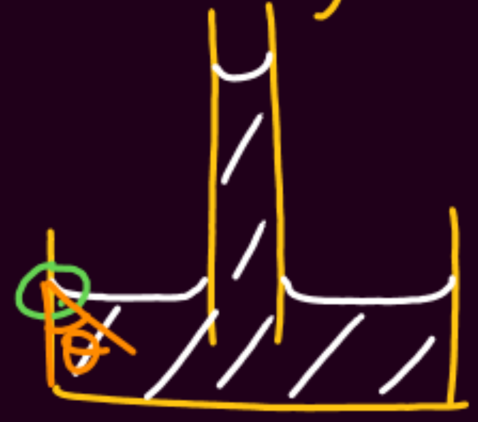
$\dfrac{r_1}{r_2} = \dfrac{3}{2}$

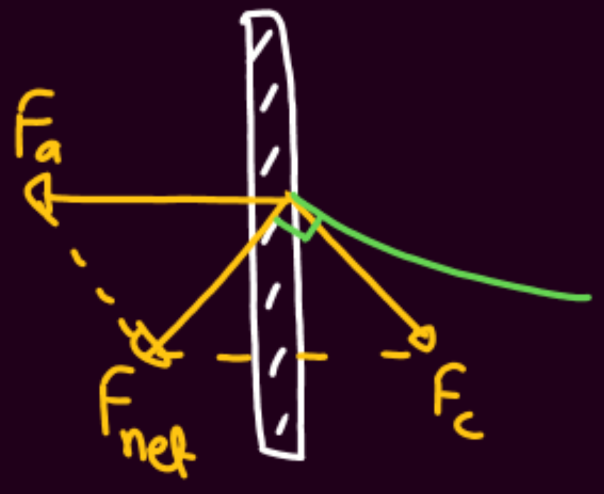
Angle of Contact

$\theta < 90^\circ$

water-glass

$\boxed{0^\circ}$



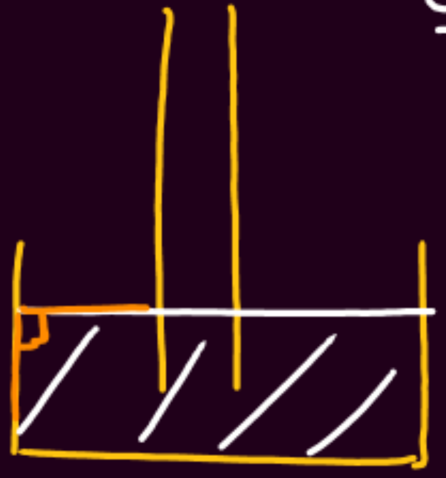


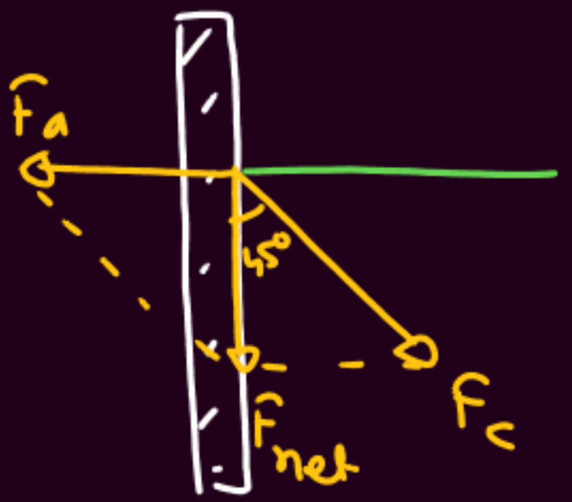
Wetting

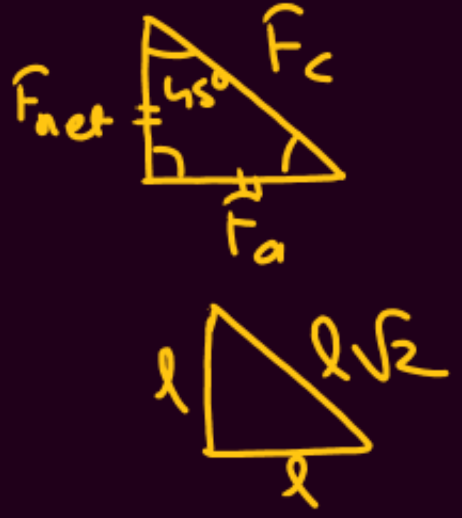
Capillary rise

$\theta = 90^\circ$

$\boxed{\text{water - Silver}}$







$$\boxed{F_c = \sqrt{2} F_a}$$

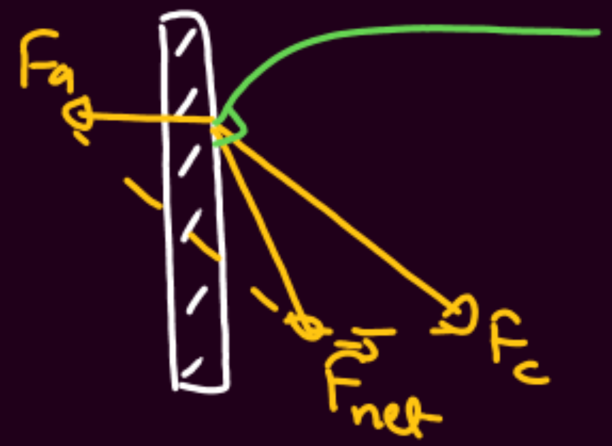
Non-wetting

neither rise nor fall

$\theta > 90^\circ$

Hg-glass.

$\boxed{140^\circ}$



Non-wetting

fall

Temp. ↑

S.T. ↓

θ ↓

Wettability ↑

...niscus | ② **Force Method**

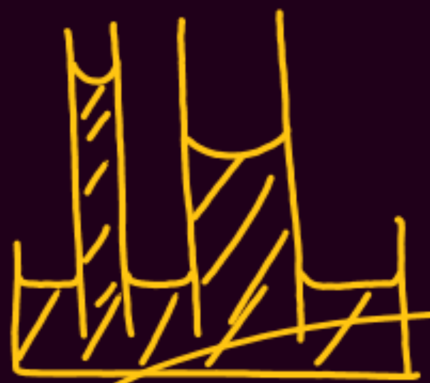
$(\ldots\cos\theta)(2\pi r) = \text{wt. of liq. col.}$

T

h

$$\boxed{h = \frac{2T\cos\theta}{\rho g r}}$$

Jurin's law

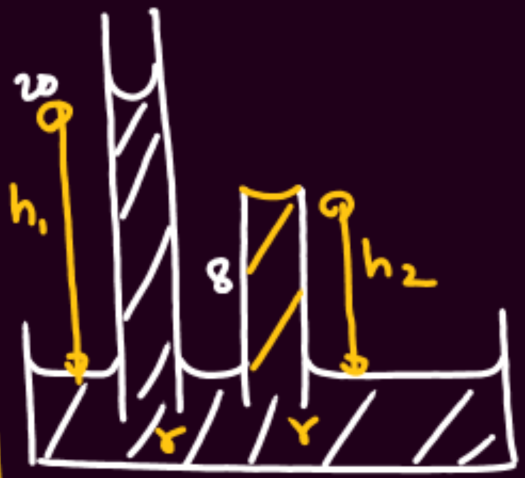


① $h_1 r_1 = h_2 r_2$

② $\frac{m_1}{m_2} = \frac{r_1}{r_2}$ ✓ | $\frac{m_1 g h_1}{m_2 g h_2} = \frac{r_1}{r_2} \cdot \frac{r_2}{r_1}$

③ $\frac{PE_1}{PE_2} = \frac{1}{1}$ ✓

Insuff. length

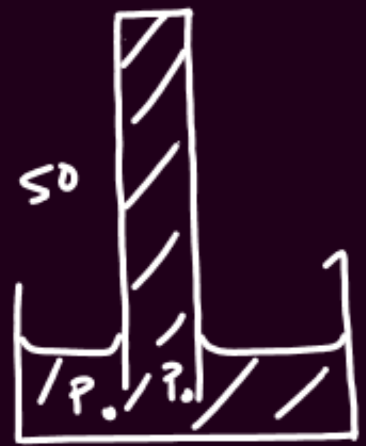


→ liq. till top

→ R ? Q ?

$h_1 = \frac{2T}{\rho g R_1}$ $h_2 = \frac{2T}{\rho g R_2}$

Free fall / Satellite

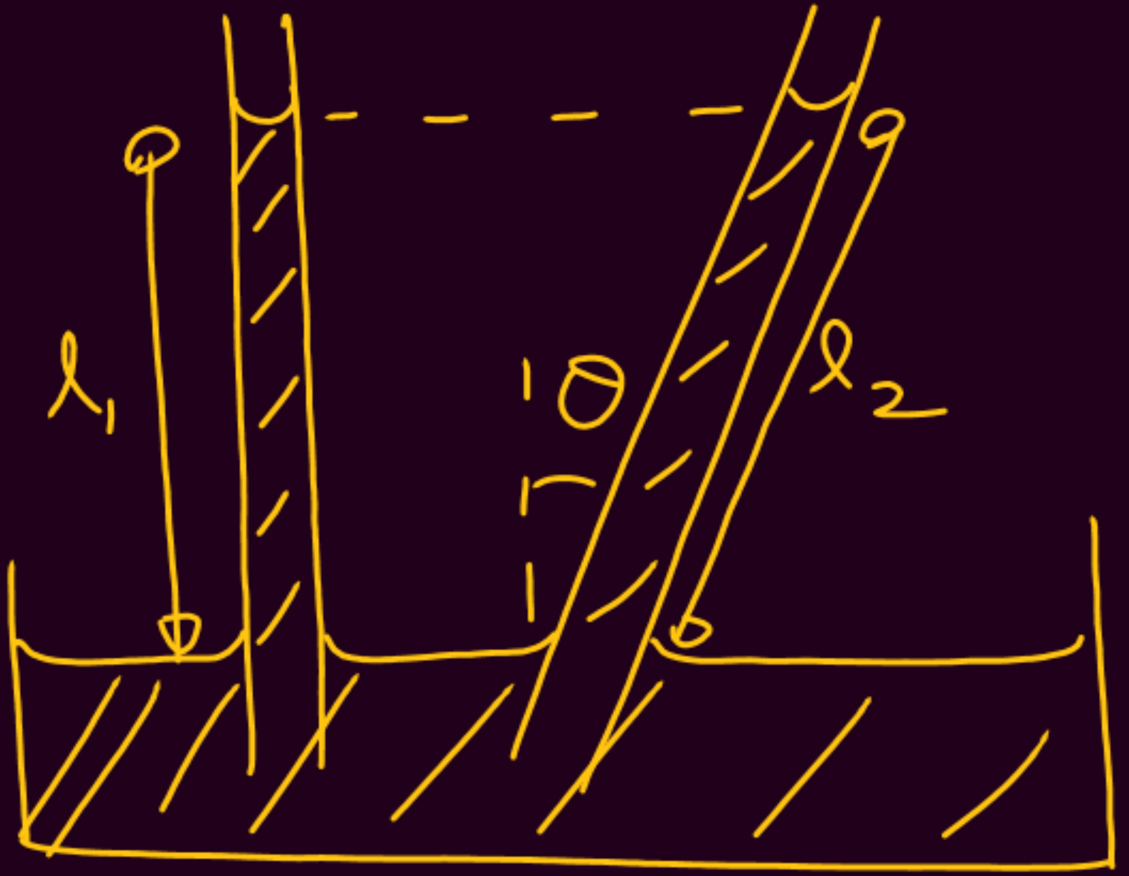


→

→

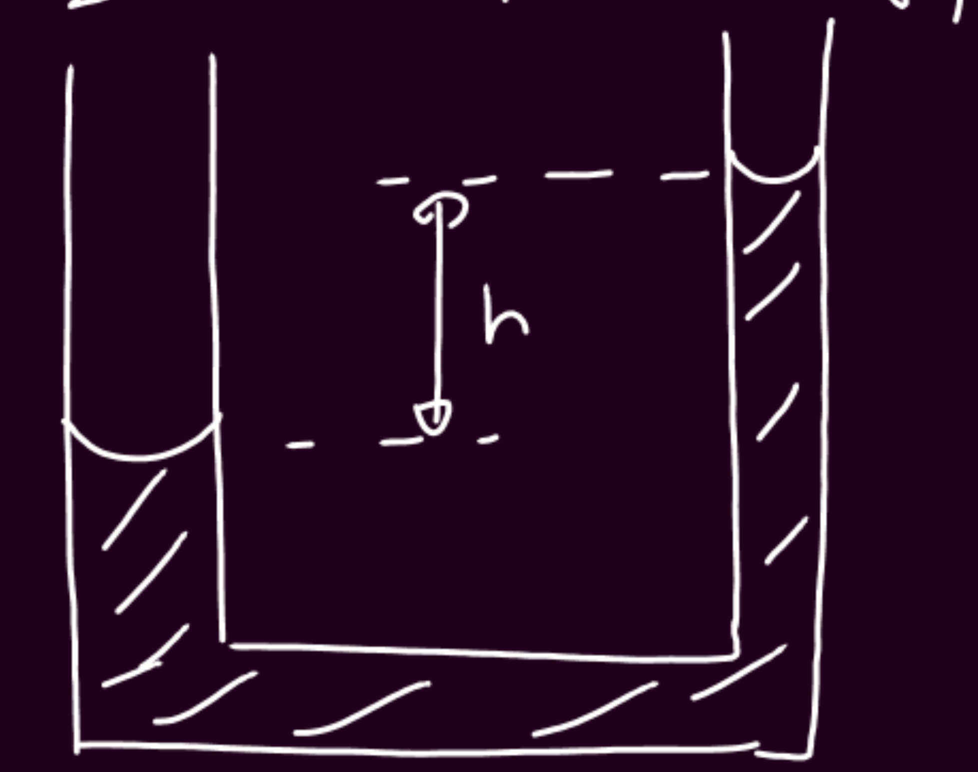
→ flat

Tilted Capillary



$$h = l_1 = l_2 \cos\theta$$

U-Shaped



$$h = \frac{2T\cos\theta}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

① $r = 3\,mm$

$$\frac{2T}{\rho g r} = 4\,mm$$

Correction ✓

$h = 4 - 1 = 3\,mm$

① $r = 3\,mm$

$$\frac{2T}{\rho g r} = 4\,cm$$

$$\boxed{h \simeq 4\,cm}$$

Correction in height

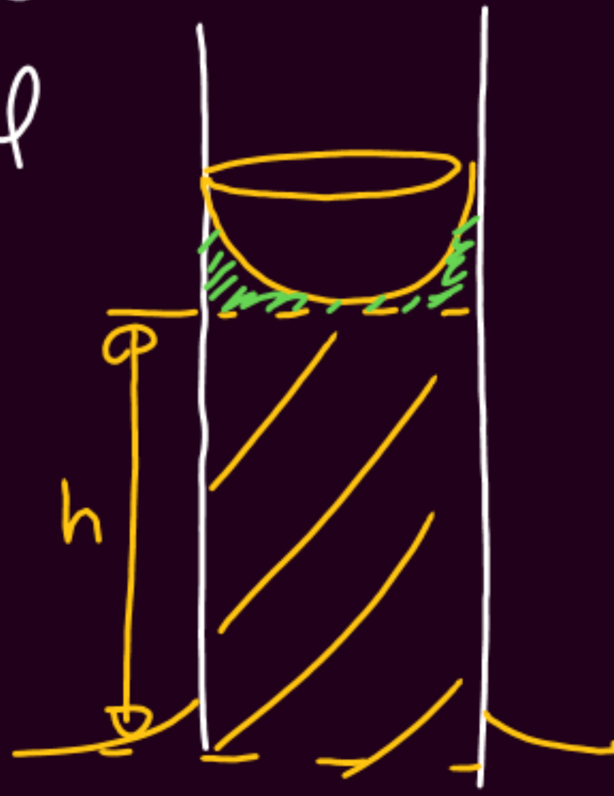
→ water & glass

→ Hemispherical

→ $\theta = 0^\circ$

→ $\cos\theta = 1$

→ $R = r$

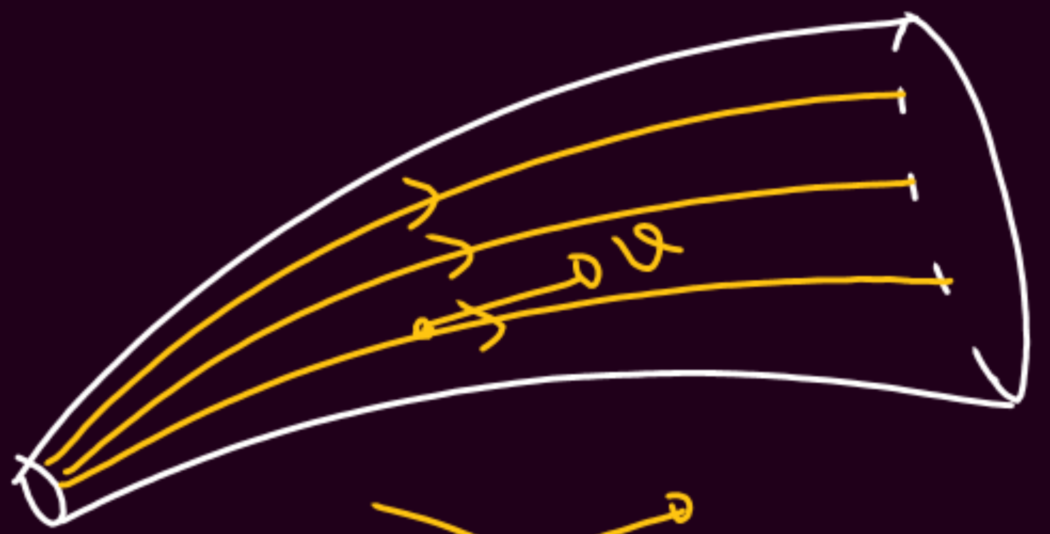


$$\boxed{h = \frac{2T}{\rho g r} - \frac{r}{3}}$$

Fluid Dynamics

Ideal Fluid → Incompressible
→ Non-viscous
→ Irrotation

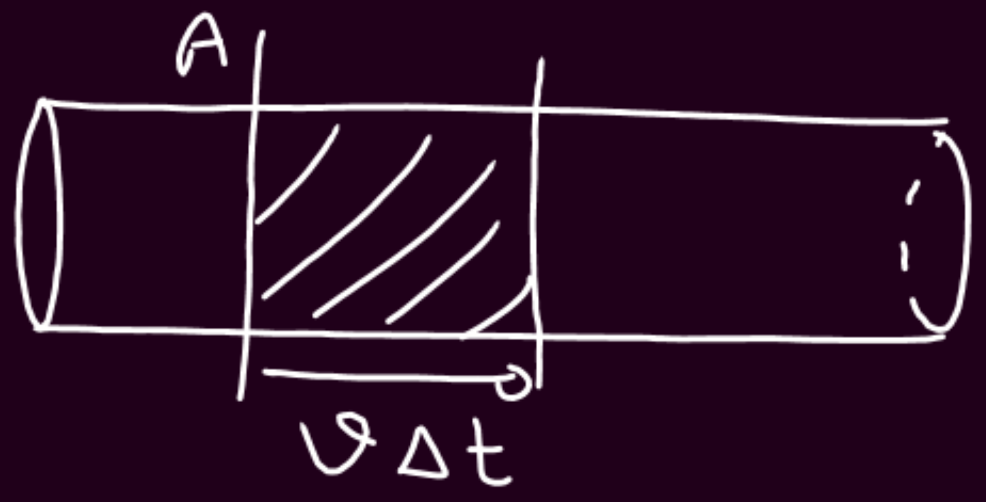
Streamline Flow (Steady)
(Laminar)



→ →
→

→ closer $v \uparrow$

Volume flow Rate



$$\Delta V = A(v\Delta t)$$

Rate of flow $\boxed{\frac{dV}{dt} = Av}$ ✓

Mass flow Rate $\boxed{\frac{dm}{dt} = \rho A v}$

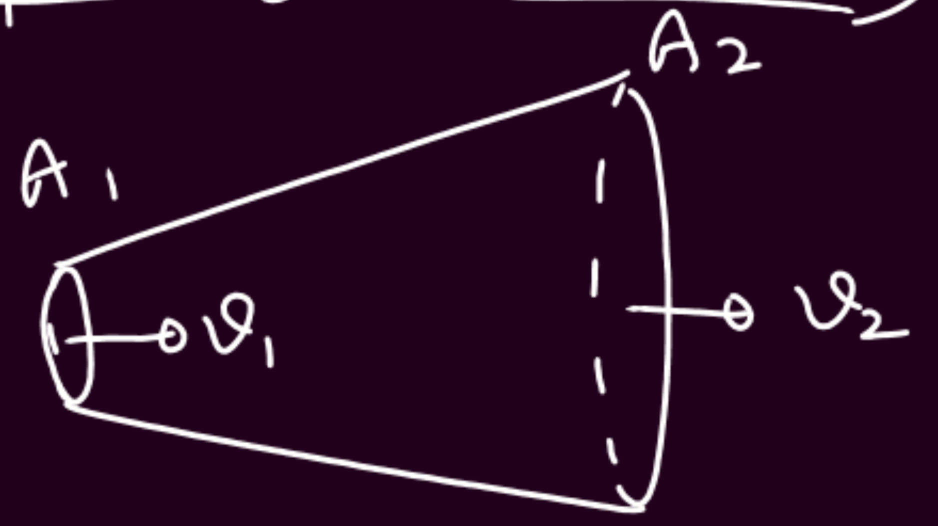
Page - 114
1 to 10

Page 117
1 to 6, 8, 9

Page - 142
1 to 10, 12 to 22

Page - 149
1 to 12, 13 to 28

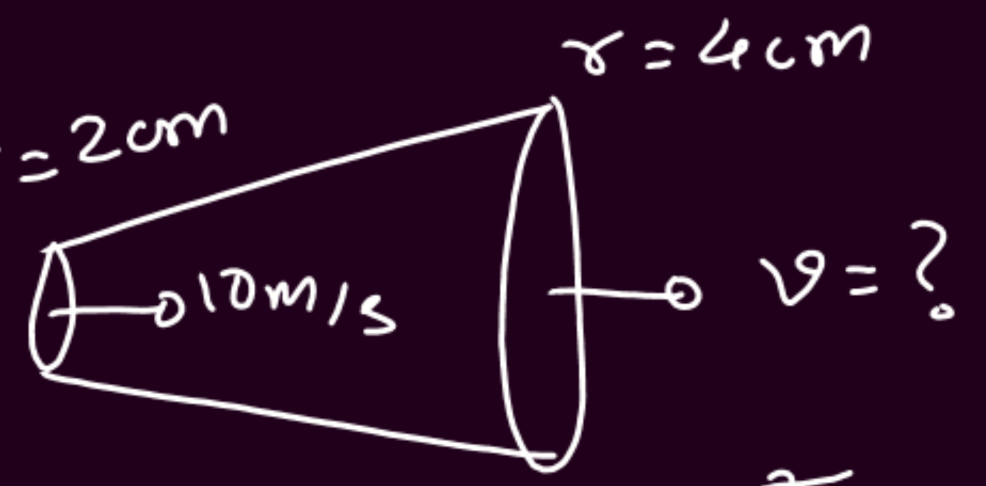
Eqⁿ of Continuity



$$\boxed{\frac{dV}{dt} = A_1 v_1 = A_2 v_2}$$

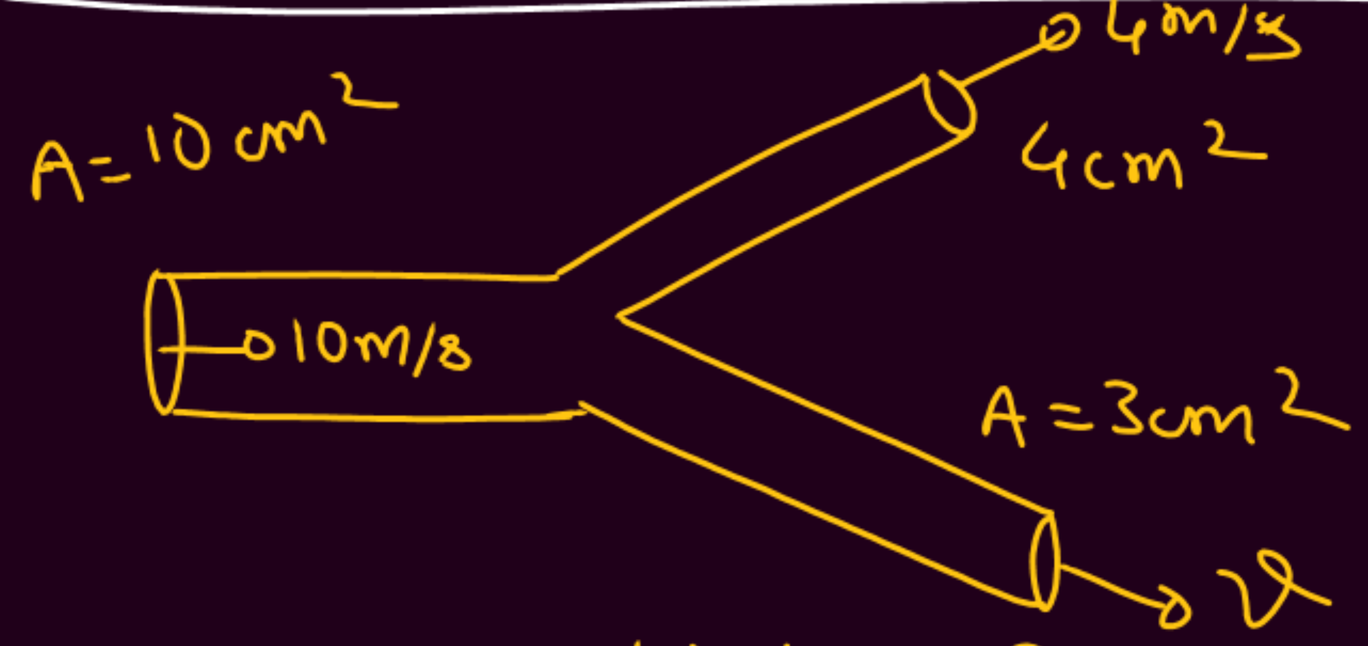
$$\boxed{r_1^2 v_1 = r_2^2 v_2}$$

Q.



$$(\cancel{2})^{\cancel{2}}(10) = (4)^2 \, \underline{v}$$

$$v = \frac{10}{4} = 2.5 \ m/s$$



$$10 \times 10 = 4 \times 4 + 3v$$

$$84 = 3v$$

$$v = 28 m/s$$

eem

$_2$

P_2

2

Constant

not

el Head

Apps

① 

② 

③ 

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 > v_2$$

$$P_1 < P_2$$

$$\boxed{P_2 - P_1 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)}$$

④ 

⑤ Atomiser/spray

⑥ 

$$\boxed{\text{Upthrust} = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A}$$

⑦ Aerofoil / Aeroplane

$$F = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A > \left(\text{wt. of plane}\right)$$

Q. $\rho = 1.6 \ kg/m^3$

$A = 10 m^2$

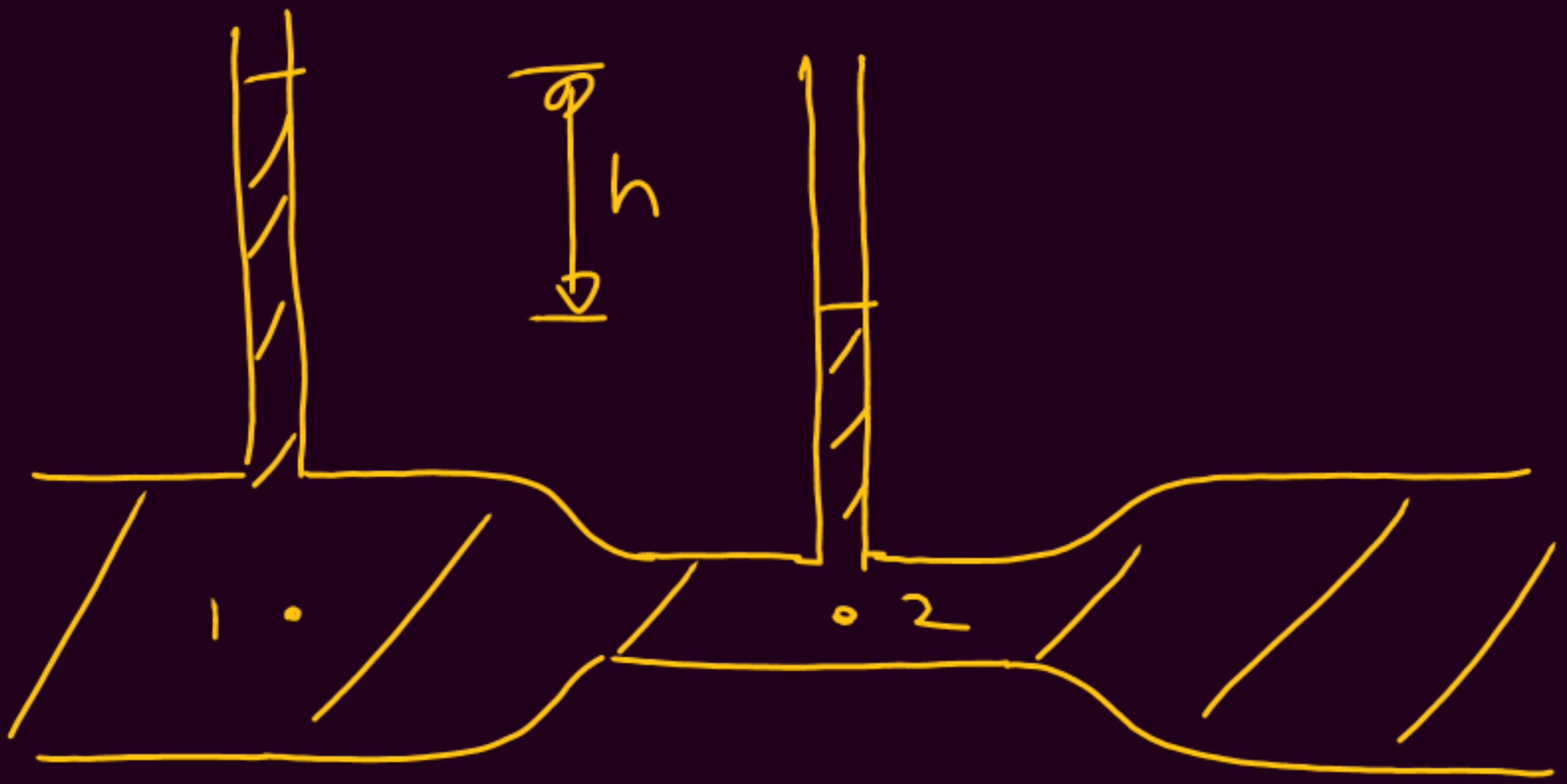
$$F = \frac{1}{2}\, 1.6\left(120^2 - 80^2\right)$$

$$= 8\,(200)(40)$$

$$= 64000 \text{ N}$$

$$\boxed{6400 \text{ kg}}$$

Venturimeter (flow meter)



① $A_1 > A_2$

② $v_1 < v_2$ $\quad (A_1 v_1 = A_2 v_2)$

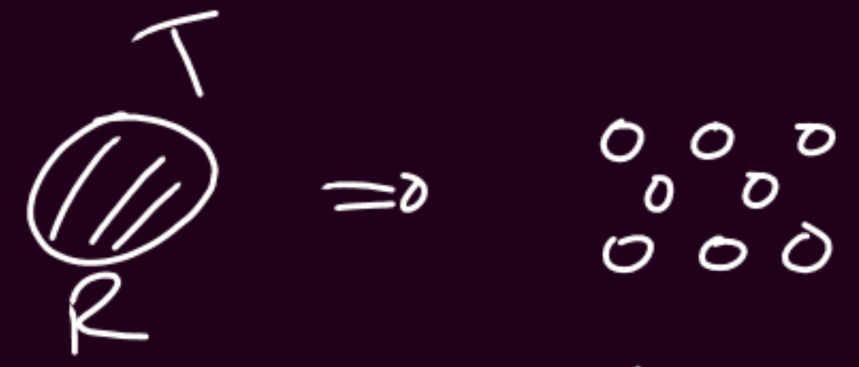
③ $P_1 > P_2$ $\quad \left(P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2\right)$

$$\boxed{P_1 - P_2 = \rho g h}$$

Extra

$$\boxed{\frac{dV}{dt} = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

$$-4\pi R^2 T$$



$$\frac{SE_n}{SE_1} = (10^6)^{1/3} = 100$$

$$W = (100-1)\, SE_1$$

$$= 99\,(4\pi R^2)\,T$$

(5)

$$(8)^{1/3} = 2$$

$$2(4\pi R^2 T) - 4\pi R^2 T$$

$$2(4\pi R^2)T$$

$$i = 2\pi D^2 T$$

$$f = 2\pi (2D)^2 T$$

$$f - i = 6\pi D^2 T$$

Fluid Statics

→ Ideal

→ Density = $\boxed{\rho = \frac{m}{V}}$

unit $\frac{g}{cc} = \frac{g}{cm^3}$

$\frac{kg}{m^3}$

$\rho_w = 1\ g/cc = 1000\ kg/m^3$

Case 1 Two liq. in Equal Volume

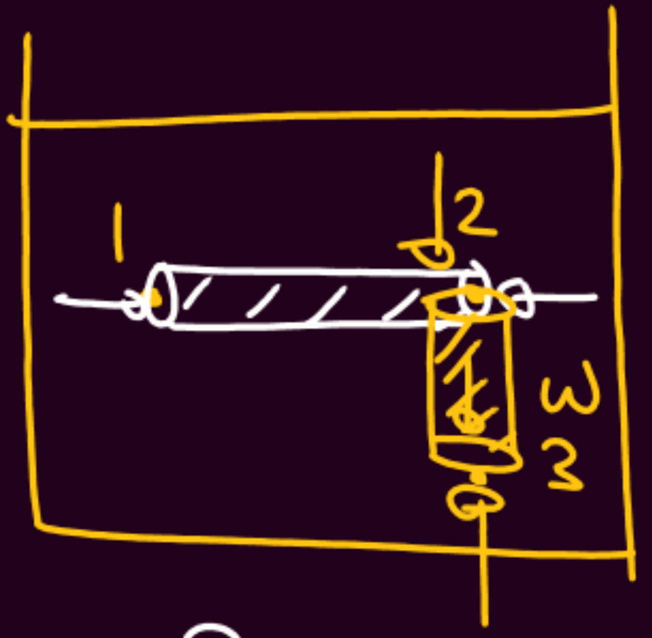
$$\rho_{mix} = \frac{\rho_1 V + \rho_2 V}{V + V} = \frac{\rho_1 + \rho_2}{2}$$

Case-2 Two liq & Equal mass

$$\rho_{mix} = \frac{m + m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

Pressure

$1\ atm = 10^5 \frac{N}{m^2} = 10^5\ Pa$



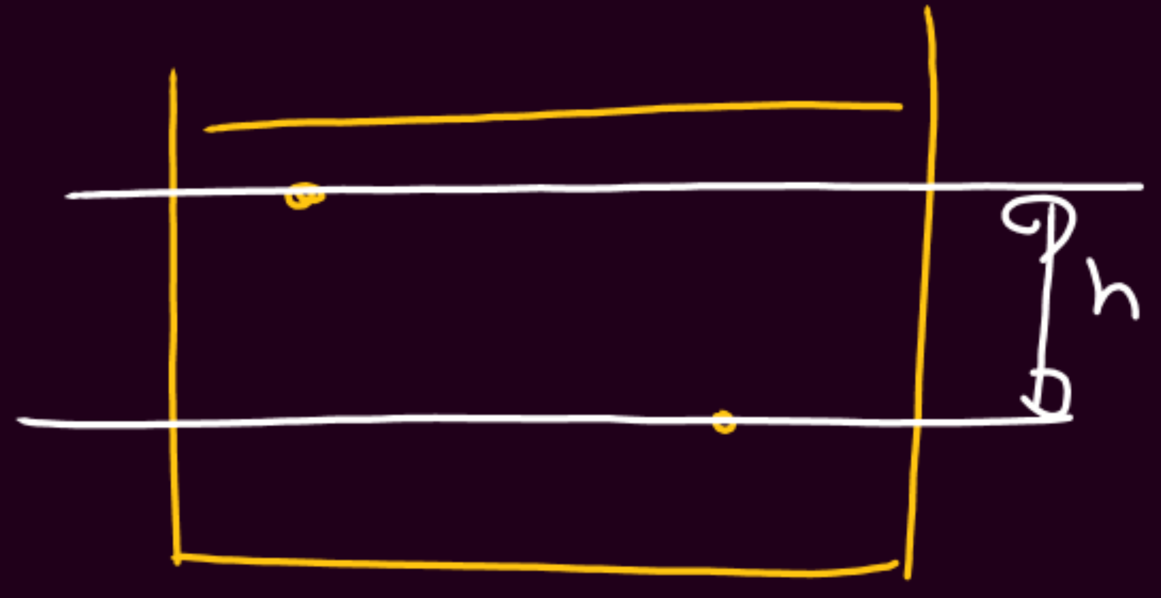
$\boxed{\Delta P = \rho g h}$

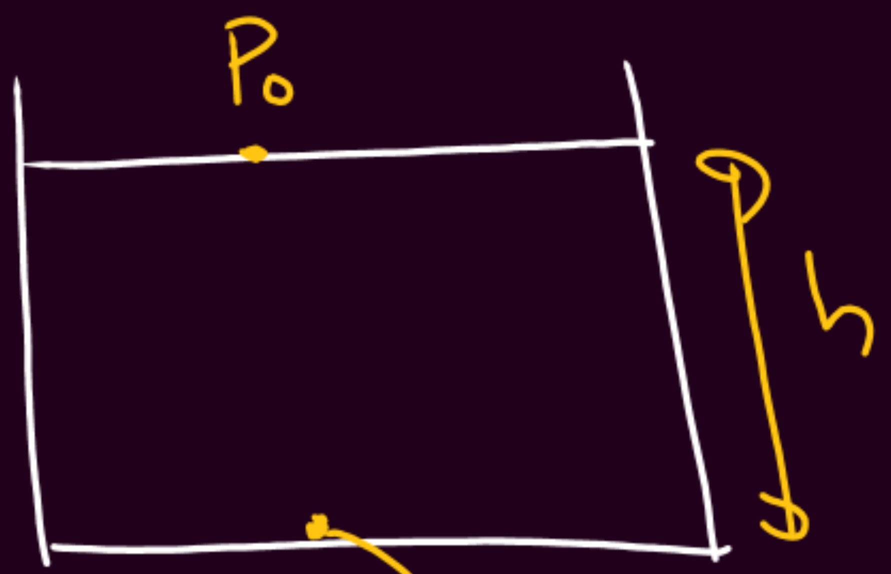
→ Pressure diff. due to height

$P_1 = P_2$

Same H-level then Pressure Same

- → Rest
- → Same fluid
- → Connected





→ $P_0 + \rho g h$

$$1\ atm = 10^5 \frac{N}{m^2} = 10^5\ Pa$$

$$1\ atm = 76\,cm \text{ of } Hg = 760\,mm \text{ of } Hg$$

P is 60 cm of Hg

$76\,cm \rightarrow 1\ atm$

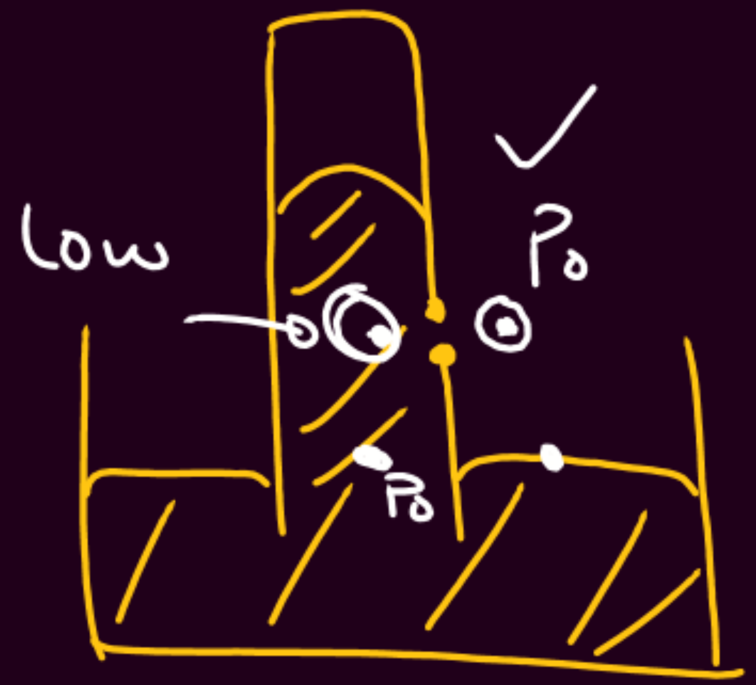
$60\,cm \rightarrow P = \frac{60}{76}\ atm$

$$P = \rho g h = (13.6\times10^3)(10)(0.60)$$

P is 6 m of water

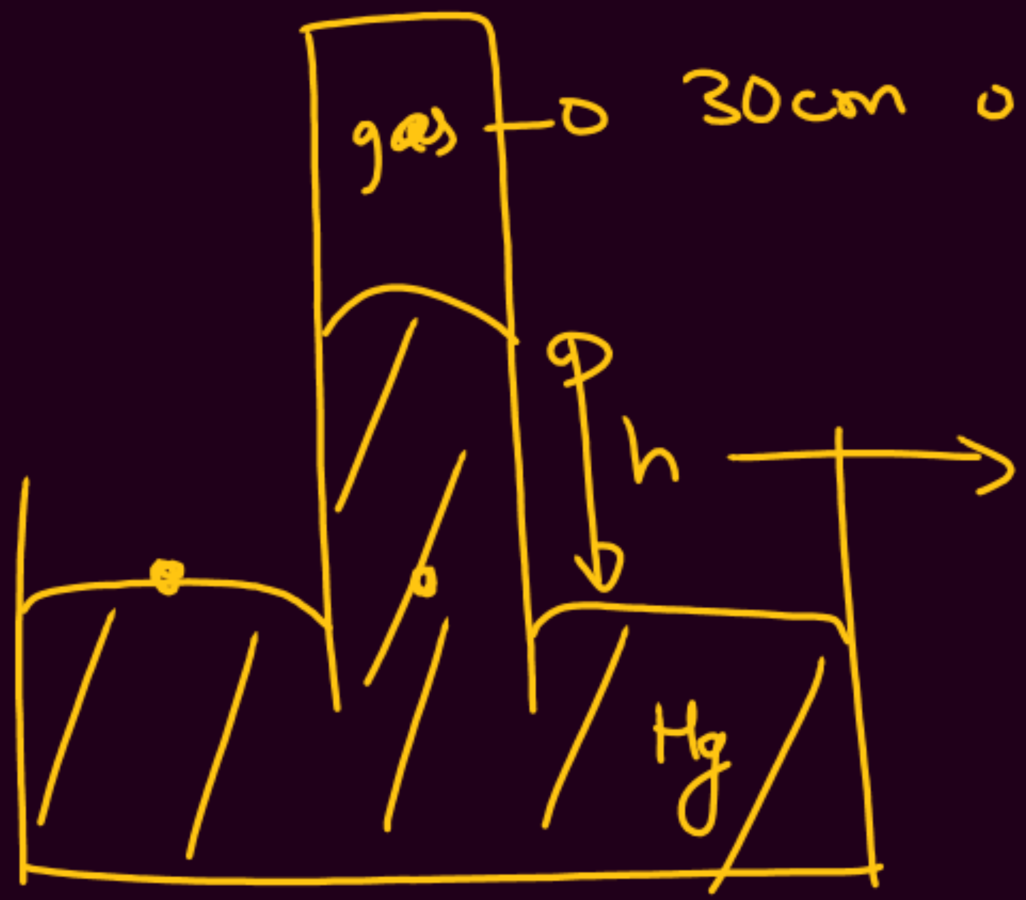
$$P = \rho g h = (1000)(10)6 = 6\times10^4\ N/m^2$$

$$= \frac{6\times10^4}{10^5}\ atm = \underline{0.6\ atm}$$

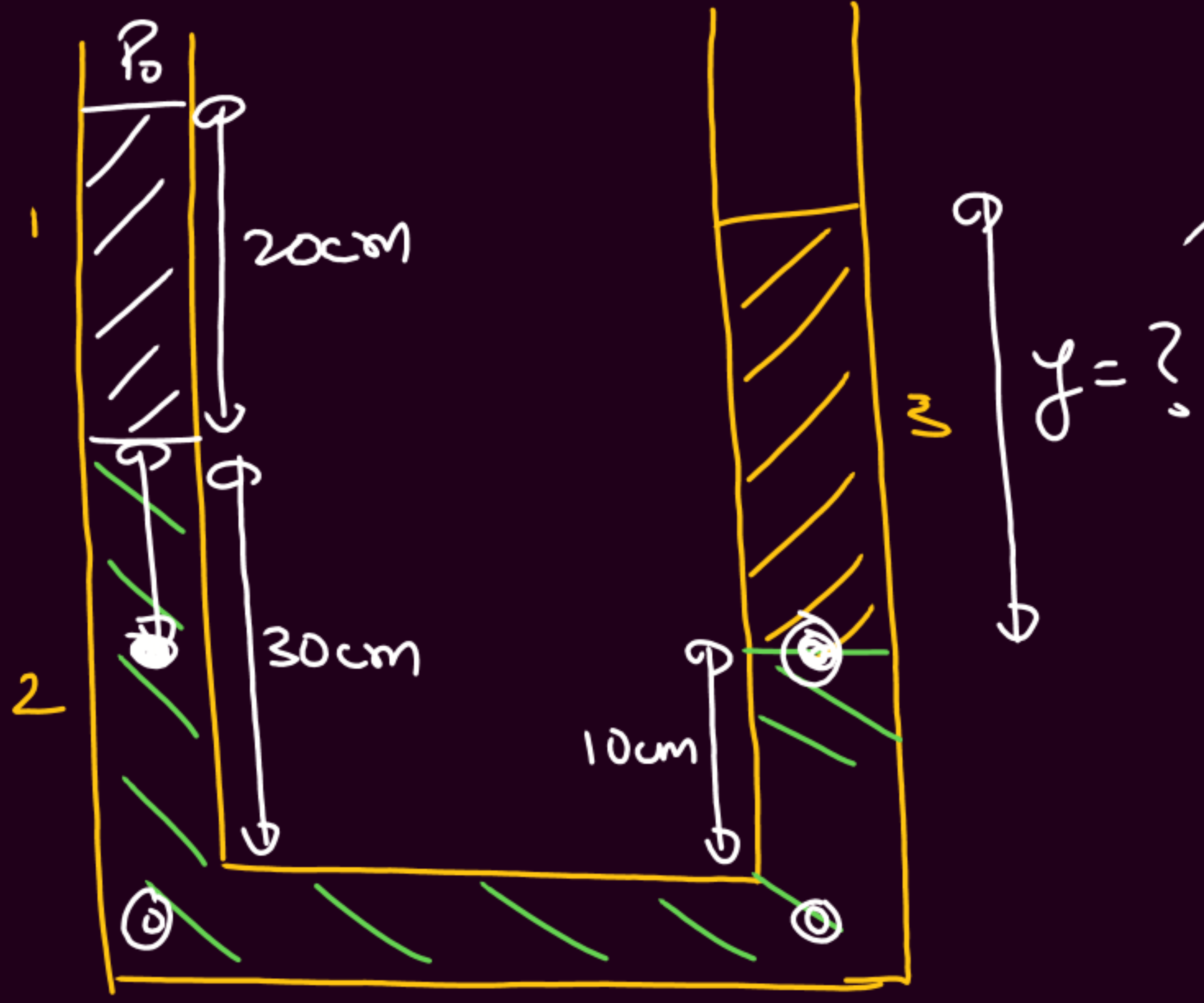


Relative Density (R.D.) [Specific gravity] $= \frac{\rho}{\boxed{\rho_w}}$ (unitless / Dimensionless)

(R.D.) → [ρ in g/cc]



$76 - 30 = 46\,cm$



$$\cancel{P_0} + 2g(20) + 4g(20) = \cancel{P_0} + 3gy$$

$$40 + 80 = 3y$$

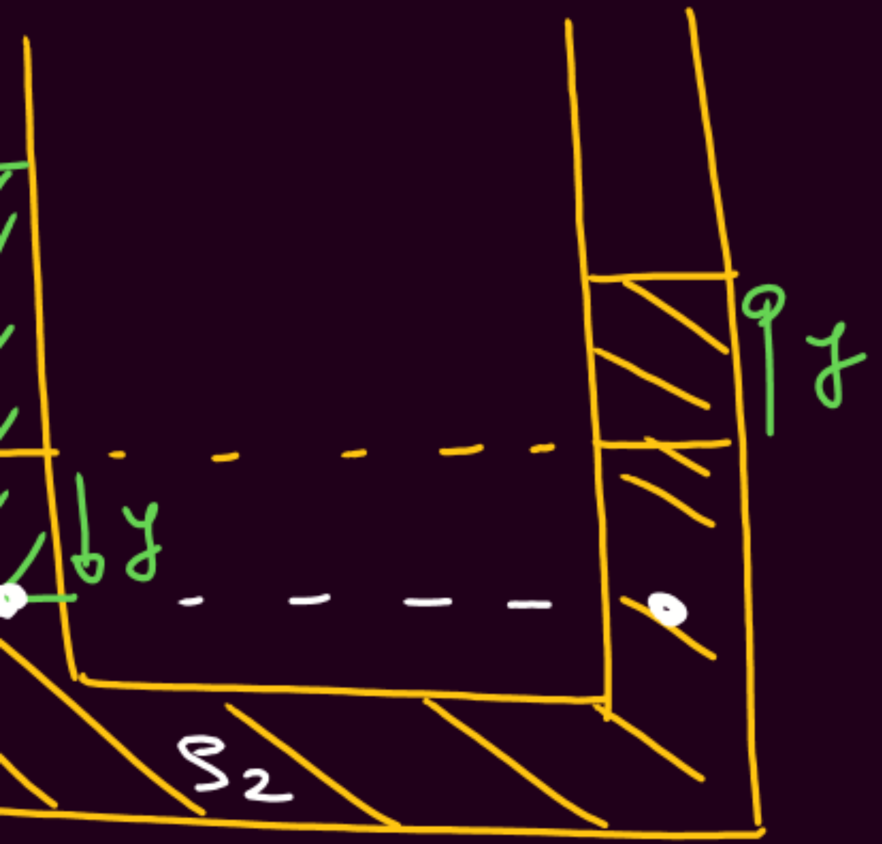
$$\boxed{y = 40\,cm}$$

$\rho_1 = 2$ g/cc

$\rho_2 = 4$ g/cc

$\rho_3 = 3$ g/cc

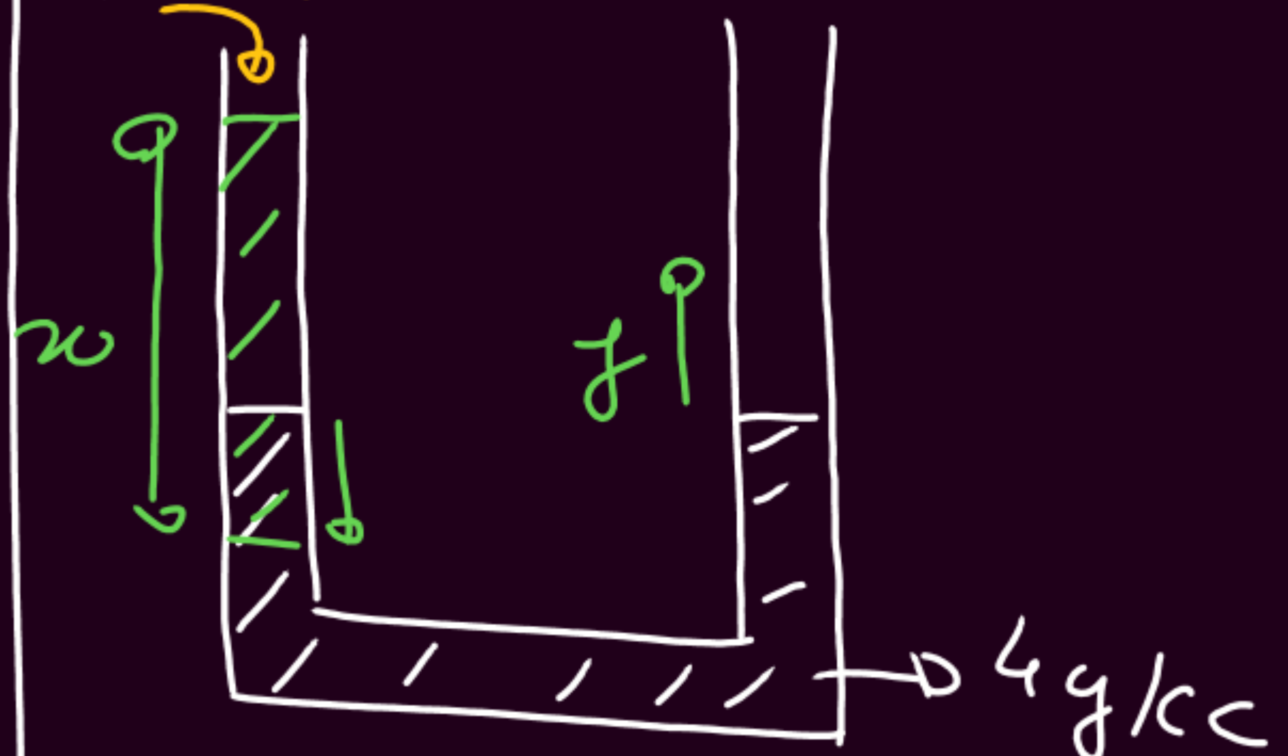
niform Cross section



$$\rho_1 g h = \rho_2 g (2y)$$

$$\boxed{\rho_1 (h) = \rho_2 (2y)}$$

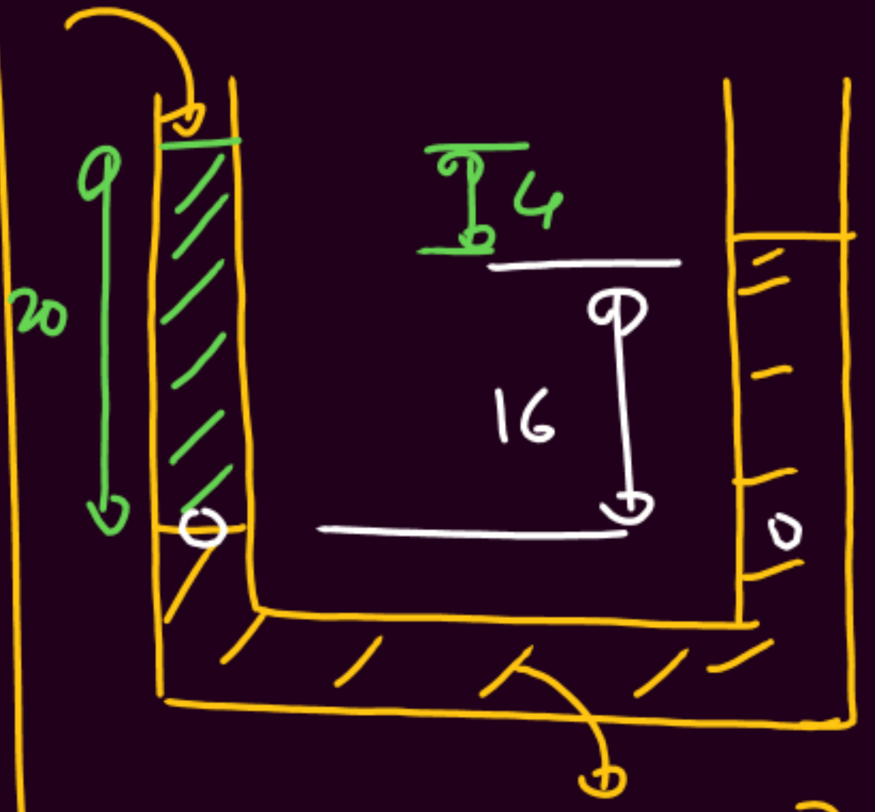
Q $\rho_1 = 2g/cc$



$$2(20) = 4(2y)$$

$$\boxed{y = 5cm}$$

2g/cc



$\rho = ?$

$$(2)\, g(20) = \rho\, g(16)$$

$$\rho = \frac{40}{16} = 2.5 \text{ g/cc}$$

Q:- Barometer

$$\underline{\underline{P_0}} = \underline{\cancel{\rho} g(76cm)} = \cancel{\rho}\left(g + g/3\right) h$$

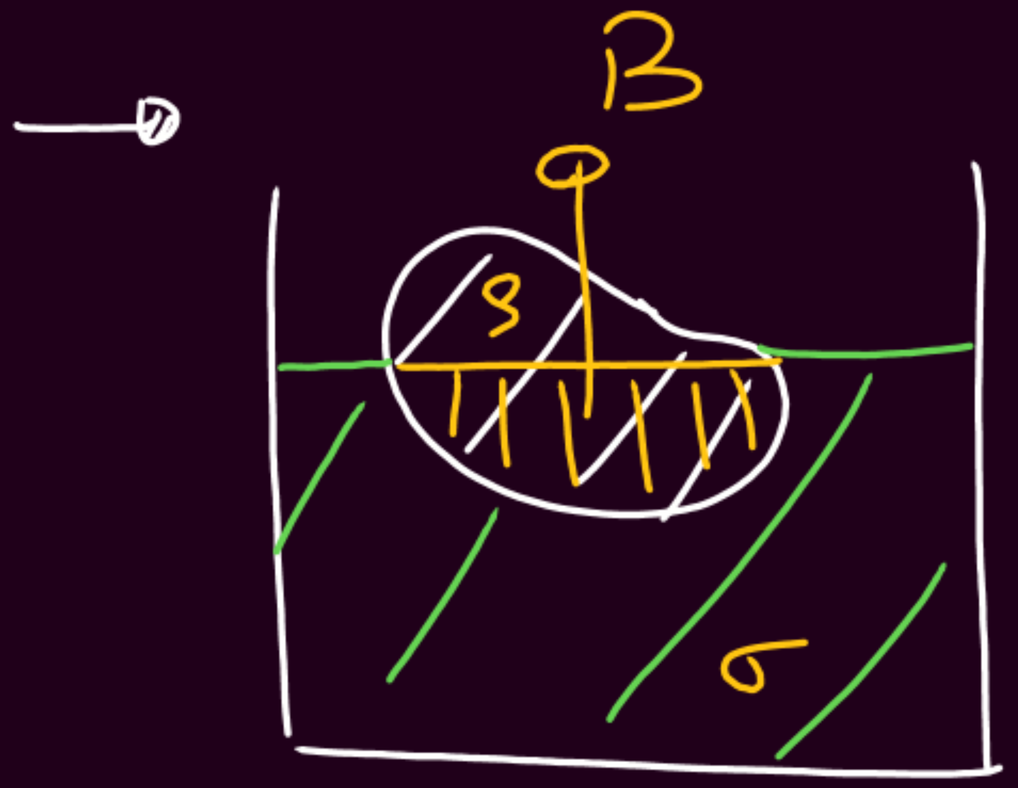
Rest

$\uparrow a = g/3$

$$g(76) = \left(\frac{4g}{3}\right) h$$

$$\boxed{h = 57 cm}$$

Buoyancy (Archimedes Principle)

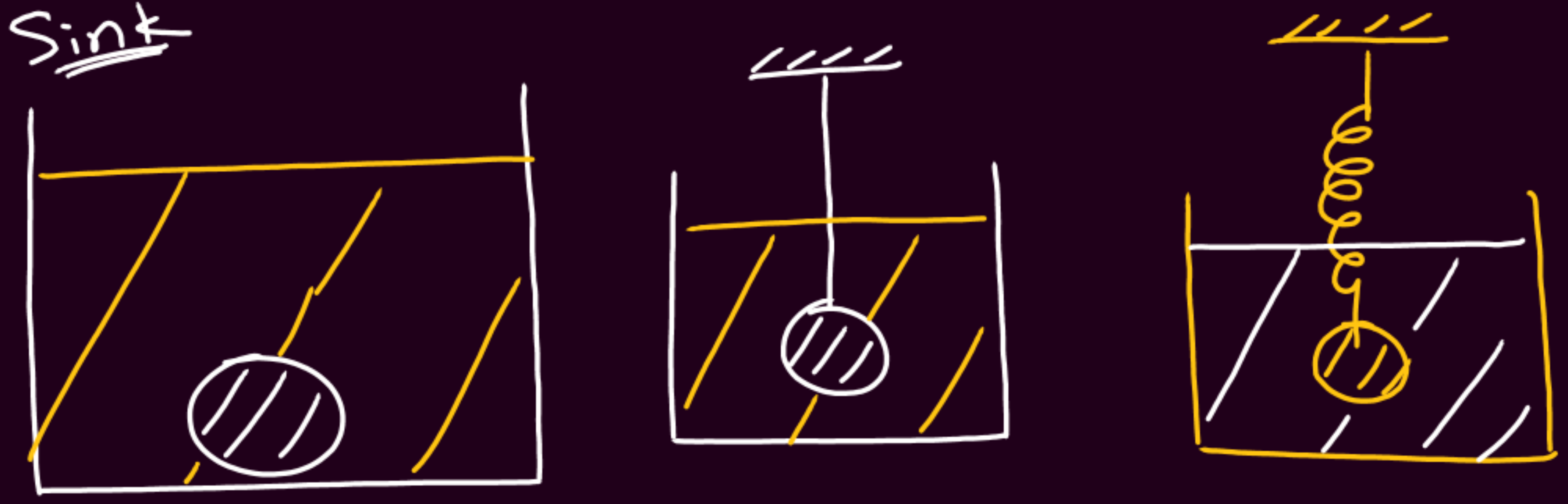


$$\boxed{B = \sigma V_{sub}\, g}$$

ρ = Density of Solid

σ = —— " —— Liq.

① $\rho > \sigma$ (Compact Size)



Tension = kx = App. wt.

① $B = \sigma V g$

② $W = \rho V g$

③ $\boxed{\text{Apparent Weight} = W - B = W\left(1 - \frac{B}{W}\right) = W\left(1 - \frac{\sigma}{\rho}\right)}$

Q 10kg $\rho = 5\ g/cc$

① App wt $= 100\left(1 - \frac{1}{5}\right)$

$(W - B) = 80\ N$ (8 kg)

② $\frac{B}{W} = \frac{\sigma}{\rho} = \frac{1}{5}$

$\frac{B}{100} = \frac{1}{5}$

$\boxed{B = 20\ N}$

Q 15 kg weighs 10kg in water

App wt $= W\left(1 - \frac{\sigma}{\rho}\right)$

$10g = 15g\left(1 - \frac{1}{\rho}\right)$

$\frac{2}{3} = 1 - \frac{1}{\rho}$

$\frac{1}{\rho} = 1 - \frac{2}{3} = \frac{1}{3}$

$\boxed{\rho = 3\ g/cc}$

Q. 10 kg → 6 kg in water

→ 2 kg in liq

① $6 = 10\left(1 - \frac{1}{\rho}\right)$ $\boxed{\rho = 2.5\ g/cc}$

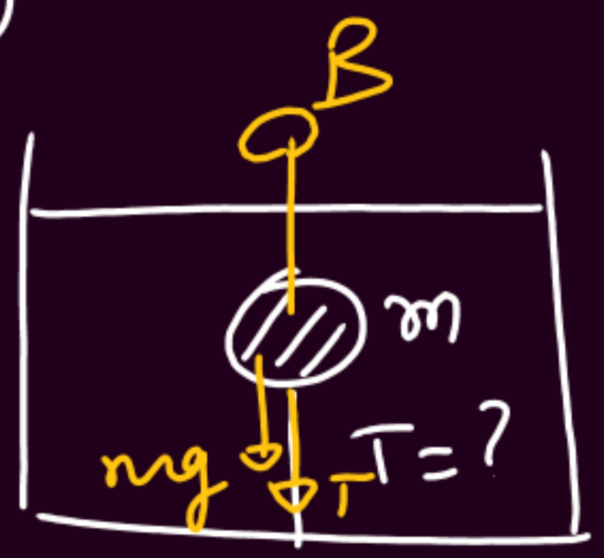
② $2 = 10\left(1 - \frac{\rho_{liq}}{\rho}\right)$

$0.2 = 1 - \frac{\rho_{liq}}{2.5}$

$\frac{\rho_{liq}}{2.5} = 0.8$

$\boxed{\rho_{liq} = 2\ g/cc}$

37



ρ_{sphere} is η times lighter than water

$$\text{App. wt.} = W - B = W\left(1 - \frac{B}{W}\right)$$

$\rho_w = 1$

$\rho_{sph} = \frac{1}{\eta}$

$$T + mg = B$$

$$T = B - mg$$

$$T = mg\left(\frac{B}{mg} - 1\right) = mg\left(\frac{1}{1/\eta} - 1\right) = (\eta - 1)\,mg$$

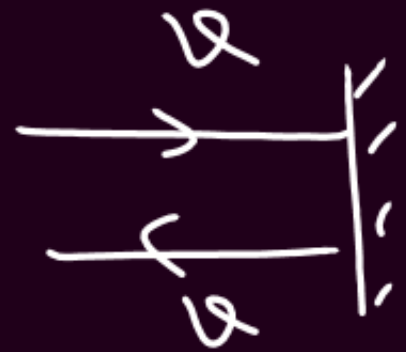
Dynamics.

1

$$F = \frac{v(dm)}{(dt)} = v(\rho A v)$$

$$\boxed{F = \rho A v^2}$$

2



$$F = 2\rho A v^2$$

Floation

① $W = B$

② App. wt. $= 0$

Case: $\boxed{\rho < \sigma}$

"It necessarily floats"

Case → "compact Size"

$$\rho V g = \boxed{\sigma V_{sub} g}$$

$$\boxed{\frac{V_{sub}}{V} = \frac{\rho}{\sigma}}$$

e.g. Ice R.D. $= 0.9$

$$\rho = 0.9 \text{ g/cc} = 900 \text{ kg/m}^3$$

Fraction of Ice Sub. in water

$$\frac{V_{sub}}{V} = \frac{0.9}{1} = 90\%$$

Q

water $0.4 = \frac{\rho}{1}$

liq $0.6 = \frac{0.4}{\rho_{liq}}$

$$\boxed{\rho_{liq} = 2/3 \text{ g/cc}}$$

Q.

$$\frac{3}{4} = \frac{600}{\rho_{liq}}$$

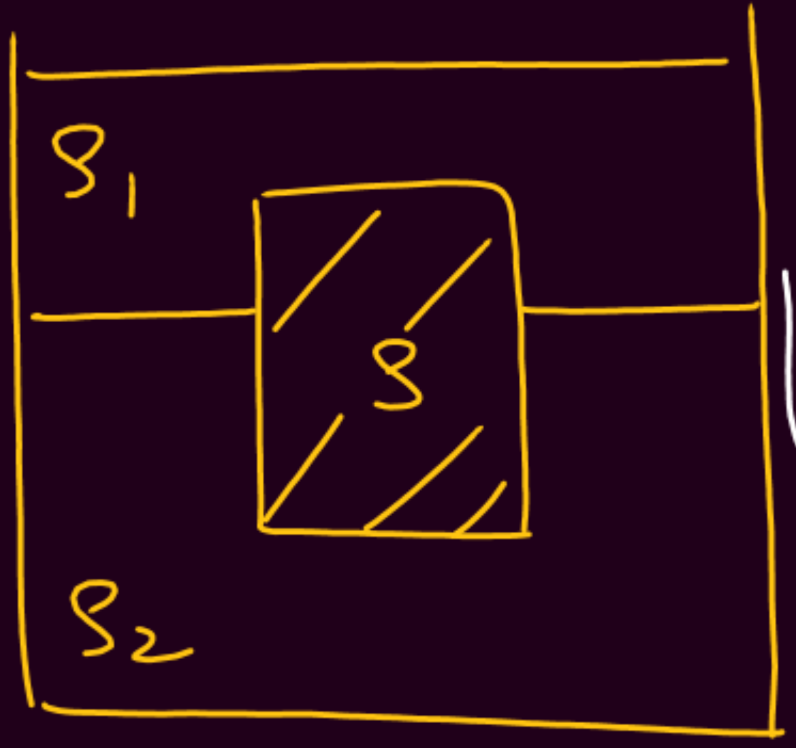
$$\rho_{liq} = 800 \text{ kg/m}^3$$

① $B = 120g = 1200\text{ N}$

② $1200 = 1000 V_{sub} g$

$V_{sub} = 0.12 \text{ m}^3 = 0.12 \times 10^6 \text{ cm}^3$

Floation on Interface



Same Area

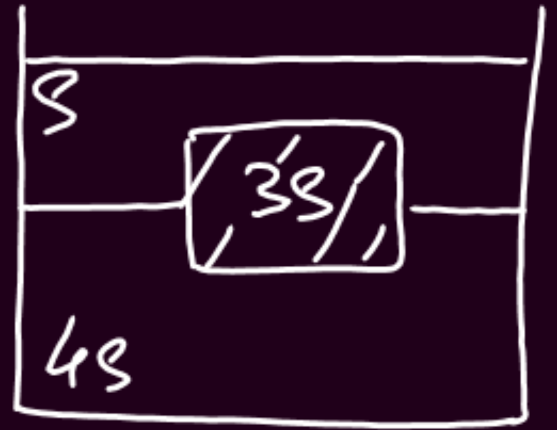
$$\boxed{\rho_1 < \rho < \rho_2}$$

$$W = B_1 + B_2$$

$$\rho V = \rho_1 V_1 + \rho_2 V_2$$

$$\rho L = \rho_1 L_1 + \rho_2 L_2$$

$$V = V_1 + V_2$$

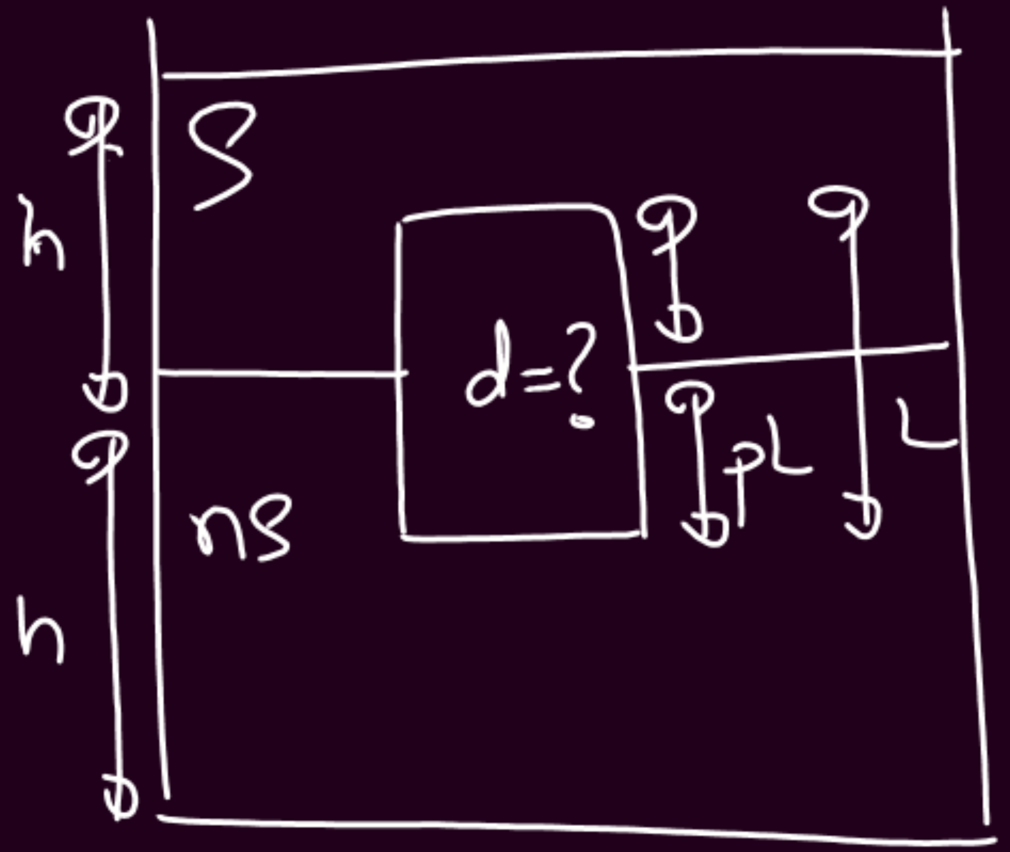


$$3\rho V = \rho V_1 + 4\rho V_2$$

$$3V = (V - V_2) + 4V_2$$

$$2V = 3V_2$$

$$\boxed{\frac{V_2}{V} = \frac{2}{3}}$$



$$\rho L = \rho_1 L_1 + \rho_2 L_2$$

$$d\,L = \rho(L - pL) + n\rho\,(pL)$$

$$d = \rho(1-p) + n\rho p$$

$$d = \rho\left[1 - p + np\right]$$

$$d = \rho\left[1 + (n-1)p\right]$$

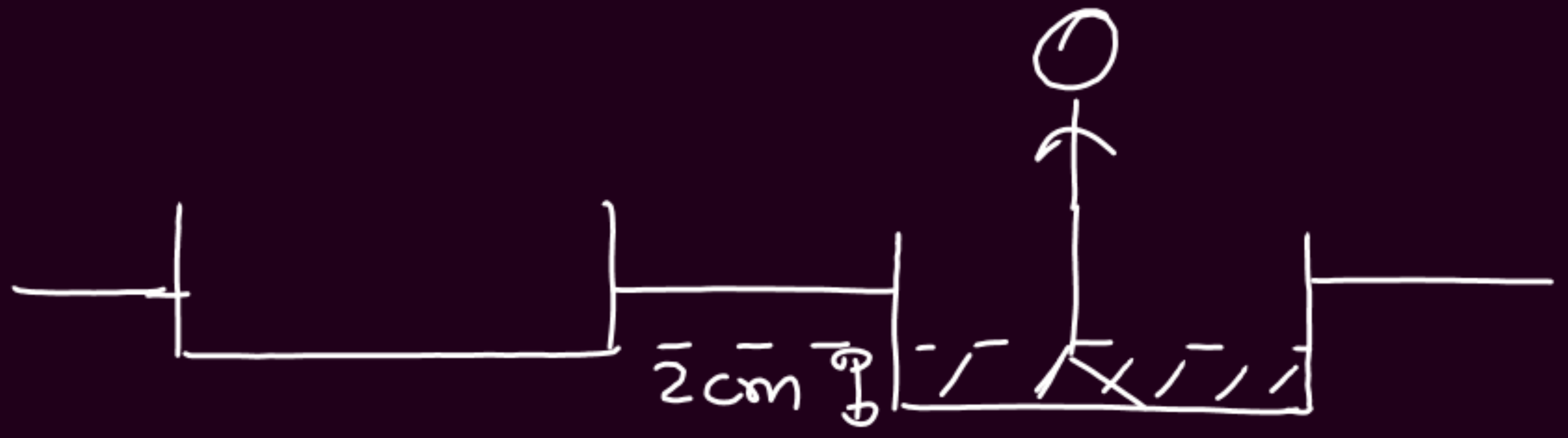
$$V = \frac{100}{600}$$

$$W = B$$

$$(m+100)\cancel{g} = (1000)\left(\frac{10\cancel{0}}{6\cancel{00}}\right)\cancel{g}$$

$$m + 100 = \frac{500}{3}$$

$$\boxed{m = \frac{200}{3}\,\text{kg}}$$



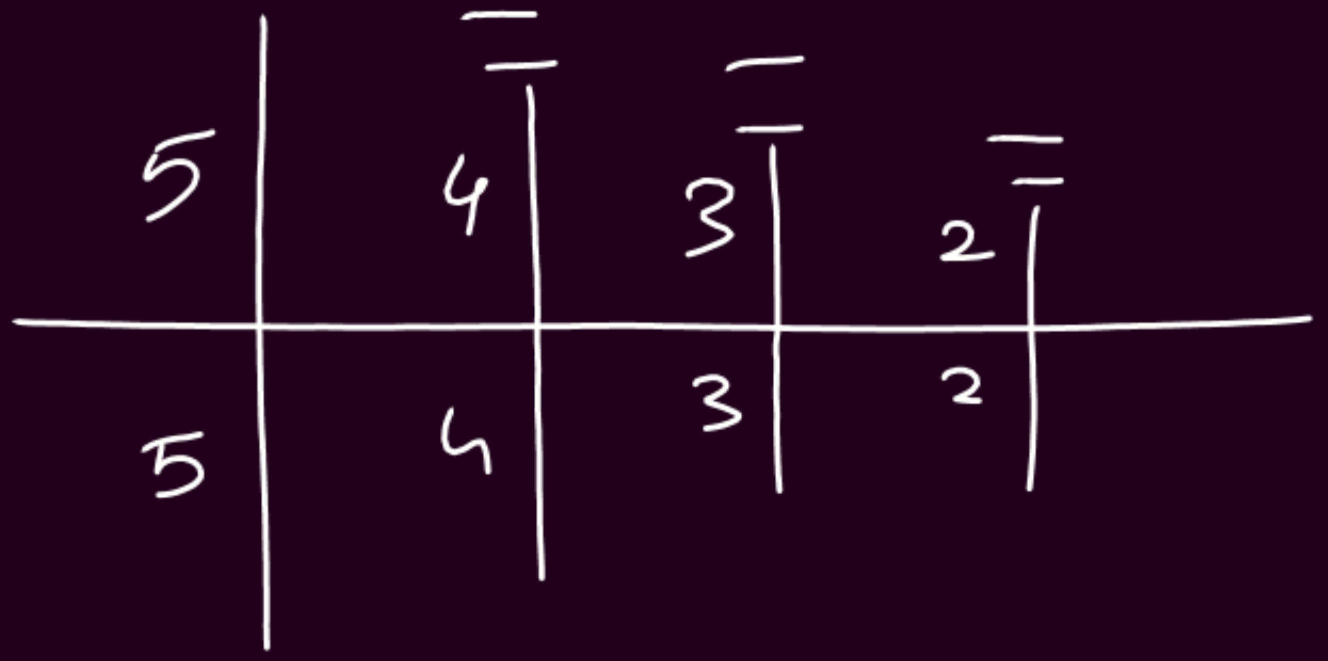
$$W_{extra} = B_{extra}$$

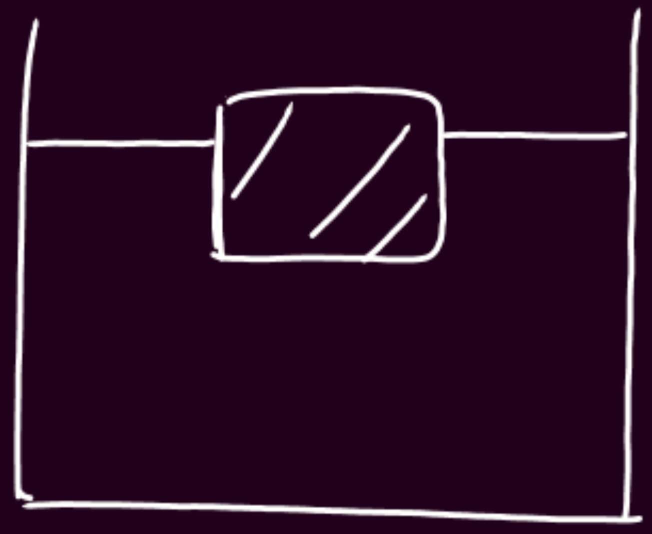
$$m\cancel{g} = (1000)\left(4 \times \frac{2}{100}\right)\cancel{g}$$

$$\boxed{m = 80\ kg}$$

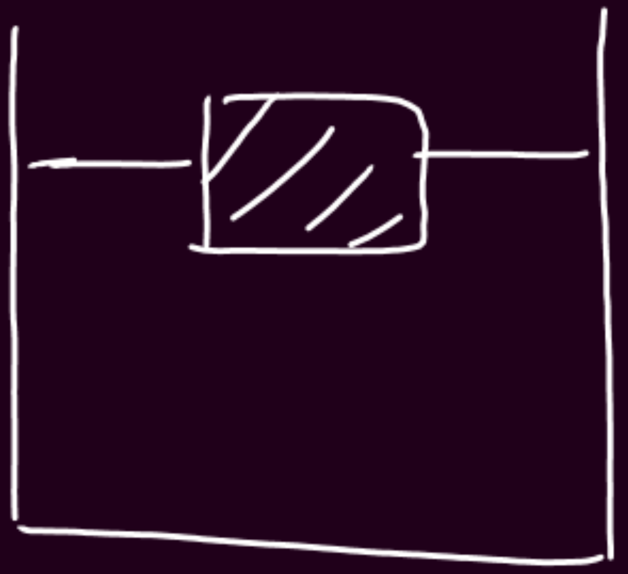
Page-145

39





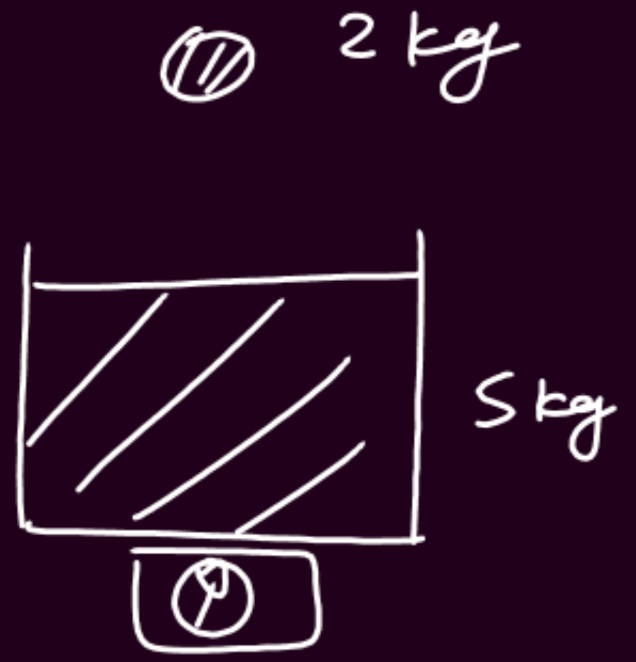
Rest

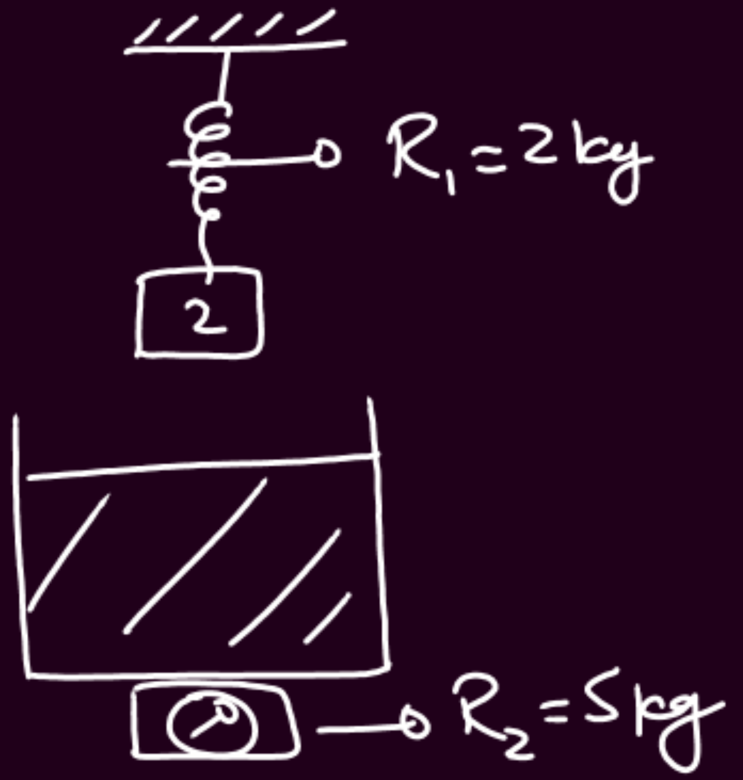


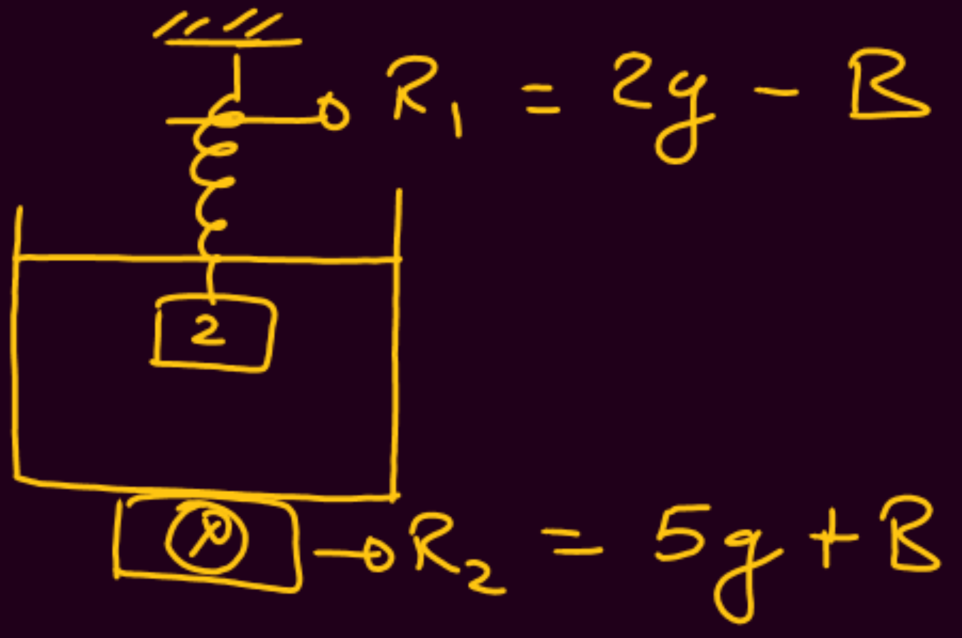
$\uparrow a = g/4$

Acc.

$2/3^{rd}$ Sub. $\longrightarrow$ $2/3^{rd}$ Sub

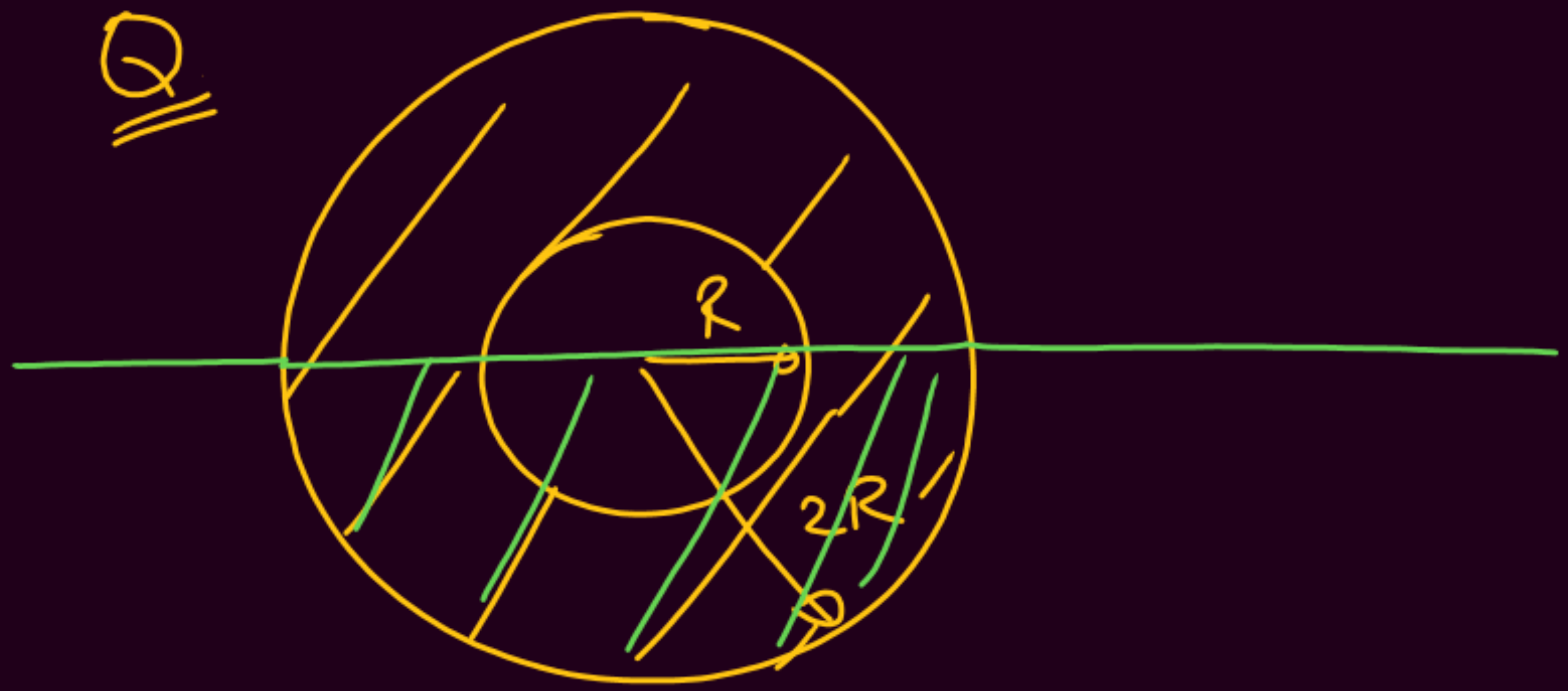






$$R_1 + R_2 = 7\,kg$$

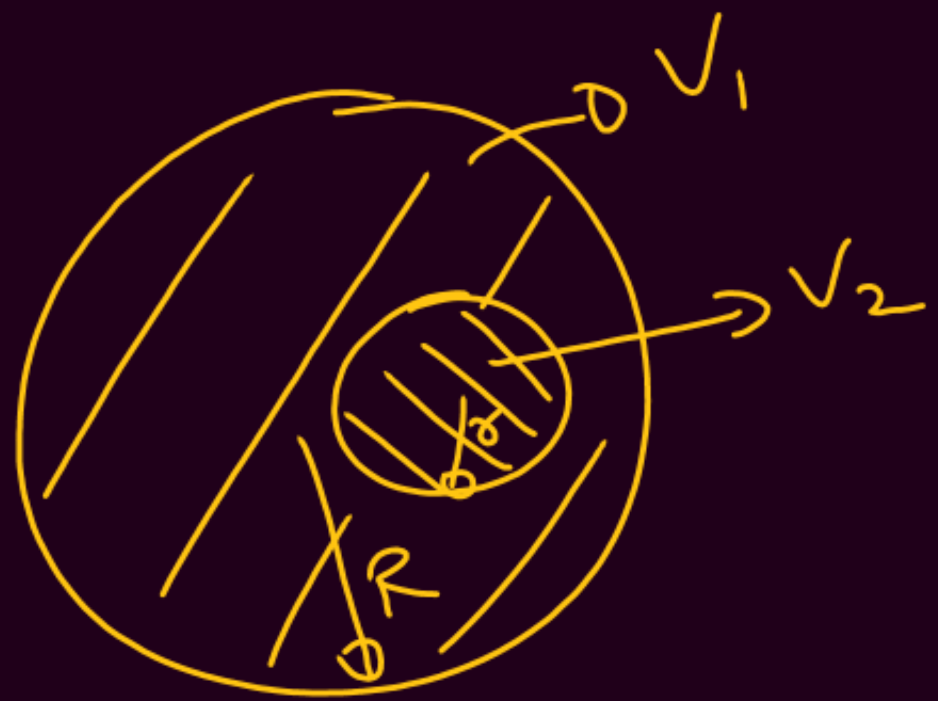
Q



$$\rho \frac{4}{3}\pi\left[(2R)^3 - R^3\right] g = \rho_w \left(\frac{2}{3}\pi (2R)^3\right) g$$

$$\frac{\rho}{\rho_w} = \frac{8}{2\times 7} = \frac{4}{7}$$

Q



$$(2.4)V_1 + (0.3)V_2 = 1(V_1 + V_2)$$

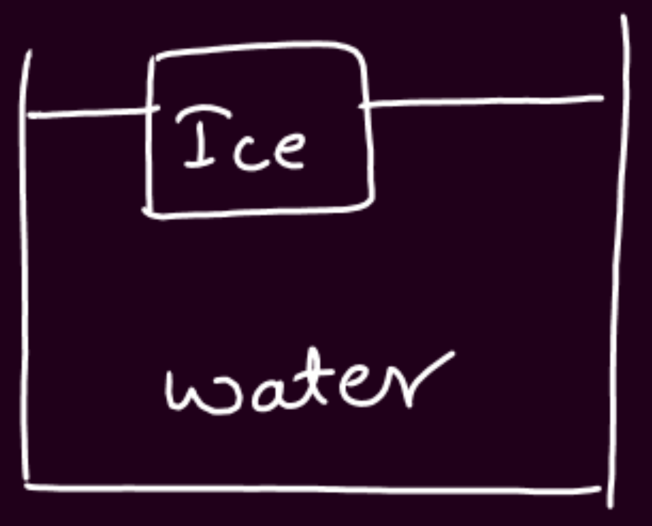
$$2.4V_1 - V_1 = V_2 - 0.3V_2$$

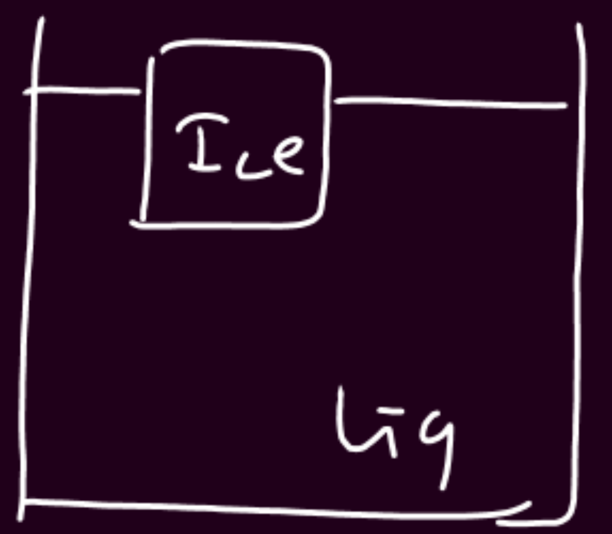
$$1.4V_1 = 0.7V_2$$

$$\frac{V_1}{V_2} = \frac{1}{2}$$

$\boxed{\text{Extra}}$

$$\frac{m_1}{m_2} = \frac{2.4\ V_1}{0.3\ V_2} = 8 \times \frac{1}{2} = 4$$

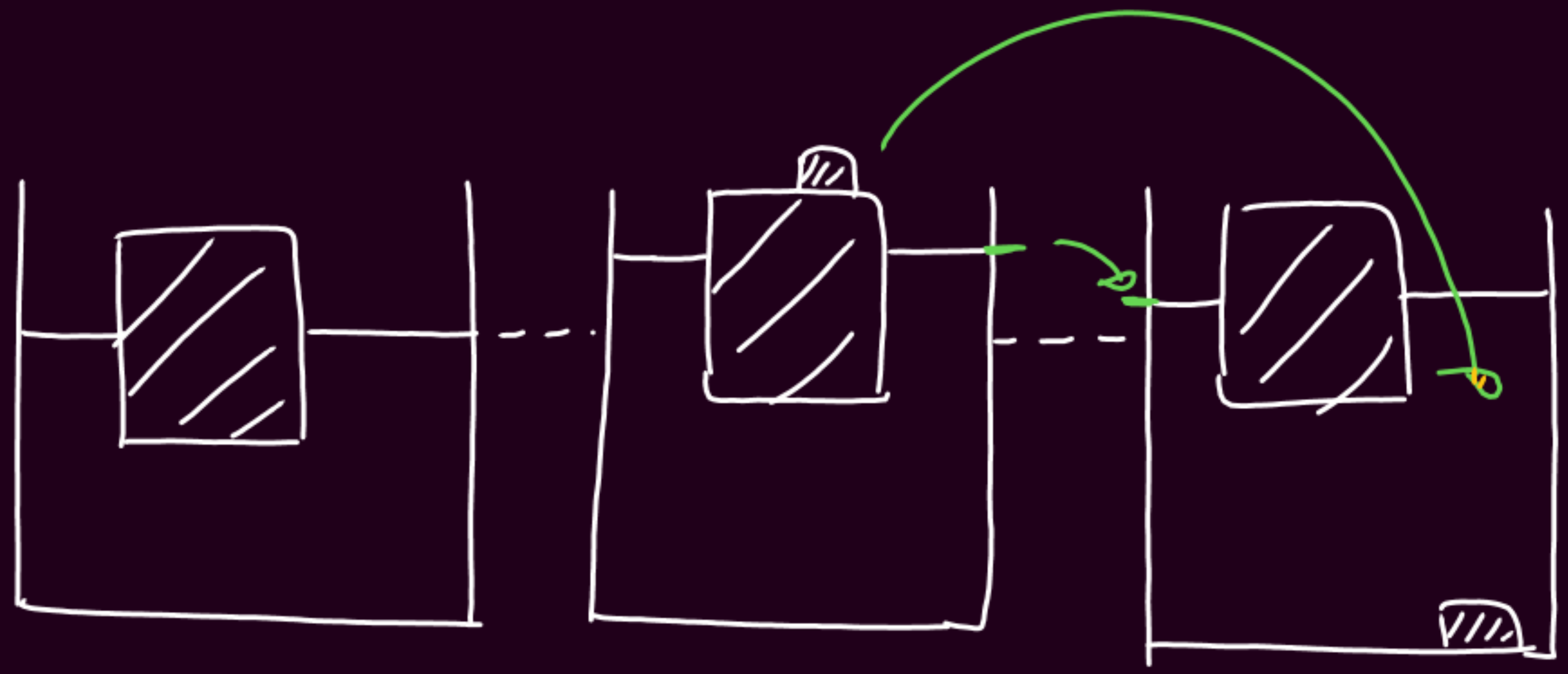




$\rho_{liq} < \rho_w$ $\rho_{liq} > \rho_w$

Same

$\boxed{\text{level } \downarrow}$ $\boxed{\text{level } \uparrow}$



coin float

water displaced is equal to wt of coin

coin sink

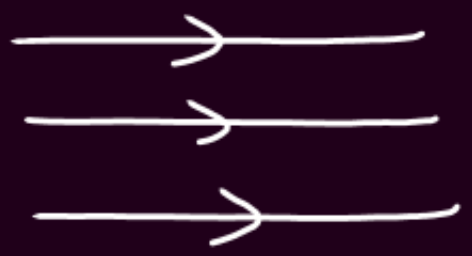
water displaced is equal to vol. of coin

Ray Optics

Light →

Ray → [Rectilinear]
→ [Reversibility]

parallel



Diverging

Converging

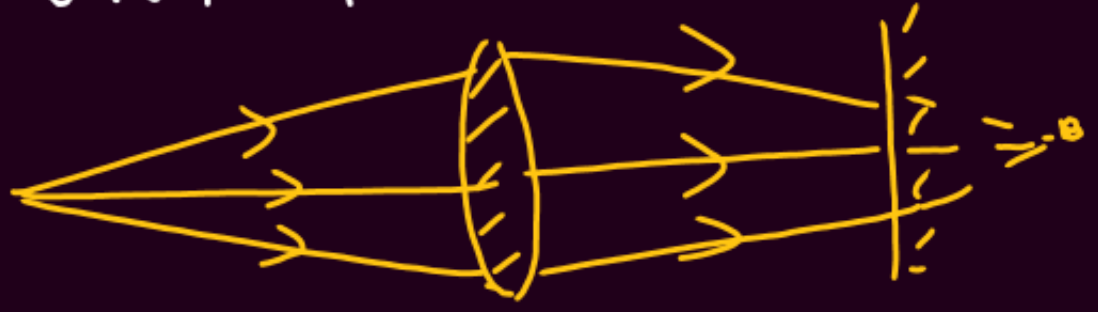
Source → Self luminous [Sun / Fire]
→ Due to Reflected [Moon]

Real

RO

RO

Virtual



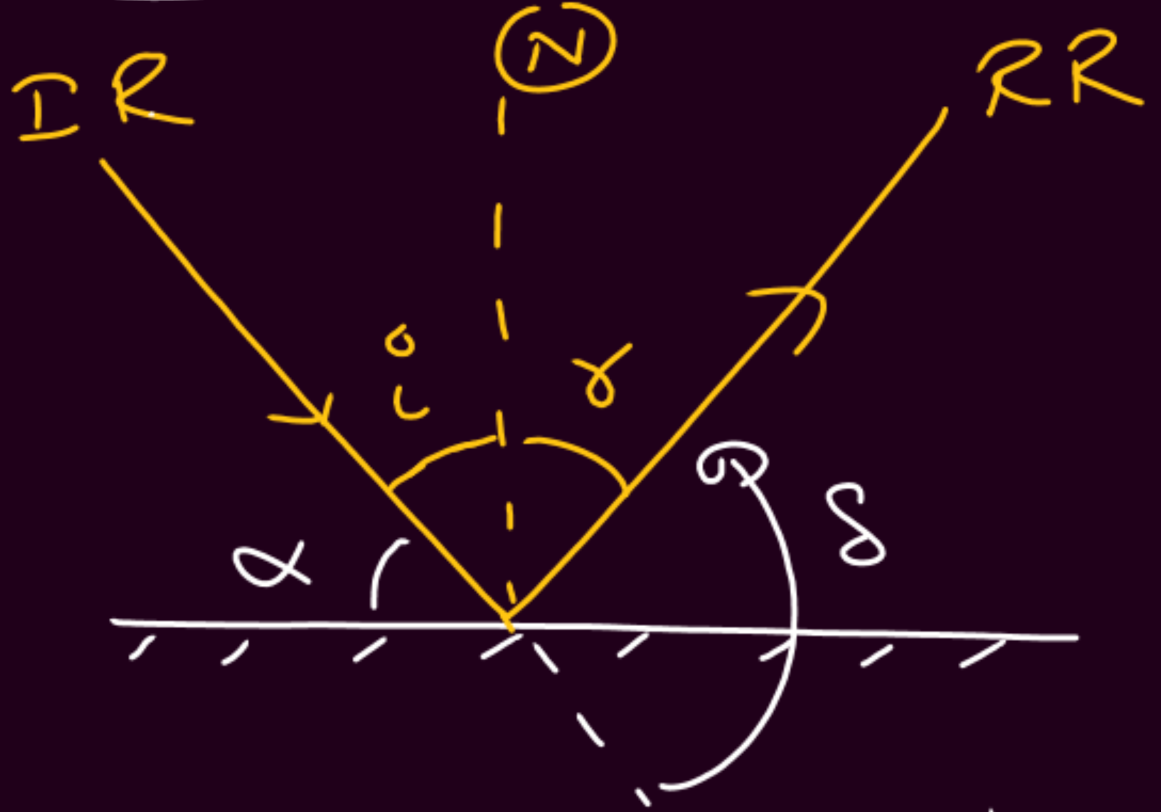
VO

VO

IR → intersect → Object

Ref → — " —— → image

Laws of Reflection

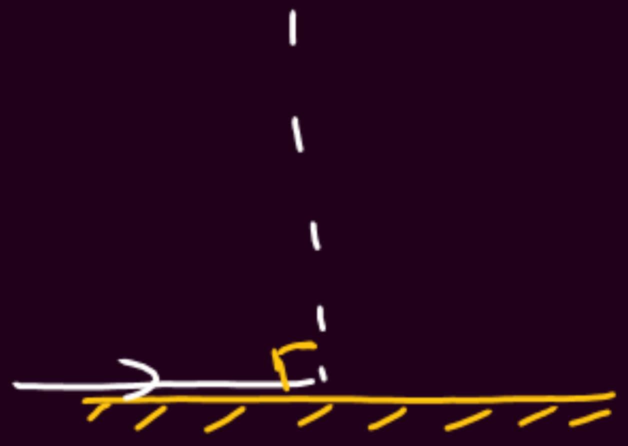


α = glancing angle

① IR, RR, & N lie in same plane

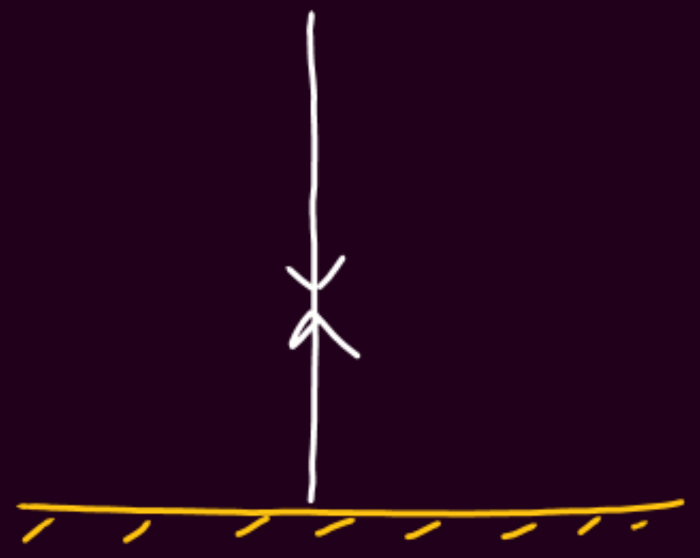
② $\angle i = \angle r$

$\delta = 180^\circ - 2i$ (Deviation)



grazing incidence

$i = 90^\circ$

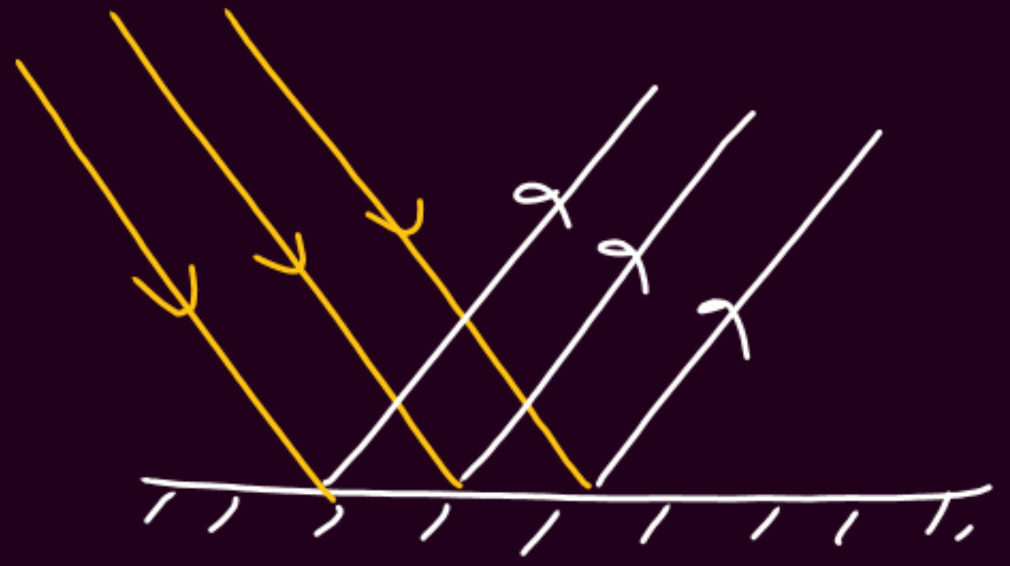


$\boxed{\text{Normal}}$ Incidence

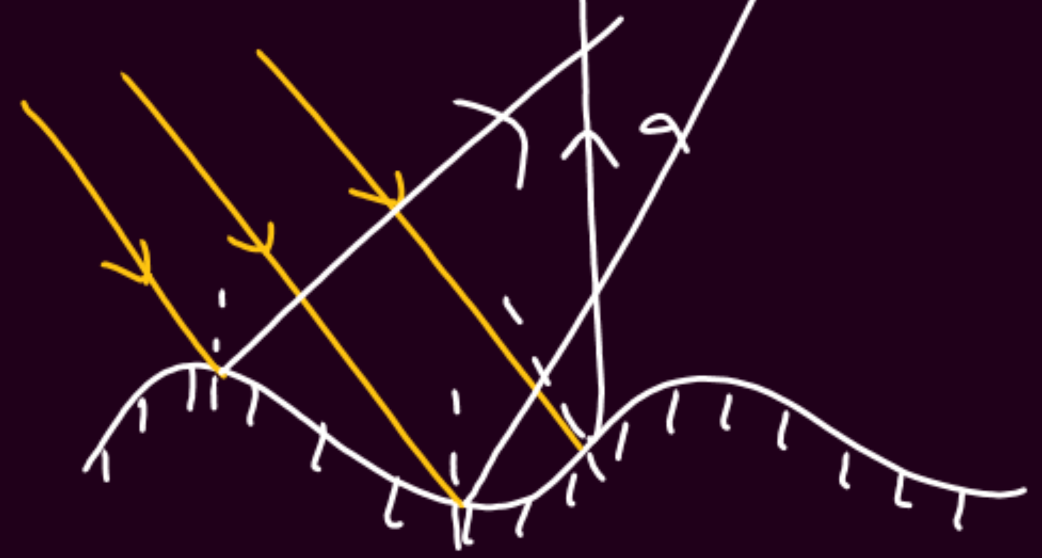
$i = 0$

"Retraces the path"

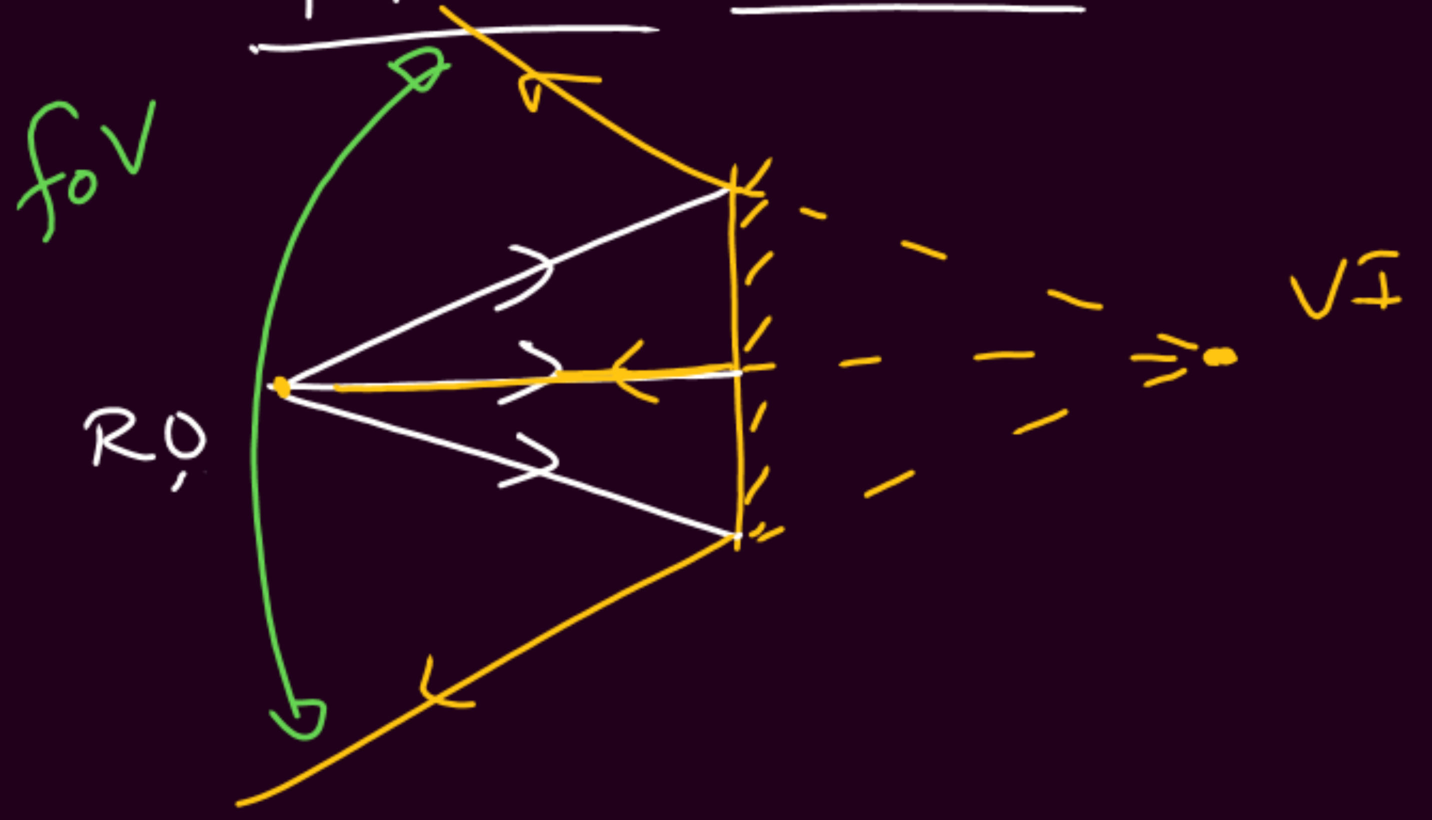
Regular / Specular Ref.

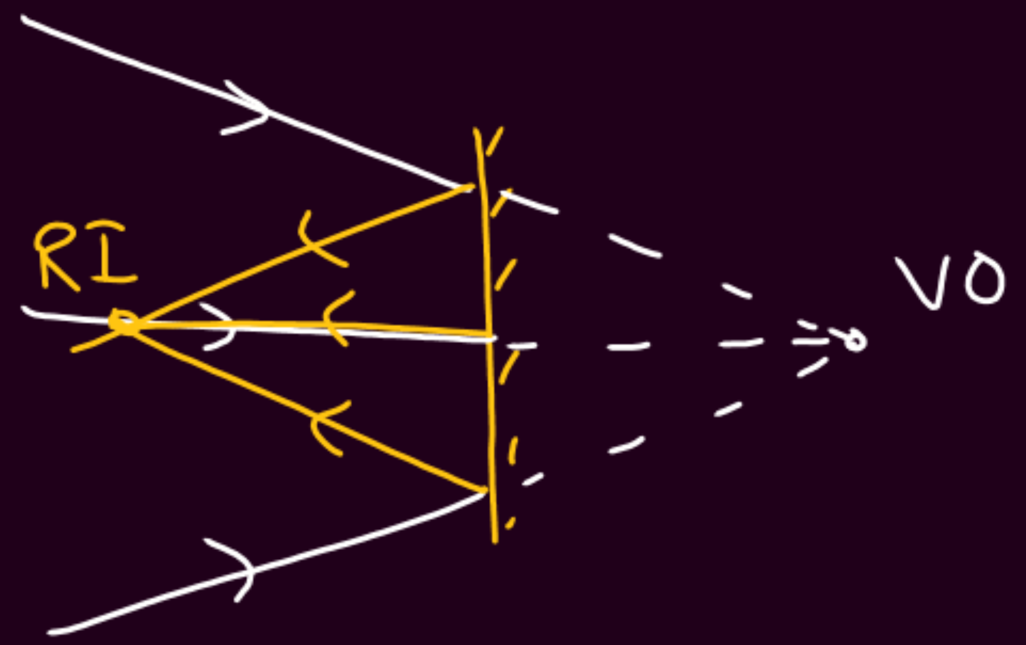


Irregular / Diffused Ref



Plane Mirror





foV → field of View

$\Rightarrow \begin{cases} RO \rightarrow VI \\ VO \rightarrow RI \end{cases}$

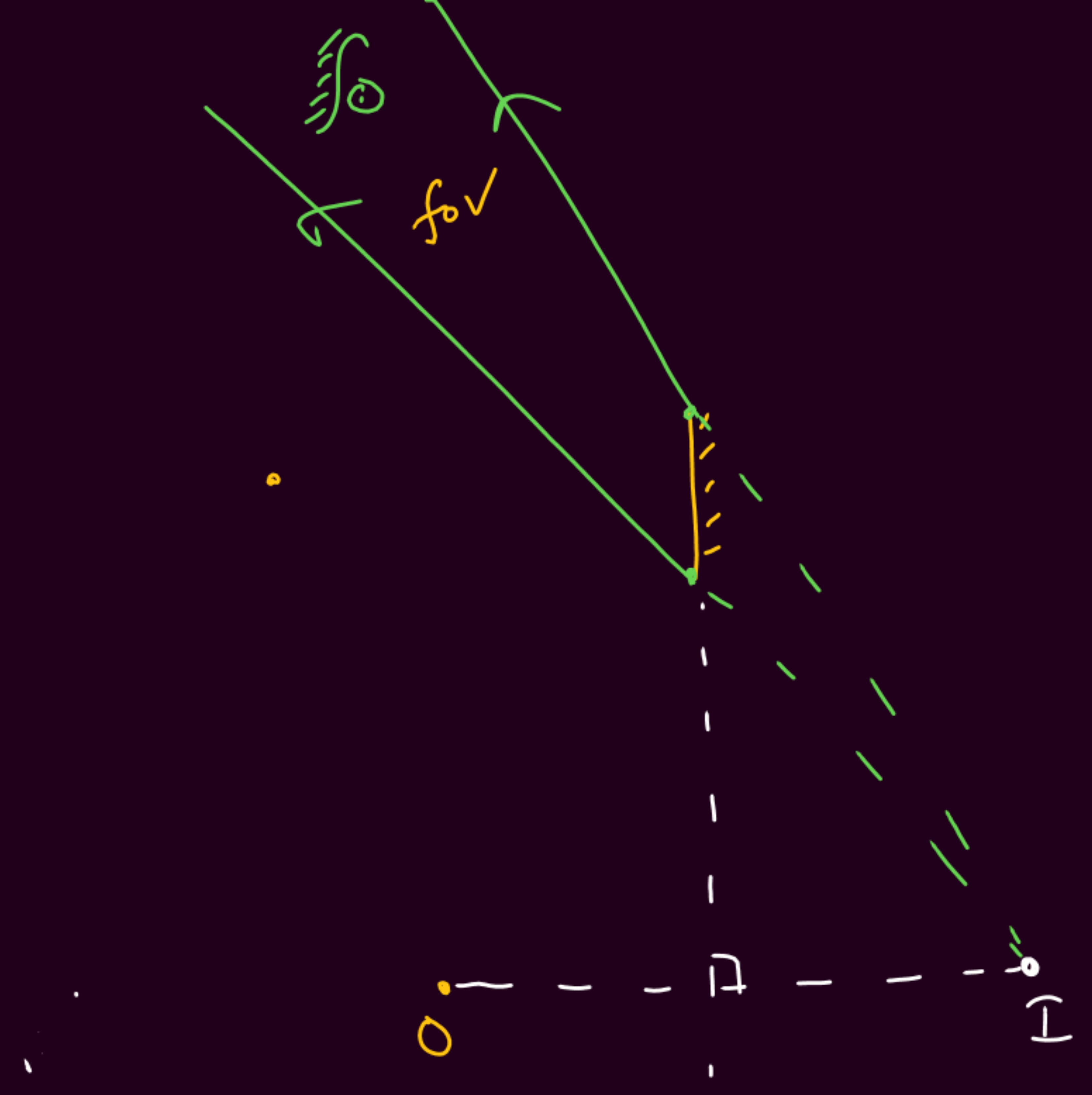
⇒ $\perp^r$ bisects (O & I are equidistant from plane mirror)

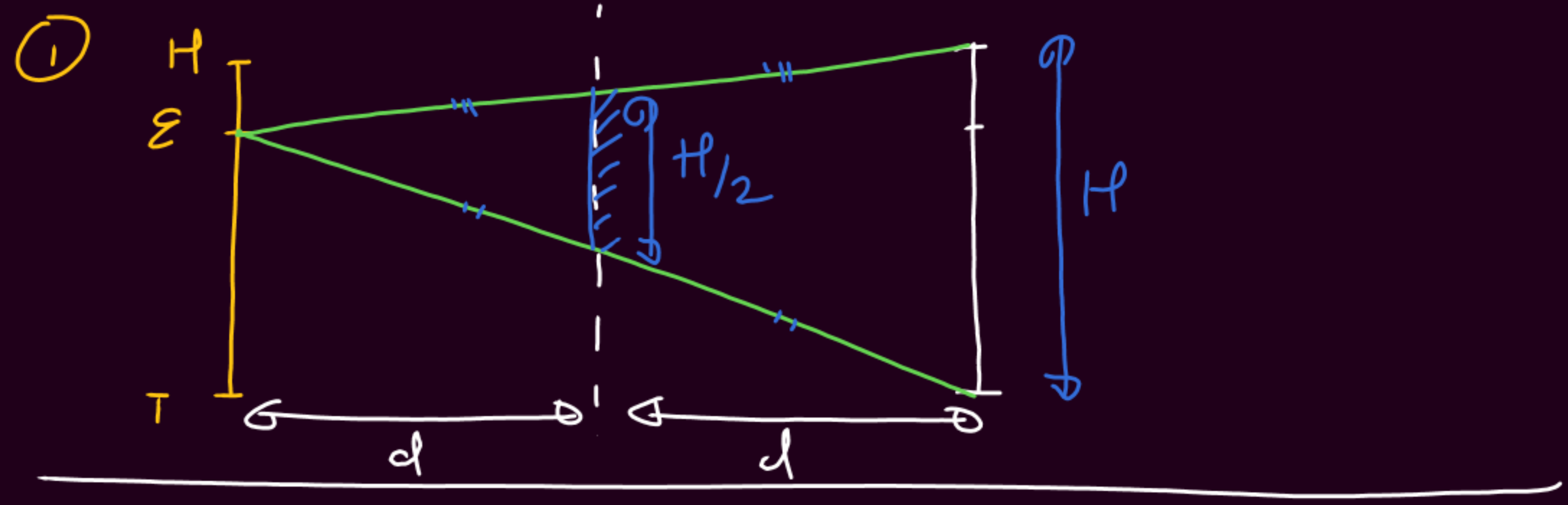
⇒ Same Size

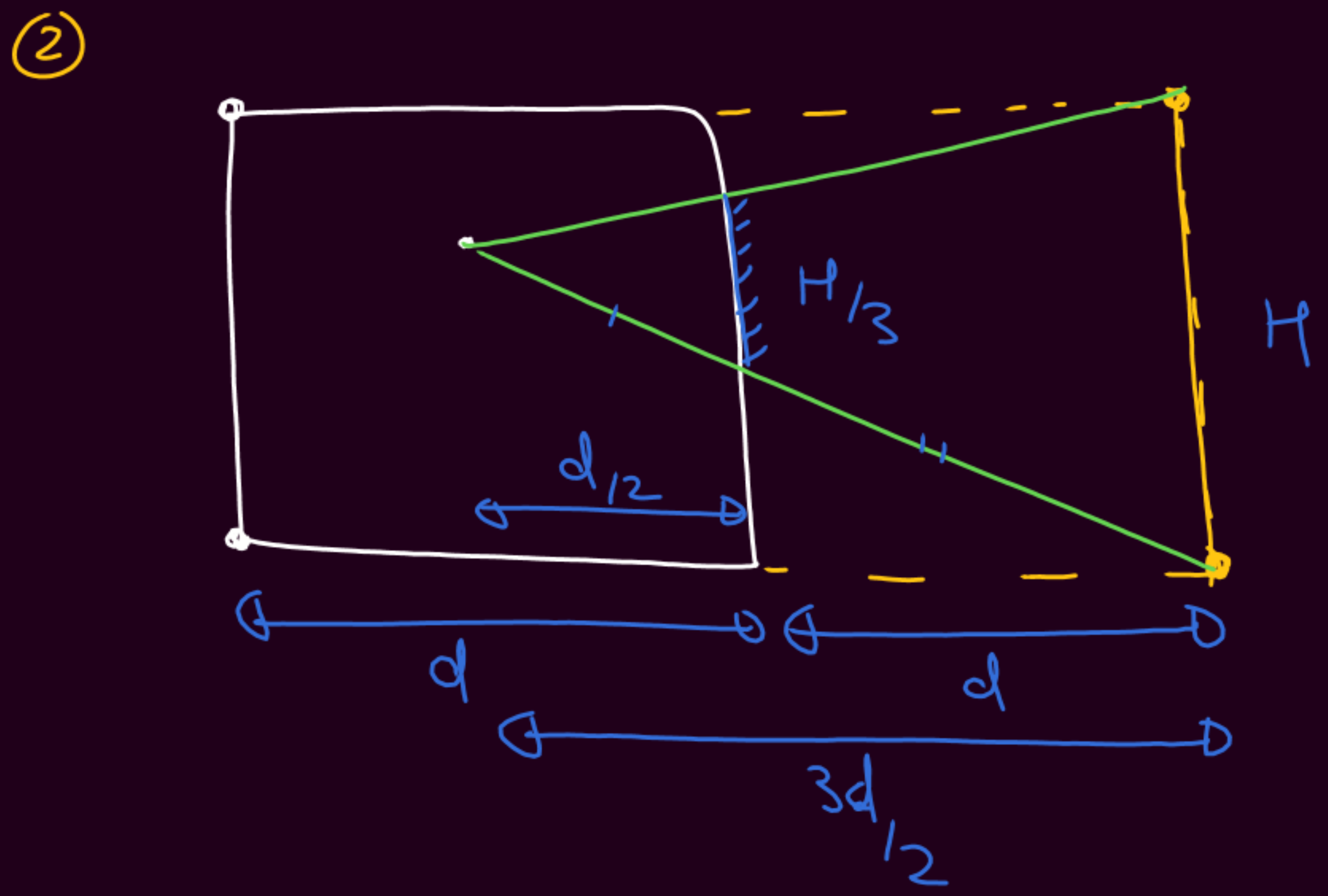
⇒ Upright

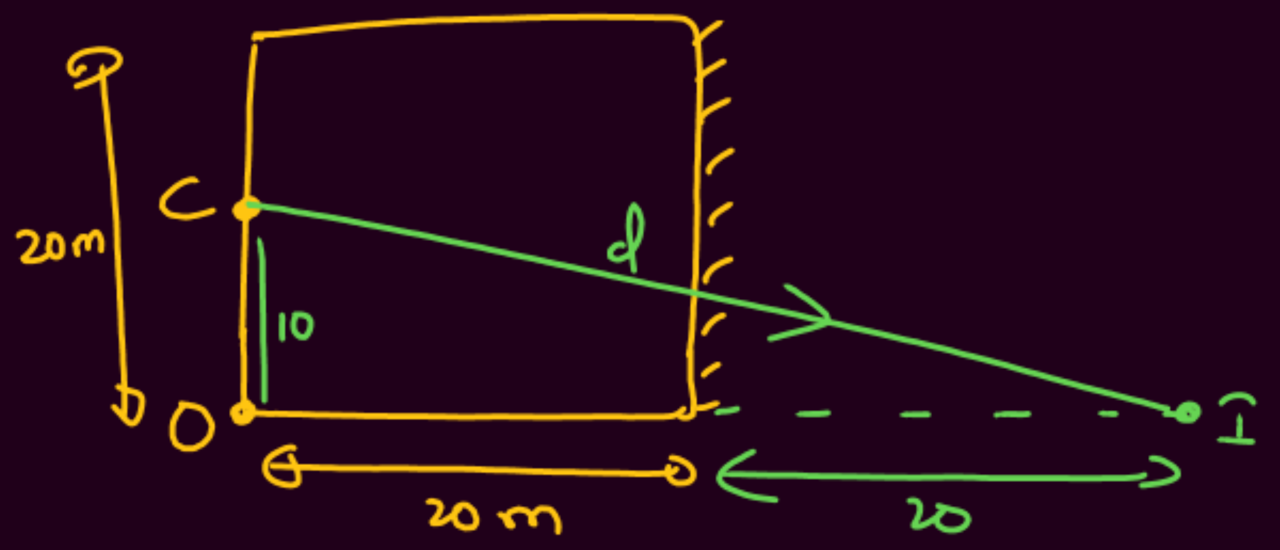
⇒ Lateral inversion [Left → Right]

Clock $\boxed{3:20}$ → image $\boxed{8:40}$


fov
O
I

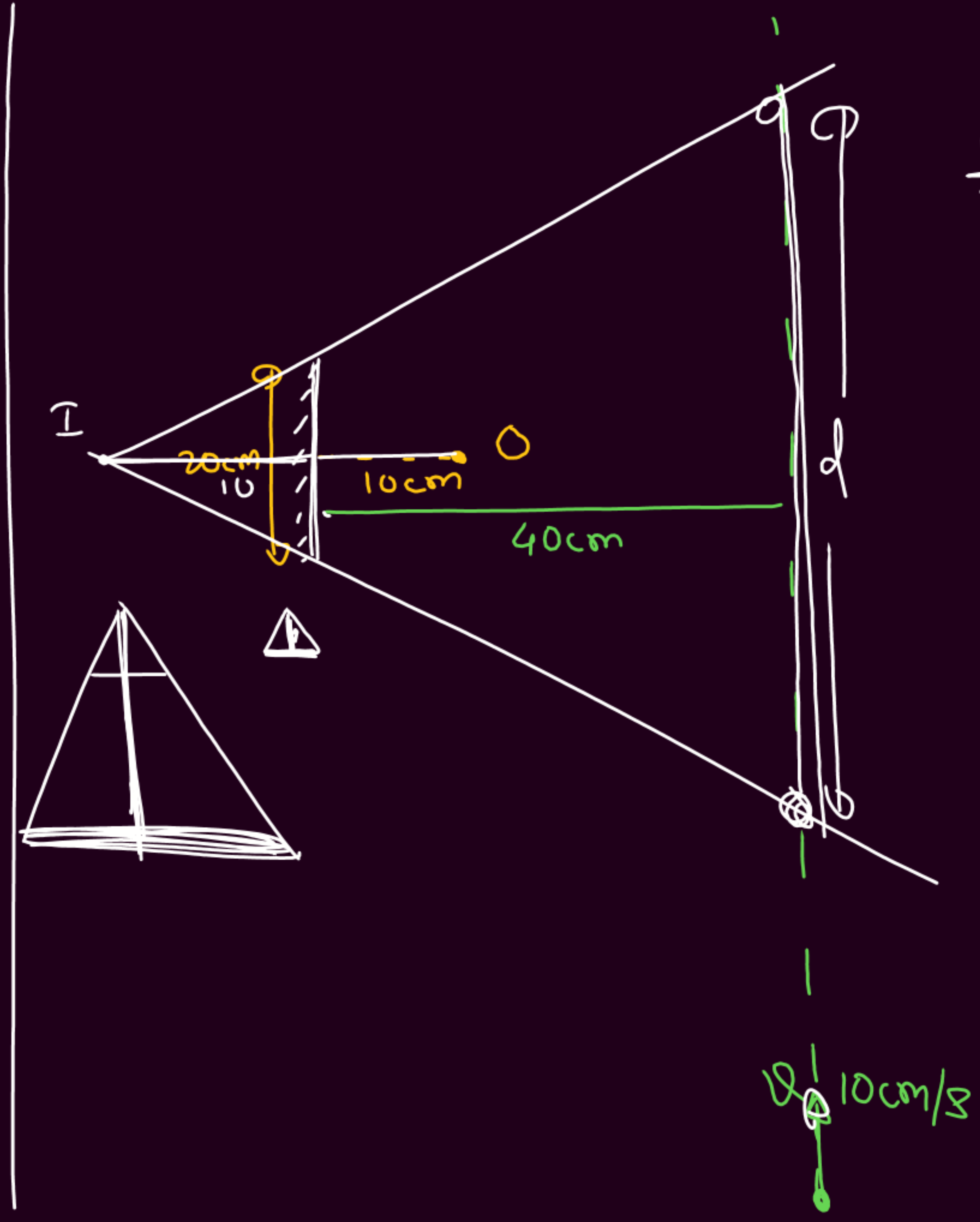

①
H
E
T
H/2
H
d
d


②
H/3
H
d/2
d
d
3d/2



$$d = \sqrt{10^2 + 40^2}$$

$$\boxed{d = 10\sqrt{17}\ m}$$



$\triangle$

$$\frac{10}{20} = \frac{40+10}{d}$$

$$d = 100\,cm$$

$$t = \frac{d}{v} = \frac{100}{10} = \underline{\underline{10\,s}}$$

Pg -153

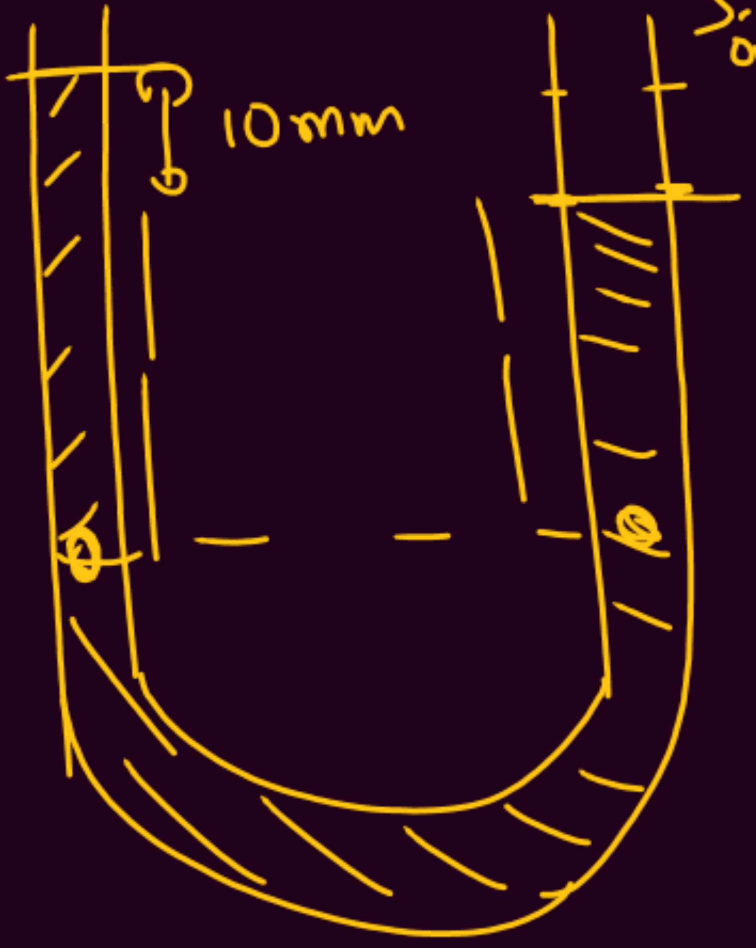
7

$W.D. = T\,\Delta A$

$3\times10^{-4} = T\,(20-8)\times10^{-4}\times 2$

8

$\rho_{oil}(65+65+10) = \rho_w(65+65)$



22

$l\cos 60° = \frac{2T(1)}{\rho g r}$

4

$W.D. = SE_n - SE_1 = 2(SE_1) - (SE_1)$

$= SE_1 = 4\pi r^2 T$

$\frac{SE_n}{SE_1} = n^{1/3} = 8^{1/3} = 2$

25

Ice w

$\left[\frac{M}{\rho} - \frac{M}{\sigma}\right]$

$T4\pi R^2 - T4\pi r^2(n)$

1

2 d

$mg = 2Tl$

26

$$10\,cm = \frac{2T_1(1)}{(1)g\gamma}$$

$$-3.112 = \frac{2T_2\cos 135^\circ}{(13.6)g\gamma}$$

27

$$mg = \boxed{2T\ell}$$

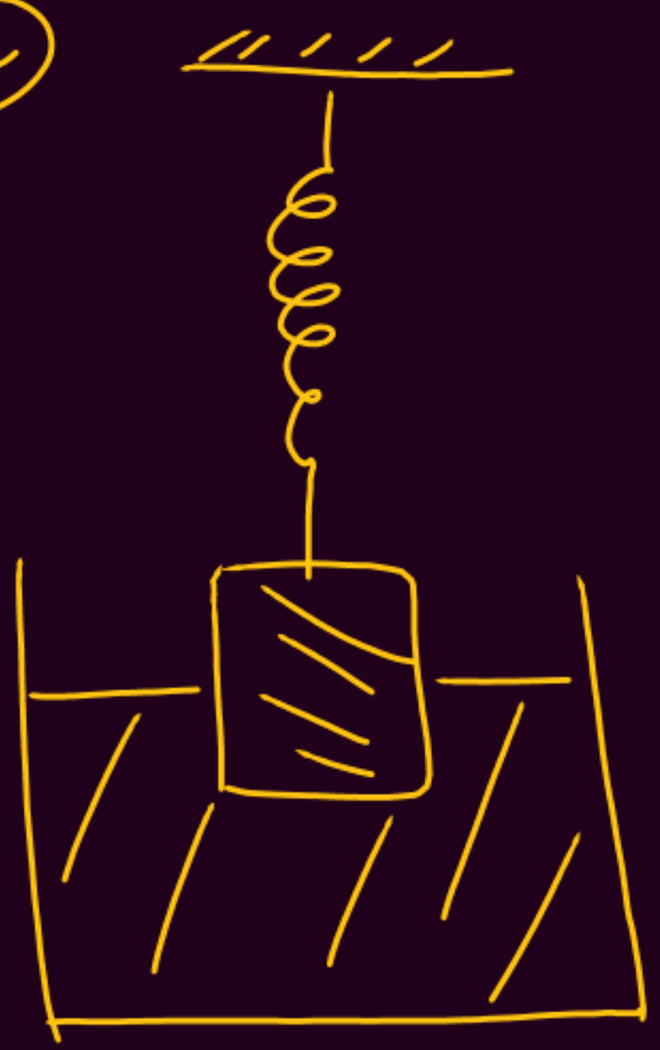
36

$$\frac{V_{sub}}{V} = \frac{\alpha}{\beta}$$

$$1 - \frac{\alpha}{\beta}$$

$$\cos 135^\circ = -\frac{1}{\sqrt{2}} = -0.7$$

42



$$kx_0 = w - B$$

$$kx_0 = Mg - \sigma\frac{V}{2}g$$

$$kx_0 = Mg\left(1 - \frac{\sigma AL\cancel{g}}{2M\cancel{g}}\right)$$

$$w = 8\pi R^2 T$$