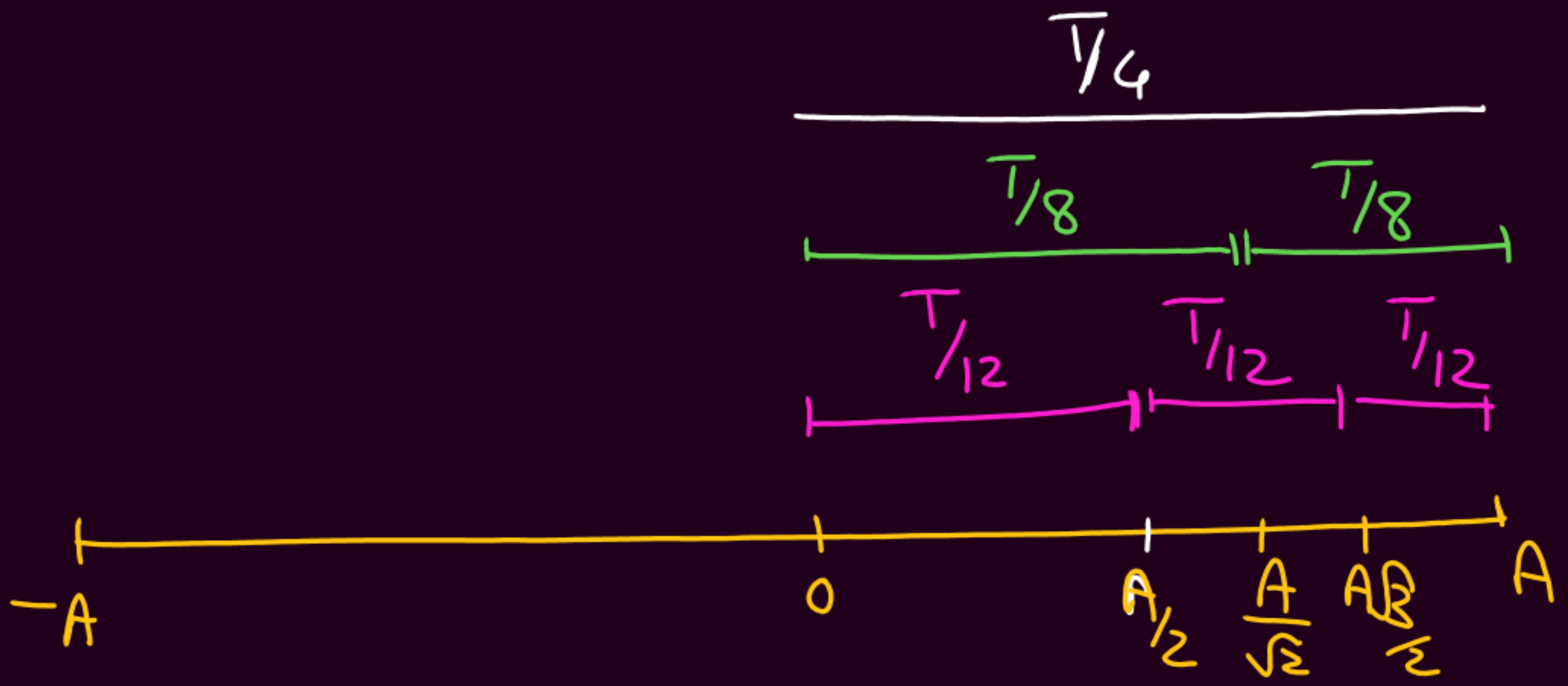
SHM

Phase / Time

$$x = A \sin(\omega t)$$



Q $T = 12\,s$

① M.P. to Ext. Pos

$$\frac{T}{4} = \frac{12}{4} = 3\,s$$

② M.P. to $x = A/2$

$$\frac{T}{12} = \frac{12}{12} = 1\,s$$

③ Ext. to Ext.

$$\frac{T}{4} + \frac{T}{4} = \frac{T}{2} = \frac{12}{2} = 6\,s$$

Q. Second's Pendulum

$$\boxed{T = 2\,s}$$

$$\frac{T}{4} = \frac{2}{4} = 0.5\,s$$

Q. $x = 10 \sin\left(\frac{\pi}{3} t\right)$

① Time Period $T = \dfrac{2\pi}{\pi/3} = 6\,s$

② $x = 5\,cm$ $\quad t = ?$

$A/2 \qquad \dfrac{T}{12} = \dfrac{6}{12} = 0.5\,s$

Q $x = 20 \sin\left(\frac{\pi}{4} t + \frac{\pi}{3}\right)$

① $t = 0$

$T = \frac{2\pi}{\pi/4} = 8\,s$

$\frac{T}{12} = \frac{8}{12}$

②

$t = \frac{T}{4} + \frac{T}{12}$

③

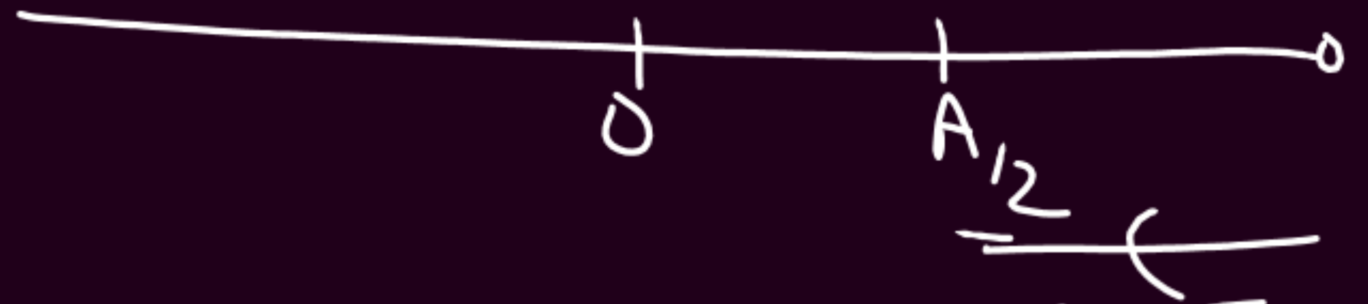
$t = \frac{T}{12} + \frac{T}{12} = \frac{T}{6}$

$x = A\sin(\underline{\omega t})$ — Starting from M.P.

$x = A\cos(\omega t) = A\sin(\underline{\omega t + \frac{\pi}{2}})$ — " — Ext. Posn

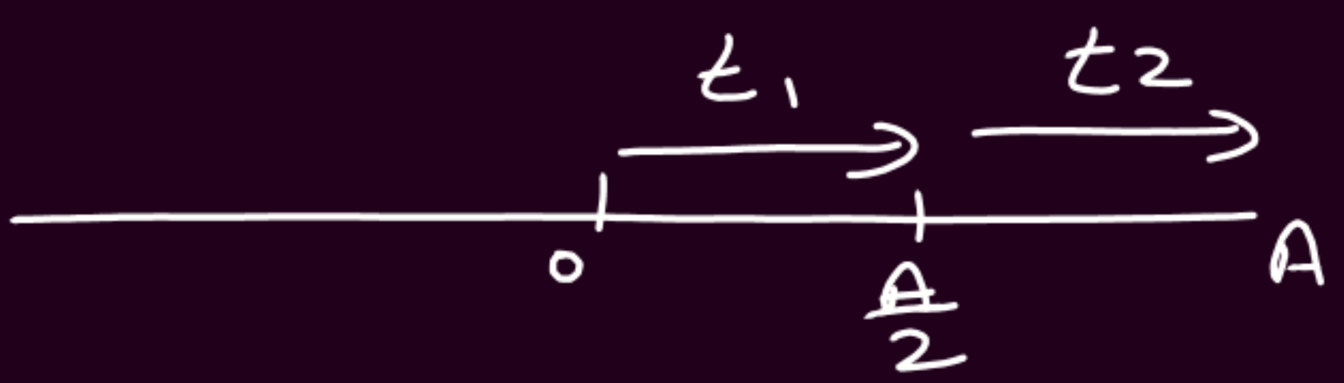
Q $x = 20\cos\left(\frac{\pi}{2}t\right)$ cm

Find time when $x = 10$ cm



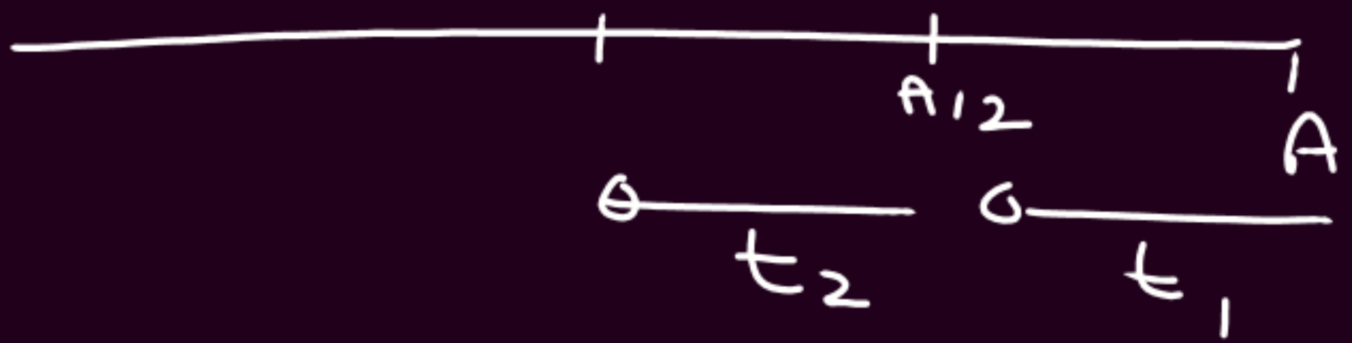
$\frac{T}{12} + \frac{T}{12} = \frac{T}{6}$

Q.



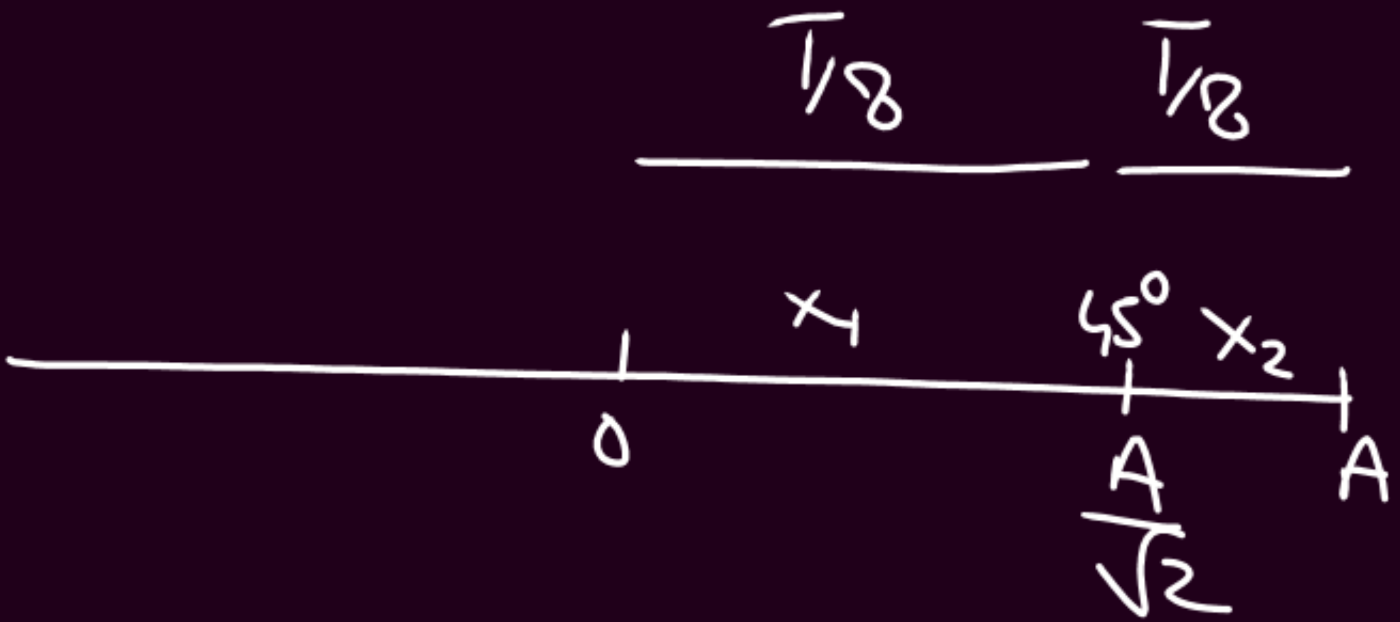
$$\frac{t_1}{t_2} = \frac{T/12}{\frac{T}{12} + \frac{T}{12}} = \frac{1}{2}$$

Q.



$$\frac{t_1}{t_2} = \frac{\frac{T}{12} + \frac{T}{12}}{\frac{T}{12}} = \frac{2}{1}$$

Q.

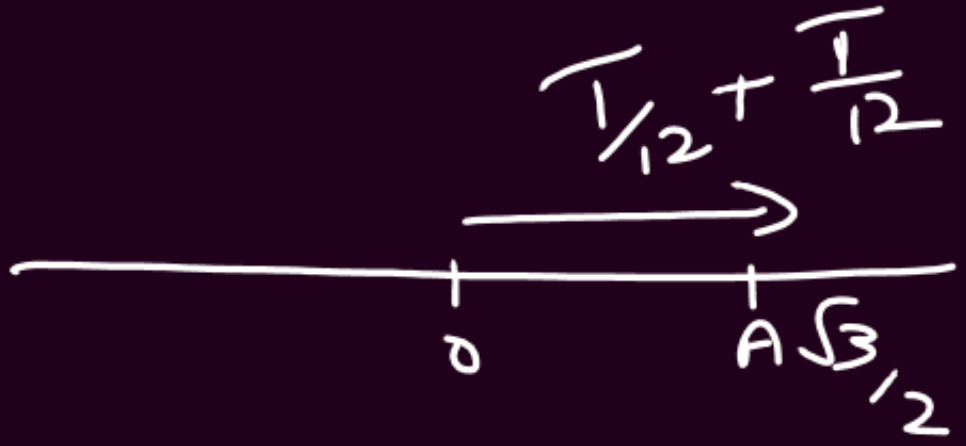


$$\frac{x_1}{x_2} = \frac{A/\sqrt{2}}{A - A/\sqrt{2}} = \frac{1}{\sqrt{2} - 1}$$

Q. $T = 6s$

Starting from M.P.

$$\boxed{\begin{array}{l} v \longrightarrow \dfrac{v_{max}}{2} \\[2ex] x \longrightarrow \dfrac{A\sqrt{3}}{2} \end{array}}$$



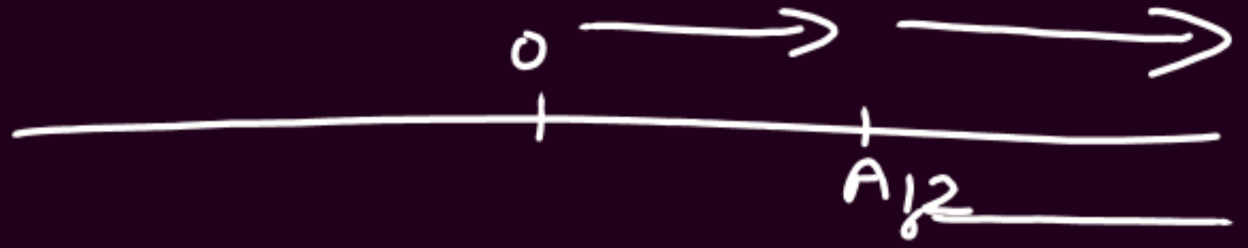
$$t = \frac{T}{12} + \frac{T}{12} = \frac{T}{6} = \frac{6}{6} = 1s$$

$$\frac{\omega A}{2} = \omega\sqrt{A^2 - x^2}$$

$$\boxed{x = \frac{A\sqrt{3}}{2}}$$

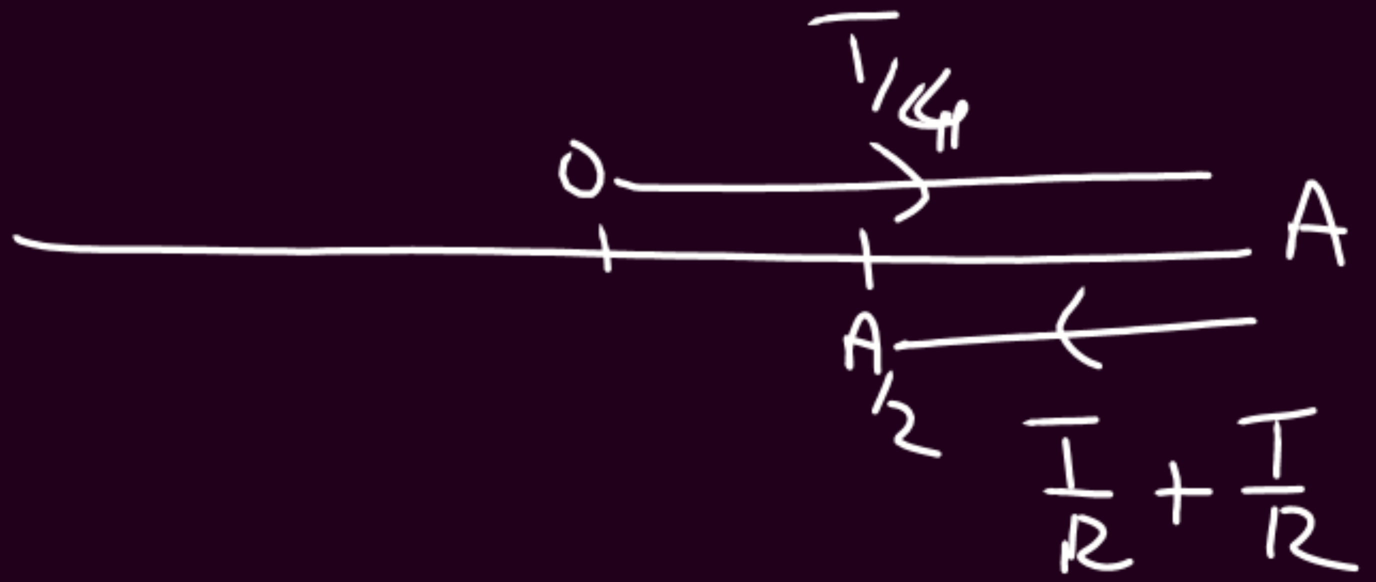
Q. $T = 8s$

time to complete $\boxed{\frac{3}{8}^{th}}$ oscillation?



$$(4A)\,\frac{3}{8} = \frac{3A}{2} = 1.5\,A$$

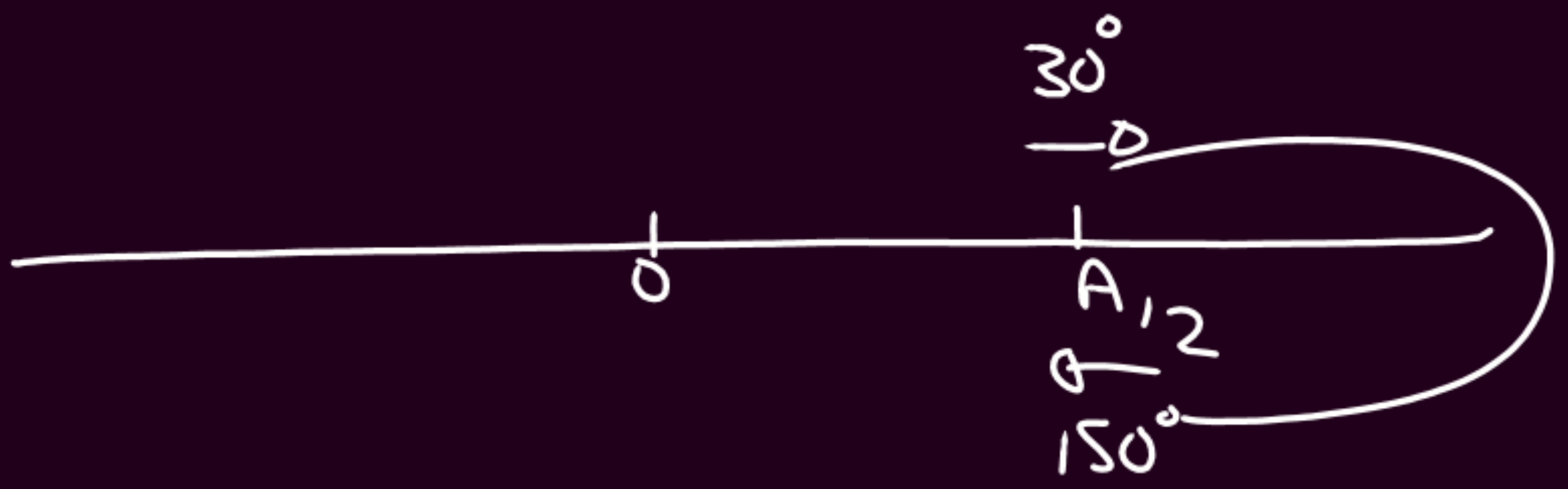
$$t = \frac{T}{4} + \left(\frac{T}{12} + \frac{T}{12}\right)$$



Q. Two objects

- Same f
- Same A
- Along Same line

$$x = A/2$$



$$\Delta\phi = 150^\circ - 30^\circ = \underline{\underline{120^\circ}} = \frac{2\pi}{3}$$

Extra Case

Two objects → Same f

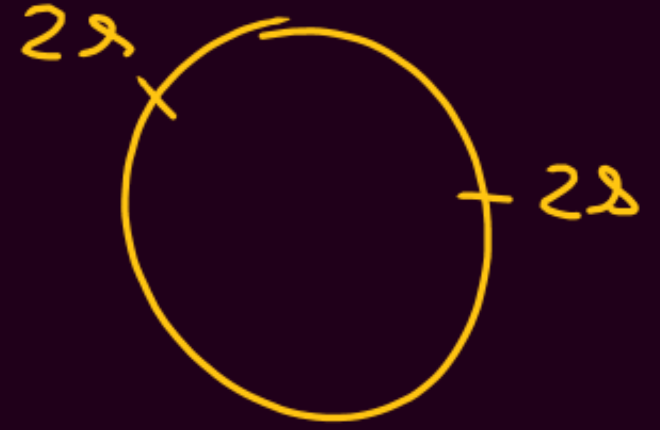
→ Same A

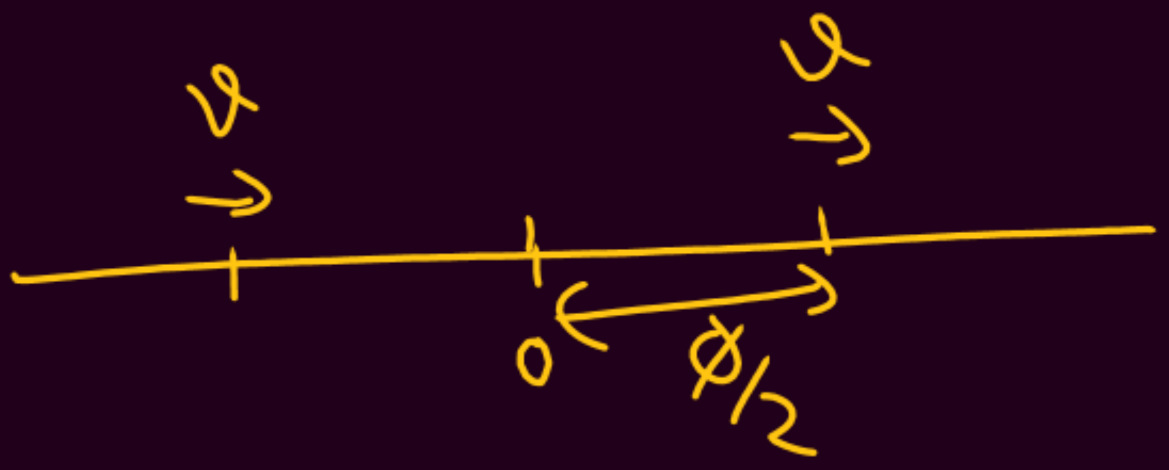
→ Same line

→ with a phase difference $= \phi$

Find max. possible separation between them

$$\boxed{\text{max separation} = 2A\sin\left(\frac{\phi}{2}\right)}$$





Spring is cut into parts

l

$$K_{eq} = \left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

$K_{eq} < K_1$

$K_{eq} < K_2$

Same Tension

$$K_1 x_1 = K_2 x_2 = Kx$$

$$K_1 = \frac{Kx}{x_1}$$

✓ $$\boxed{K_1 = \frac{Kl}{l_1}}$$

✓ $$\boxed{K_2 = \frac{Kl}{l_2}}$$

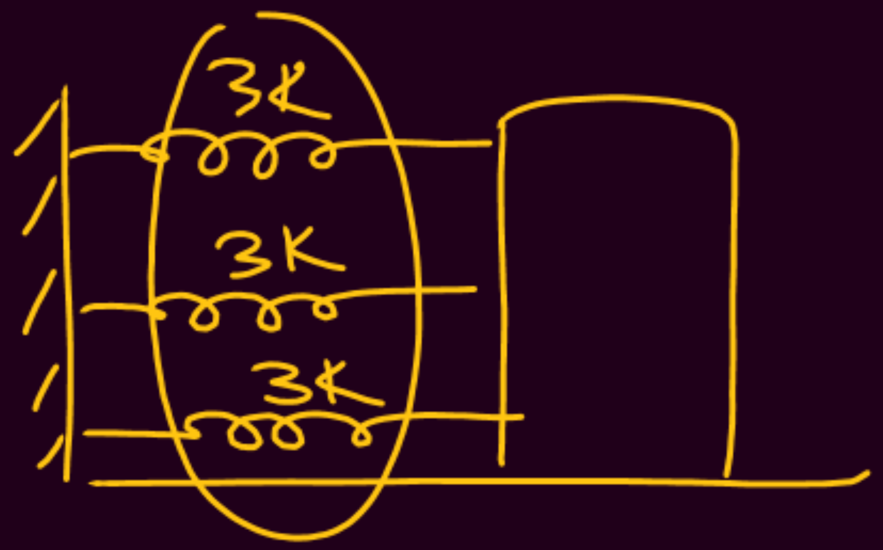
$K \rightarrow n$ equal parts

$\Downarrow$

$$K_{new} = \frac{Kl}{l/n}$$

$$\boxed{K_{new} = nK}$$

Q. $K_{new} = 3K$



$K_{eq} = 3K + 3K + 3K = 9K$

$T_0 = 2\pi\sqrt{\frac{m}{K}}$

$T_{new} = 2\pi\sqrt{\frac{m}{9K}} = \frac{T_0}{3}$

Q.

5

2 : 3

$K_1 = \frac{K(5)}{2}$ $K_2 = \frac{K(5)}{3}$

$K_{eq} = K_1 + K_2 = \frac{5}{2}K + \frac{5}{3}K = \frac{25}{6}K$

$T = 2\pi\sqrt{\frac{6m}{25K}} = T_0\sqrt{\frac{6}{25}} = \frac{T_0\sqrt{6}}{5}$

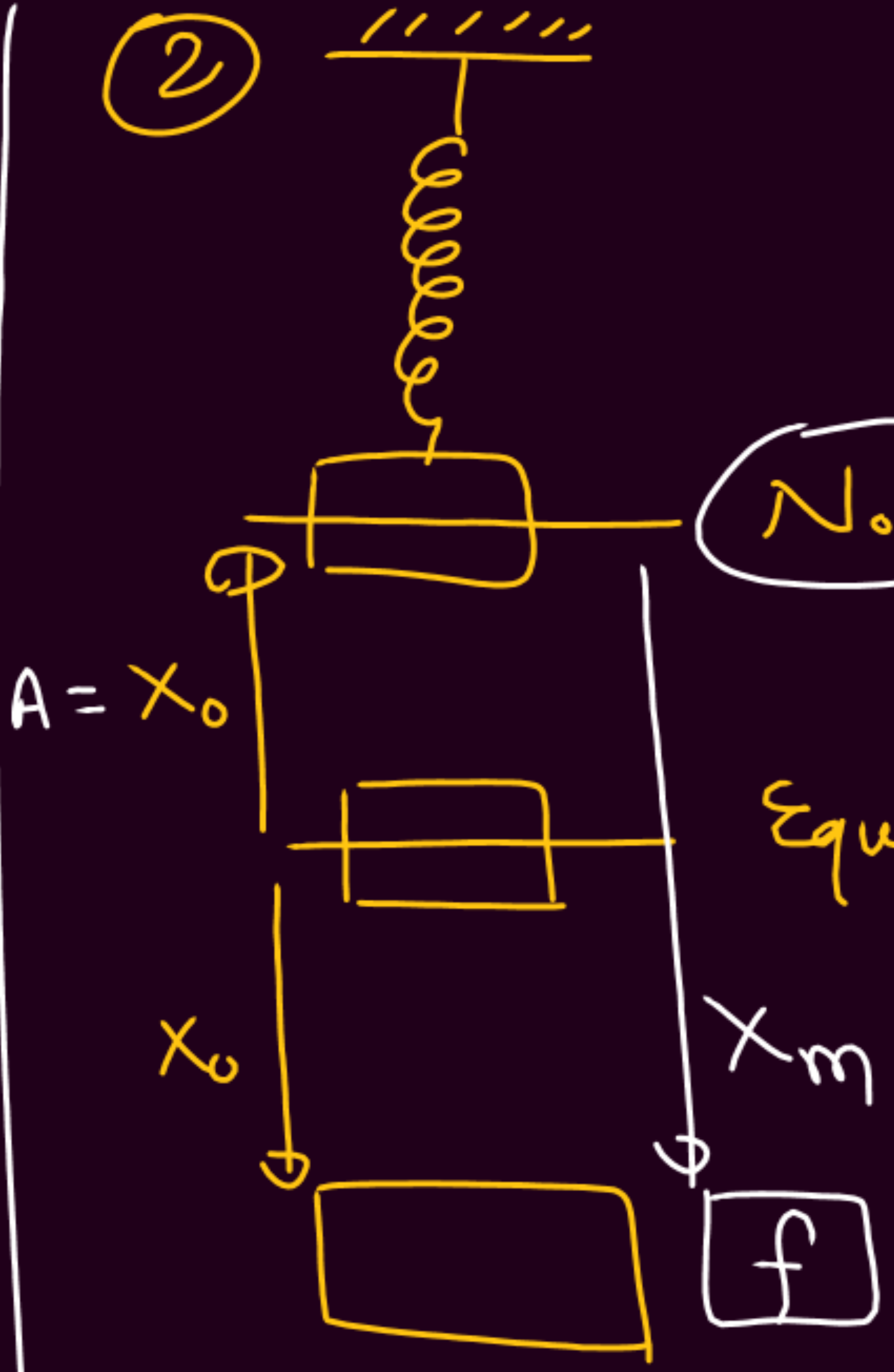
Case

① 

[Equilibrium]

$$Kx_0 = mg$$

Extension in Equilibrium $\boxed{x_0 = \frac{mg}{k}}$

② 

N.L. — i — $y = -A$

Equilibrium (M.P.) — $y = 0$

$y = A$

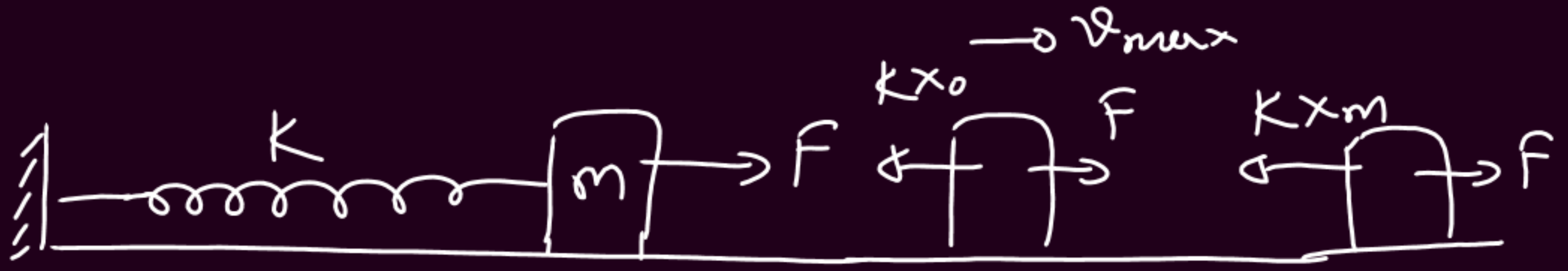
Max. Extension $\boxed{x_m = 2x_0 = \frac{2mg}{k}}$

COME

$$mg\,x_m = \frac{1}{2} k x_m^2$$

$$\boxed{x_m = \frac{2mg}{k}}$$

Case



① Equilibrium

$$F = kx_0$$

$$\boxed{x_0 = \frac{F}{K}}$$

② Max Ext.

$$\boxed{x_m = 2x_0 = \frac{2F}{K}}$$

$$Fx_m = \frac{1}{2}kx_m^2$$

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47 to 52,

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21, 23, 32,

Angular SHM

$$\tau = -K\theta$$

$$\alpha = -\omega^2 \theta$$

$$\theta = \theta_0 \sin(\omega t)$$

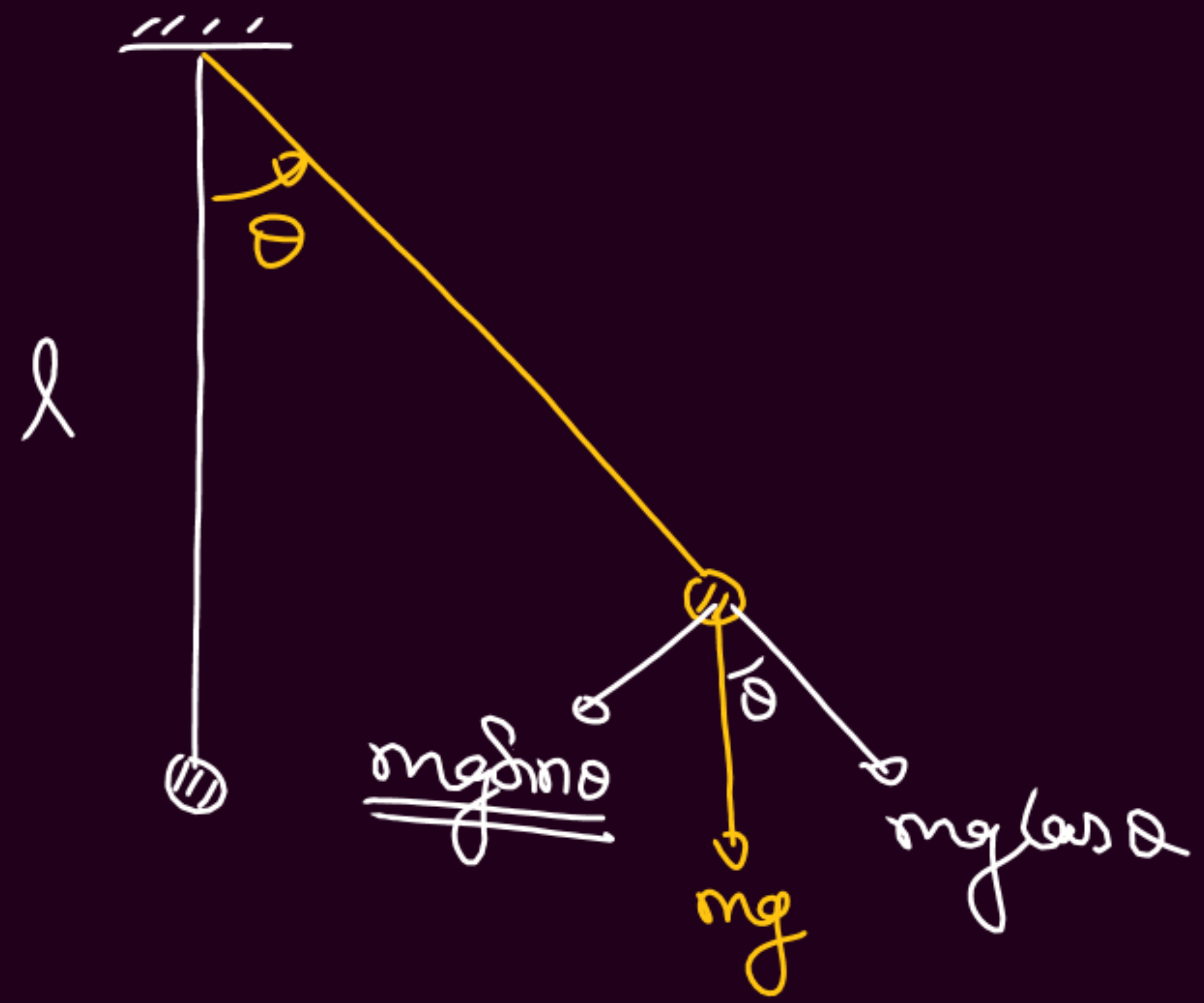
Linear SHM

$$F = -kx$$

$$a = -\omega^2 x$$

$$x = A \sin(\omega t)$$

Simple Pendulum → Oscillation



⇓

SHM (if θ is small)

⇓

(less than 5°)

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

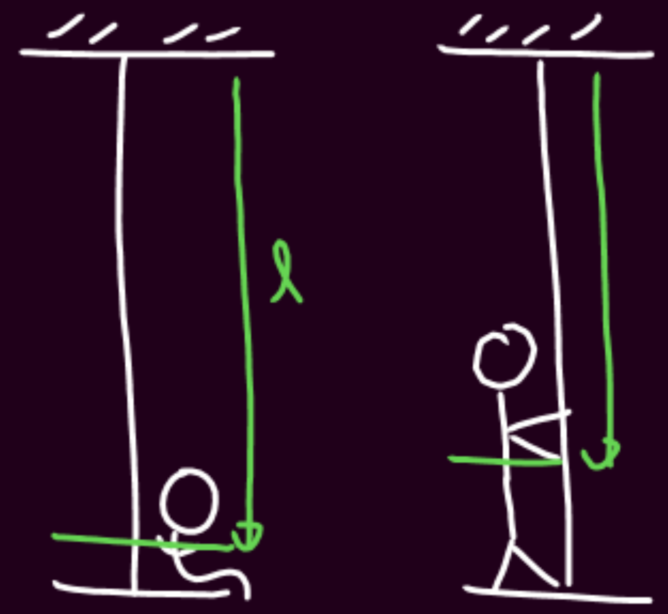
'Independent of mass'

① Second's Pendulum

$T = 2s$

$l \approx 1m$

②



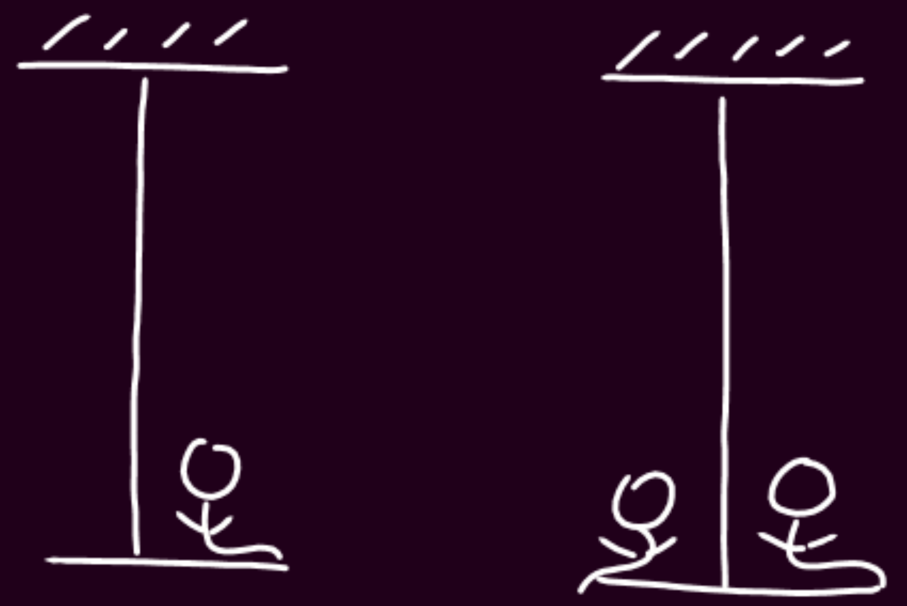
$l \downarrow$

$T \downarrow$

fast

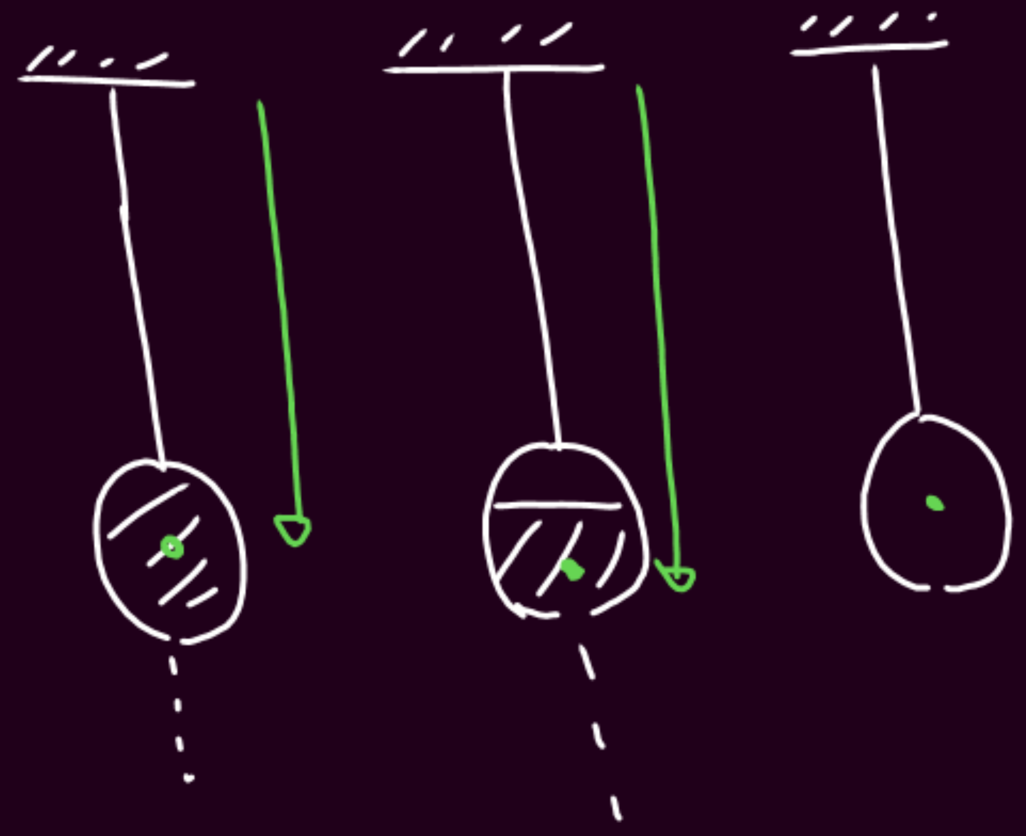
$T = 2\pi\sqrt{\frac{l}{g}}$

②



$T_1 = T_2$

③



1) Inc. continuously
2) Dec. —"—
3) ↑ then ↓
4) ↓ then ↑

Common Time Period

$$\boxed{T_c = n_1 T_1 = n_2 T_2}$$

$\boxed{\text{LCM}}$

2	3
4	6 $\boxed{2}$
$\boxed{3}$ 6	9
8	12
10	15
12	18
14	

Q. $T_1 = 2s$

$T_2 = 3s$

$$T_c = n_1 T_1 = n_2 T_2$$

$$n_1(2) = n_2(3)$$

$$\boxed{\frac{n_1}{n_2} = \frac{3}{2}}$$

$$T_c = 3(2) = \boxed{6s}$$

Q. $l_1 = 1m$

$l_2 = 1.21m$

$$n_1 T_1 = n_2 T_2$$

$$\frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{1.21}{1}}$$

$$\frac{n_1}{n_2} = \sqrt{\frac{121}{10}} = \frac{11}{10}$$

Q. $y = 3\sin(6\pi t) + 4\cos\left(\frac{3\pi}{2}t\right)$

$T_1 = \frac{2\pi}{6\pi}$ $\quad$ $T_2 = \frac{2\pi}{3\pi/2}$

$T_1 = \frac{1}{3}s$ $\quad$ $T_2 = \frac{4}{3}s$

$n_1 T_1 = n_2 T_2$

$n_1\left(\frac{1}{3}\right) = n_2\left(\frac{4}{3}\right)$

$\boxed{\frac{n_1}{n_2} = \frac{4}{1}}$

$T_c = n_1 T_1 = (4)\left(\frac{1}{3}\right) = \underline{\underline{\frac{4}{3}s}}$

$T_c = n_2 T_2 = (1)\left(\frac{4}{3}\right) =$

Q. $l_1 \longrightarrow T_1$

$l_2 \longrightarrow T_2$

$(l_1 + l_2) \longrightarrow T = ?$

1) $T = T_1 + T_2$

2) $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

✓3) $T^2 = T_1^2 + T_2^2$

4) $\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{l_2}{g}}$$

$$T = 2\pi\sqrt{\frac{l_1+l_2}{g}}$$

$$T_1^2 = 4\pi^2\frac{l_1}{g}$$

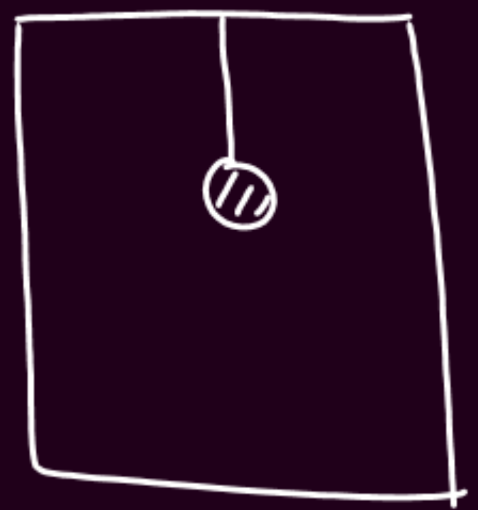
$$T_2^2 = 4\pi^2\frac{l_2}{g}$$

$$T_1^2 + T_2^2 = \frac{4\pi^2(l_1+l_2)}{g} = T^2$$

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

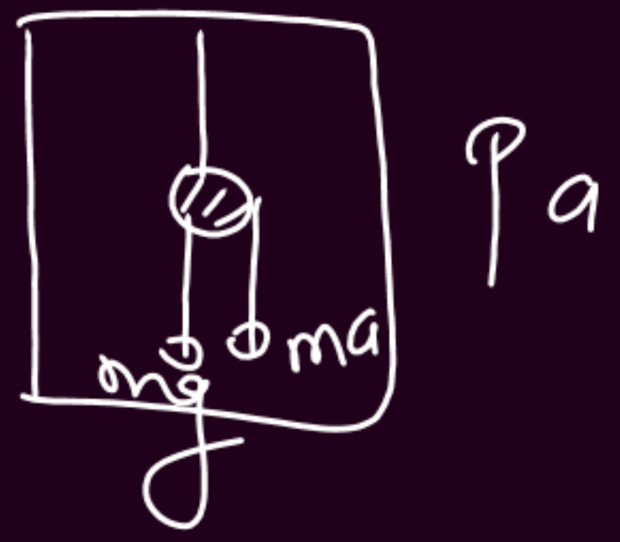
$$g_{eff} = \frac{\text{Tension in Equilibrium in NIF}}{\text{mass}}$$

Lift



1) Const. Vel. $\quad T = 2\pi\sqrt{\frac{l}{g}}$

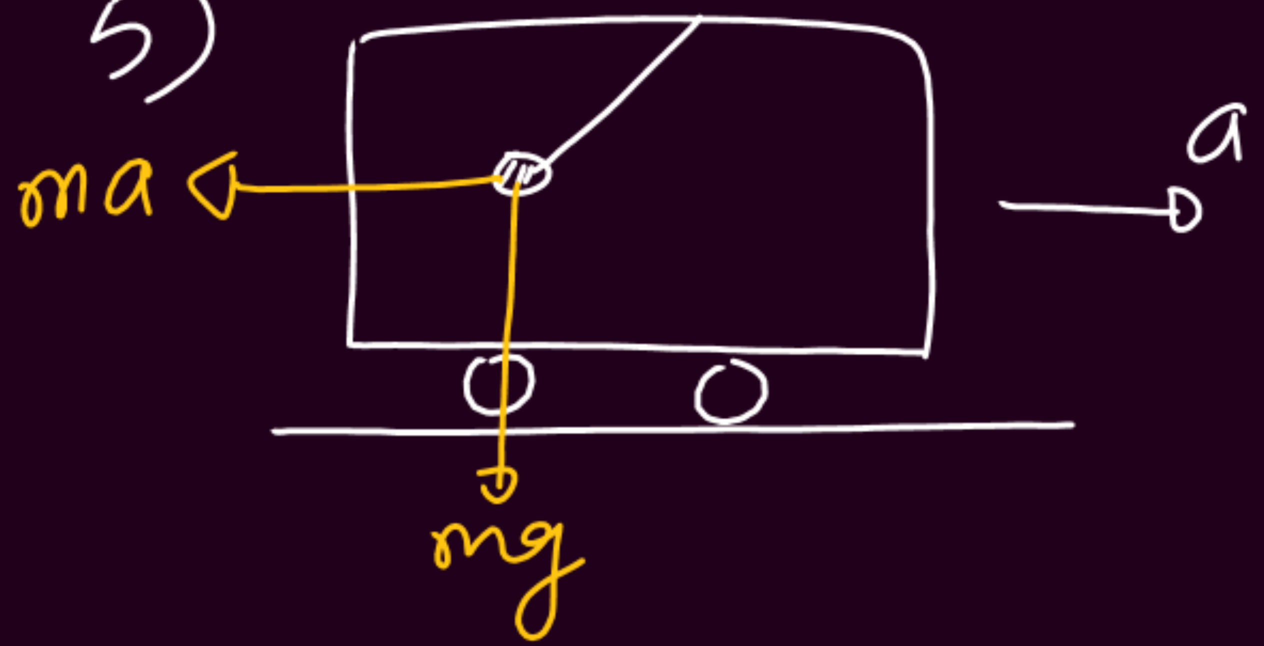
2) $\uparrow a$ $\quad T = 2\pi\sqrt{\frac{l}{g+a}}$



3) $\downarrow (a<g)$ $\quad T = 2\pi\sqrt{\frac{l}{g-a}}$

4) $\downarrow a=g$ "free fall" "satellite" $\quad T \to \infty$ (Does not oscillate)

5)

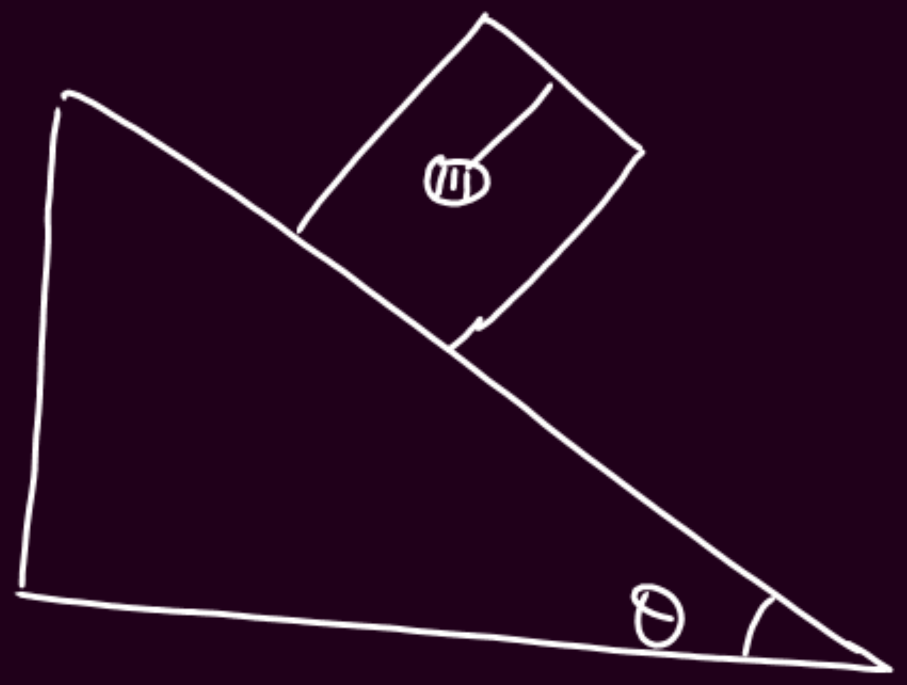


$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$$

6) Moon $T_m = 2\pi\sqrt{\frac{l}{g/6}}$

$$\boxed{T_m = \sqrt{6}\, T_{earth}}$$

7)

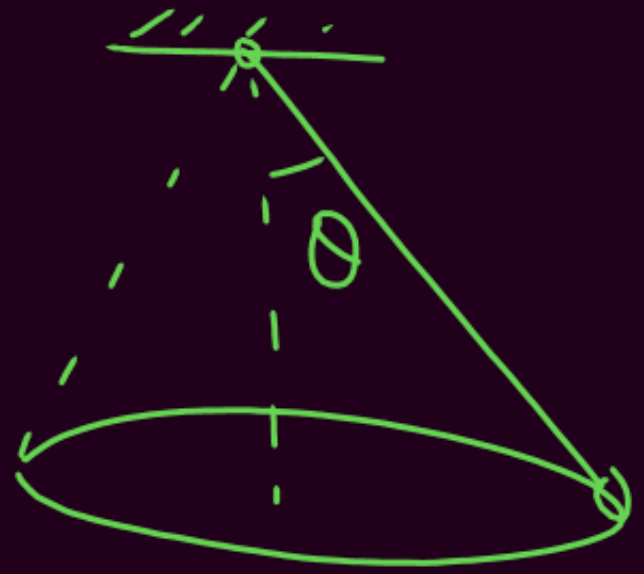


$$T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$$

Conical Pendulum → Periodic Motion

→ UCM

→ "Not SHM"



$$\boxed{T = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$$

Q.

$$\frac{3 = 2\cancel{\pi}\sqrt{\frac{\cancel{\ell}}{g}}}{2 = 2\cancel{\pi}\sqrt{\frac{\cancel{\ell}}{g+a}}}$$

$$\frac{3}{2} = \sqrt{\frac{g+a}{g}}$$

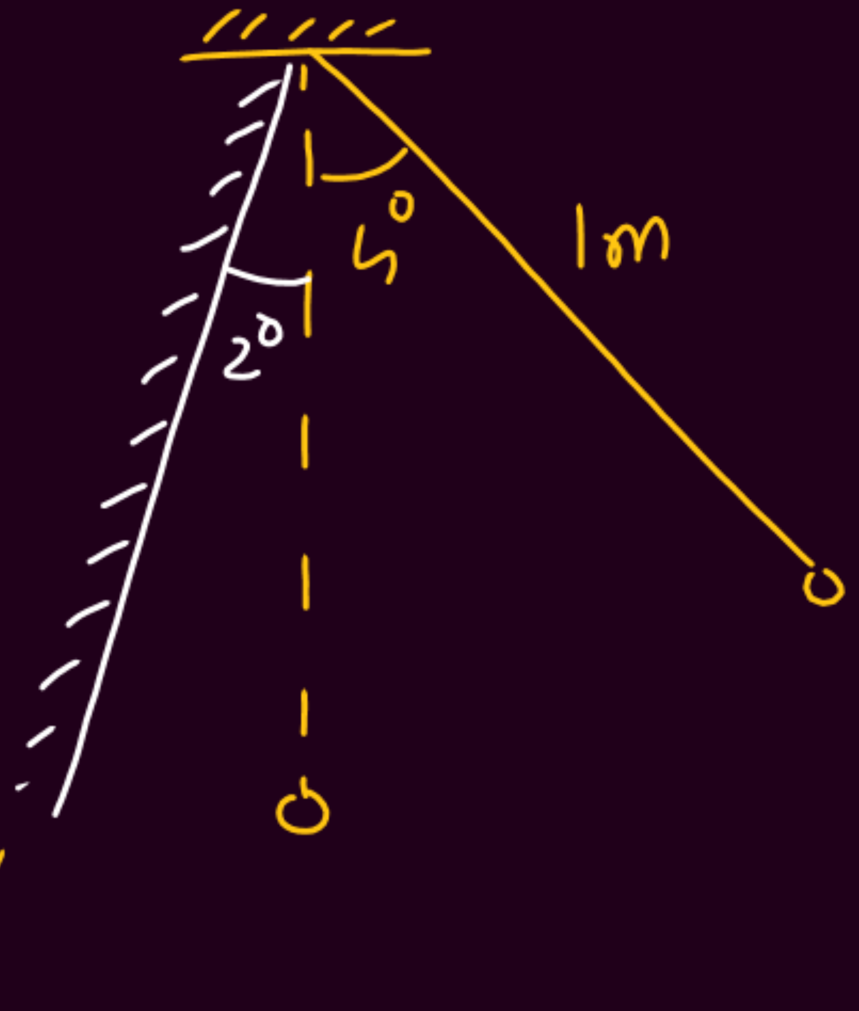
$$\frac{9}{4} = \frac{g+a}{g}$$

$$9g = 4g + 4a$$

$$5g = 4a$$

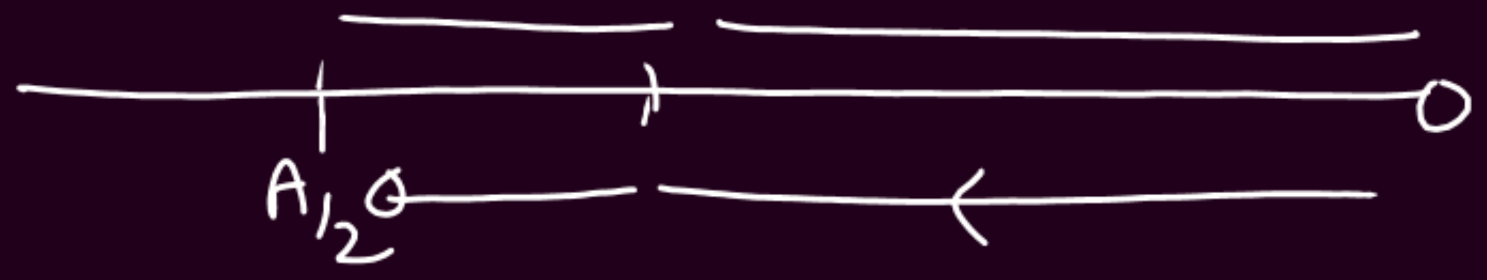
$$\boxed{a = \frac{5g}{4} = 12.5 \ m/s^2}$$

Extra



Elastic Collision

Time Period ?

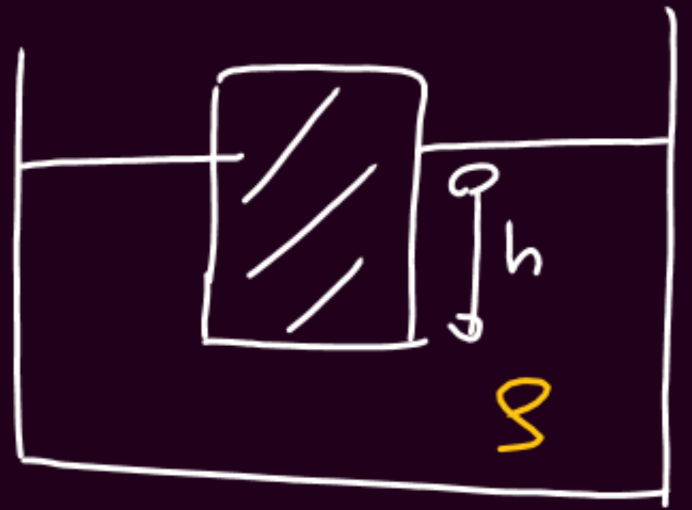


$$2\left(\frac{T}{4}+\frac{T}{12}\right)=\frac{8T}{12}=\frac{2T}{3}$$

$$=\frac{2\times 2}{3}$$

$$=\frac{4}{3}\,s$$

Extra



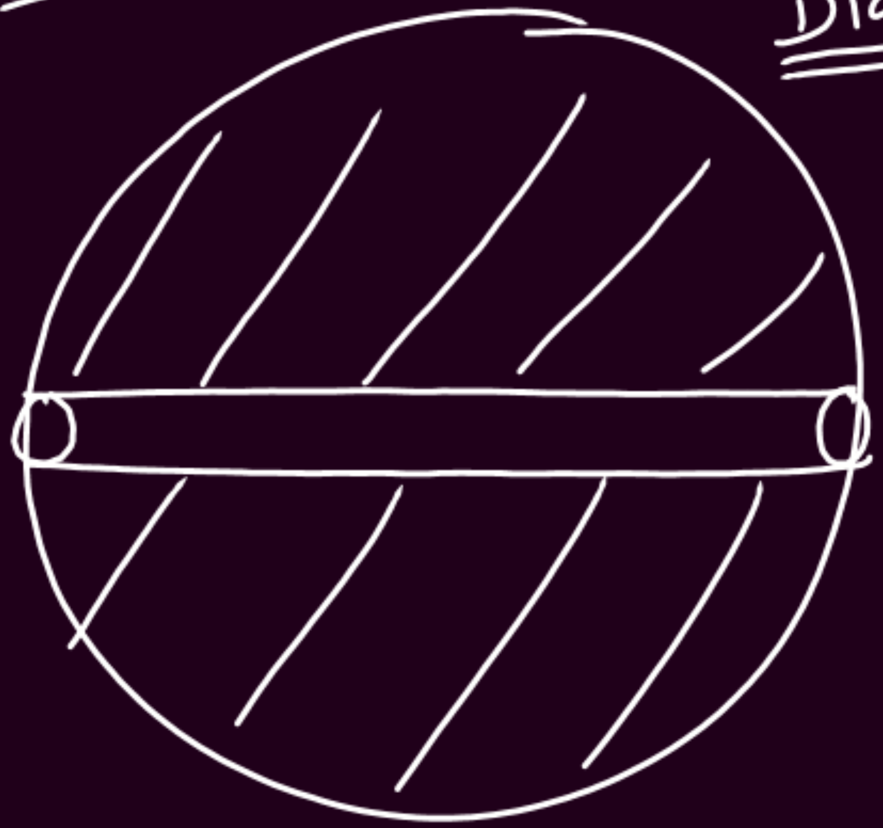
$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{m}{\rho A g}}$$

$$mg = B = \rho(Ah)g$$

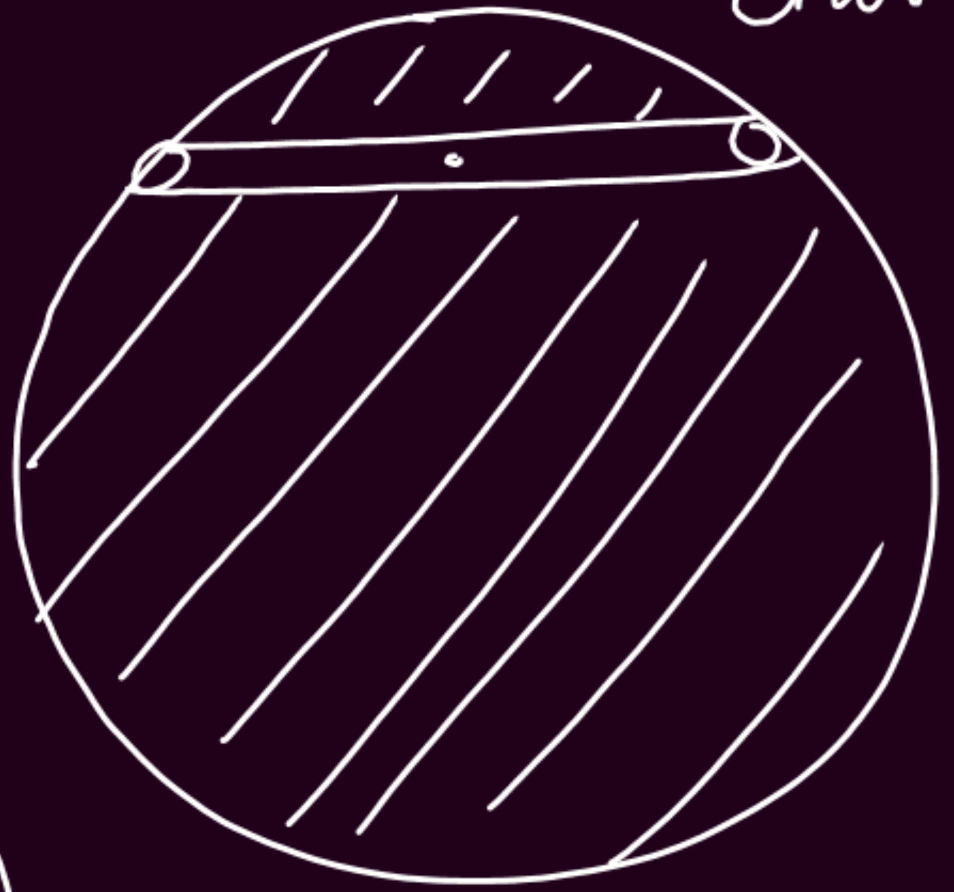
$$m = \rho A h$$

Earth

Dia



chord



$$\boxed{T = 2\pi\sqrt{\frac{R}{g}} \approx 84.6\,\text{min}}$$

$$T = 2\pi\sqrt{\frac{m_1 m_2}{(m_1+m_2)K}}$$

Pg-110

2

l

$l_1 \qquad l_2$

$$\boxed{l_1 = n l_2}$$

$$\frac{l_1}{l_2} = \frac{n}{1}$$

$$K_1 = \frac{Kl}{l_1} = \frac{K(n+1)}{n}$$

$$K_1 = \frac{Kl}{l_1} = \frac{K \cancel{l_1}\left(\frac{n+1}{n}\right)}{\cancel{l_1}}$$

$$l_1 + l_2 = l$$

$$l_1\left(1+\frac{1}{n}\right) = l_1 + \frac{l_1}{n} = l$$

$$l_1\left(\frac{n+1}{n}\right) = l$$

$$\frac{1}{K_{eq}} = \left[\frac{1}{2K_1} + \frac{1}{K_2}\right]$$

$$1g = K\left(\frac{5}{100}\right)$$

$$v_{max} = \sqrt{\frac{K}{m}}\,A$$

41 Extra

$$K_1 x_1 = K x$$

$$\cancel{K_1} x_1 = \frac{\cancel{K_1} K_2}{K_1 + K_2} A$$

$$F_1 = K_1 x$$

$$F_2 = K_2 x$$

$$F_1 + F_2 = (K_1 + K_2) x$$

$$T_1 = 2\pi \sqrt{\frac{m}{K_1}}$$

$$T_2 = 2\pi \sqrt{\frac{m}{K_2}}$$

$$T = 2\pi \sqrt{\frac{m}{K_1 + K_2}}$$

$$\frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{1}{T^2}$$

$$\frac{1}{2} + \frac{1}{3} \neq \frac{1}{5}$$

$$2 + 3 = 5$$

Equations Representing SHM / AC

1) $x = A\sin(\omega t)$

$x = A\cos(\omega t)$

$x = A\sin(\omega t \pm \phi)$

$x = A\cos(\omega t \pm \phi)$

$x = x_0 \pm A\sin(\omega t \pm \phi)$

Amp.	freq.
A	$\frac{\omega}{2\pi}$

$x = A\sin(\omega t)\cos(\omega t) = \frac{A}{2}\sin(2\omega t)$

$x = A\sin^2(\omega t) = \frac{A}{2}\left[1 - \cos(2\omega t)\right]$

$x = A\cos^2(\omega t) = \frac{A}{2}\left[1 + \cos(2\omega t)\right]$

Amp.	freq.
$\frac{A}{2}$	$\frac{2\omega}{2\pi}$

① $\sin(2\theta) = 2\sin\theta\cos\theta$

② $\cos(2\theta) = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta$

$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$

$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$

Superposition of SHM

1D $x_1 = A_1 \sin(\omega_1 t)$

$x_2 = A_2 \sin(\omega_2 t)$

$X = x_1 + x_2$

$x = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$

$a = -\omega_1^2 x_1 - \omega_2^2 x_2$

1. if $\omega_1 \neq \omega_2$ then [Periodic But not SHM]
   ↳ $(n_1 T_1 = n_2 T_2)$

2. if $\omega_1 = \omega_2 = \omega \rightarrow \boxed{\text{SHM}}$

① $\Delta\phi = 0$ (Same phase)

$$\begin{cases} x_1 = A_1 \sin(\omega t) \\ x_2 = A_2 \sin(\omega t) \end{cases} \quad \text{or} \quad \begin{cases} x_1 = A_1 \cos(\omega t) \\ x_2 = A_2 \cos(\omega t) \end{cases}$$

$$\boxed{A = A_1 + A_2}$$

② $\Delta\phi = \pi/2$

$$x_1 = A_1 \sin(\omega t)$$

$$x_2 = A_2 \cos(\omega t)$$

$$\boxed{A = \sqrt{A_1^2 + A_2^2}}$$

③ $\Delta\phi = \phi$

$\phi = \pi/2$

$\phi = 0$

$$X = A_1 \sin(\omega t) + A_2 \sin(\omega t + \phi)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Q.

$$X = 3\cos(\pi t) + 2\sin(\pi t)$$

$$A = \sqrt{3^2 + 2^2} = \sqrt{13}$$

Q.

$$X = 4\sin\left(\frac{\pi}{2}t\right) + 3\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}\right)$$

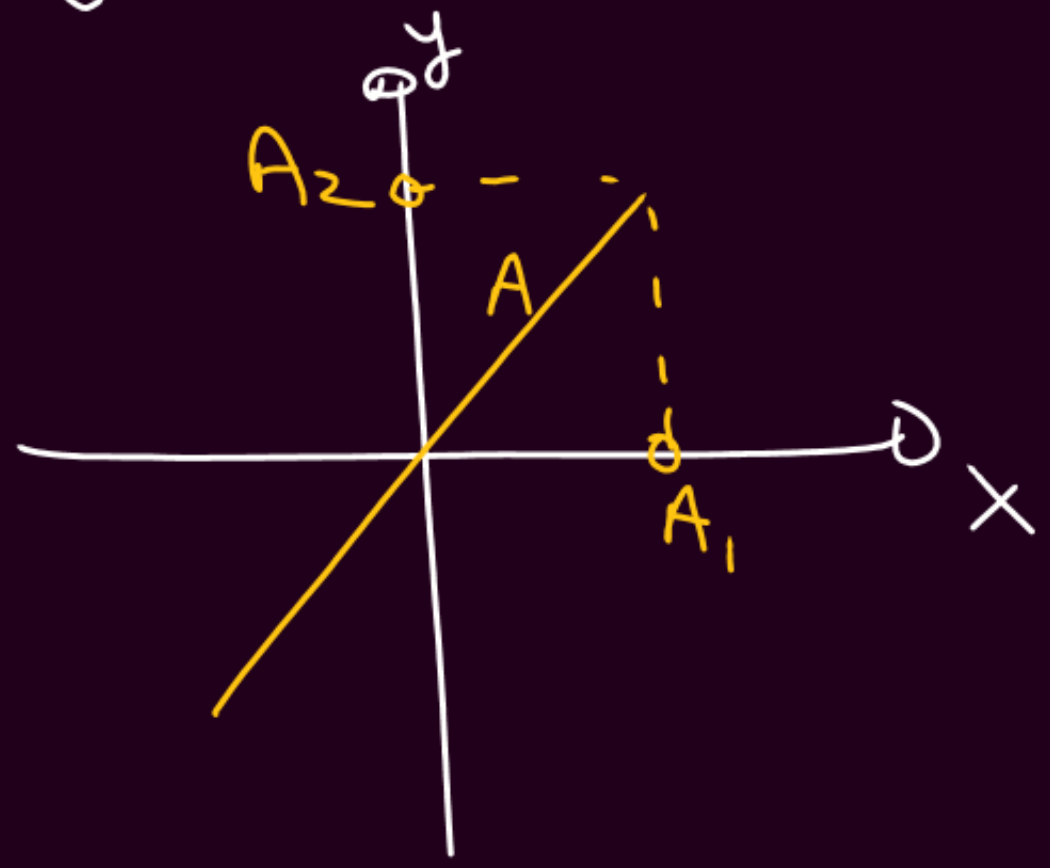
$$A = \sqrt{4^2 + 3^2 + \cancel{2}(4)(3)\cancel{\cos(\pi/3)}} = \sqrt{37}$$

2D

$$x = A_1 \sin(\omega_1 t)$$
$$y = A_2 \sin(\omega_2 t)$$

$\omega_1 \neq \omega_2 \Rightarrow$ Not SHM

① $x = A_1 \sin(\omega t)$
$y = A_2 \sin(\omega t)$
Same phase

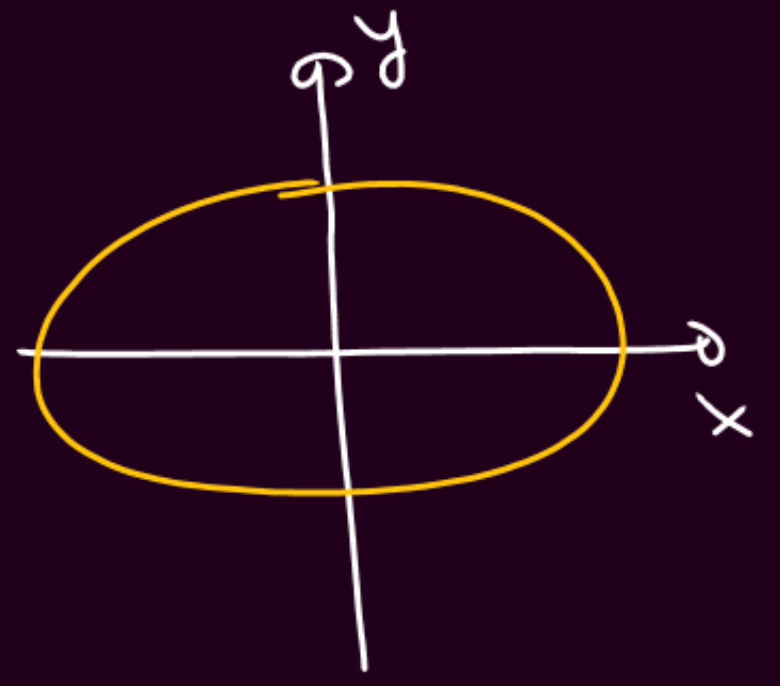


$\boxed{\text{SHM}}$

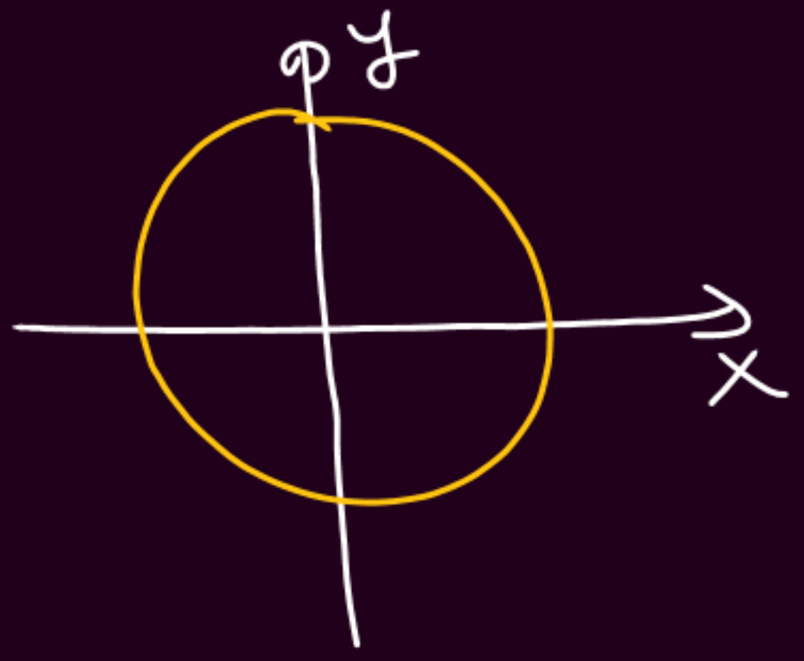
path → st. line

$$A = \sqrt{A_1^2 + A_2^2}$$

② $x = A_1 \sin(\omega t)$
$y = A_2 \cos(\omega t)$



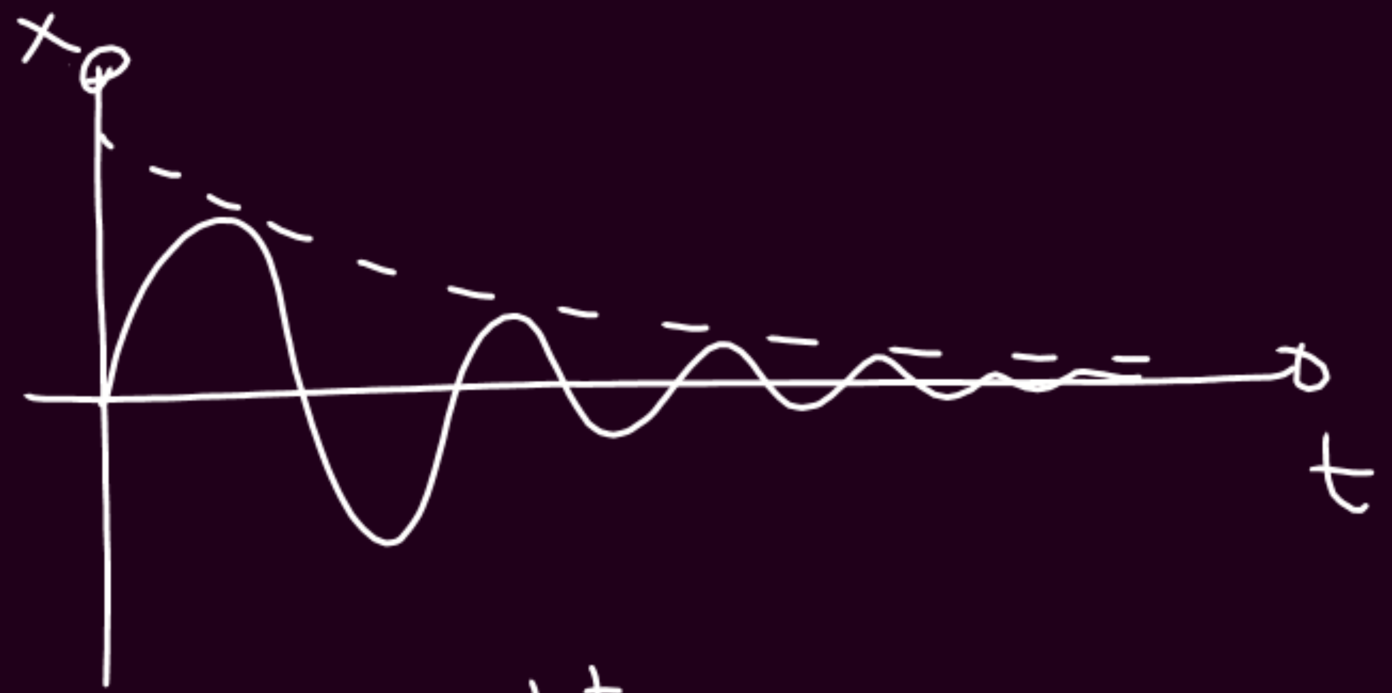
$x = A \sin(\omega t)$
$y = A \cos(\omega t)$



Periodic but not SHM

① Free Oscillations $F_R = -kx$

② Damped oscillations



$F = -kx - bv$

$A = A_0 e^{-\frac{bt}{2m}}$

$\mathcal{E} = \mathcal{E}_0 e^{-\frac{bt}{m}}$

$\mathcal{E}_0 = \frac{1}{2} K A_0^2$

$A_0 \xrightarrow{1\,hr} \frac{A_0}{3} \xrightarrow{1\,hr} \frac{A_0}{3^2} \xrightarrow{1\,hr} \frac{A_0}{3^3}$

$10\,cm \xrightarrow{10\,min} 2\,cm \xrightarrow{10\,min} \frac{2}{5}\,cm$

$10\,cm \xrightarrow{5\,min} 4\,cm \xrightarrow{5\,min} 4 \times \frac{2}{5} = 8/5\,cm$

③ Forced Oscillation

$$F = -kx - bv + F_0 \sin(\omega_d t)$$

$\Rightarrow \boxed{\omega_d}$

Resonance $\rightarrow$