Electrostatics

Properties of Charge

- ⊕ ⊖
- Like point charges → repel

- Like charged bodies may repel / Attract
- Unit $1\,C = 3 \times 10^9$ statcoulomb (esu)
- $[C] = [AT]$
- Scalar / Additive

- Charged Bodies
  - ⊕ → Deficiency of e^-
  - ⊖ → Excess of e^-
- Associated with mass
- Invariant → independent of v
-

	$\vec{E}$	$\vec{B}$	EMW
Rest	✓		
Const. $\vec{v}$	✓	✓	
Acc.	✓	✓	✓

→ Conservation of Charge

→ for isolated system
net q is conserved

$+4 \quad -2 \quad = 0 \quad +1 \quad +1$

→ Repulsion is true test

A B

→ Quantised → Millikan

$$\boxed{q = \pm ne}$$

① 1.6×10^{-20} C

② 1.6×10^{-18} C

③ 2×10^{-17} C

④ $2 \mu C$

$1.6 \times 10^{-20} = ne$

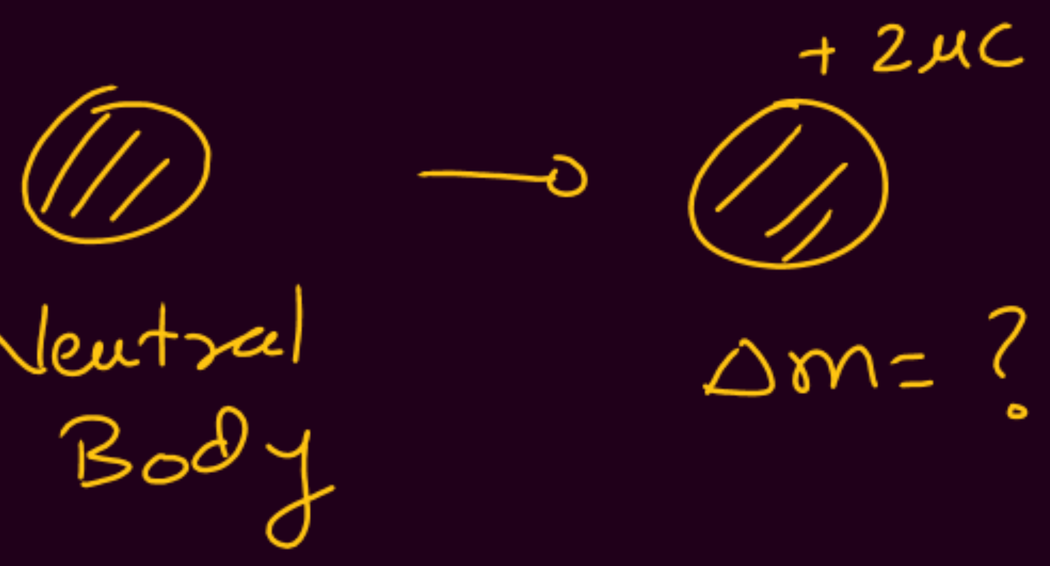
$n = 10^{-1}$ ✗

$n = 10$ ✓

$n = \frac{200}{1.6} = 125$ ✓

✓

$\frac{2000}{16} = 125$



$$\Delta m = n\, m_e$$

$$= \frac{q}{e} m_e$$

$$= \frac{2\times10^{-6}}{1.6\times10^{-19}}\left(9.1\times10^{-31}\right)$$

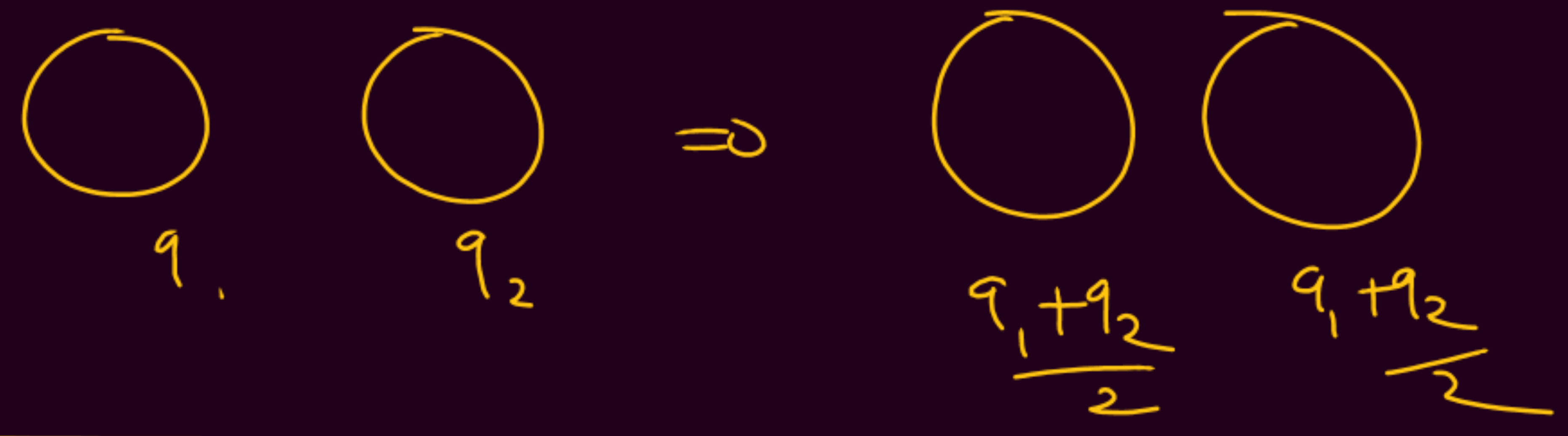
$$\simeq 10^{-17}\ \text{kg}$$

Methods of charging

① Friction / Rubbing

② Conduction [q flow → P.D.]

Identical conductors



Soap Bubble

RP RP

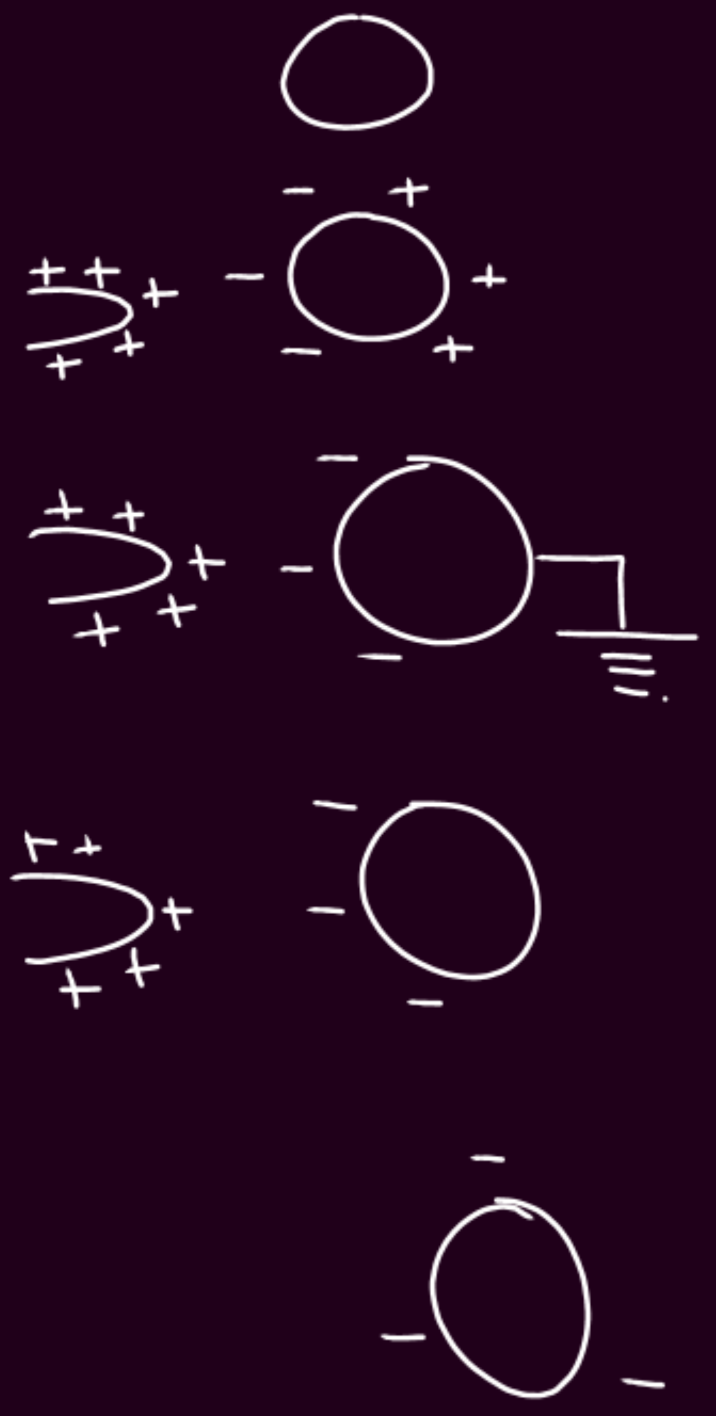
Indentical Spheres

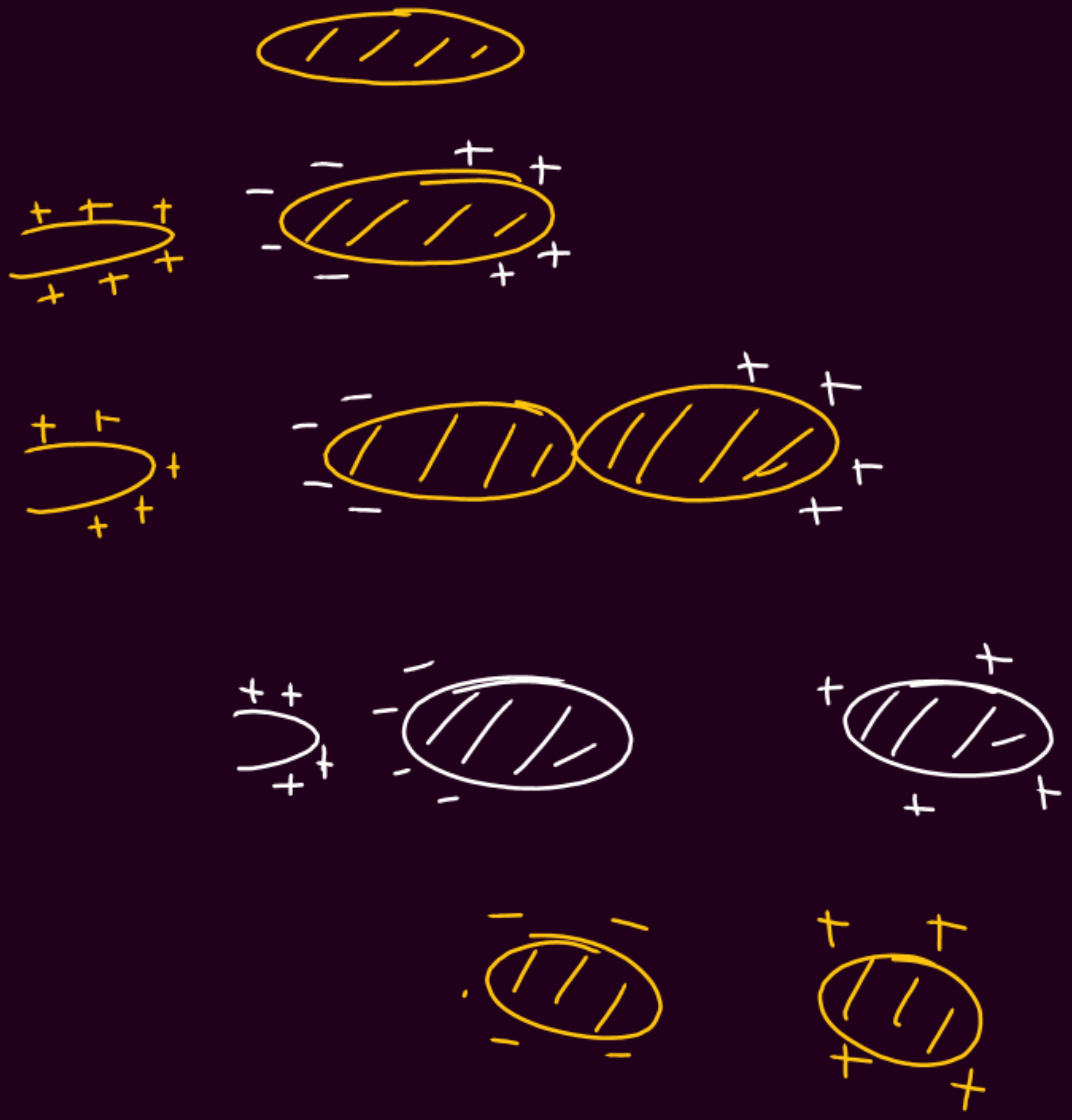
	A	B	C
	+4	−2	+6
A & B	+1	+1	+6
A & C	3.5	+1	3.5
C & B	3.5	2.25	2.25

Earthing

$\boxed{V = 0}$

Charging By Induction





Extra

Gold leaf Electroscope

Diverge

more Diverge

- converge
- (collapse)

initially neutral

X - Rays

Coulomb's law

$$\boxed{F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}}$$

→ only for point charges

→ long range

$$\frac{1}{4\pi\epsilon_0} = k = 9\times10^{9} \ \frac{N\text{-}m^2}{C^2}$$

$$\epsilon_0 = 8.85\times10^{-12} \ \frac{C^2}{N\text{-}m^2}$$

$$[\epsilon_0] = \frac{A^2T^2}{MLT^{-2}L^2} = [M^{-1}L^{-3}T^4A^2]$$

$$\boxed{\epsilon = \epsilon_0\epsilon_r = \epsilon_0 K}$$

$\epsilon_r = K =$ Dielectric Constant

= Relative permittivity of medium

$K = 1$ (vac.)

$K \simeq 1$ (Air)

$K_w = 80$ (water)

$K_{metal} \to \infty$

$$k = \frac{1}{4\pi\epsilon_0} = 1 \text{ esu}$$

Q.

2 mm

2 μC ———————— 1 μC

$$F = \frac{(9\times10^{9})(2\mu)(1\mu)}{(2\times10^{-3})^{2}}$$

$$\boxed{F = 4.5\ kN}$$

Q. Identical charges F

20% → transfer

$$F = \frac{kQQ}{r^2}$$

$$F' = \frac{k(0.8Q)(1.2Q)}{r^2}$$

$$F' = 0.96\ F$$

Q. Equal & opposite F

20% → transfer

$Q - 0.2Q$

$-Q + 0.2Q$

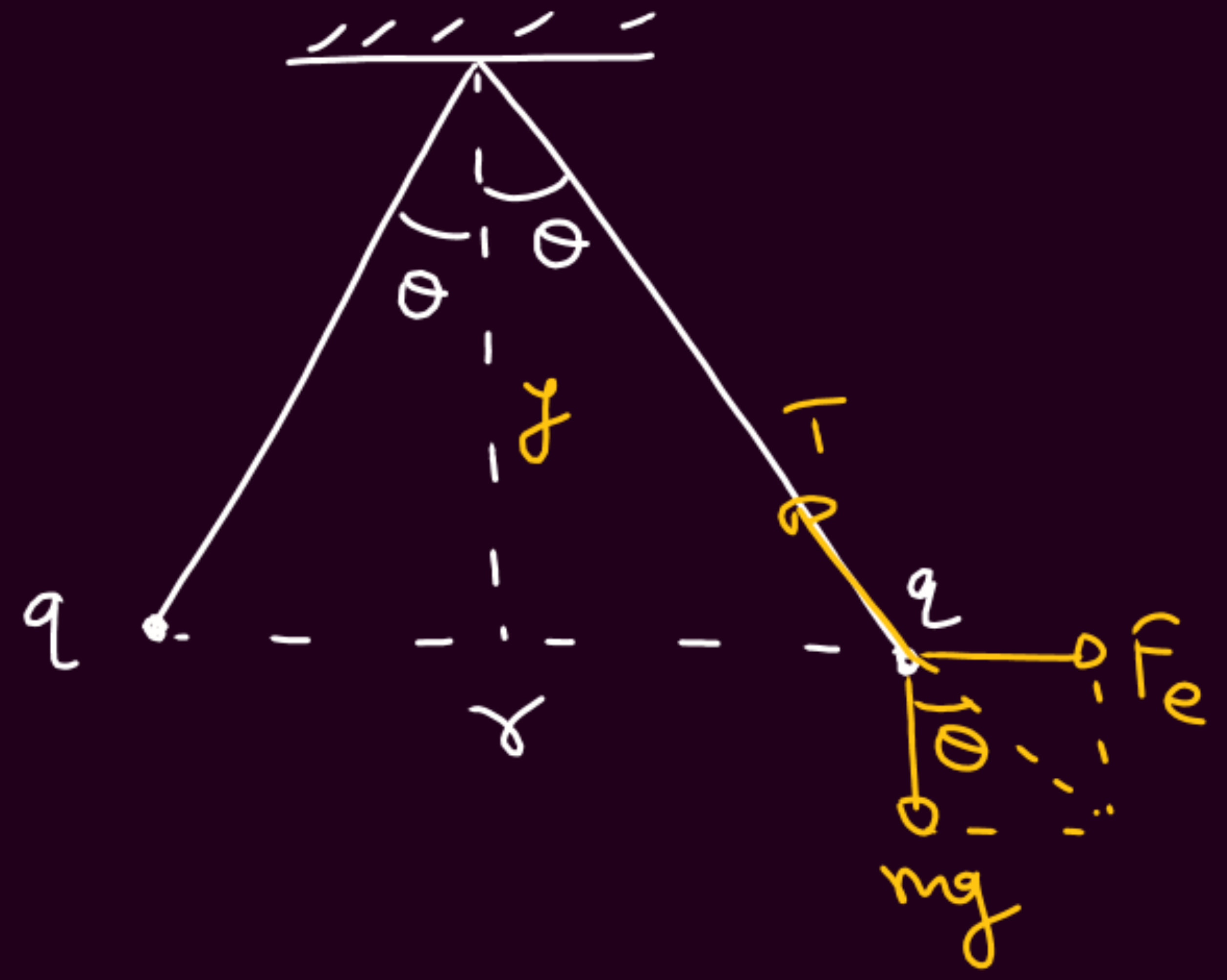
$$F' = \frac{k(0.8Q)(-0.8Q)}{r^2}$$

$$\boxed{F' = 0.64\ F}$$

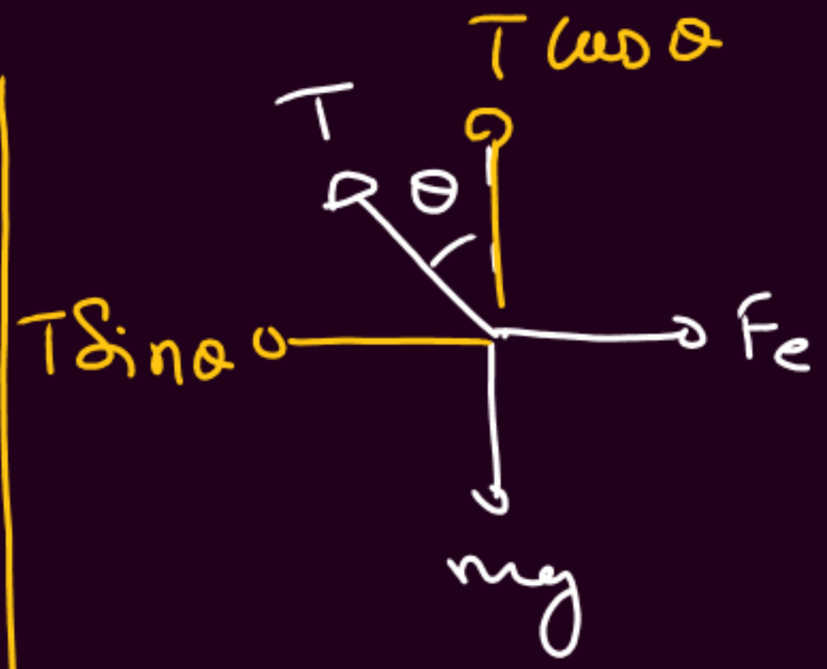
Q. F_{max}

$q_1 = q_2 = \frac{Q}{2}$

Case



$\frac{r/2}{y} = \tan\theta = \frac{F_e}{mg} = \frac{kq^2}{r^2 mg}$



$T\cos\theta = mg$

$T\sin\theta = F_e$

Q.

① F_{net} on q

$$F_{net} = F_1 - F_2$$

$$= \frac{k(4q)(q)}{(3r)^2} - \frac{kqq}{r^2}$$

$$= -\frac{5kq^2}{9r^2}$$

② F_{net} on $-q$

$$F_{net} = F_2 - F_1$$

$$= \frac{kq^2}{r^2} - \frac{k(4q^2)}{(2r)^2}$$

$$\boxed{F_{net} = 0}$$

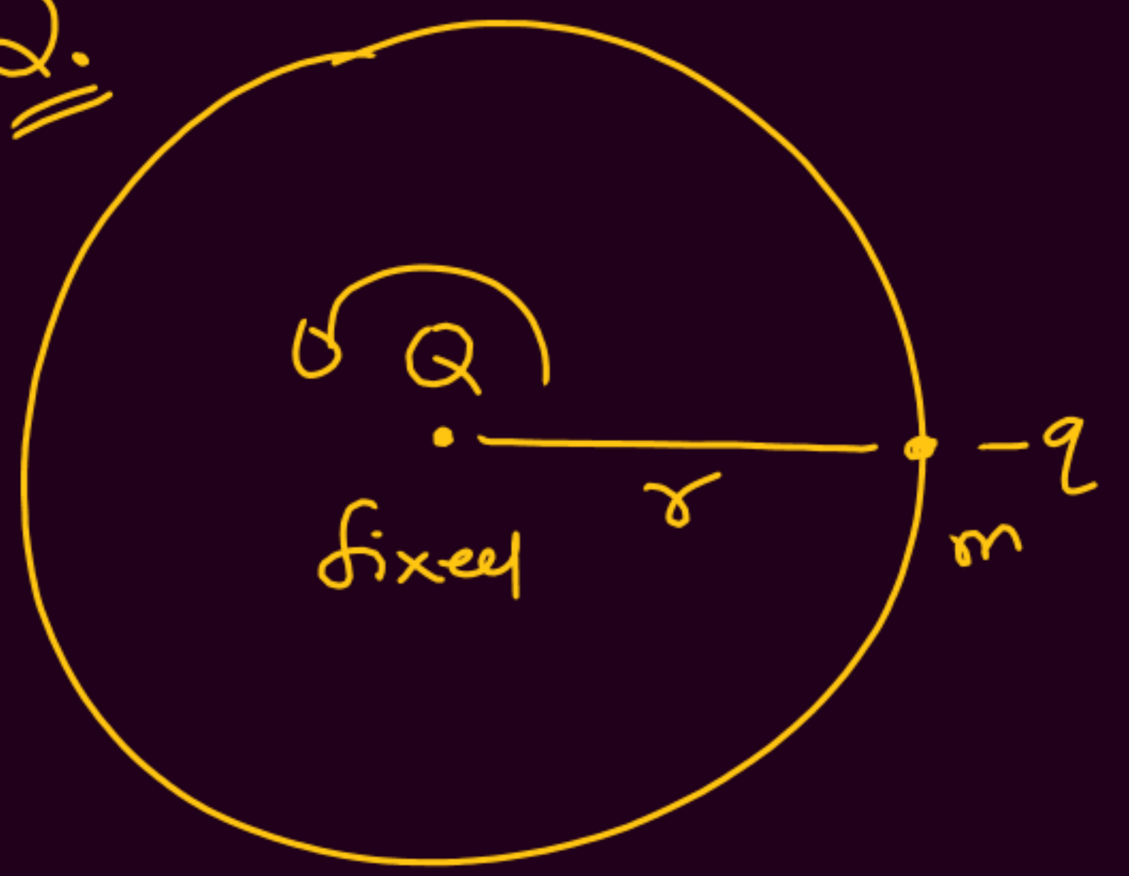
Q. Water drop. $r = 2mm$

(q) $E = 10\,N/C$ ↑

Equilibrium

$$q = \frac{mg}{E} = \frac{10^3 \, \frac{4}{3}\pi (2\times10^{-3})^3 \times 10}{10}$$

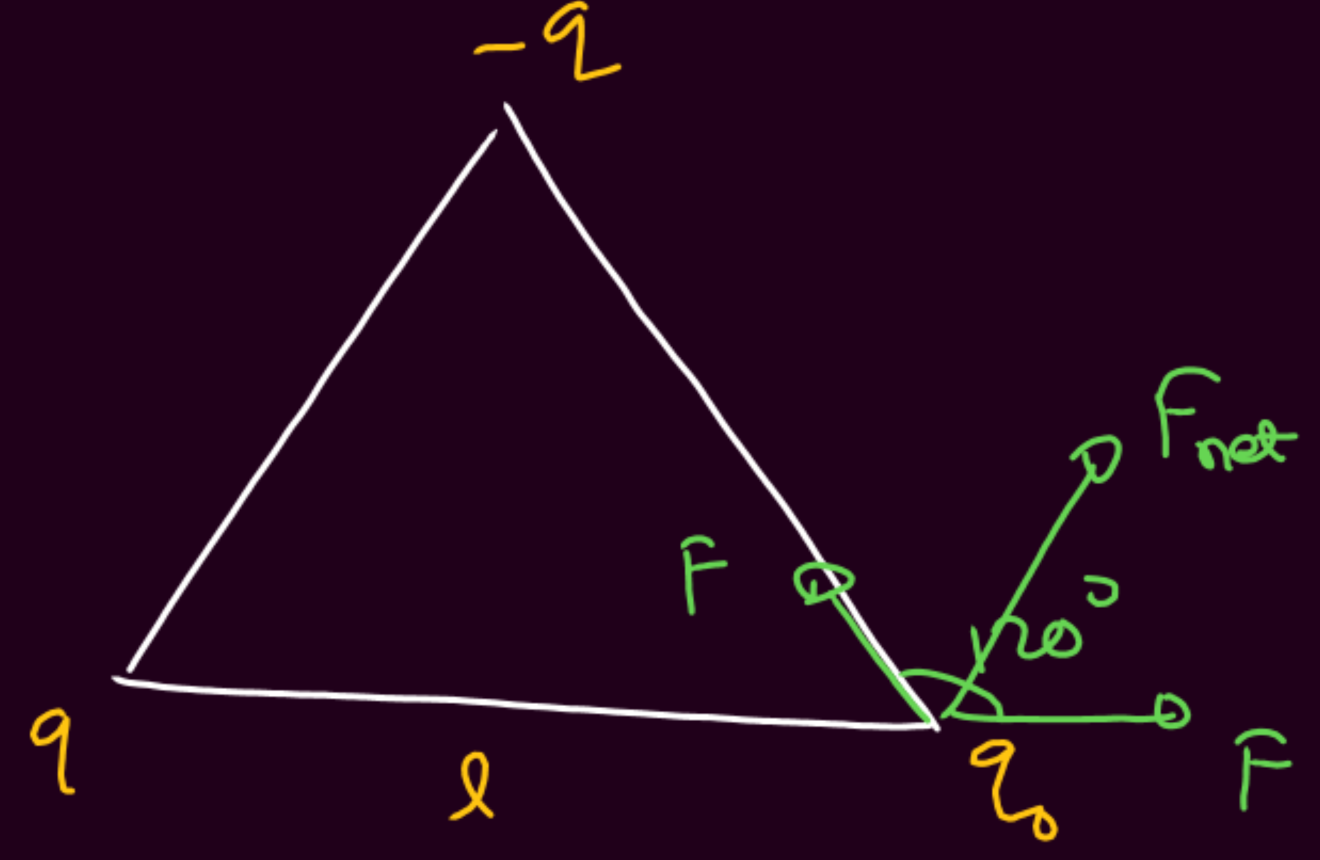
Q.



$$m\omega^2 r = \frac{kQq}{r^2}$$

$$\frac{2\pi}{T} = \omega = \sqrt{\frac{kQq}{mr^3}}$$

Q.



$$F_{net} = 2F\cos\left(\frac{120^\circ}{2}\right) = F = \frac{kqq_0}{l^2}$$

Q

1) $F = F_0$

2) $F > F_0$

3) $F < F_0$

Electric Field ($\vec{E}$)

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

$N/C = V/m$

$$\boxed{\vec{F} = q_0 \vec{E}}$$

① $q_0 = \oplus$ $\vec{F}$ & $\vec{E}$ same dir

② $q_0 = \ominus$ —— " ——— opp. dir

Q. (m, q) Equilibrium $q\,E$

if dir of field is reversed then Find $a = ?$

$qE = mg$

$$a = \frac{F_{net}}{m}$$

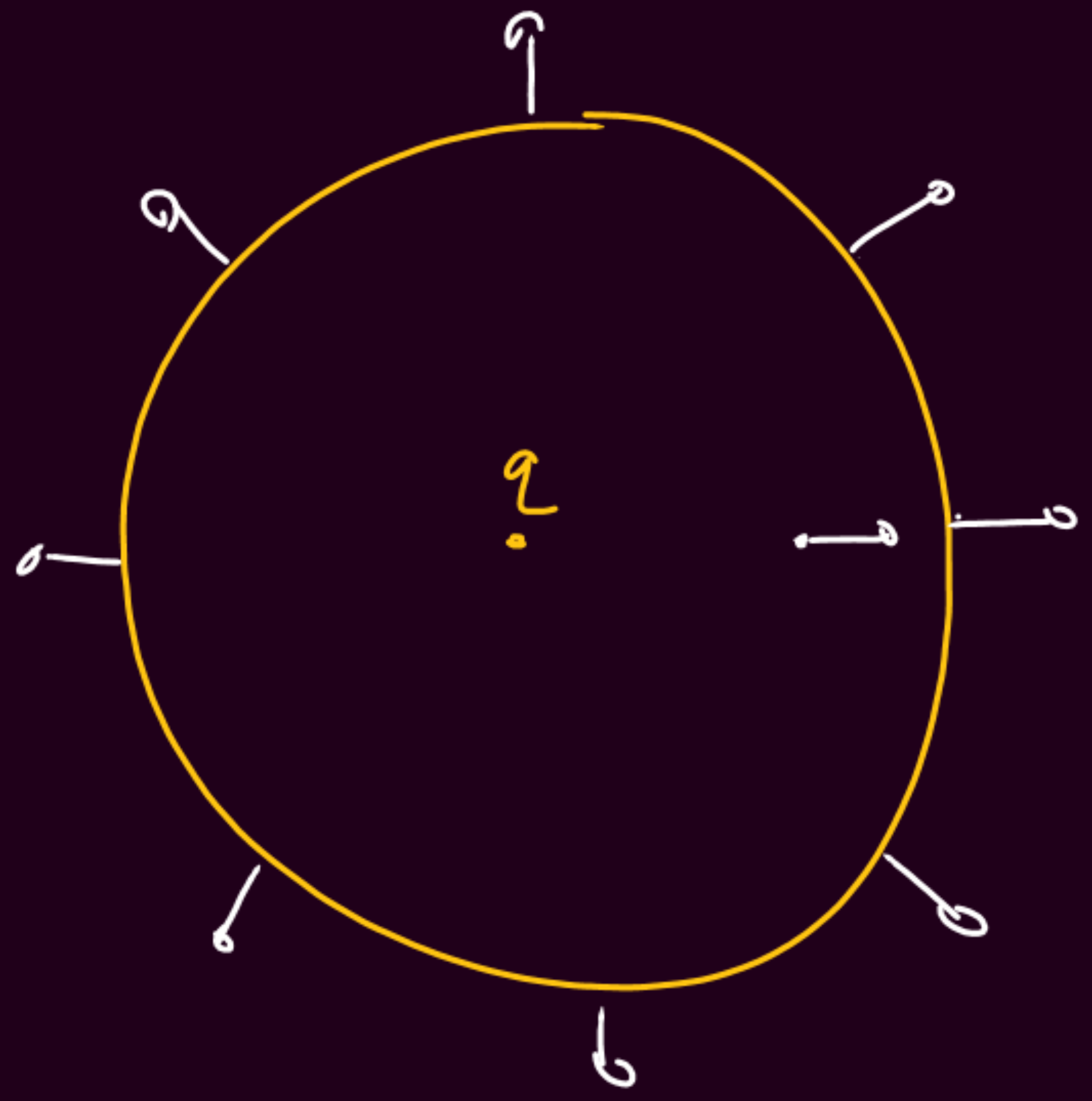
$$= \frac{mg + mg}{m}$$

$$= 2g$$

Field due to point charge

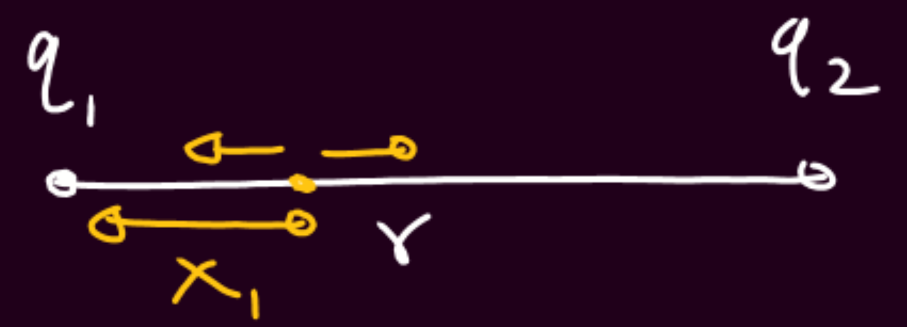
$$\frac{F}{q_0} = \frac{kqq_0}{r^2 q_0}$$

$$\boxed{E = \frac{kq}{r^2}}$$



Null point $\boxed{\vec{E}_{net} = 0}$

① Like charges



Concept
$$\frac{k q_1}{x_1^2} = \frac{k q_2}{(r - x_1)^2}$$

$$\boxed{x_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}} + 1}}$$

② Equal & opposite $\boxed{\text{dipole}}$

No null point

③ Unequal & Opposite

$$\boxed{x_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}} - 1}}$$

Q. $4\mu C$ —— $9\mu C$

25 cm

$$X_1 = \frac{25}{\sqrt{\frac{9}{4}}+1}$$

$$= \frac{25}{5/2} = \underline{\underline{10\,cm}}$$

from $4\mu c$

$$X_2 = \frac{25}{\sqrt{\frac{4}{9}}+1} = \frac{25}{5/3} = \underline{\underline{15\,cm}}$$

Q. $4\mu C$ —— $-9\mu C$

50 cm, 25 cm, x_2

$$X_1 = \frac{25}{\frac{3}{2}-1} = \underline{\underline{50\,cm}}$$

$$X_2 = \frac{25}{\sqrt{\frac{4}{9}}-1} = \ominus\,\underline{\underline{75\,cm}}$$

Q. $q \quad Q \quad q$

$$\frac{Q}{q} = ?$$

$$\frac{kq^2}{r^2} + \frac{kQq}{(r/2)^2} = 0$$

$$\boxed{\frac{Q}{q} = -\frac{1}{4}}$$

All are in Eq.

$q \quad Q \quad 1\mu C \quad -9\mu C$

20 cm

$$x_1 = \frac{20}{\sqrt{\frac{9}{1}} - 1} = 10\,cm$$

$$\frac{kq}{(30)^2} + \frac{k(1)}{(20)^2} = 0$$

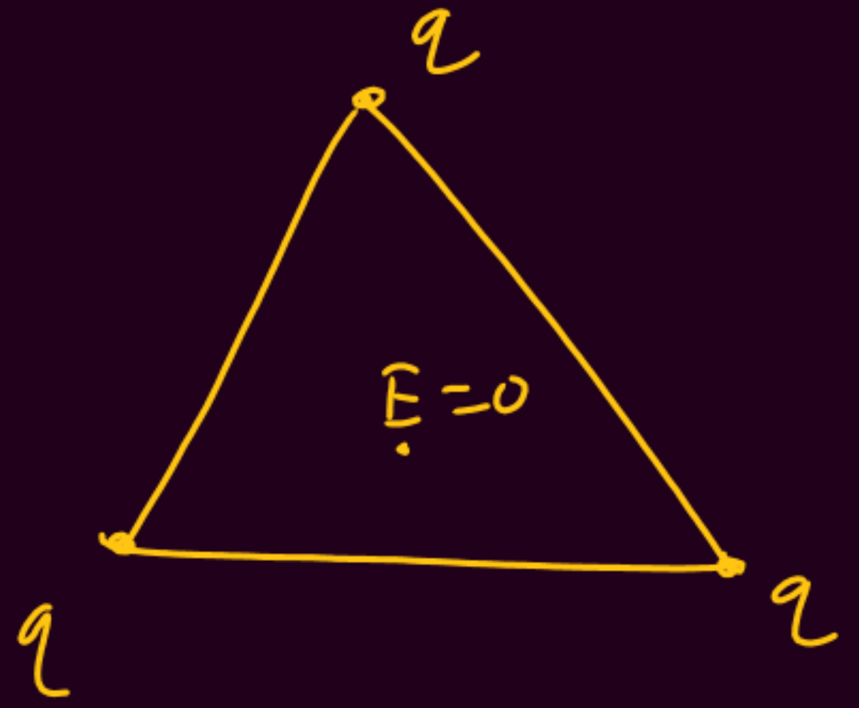
$$\boxed{q = -\frac{9}{4}\,\mu C}$$

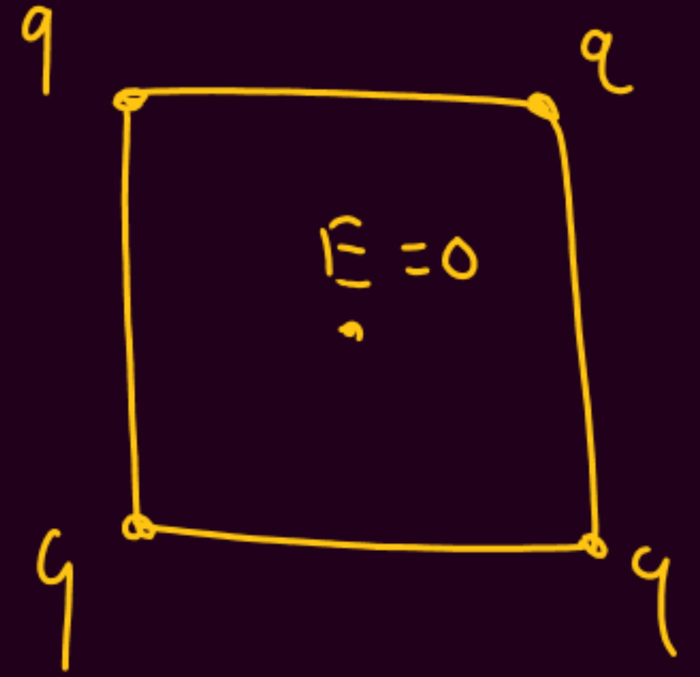
$$\frac{kq}{(10)^2} + \frac{k(9)}{(20)^2} = 0$$

$q \quad -9$

$$\boxed{q = -\frac{9}{4}\,\mu C}$$

Symmetry

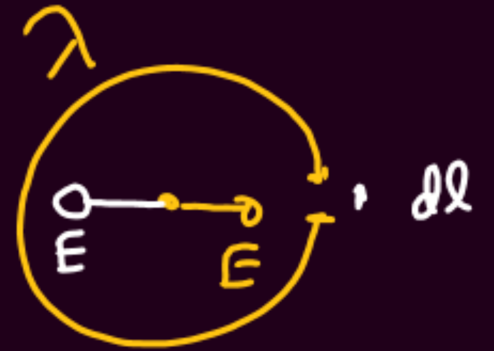




If any one charge is removed

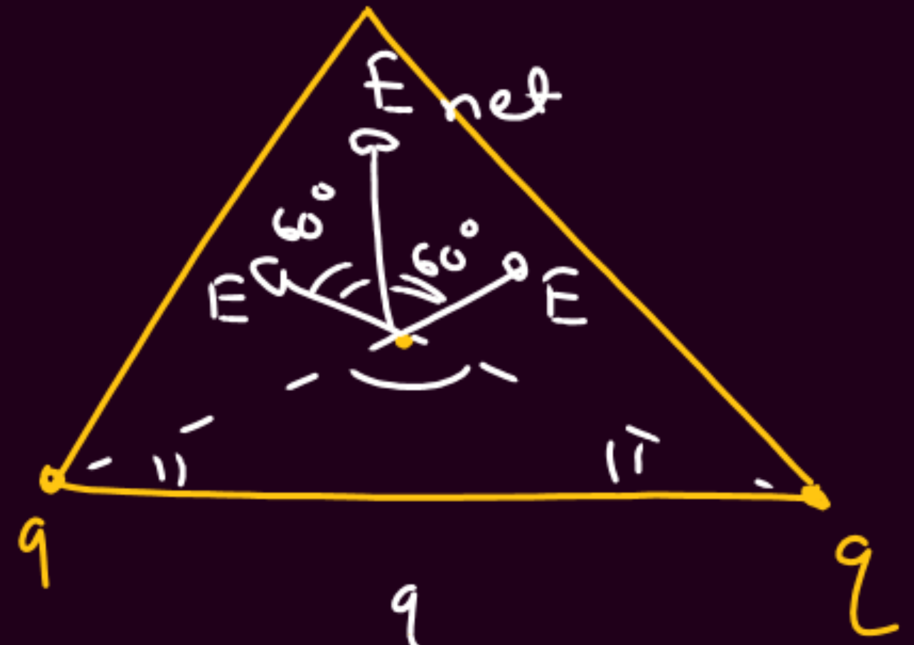
$$\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_5 = 0$$

$$\left(\vec{F}_1 + \cdots + \vec{F}_4\right) = -\vec{F}_5$$

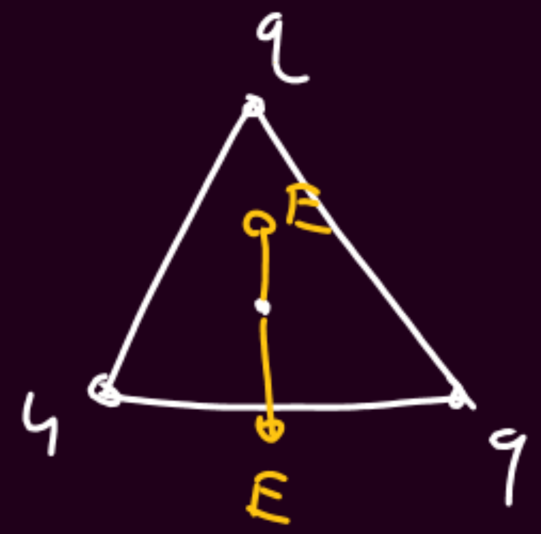


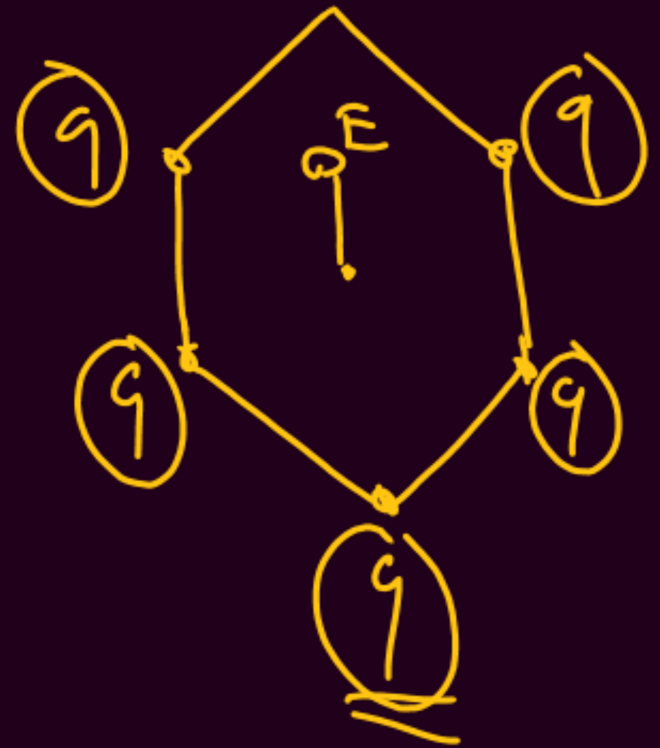
$$E = \frac{K(dq)}{R^2}$$

$$dq = \lambda(dl)$$

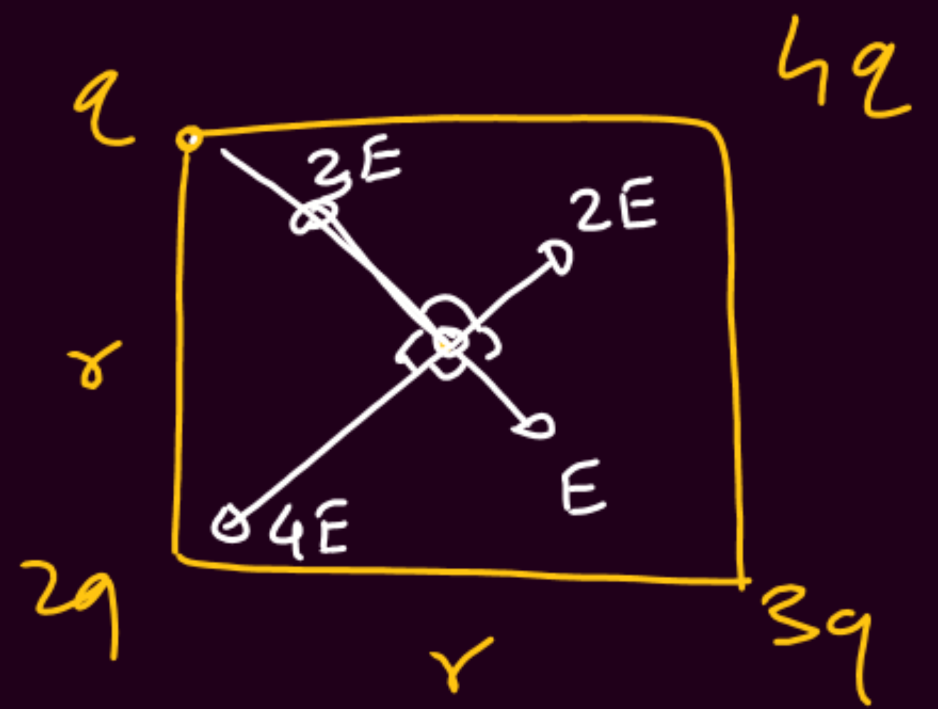


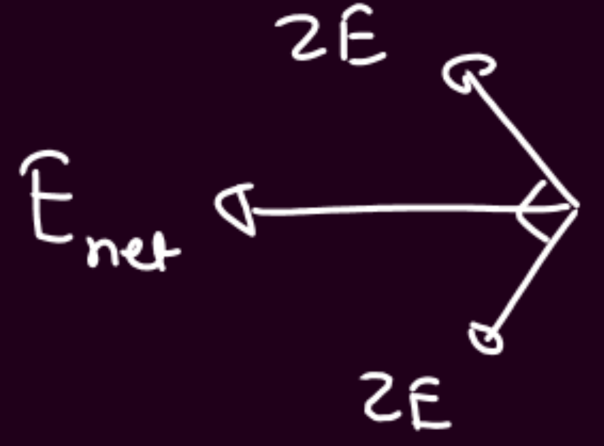
$$E_{net} = 2E\cos 60^\circ = E$$





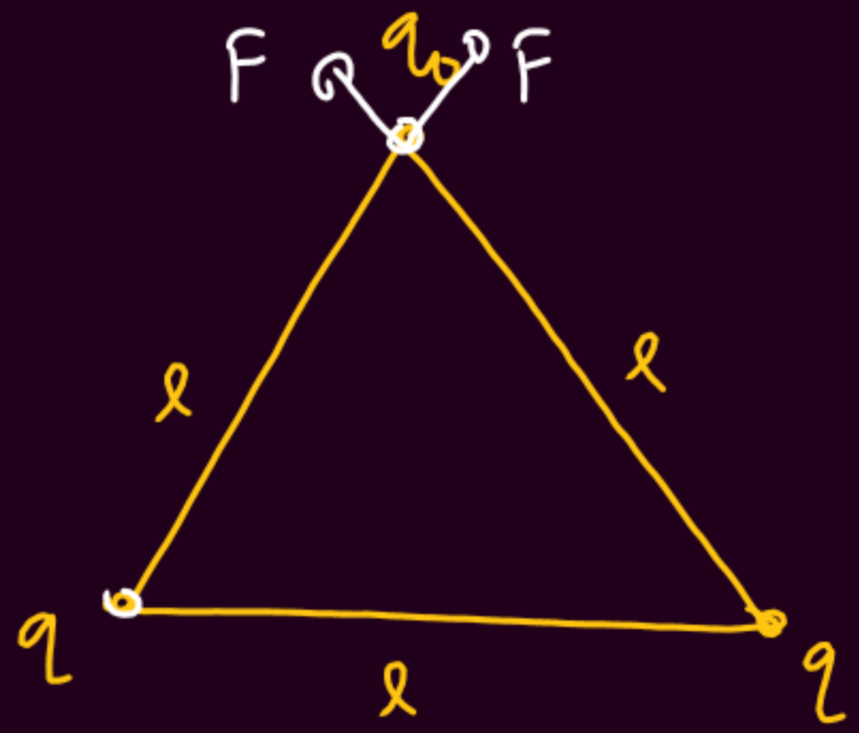
Q:





$$E_{net} = 2(2E)\cos(45^\circ)$$

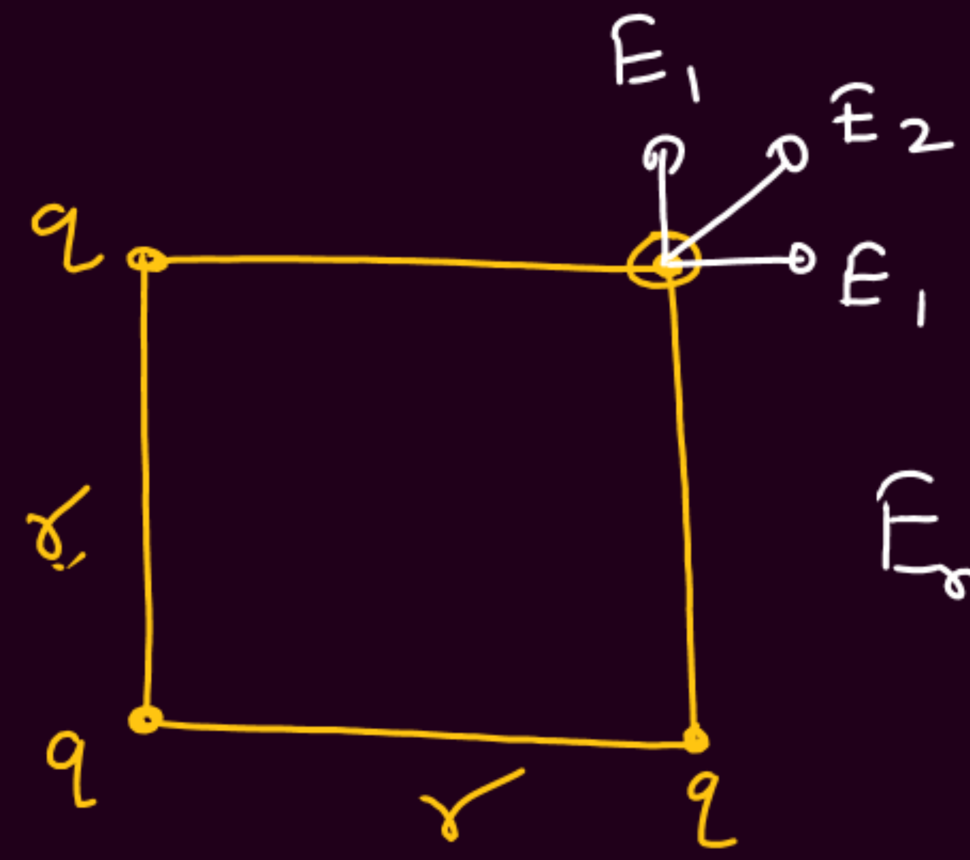
$$= 2\sqrt{2}E = 2\sqrt{2}\frac{kq}{\left(\frac{r\sqrt{2}}{2}\right)^2} = \frac{4\sqrt{2}kq}{r^2}$$

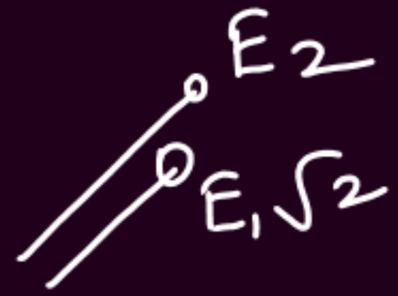


$$F_{net} = 2F\cos(30^\circ) = \sqrt{3}F$$

$$= \sqrt{3}\frac{kqq_0}{l^2}$$

Q.

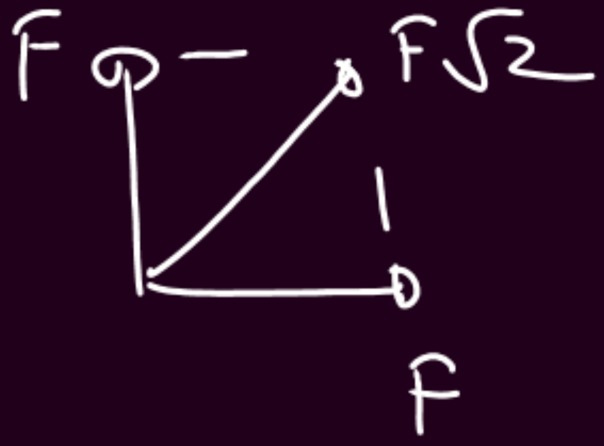




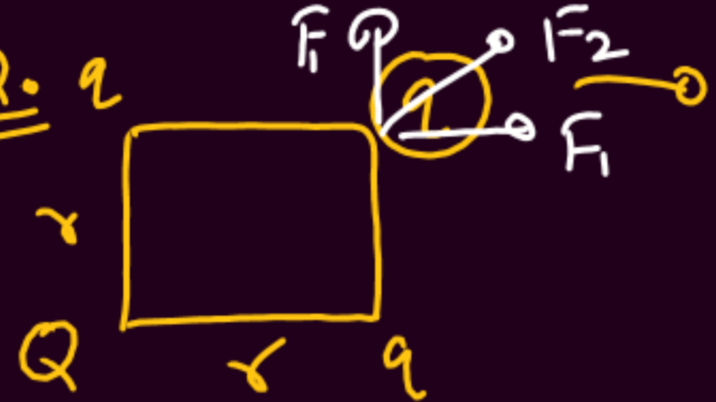
$$E_{net} = E_2 + E_1\sqrt{2}$$

$$= \frac{kq}{(r\sqrt{2})^2} + \left(\frac{kq}{r^2}\right)\sqrt{2}$$

$$= \frac{kq}{r^2}\left(\frac{1}{2} + \sqrt{2}\right)$$



Q.



$F_{net} = 0$

$\frac{Q}{q} = ?$

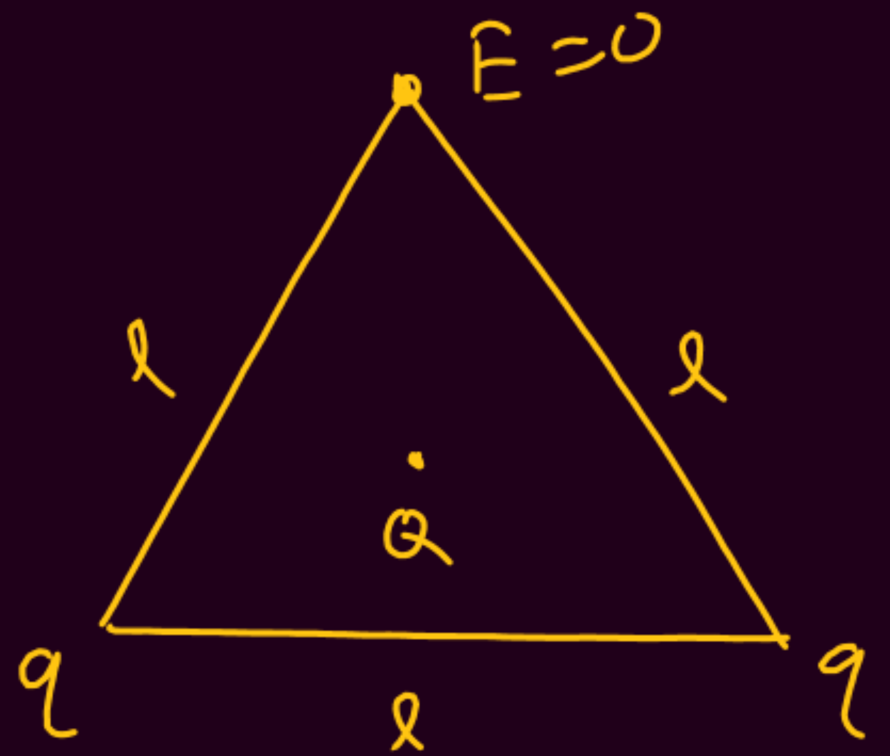
$$F_1\sqrt{2} + F_2 = 0$$

$$\frac{kqq}{r^2}\sqrt{2} + \frac{kQq}{(r\sqrt{2})^2} = 0$$

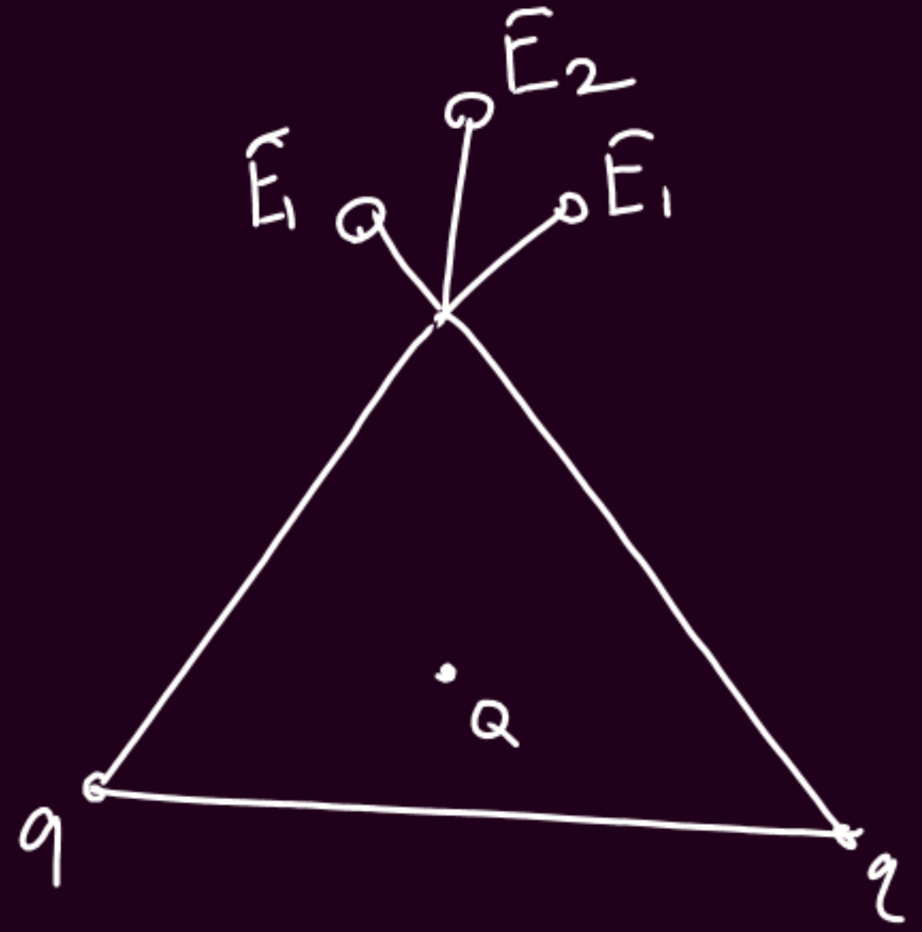
$$q\sqrt{2} + \frac{Q}{2} = 0$$

$$\frac{Q}{2} = -q\sqrt{2}$$

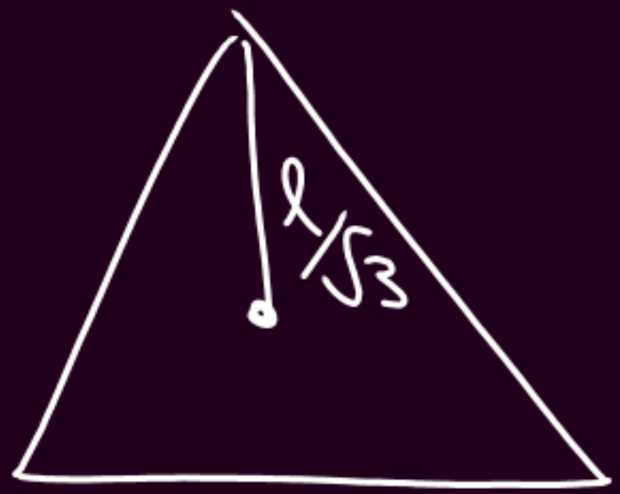
$$\boxed{\frac{Q}{q} = -2\sqrt{2}}$$



$$\frac{Q}{q} = ?$$



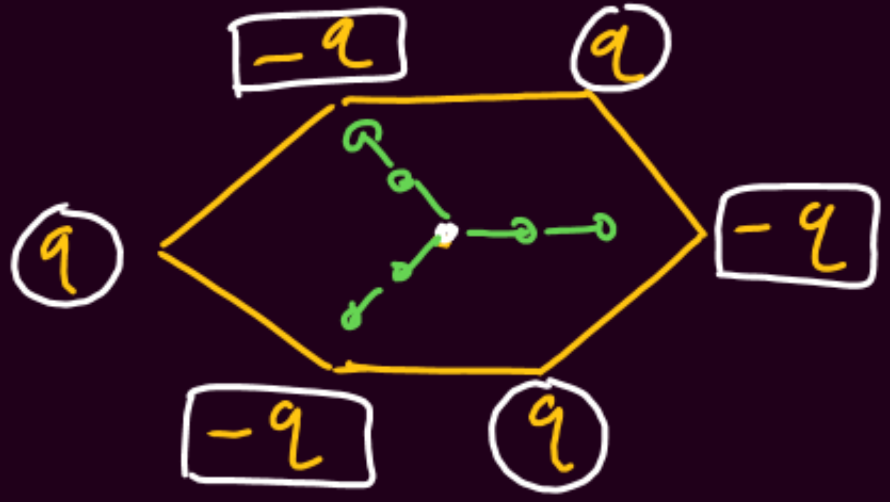
$$E_{net} = 2E_1 \cos 30^\circ + E_2 = 0$$



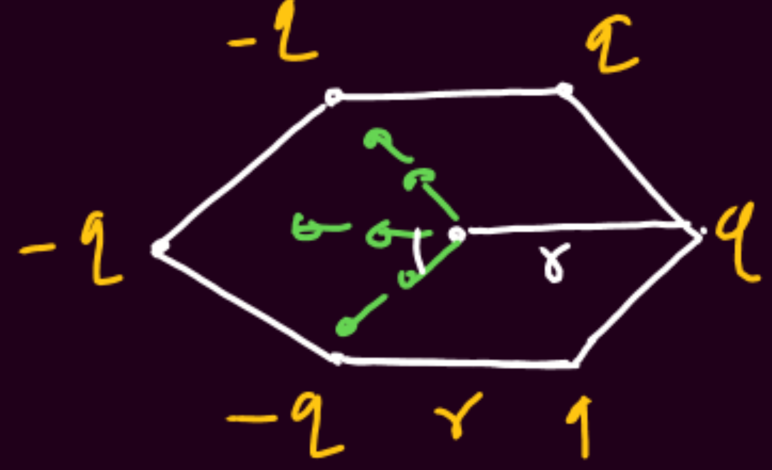
$$\sqrt{3}\,\frac{kq}{l^2} + \frac{kQ}{\left(\frac{l}{\sqrt{3}}\right)^2} = 0$$

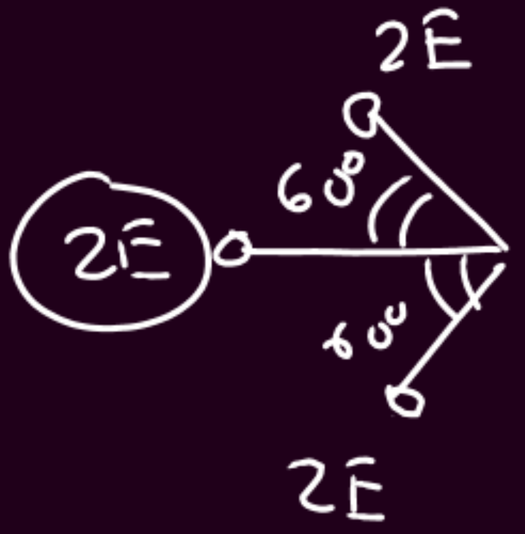
$$\sqrt{3}q + 3Q = 0$$

$$\boxed{\frac{Q}{q} = -\frac{1}{\sqrt{3}}}$$



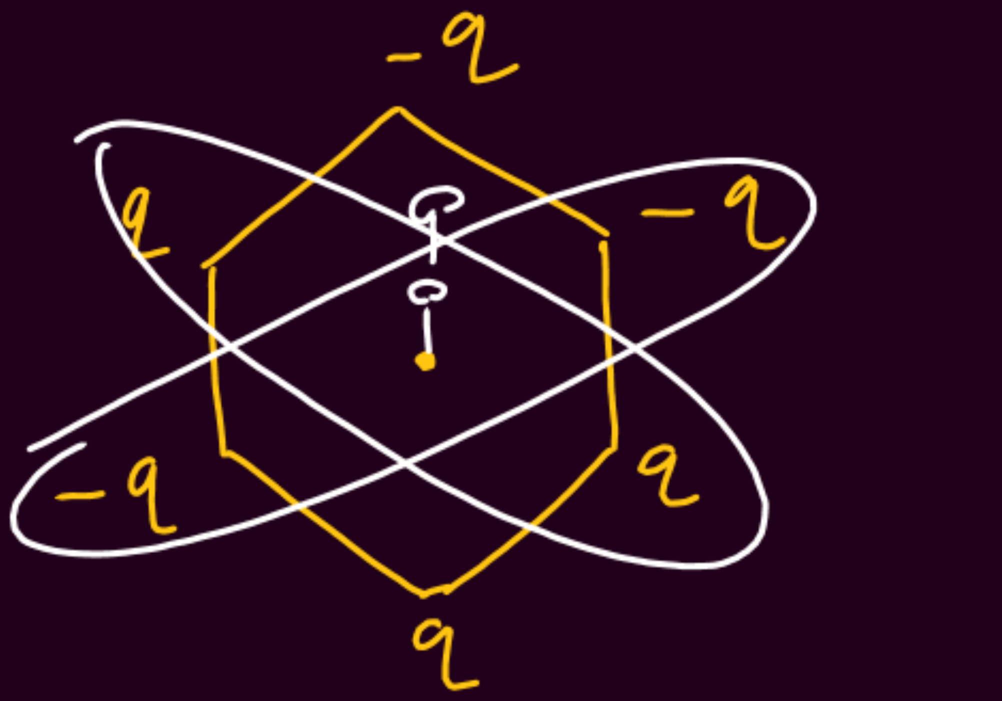
$$\boxed{E_{net} = 0}$$



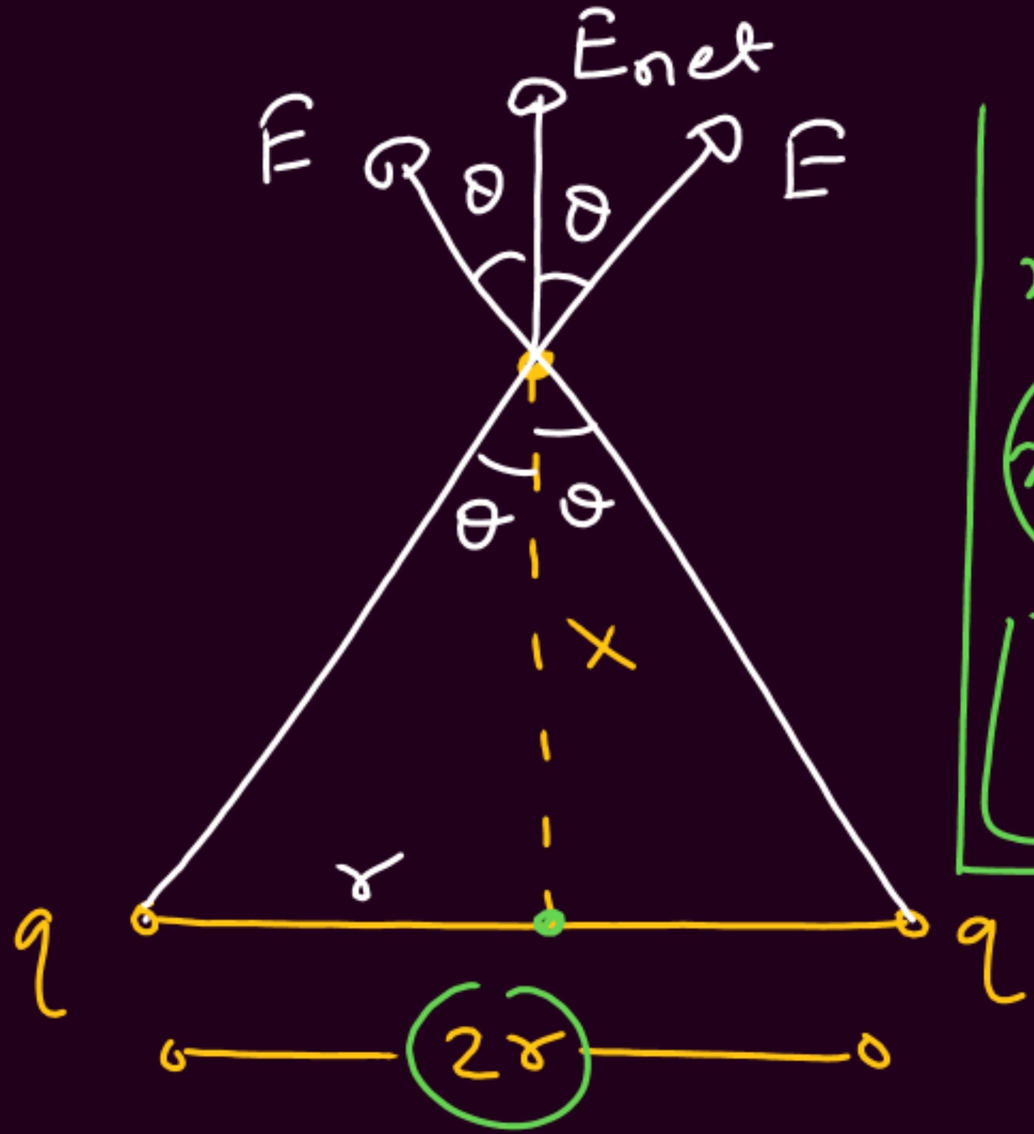


$$E_{net} = 2E + \cancel{2}(2E)\cancel{\cos 60^\circ}$$

$$= 4E = 4\left(\frac{kq}{r^2}\right)$$



$$E_{net} = 2E = \frac{2kq}{r^2}$$



$x = 0 \quad E = 0$

$x \to \infty \quad E = 0$

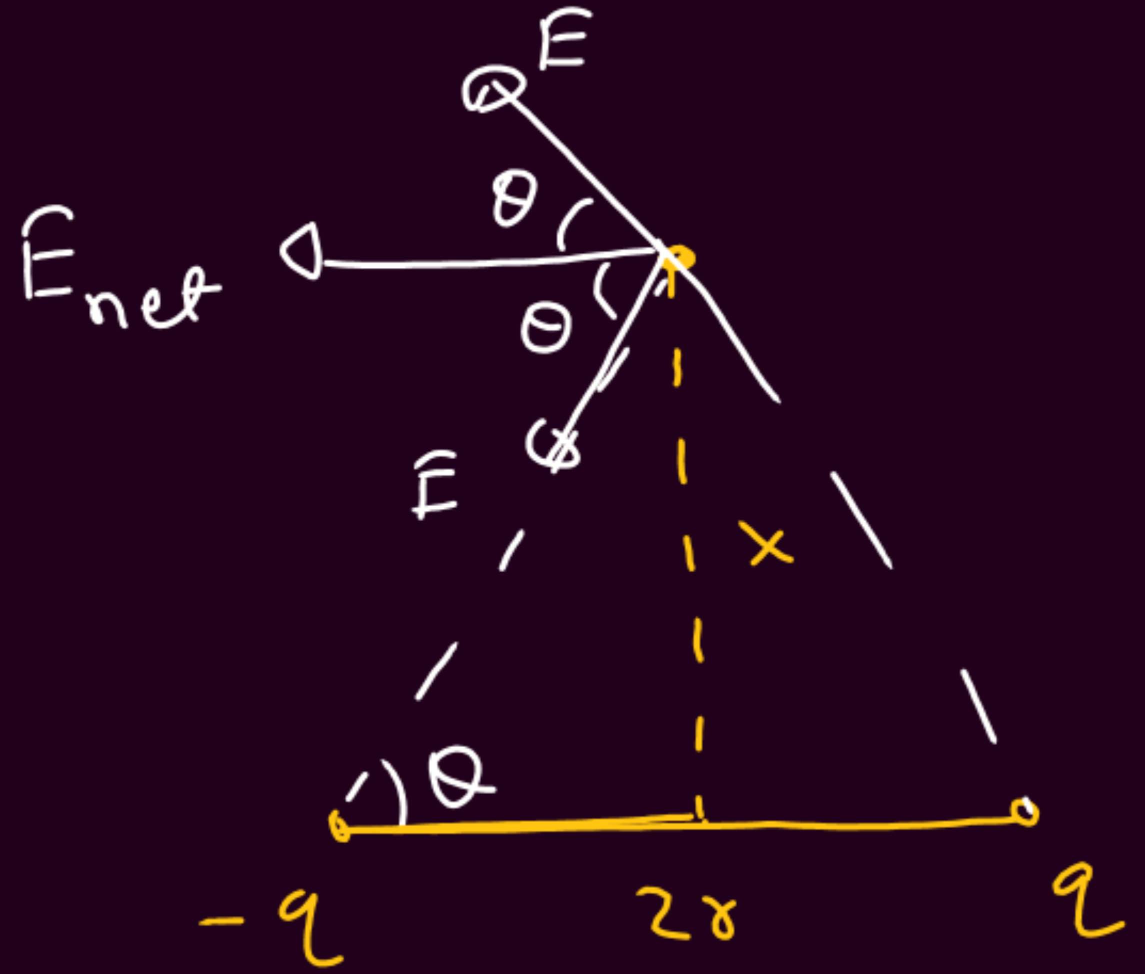
$x = \frac{r}{\sqrt{2}} \quad E_{max}$

$$\boxed{\frac{x}{(x^2+r^2)^{3/2}}}$$

$$E_{net} = 2E\cos\left(\frac{2\theta}{2}\right)$$

$$= 2\frac{kq}{(x^2+r^2)}\frac{x}{(x^2+r^2)^{1/2}}$$

$$\boxed{E_{net} = \frac{2kqx}{(x^2+r^2)^{3/2}}}$$



$x = 0 \quad E_{max}$

$x \uparrow \quad E \downarrow$

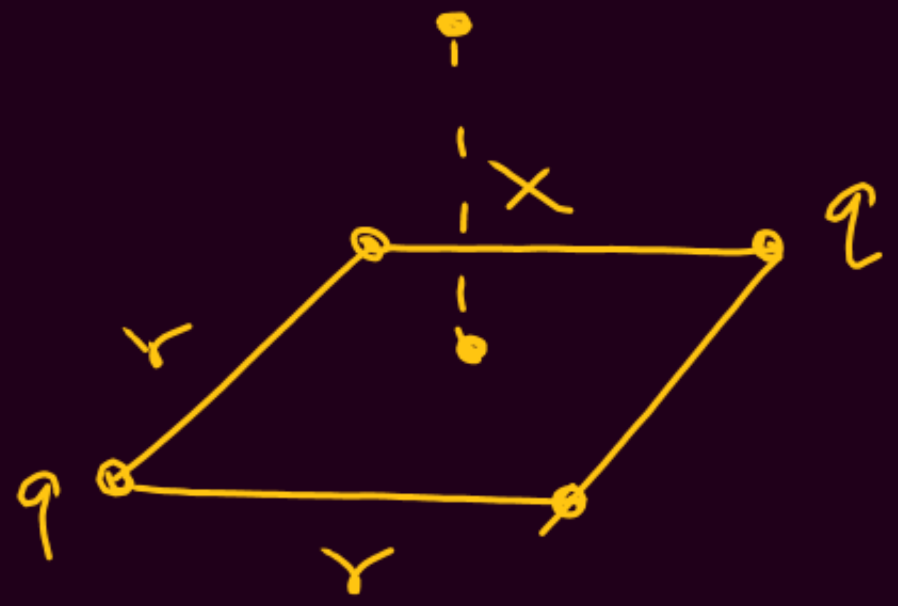
$x \to \infty \quad E = 0$

$$E_{net} = 2E\cos\theta$$

$$= 2\frac{kq}{(x^2+r^2)}\frac{r}{(x^2+r^2)^{1/2}}$$

$$\boxed{E_{net} = \frac{2kqr}{(x^2+r^2)^{3/2}}}$$

For E_{max}

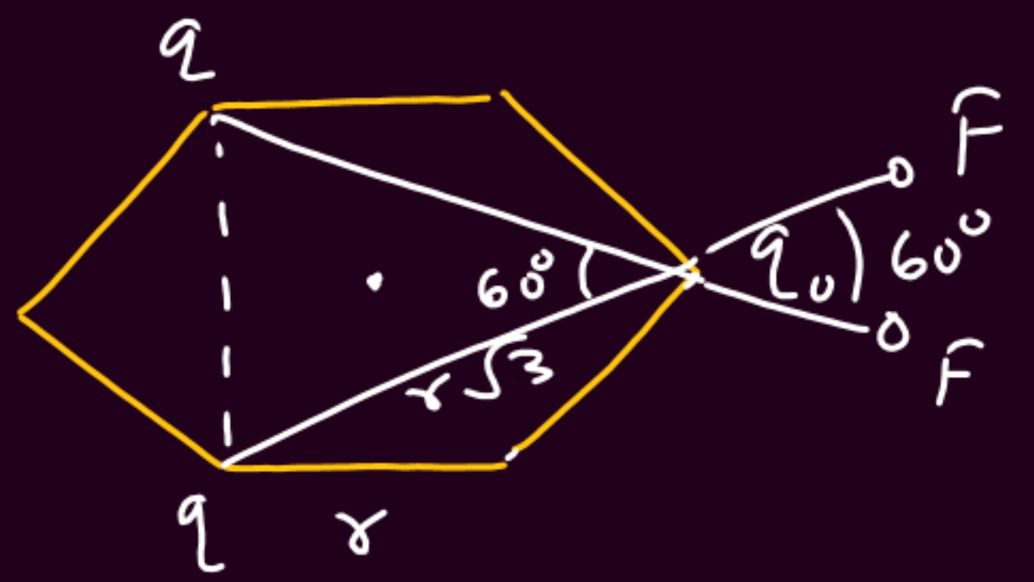


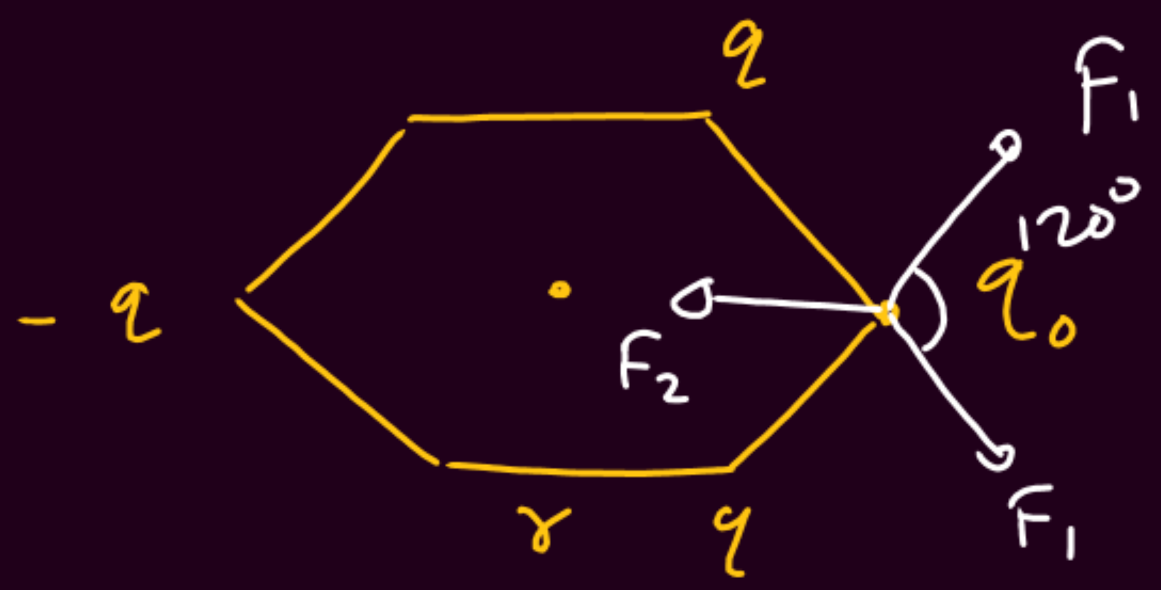
$$\boxed{x = \frac{r}{\sqrt{2}}}$$

$$x = \frac{2r}{2\sqrt{2}}$$

$$\boxed{x = \frac{r}{2\sqrt{2}}}$$

$$x = \frac{r\sqrt{2}}{2\sqrt{2}} = \frac{r}{2}$$





$$\vec{F}_{net} = 2F\cos 30^\circ = F\sqrt{3}$$

$$= \frac{Kqq_0}{(r\sqrt{3})^2}\sqrt{3}$$

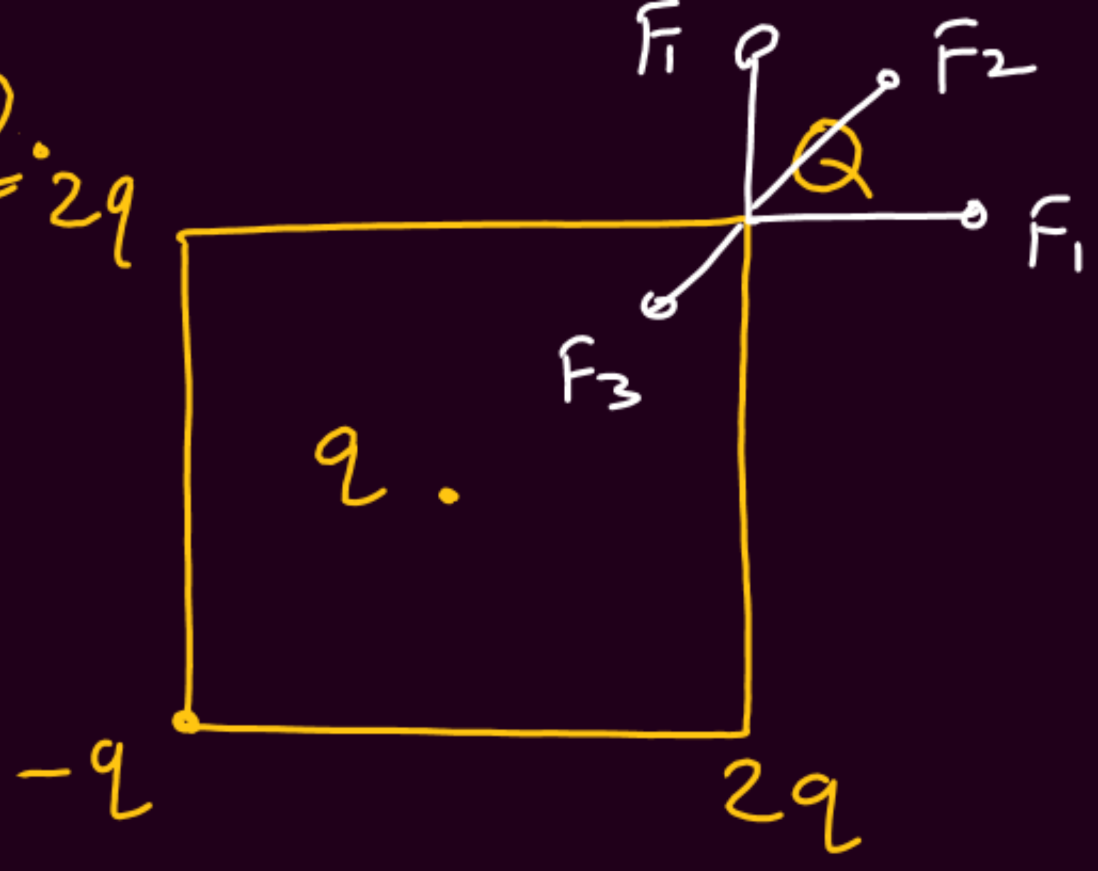
$$= \frac{Kqq_0}{\sqrt{3}r^2}$$

$$F_2 \longleftarrow\!\!\circ \qquad \circ\!\!\longrightarrow 2F_1\cos 60^\circ = F_1$$

$$F_{net} = F_1 - F_2 = \frac{Kqq_0}{r^2} - \frac{Kqq_0}{(2r)^2}$$

$$F_{net} = \frac{3Kqq_0}{4r^2}$$

Q.

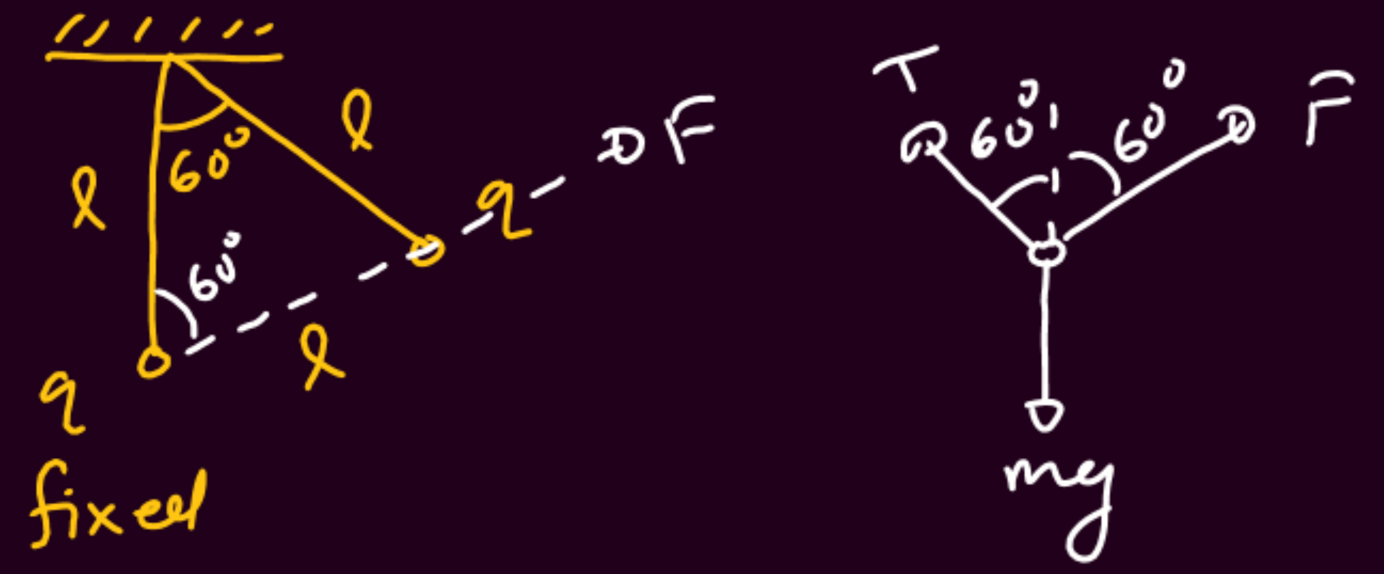


$$F_{net} = F_1\sqrt{2} + F_2 - F_3$$

$$= \frac{k(2q)Q}{r^2}\sqrt{2} + \frac{kqQ}{\left(\frac{r}{\sqrt{2}}\right)^2} - \frac{kqQ}{(r\sqrt{2})^2}$$

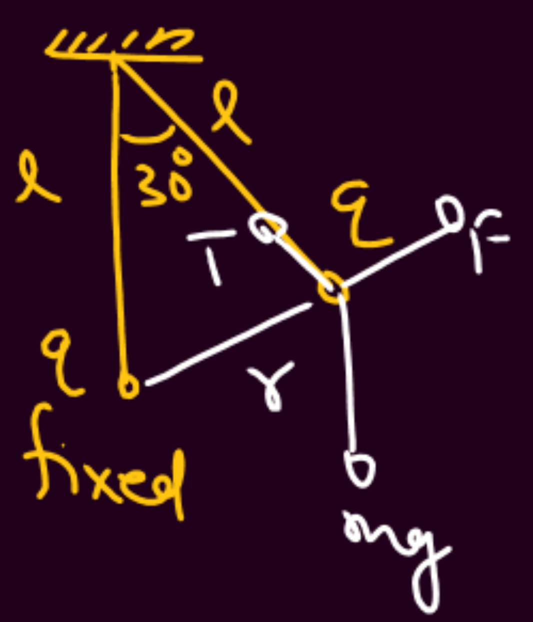
$$F_{net} = \frac{kQq}{r^2}\left(2\sqrt{2} + \frac{3}{2}\right)$$

Q.



$$T = F = \frac{kq^2}{l^2}$$

Q.



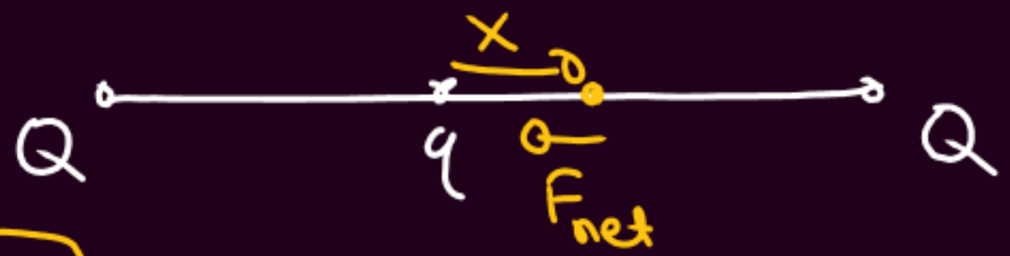
Lami's Theorem

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

$$\frac{T}{\sin(105^\circ)} = \frac{F}{\sin(150^\circ)}$$

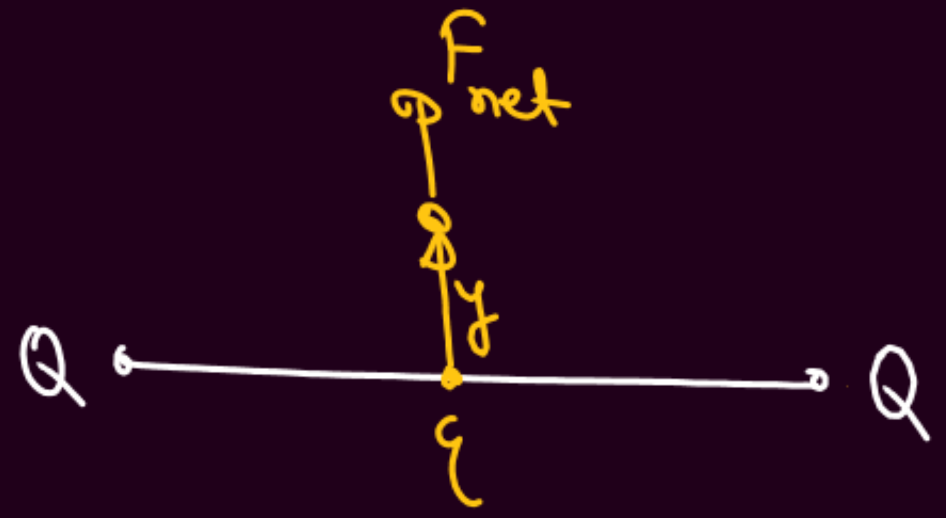
$$r = 2l\sin(15^\circ)$$

① x-dir



$\boxed{\text{Stable}}$

② y-dir



$\boxed{\text{Unstable}}$

① x-dir [unstable]

② y-dir [Stable]

Continuous charge Distribution

$\lambda \rightarrow$ linear charge density

$\sigma \rightarrow$ Surface —— " ——

$\rho \rightarrow$ Volume —— " ——

$$\lambda = \frac{Q}{L}$$

$$\sigma = \frac{Q}{A}$$

$$\rho = \frac{Q}{V}$$

$\lambda = \lambda_0 \cos\theta$

$Q = ?$

$$\boxed{Q = 0}$$

$$dQ = \lambda(R\,d\theta) = \lambda_0 \cos\theta\,(R\,d\theta)$$

$$Q = \lambda_0 R \int_0^{\pi} \cos\theta\,d\theta$$

$$Q = \lambda_0 R\left[\sin\theta\right]_0^{\pi} = \lambda_0 R\left[\sin\pi - \sin 0\right]$$

$$Q = 0$$

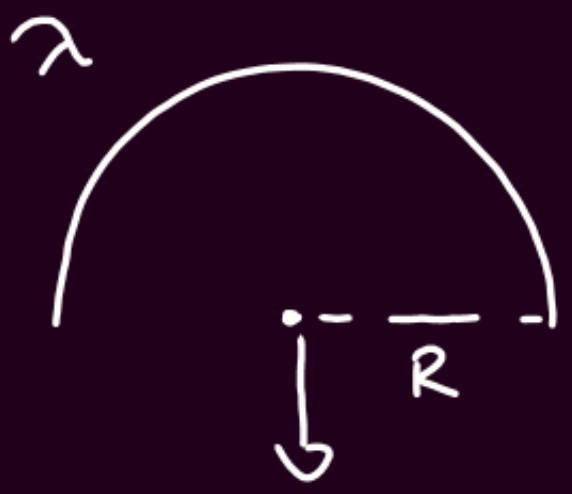
① Circular Arc

$$E = \frac{2k\lambda}{R}\sin\left(\frac{\alpha}{2}\right)$$

Case-1

$$E = 0$$

Case-2 Semicircular ($\alpha = 180^\circ$)



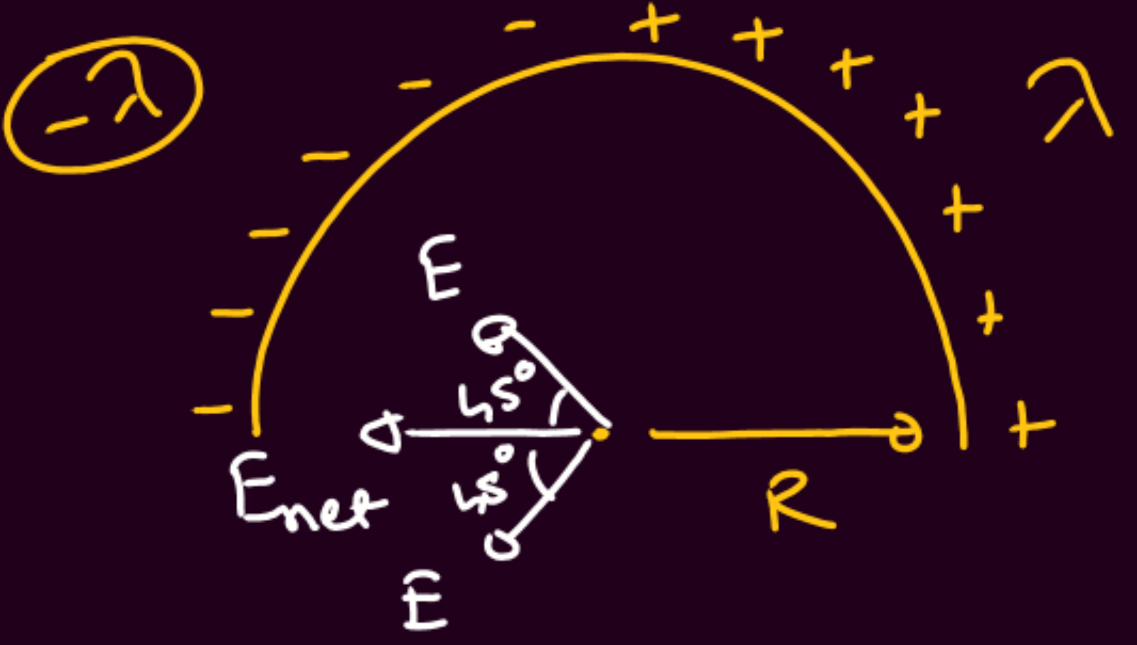
$$\lambda = \frac{Q}{L} = \frac{Q}{\pi R}$$

$$E = \frac{2k\lambda}{R}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 R}$$

$$E = \frac{Q}{2\pi^2\epsilon_0 R^2}$$

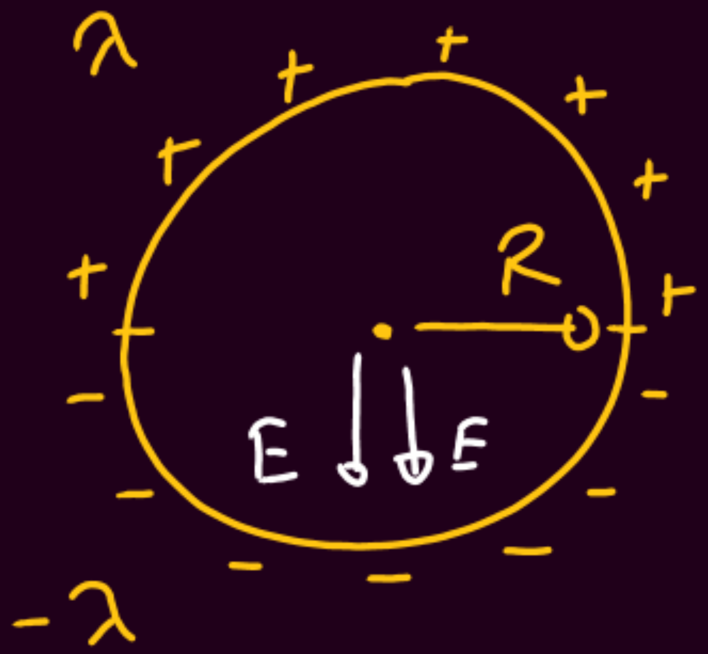
$$E = \frac{Q}{2\epsilon_0 L^2}$$



$$E_{net} = 2E\cos 45^\circ = E\sqrt{2}$$

$$= \left[\frac{2k\lambda}{R}\sin\left(\frac{90^\circ}{2}\right)\right]\sqrt{2}$$

$$= \frac{2k\lambda}{R}$$



$$E_{net} = 2E = 2\left(\frac{2k\lambda}{R}\right)$$

② Circular Ring

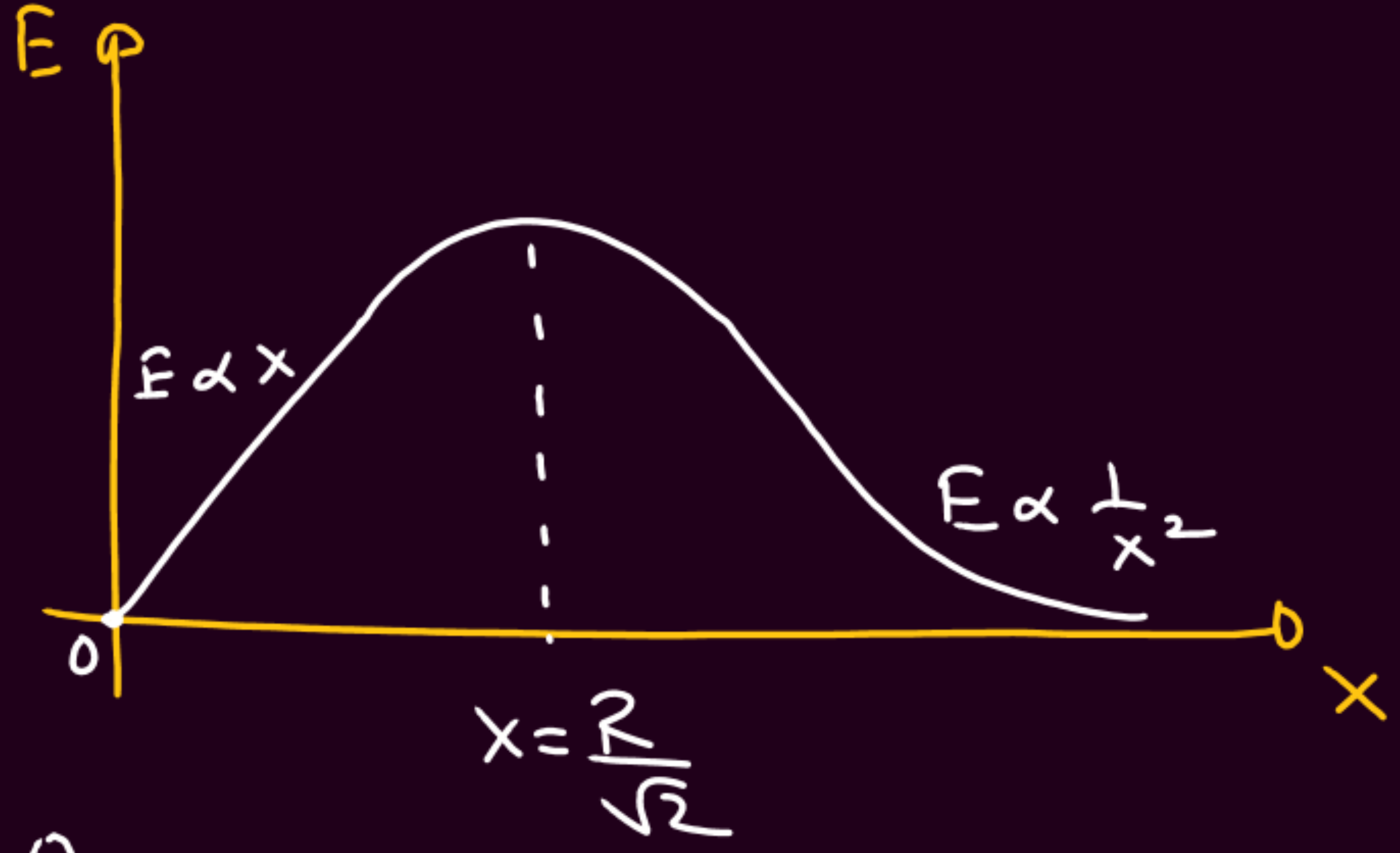
$$\boxed{E = \frac{kQx}{(x^2+R^2)^{3/2}}}$$

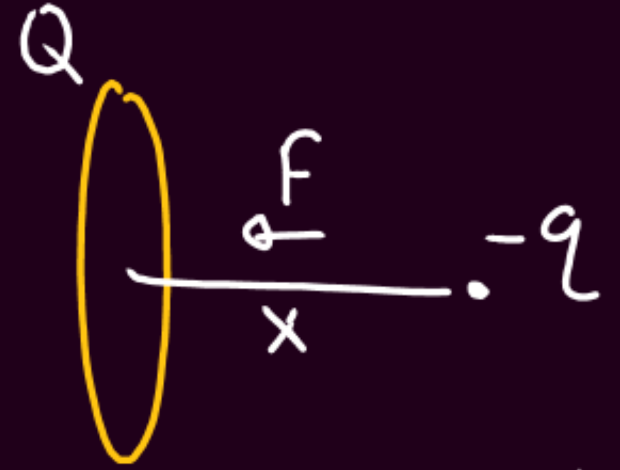
① Center $(x=0)$ $\qquad E = 0$

② $x \ll R$ $\qquad E = \frac{kQx}{R^3}$

③ $x \gg R$ $\qquad E = \frac{kQ}{x^2}$

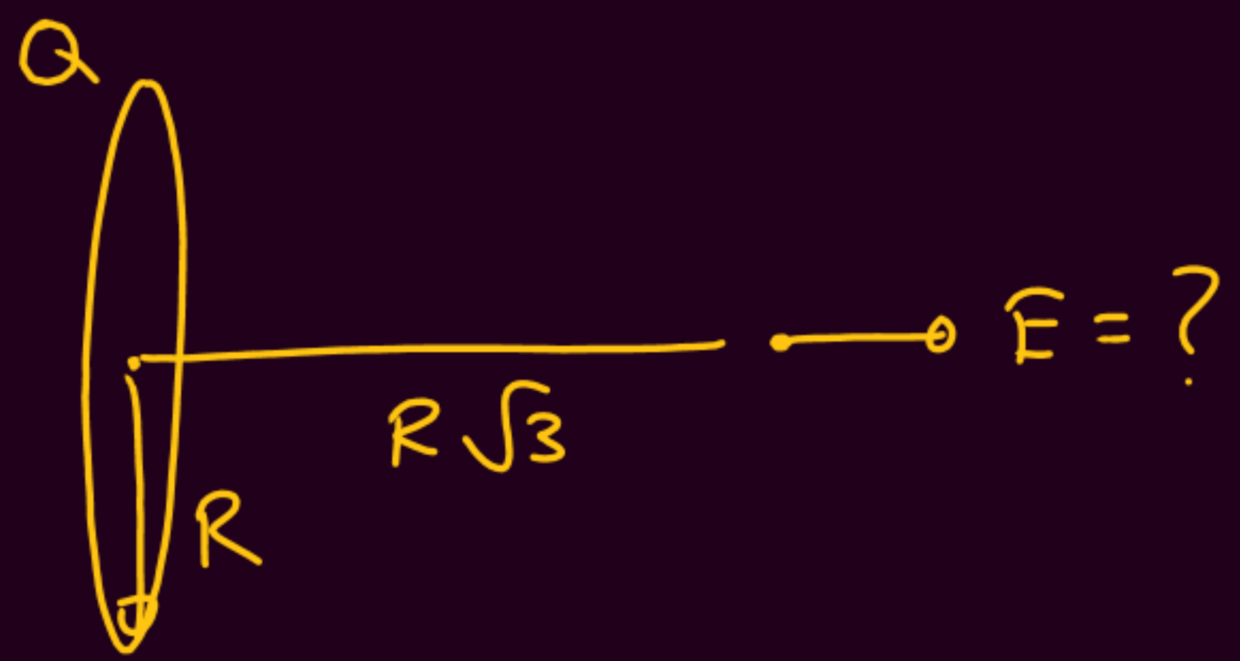
④ $x = \frac{R}{\sqrt{2}}$ $\qquad E_{max}$





Oscillation
↳ SHM if $x \ll R$

$$ma = -\left(\frac{kQq}{R^3}\right)x$$



$$E = \frac{kQ\left(R\sqrt{3}\right)}{\left(3R^2 + R^2\right)^{3/2}}$$

$$= \frac{kQR\sqrt{3}}{\left(4R^2\right)\sqrt{4R^2}} = \frac{\sqrt{3}\,kQ}{8R^2}$$

Symmetry

E_{max}

(1)

$$E = \frac{kQx}{(x^2+R^2)^{3/2}}$$

$$x = \frac{R}{\sqrt{2}}$$

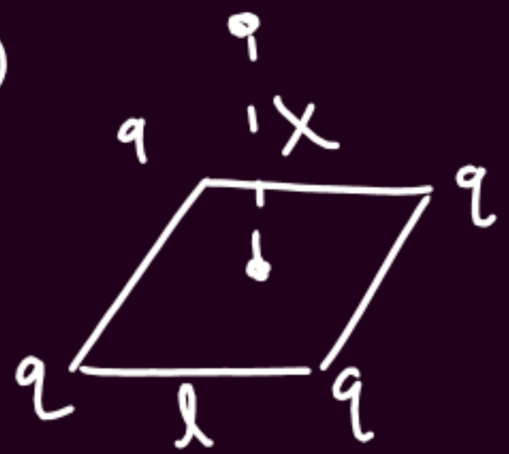
(2)

$Q = 2q$

$R = r$

$$x = \frac{r}{\sqrt{2}}$$

(3)



$Q = 4q$

$R = \frac{l}{\sqrt{2}}$

$$x = \frac{l}{2}$$

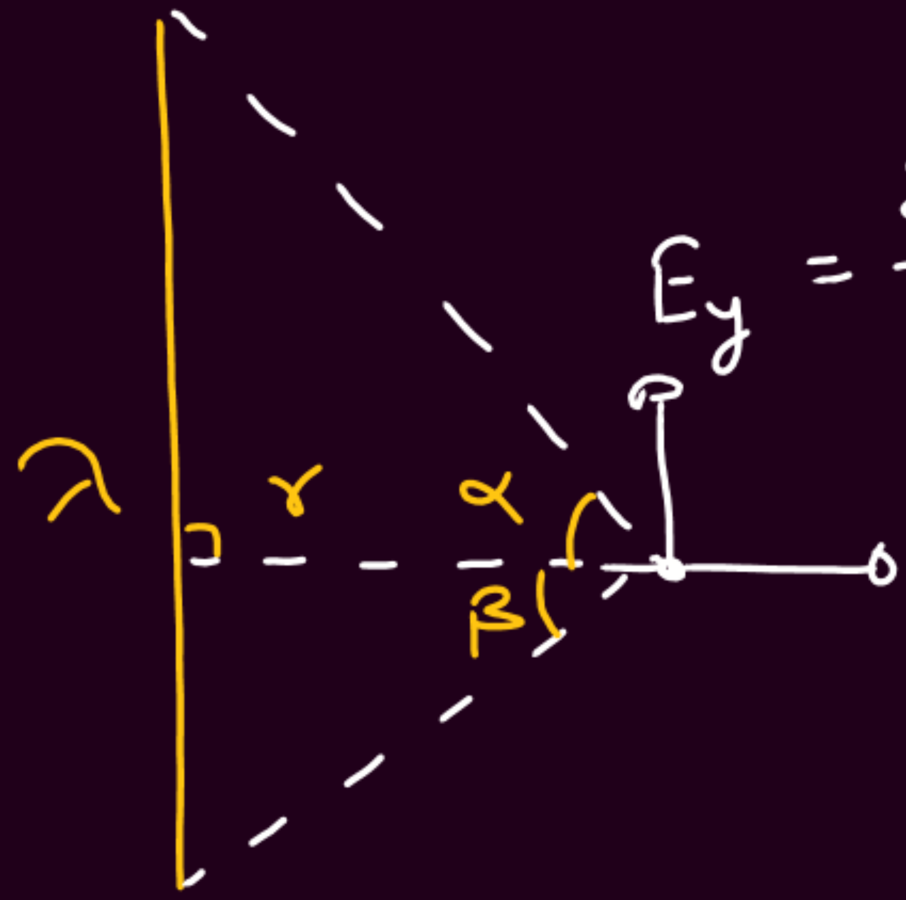
(4)

$Q = 6q$

$R = l$

$$x = \frac{l}{\sqrt{2}}$$

③ St. wire



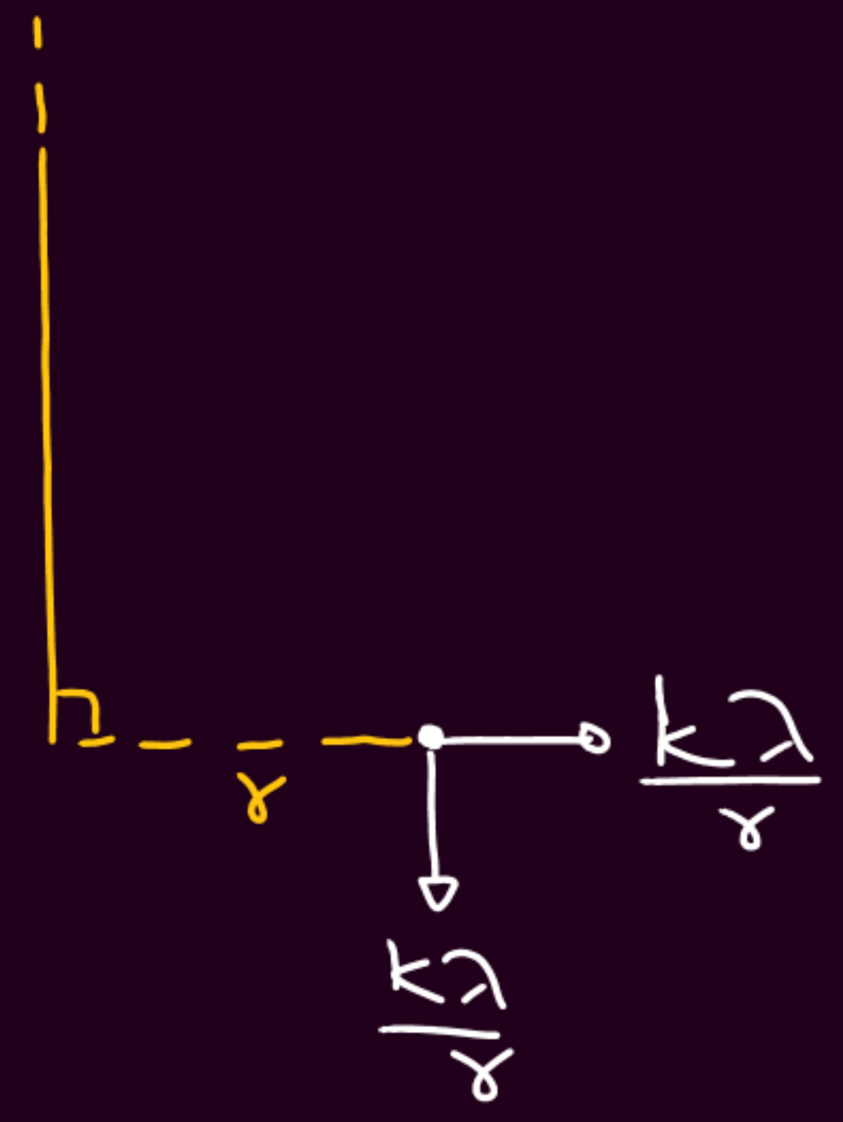
$$E_y = \frac{k\lambda}{r}(\cos\alpha - \cos\beta)$$

$$E_x = \frac{k\lambda}{r}(\sin\alpha + \sin\beta)$$

Extra

④ Semi-infinite $\alpha = 90^\circ$ $\beta = 0^\circ$

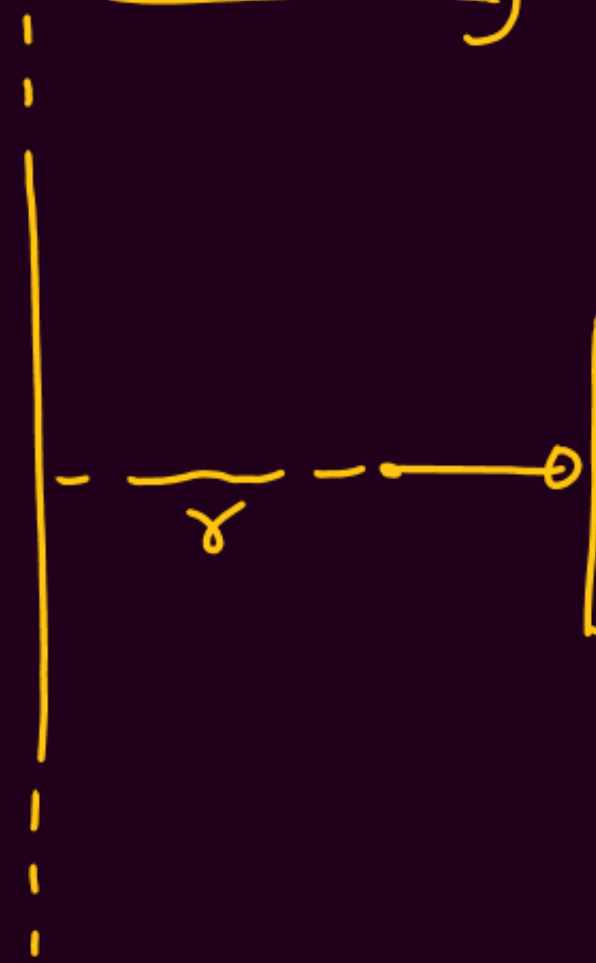
Extra



$$E_x = \frac{k\lambda}{r}\left(\sin 90^\circ + \sin 0\right) = \frac{k\lambda}{r}$$

$$E_y = \frac{k\lambda}{r}\left(\cos 90^\circ - \cos 0^\circ\right) = -\frac{k\lambda}{r}$$

(5) Infinitely long $(\alpha = \beta = 90^\circ)$

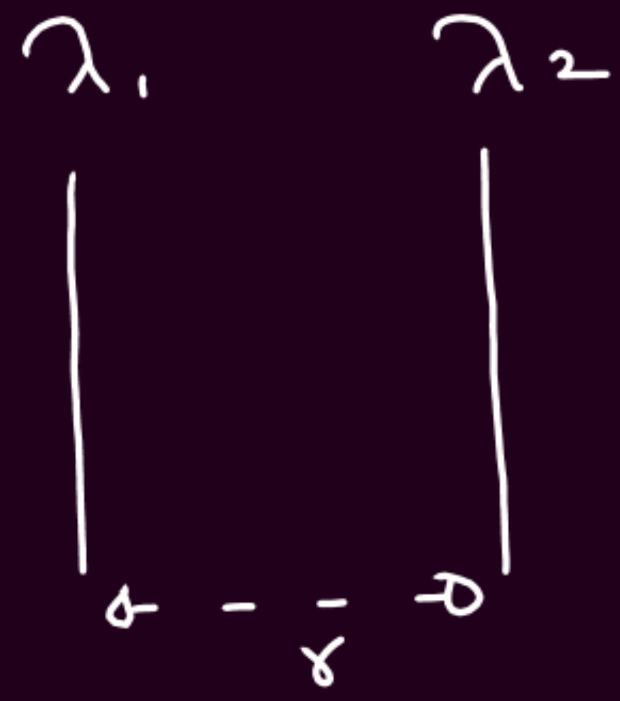


$$\boxed{E = \frac{2k\lambda}{r} = \frac{\lambda}{2\pi\epsilon_0 r}}$$

$$E_x = \frac{k\lambda}{r}(1+1)$$

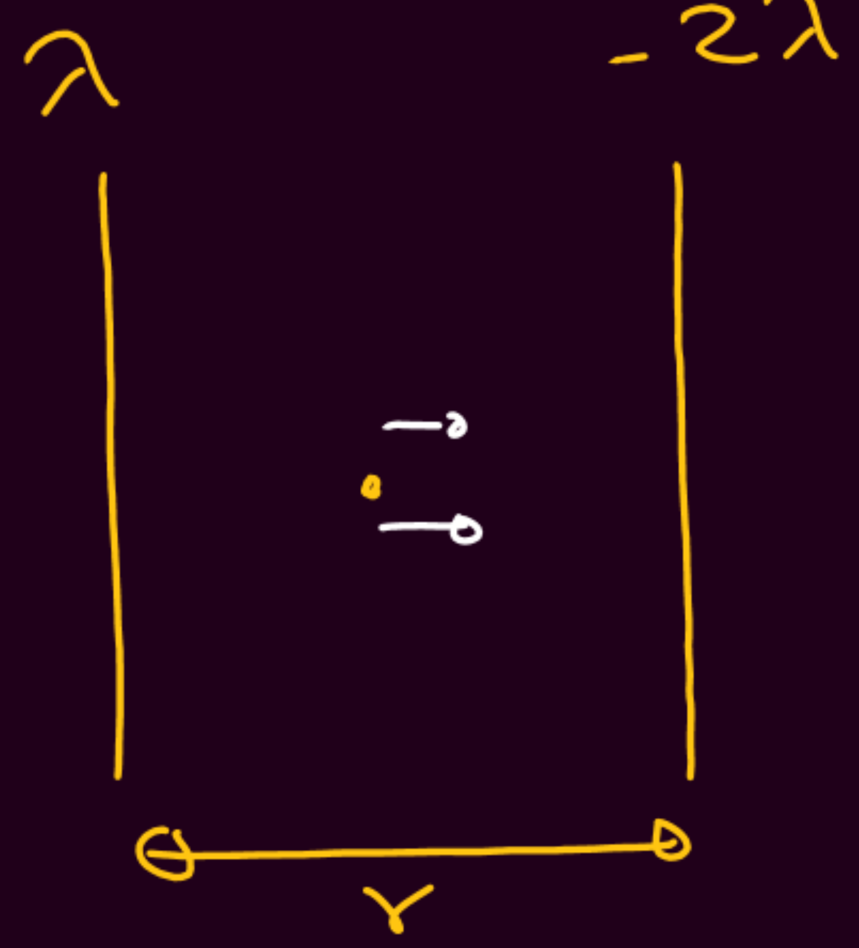
$$E_y = \frac{k\lambda}{r}(0-0) = 0$$

Force per unit length



$$\boxed{\frac{F}{l} = \frac{2k\lambda_1\lambda_2}{r}}$$

Q.

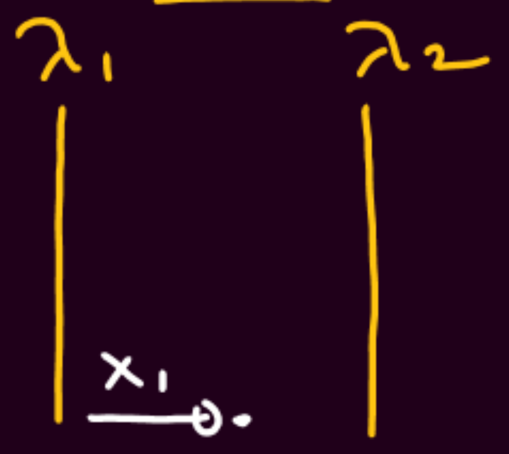


$$E = \frac{2k\lambda}{(r/2)} + \frac{2k(2\lambda)}{r/2}$$

$$E = \frac{12k\lambda}{r}$$

Null pt / Line

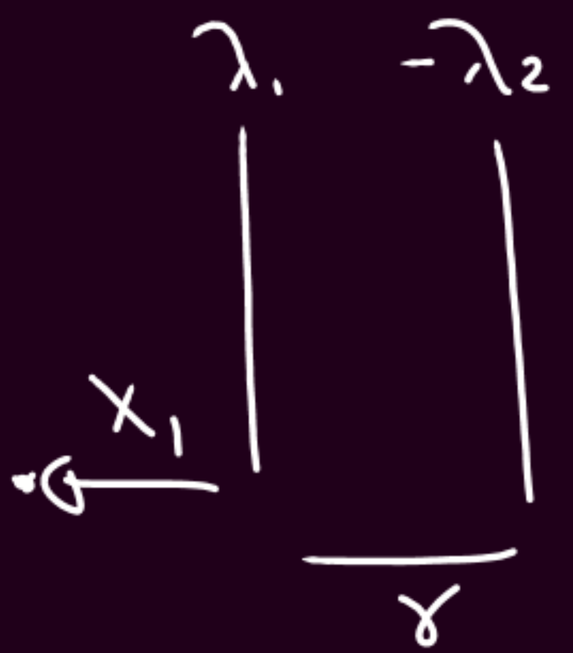
① Like



$$\boxed{x_1 = \frac{r}{\frac{\lambda_2}{\lambda_1} + 1}}$$

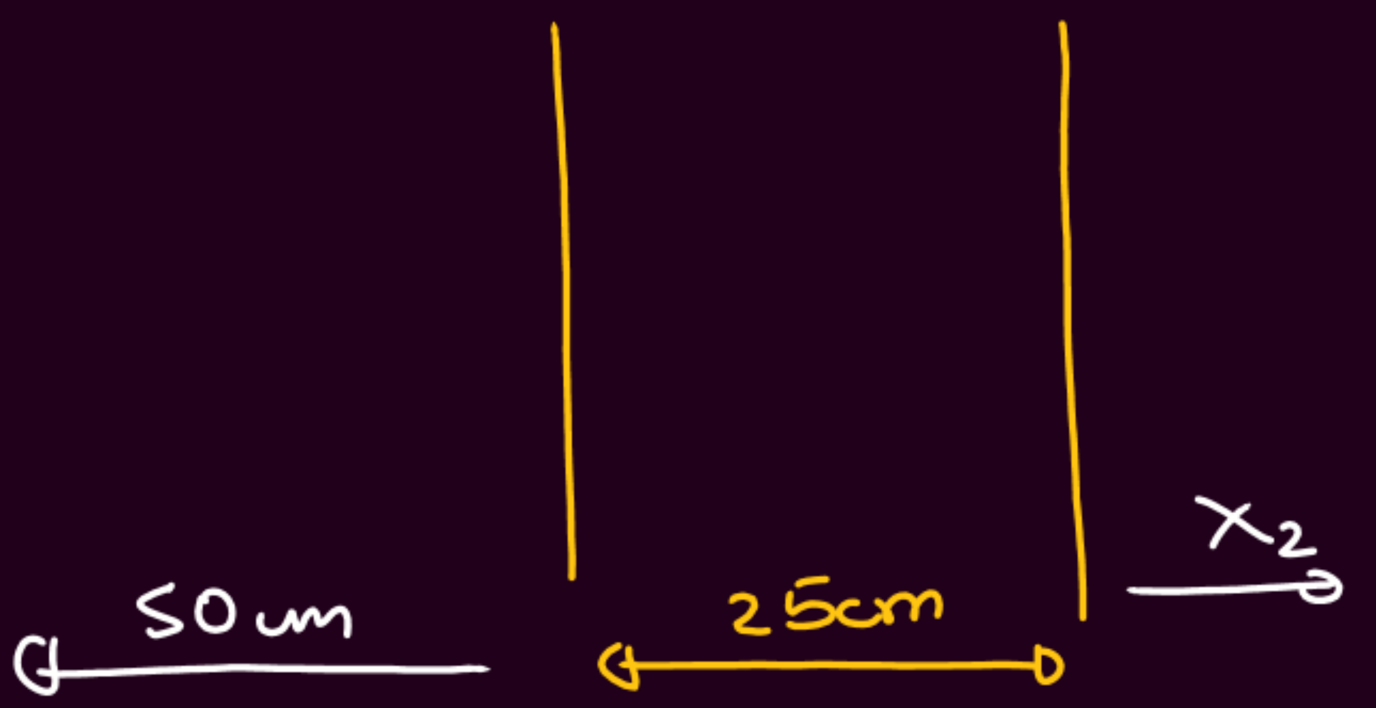
② Equal & opp. No Null pt.

③ Unlike & Unequal



$$x_1 = \frac{r}{\frac{\lambda_2}{\lambda_1} - 1}$$

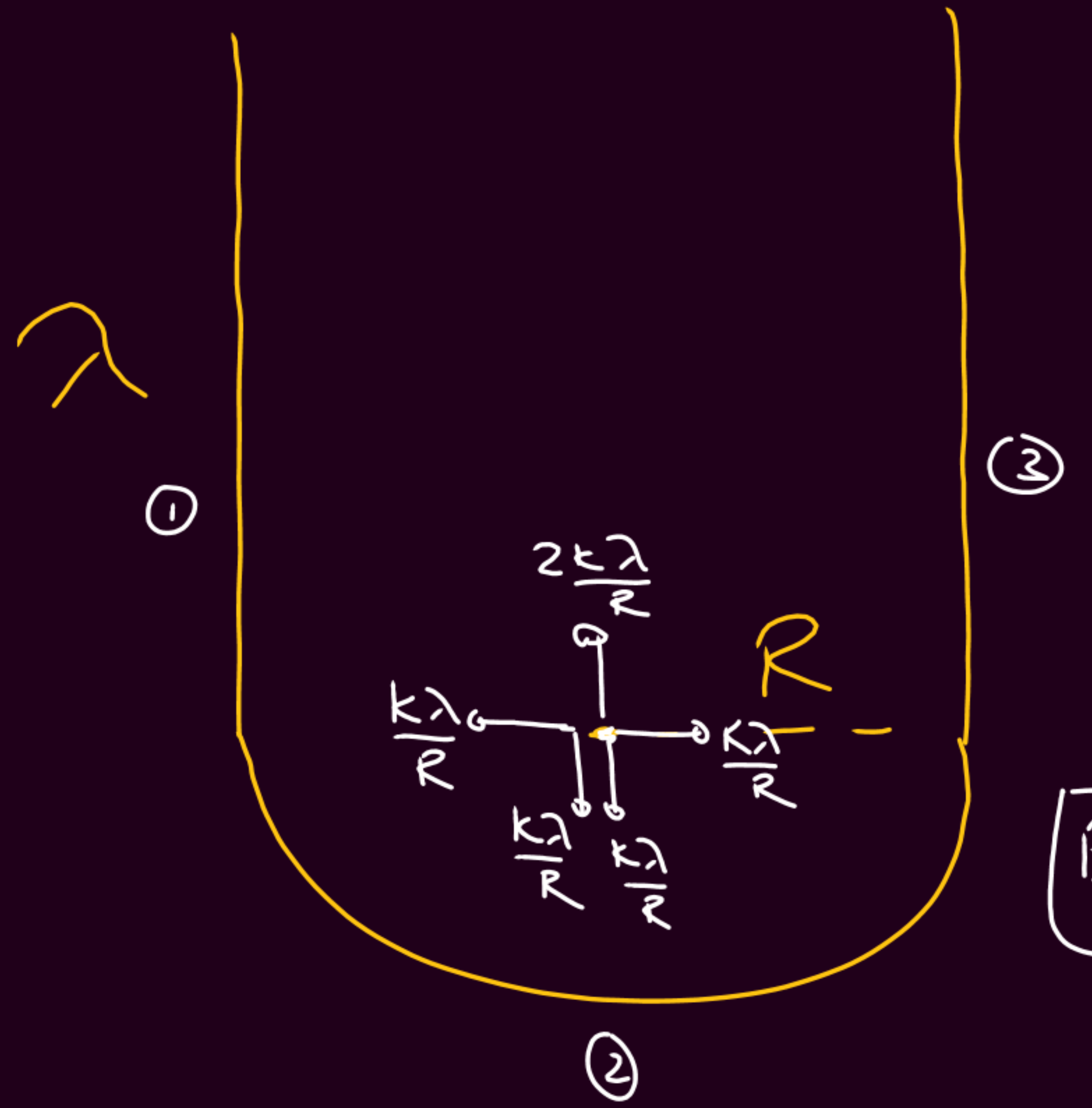
Q.



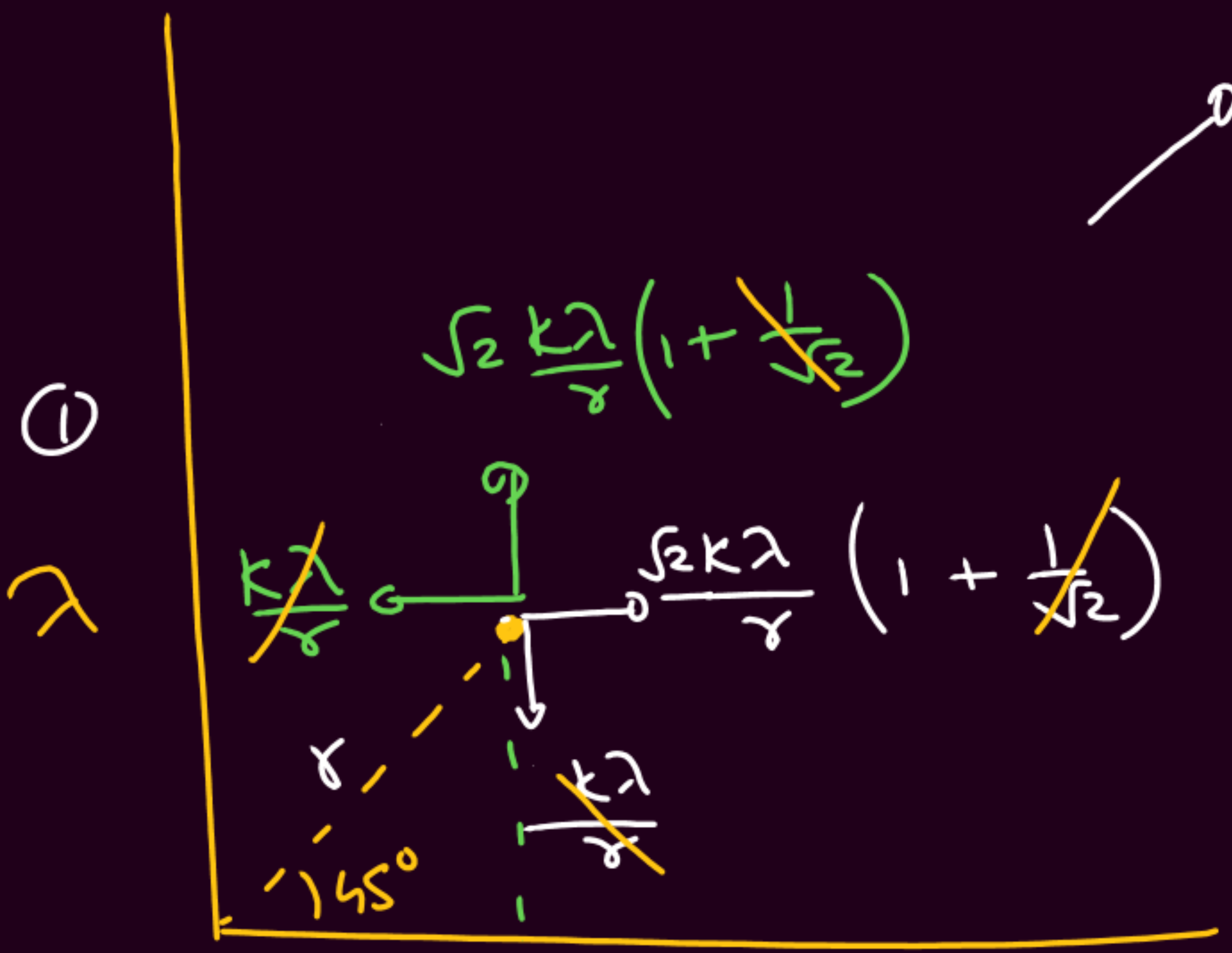
$$X_1 = \frac{25}{\frac{3\lambda}{2\lambda} - 1} = 50\,cm$$

$$X_2 = \frac{25}{\frac{2\lambda}{3\lambda} - 1} = -75\,cm$$

Q.



$$\boxed{E_{net} = 0}$$



$$E_{net} = \frac{2k\lambda}{r}$$

$$E_x = \frac{k\lambda}{\left(\frac{r}{\sqrt{2}}\right)}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$$E_y = \frac{k\lambda}{\left(\frac{r}{\sqrt{2}}\right)}\left(0 - \frac{1}{\sqrt{2}}\right)$$

Motion of charged Particle in Uniform $\vec{E}$.

$\vec{E}$ = const.

$\vec{F}$ = const

$\vec{a} = \frac{q\vec{E}}{m}$ = const.

① $v = u + at$

② $S = ut + \frac{1}{2}at^2$

③ $v^2 = u^2 + 2aS$

Path

① Rest or $\vec{u} \parallel \vec{E}$

path ⇒ st. Line

② $\vec{u} \nparallel \vec{E}$

path ⇒ parabola

Rest

$$v = \frac{qE}{m} t$$

$$x = \frac{1}{2} \frac{qE}{m} t^2$$

$$v^2 = 2 \frac{qE}{m} x$$

e^- & proton in uniform $\vec{E}$.

① $$\frac{F_e}{F_p} = \frac{1}{1}$$

② $$\frac{a_e}{a_p} = \frac{m_p}{m_e}$$

③ **v in same time**

$$\frac{v_e}{v_p} = \frac{m_p}{m_e}$$

④ **v in same dist.**

$$\frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}}$$

⑤ **t for same distance**

$$\frac{t_e}{t_p} = \sqrt{\frac{m_e}{m_p}}$$

Electrostatic PE

$$\boxed{\Delta U = -W_c}$$

① U of 2 pt. charge System

$$\boxed{U = \frac{K q_1 q_2}{r}}$$

q_1 —— r —— q_2

Q. Sep. $\uparrow$ then U

1) $\uparrow$

2) $\downarrow$

✓ 3) may $\uparrow$ or $\downarrow$

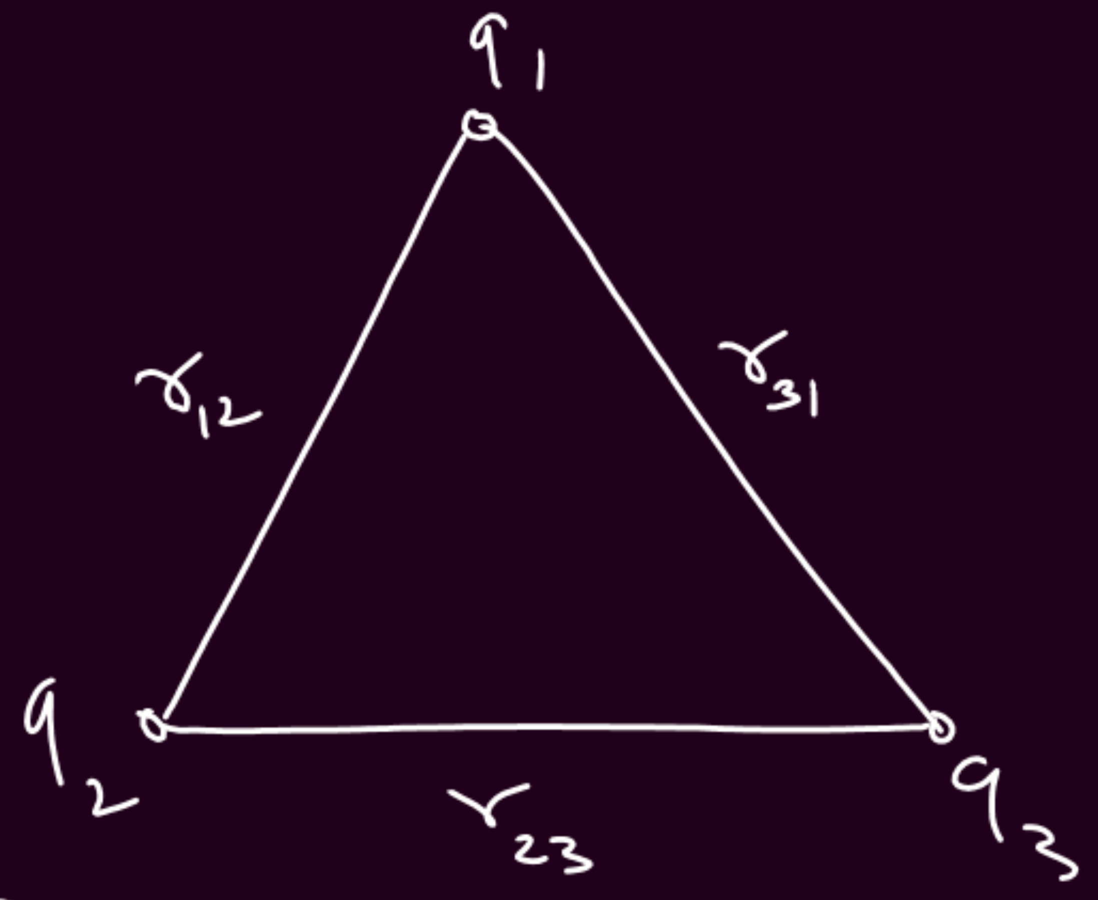
Like: Sep. $\uparrow$ U $\downarrow$

Unlike: sep. $\uparrow$ U $\uparrow$

Q. $2\mu C$ —— 10 cm —— $-1\mu C$

$$U = \frac{(9\times10^9)(2\mu)(-1\mu)}{10\times10^{-2}}$$

$$\boxed{U = -0.18\ J}$$

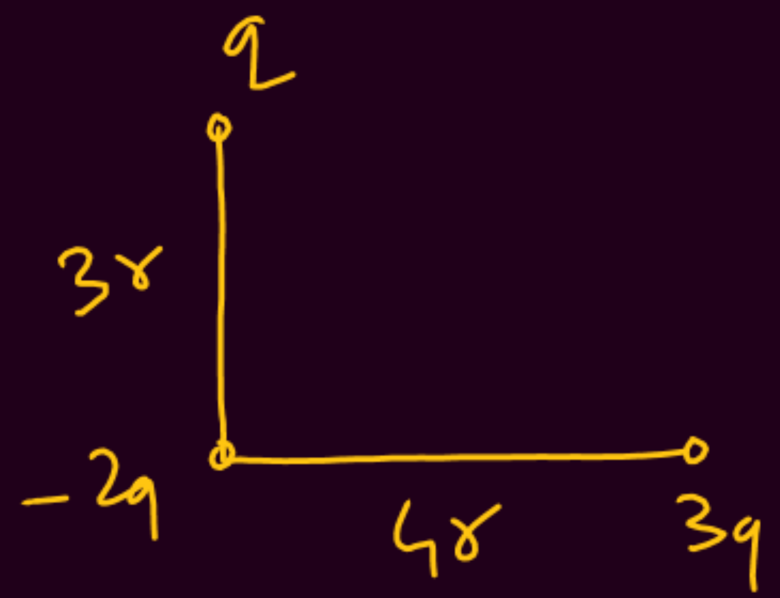


(1)

$$U_{sys} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_2 q_3}{r_{23}} + \frac{k q_3 q_1}{r_{31}}$$

(2) <u>PE of q_2</u>

$$U_{q_2} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_2 q_3}{r_{23}}$$



(1) $$U_{sys} = \frac{k\, q(-2q)}{3r} + \frac{k\,(-2q)(3q)}{4r} + \frac{k\, q(3q)}{5r}$$

(2) $$U_q = \frac{k\, q(-2q)}{3r} + \frac{k\, q(3q)}{5r}$$

Self Energy (Extra)

1. Sphere with uniform Surface charge distribution

$$U = \frac{kQ^2}{2R}$$

2. Sphere with uniform voloume charge distribution

$$U = \frac{3kQ^2}{5R}$$

① - W.D. in taking/Bringing a q_0 from A to B

- W.D against the field

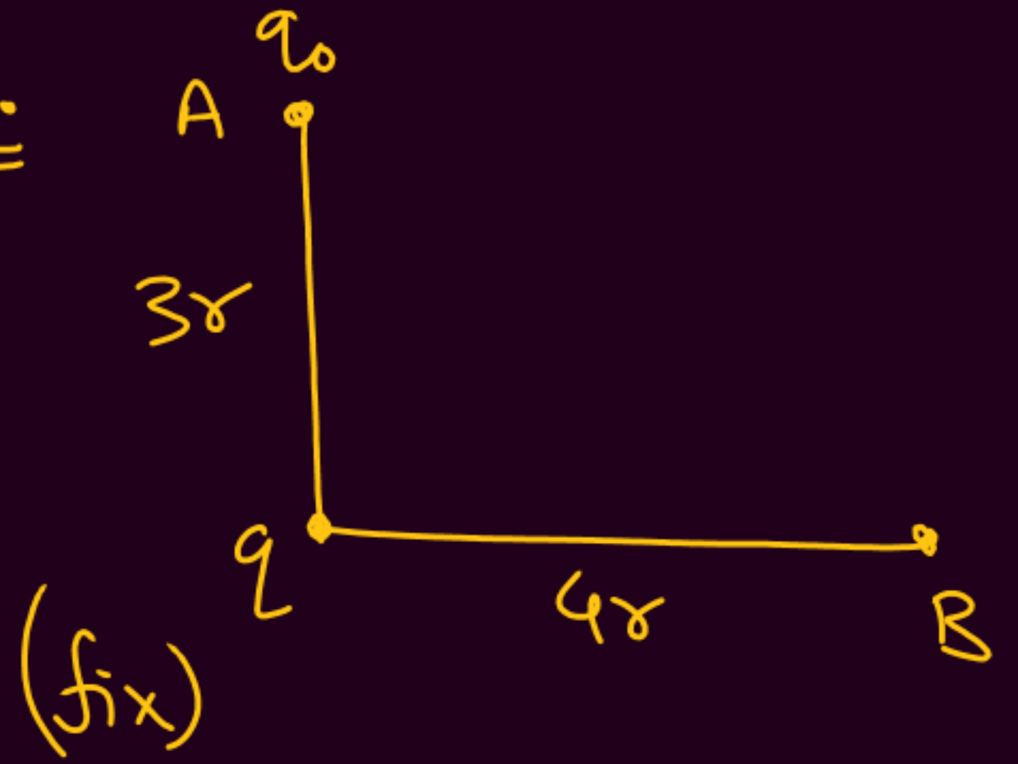
$$\boxed{W.D. = \Delta U}$$

② W.D. by field $= -\Delta U$

Q. q displaced by d $(\vec{E})$

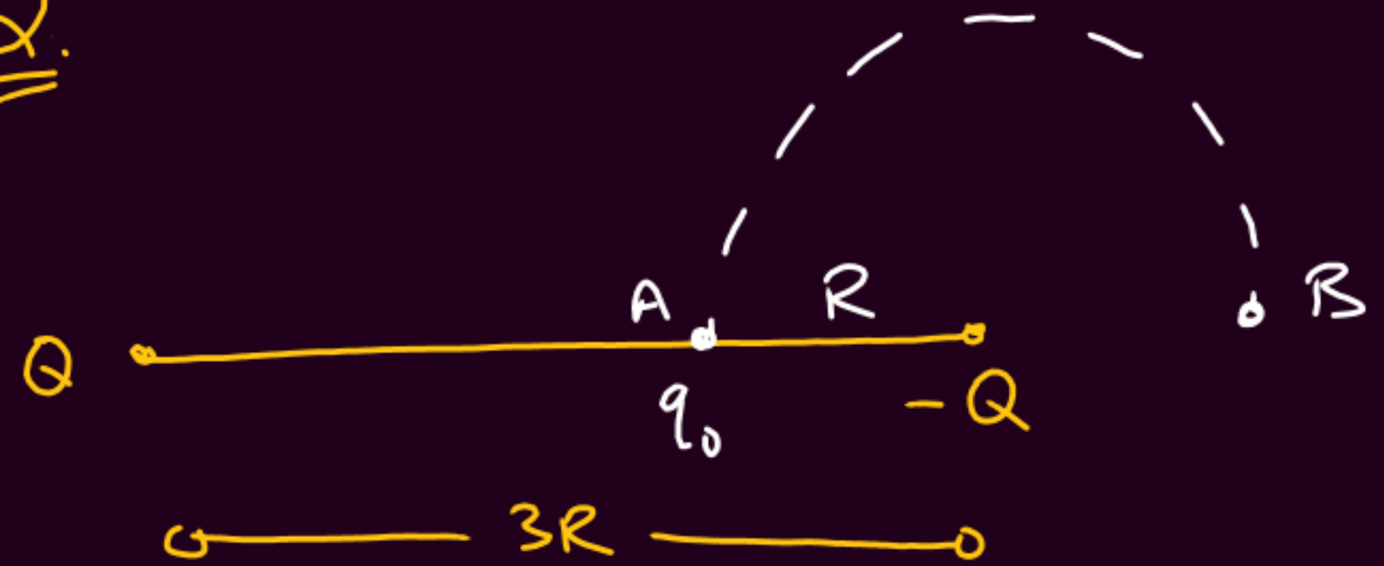
$$W_{by} = (qE)d$$

Q.



$$W = U_B - U_A = \frac{kqq_0}{4r} - \frac{kqq_0}{3r}$$

Q.

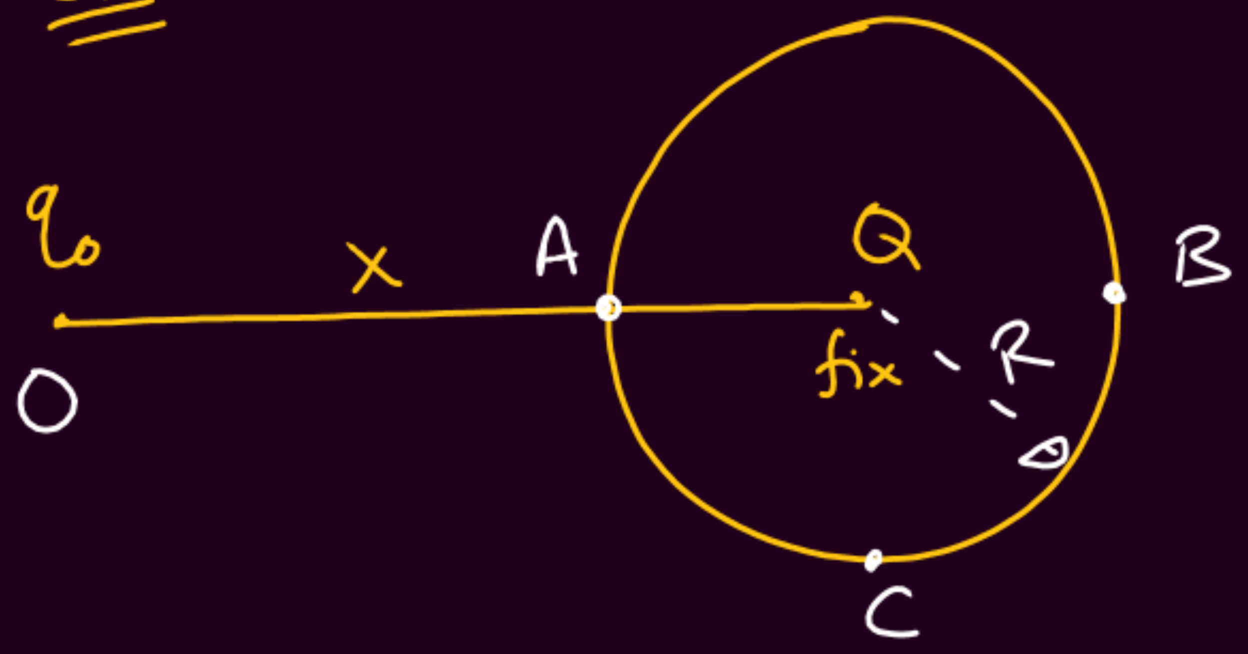


(B)

$$W = \left[\frac{KQq_0}{4R} + \cancel{\frac{K(-Q)q_0}{R}}\right] - \left[\frac{KQq_0}{2R} + \cancel{\frac{K(-Q)q_0}{R}}\right]$$

$$W = -\frac{KQq_0}{4R}$$

Q



$$W_1 = W_{O\to A}$$

$$W_2 = W_{O\to B}$$

$$W_3 = W_{O\to C}$$

$$U_i = \frac{KQq_0}{x}$$

$$U_f = \frac{KQq_0}{R}$$

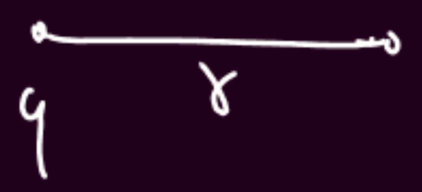
Electrostatic Potential (V) Scalar

① $V = \dfrac{U}{q_0}$ $\text{Volt} = \dfrac{\text{Joule}}{\text{Coulomb}}$

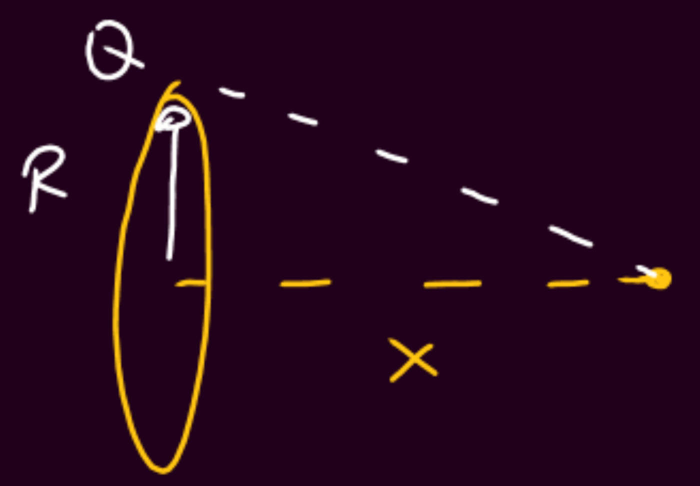
② $\Delta V = \dfrac{\Delta U}{q_0}$

① Potential due to point charge

$$V = \frac{kq}{r}$$



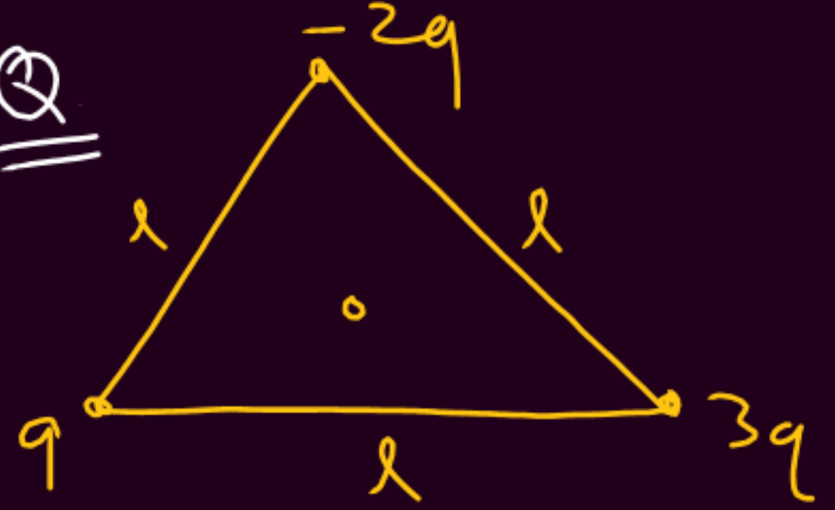
② V due to Ring



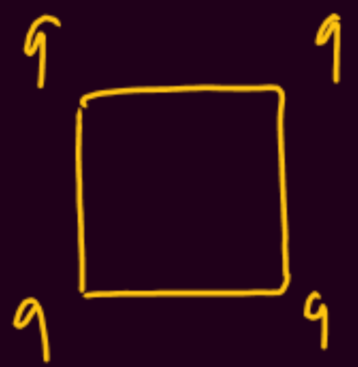
$$V = \frac{kQ}{\sqrt{x^2+R^2}}$$

$$V_{center} = \frac{kQ}{R}$$

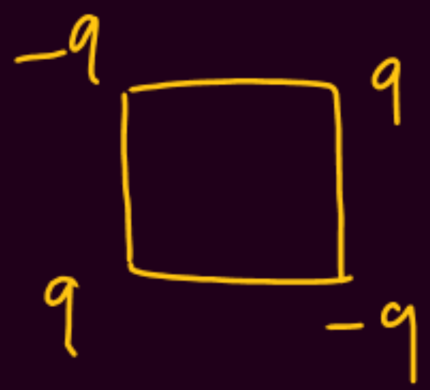
Q



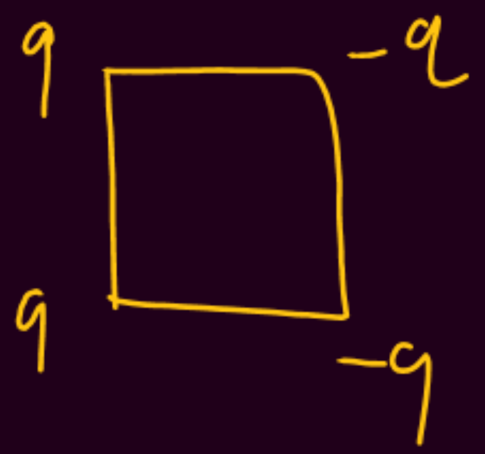
$$V = \frac{k}{\left(\frac{l}{\sqrt{3}}\right)}(q-2q+3q) = \frac{2\sqrt{3}kq}{l}$$



$E = 0$ $V \neq 0$



$E = 0$ $V = 0$



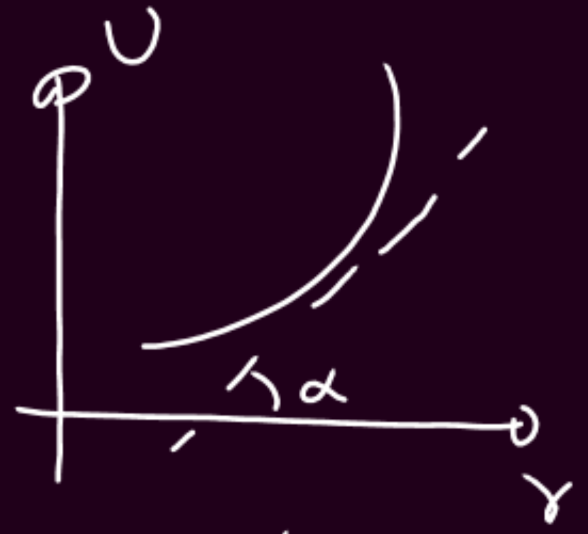
$E \neq 0$ $V = 0$

F & U

$$\Delta U = -\int \vec{F}\cdot d\vec{r}$$

1D

$$F = -\frac{dU}{dr}$$



$$F = -(\text{slope})$$

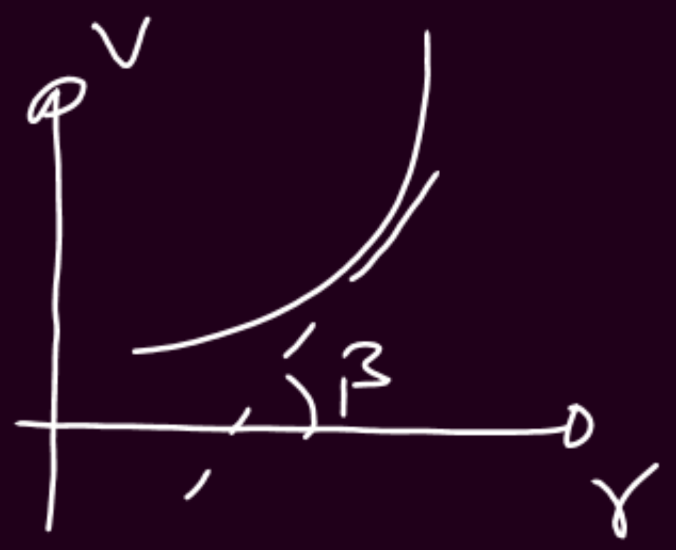
E & V

$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$E = -\left(\frac{dV}{dr}\right)$$

Potential Gradient

$$\boxed{\frac{N}{C} = \frac{V}{m}}$$



$$E = -(\text{slope})$$

3D

$$F_x = -\frac{\partial U}{\partial x}$$

$$F_y = -\frac{\partial U}{\partial y}$$

$$F_z = -\frac{\partial U}{\partial z}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

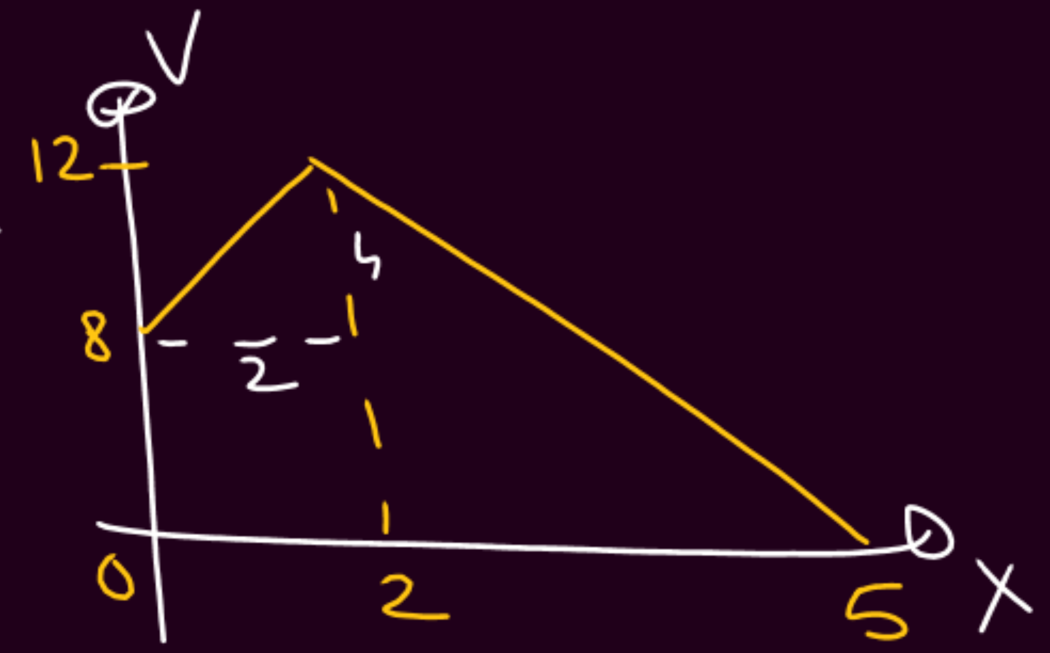
Q. $V = -2x^2$

$E = ?$ at $x = 2$ m

$$\frac{dV}{dx} = -4x$$

$$E = +4x = 8 \text{ V/m}$$

Q.



$x=1 \quad E = -\left(\frac{4}{2}\right) = -2 \text{ V/m}$

$x=4 \quad E = -\left(-\frac{12}{3}\right) = +4 \text{ V/m}$

Q. $V = 4x^2 - 8x + 2$

Find V at N.P.

$$E = 0$$

$$\frac{dV}{dx} = 0$$

$$8x - 8 = 0$$

$$x = 1 \text{ m}$$

$$V = 4 - 8 + 2 = -2 \text{ V}$$

Q. $V = 6xy^2z$ $\qquad \vec{E}(1,1,-1) = ?$

$$\frac{\partial V}{\partial x} = 6(1)y^2z = -6$$

$$\frac{\partial V}{\partial y} = 6x(2y)z = -12$$

$$\frac{\partial V}{\partial z} = 6xy^2(1) = 6$$

$$\vec{E} = 6\hat{i} + 12\hat{j} - 6\hat{k}$$

$$E = 6\sqrt{1^2+2^2+1^2}$$

$$E = 6\sqrt{6} \ V/m$$

Q. $V = 3xy^2 - 2z$ $\qquad \vec{E}(1,-1,1) = ?$

$$\frac{\partial V}{\partial x} = 3y^2 - 0 = 3$$

$$\frac{\partial V}{\partial y} = 6xy - 0 = -6$$

$$\frac{\partial V}{\partial z} = 0 - 2 = -2$$

$$\vec{E} = -3\hat{i} + 6\hat{j} + 2\hat{k}$$

$$E = \sqrt{9+36+4} = 7 \ V/m$$

Q. $V = x^2y - 2yz^2 + x^2z$ $\qquad \vec{E}(1, 0, -1) = ?$

$$\frac{\partial V}{\partial x} = 2xy + 2xz = -2$$

$$\frac{\partial V}{\partial y} = x^2 - 2z^2 = -1$$

$$\frac{\partial V}{\partial z} = -4yz + x^2 = 1$$

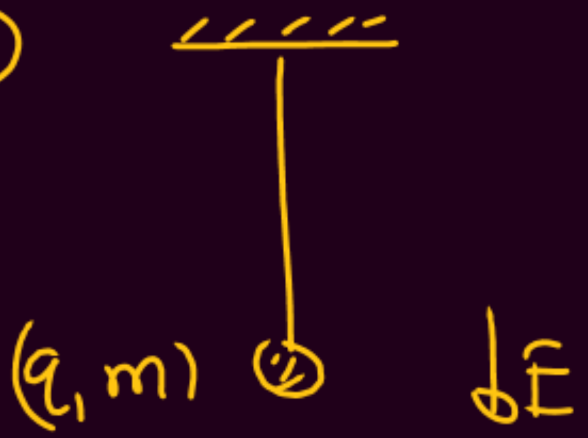
$$\vec{E} = 2\hat{i} + \hat{j} - \hat{k}$$

$$E = \sqrt{4+1+1} = \sqrt{6}\ V/m$$

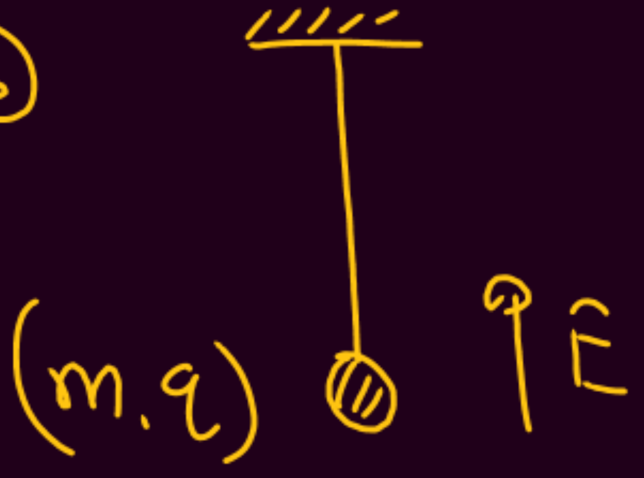
Time Period of Pendulum

① 

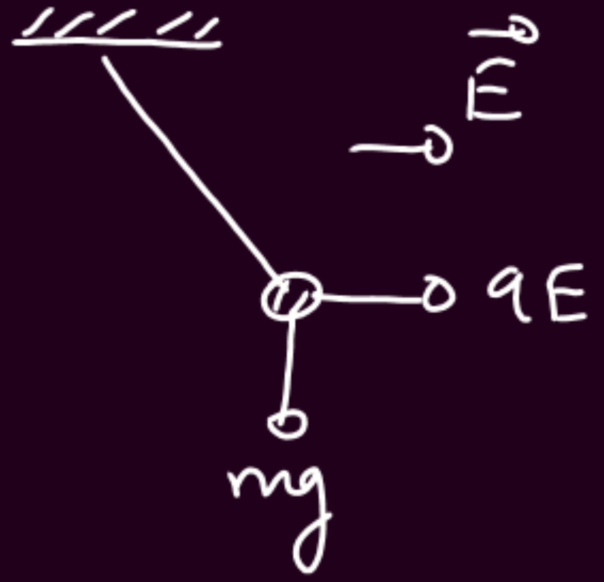
$$T = 2\pi\sqrt{\frac{l}{g}}$$

② 

$$T = 2\pi\sqrt{\frac{l}{g + \frac{qE}{m}}}$$

③ 

$$T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$$

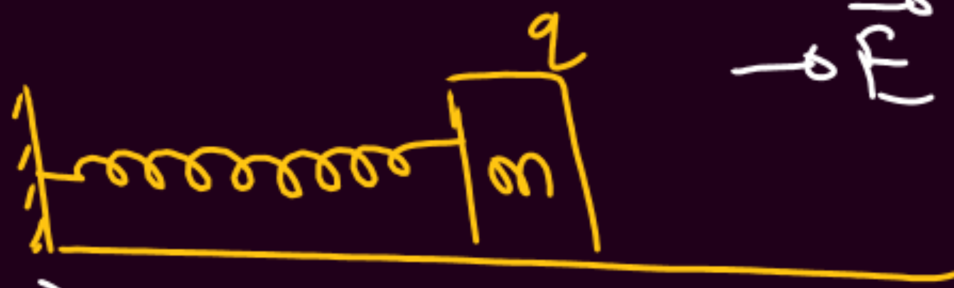
④ 

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

⑤ 

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Spring-Block



Suddenly uniform $\vec{E}$ is switched on

Max. Ext. $\quad X_m = \frac{2qE}{k}$

Dot Product (Scalar)

① $\vec{a}\cdot\vec{b} = ab\cos\theta$

$\oplus$ $\quad$ 0 $\quad$ $\ominus$

② If $\vec{a} \perp \vec{b}$ then $\vec{a}\cdot\vec{b} = 0$

③ $\vec{a}\cdot\vec{a} = a^2$

④ $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

⑤ $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

⑥ $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$$\vec{a}\cdot\vec{b} = x_1x_2 + y_1y_2 + z_1z_2$$

Ex-1 $\quad a = 12 \quad b = 4 \quad \theta = 60^\circ$

$\vec{a}\cdot\vec{b} = (12)(4)\cos 60^\circ = 24$

Ex-2 $\quad \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\vec{a}\cdot\vec{b} = (1)(2) + (2)(-1) + (-3)(-2)$

$= 6$

$$\boxed{\Delta V = -\int \vec{E} \cdot d\vec{r}}$$

① In the dir of $\vec{E}$

$V \downarrow$

② If $\vec{E} = 0$ then

$\Delta V = 0$

$V = \text{Const.}$

$W = 0$

③ In a closed path

$\Delta V = 0$

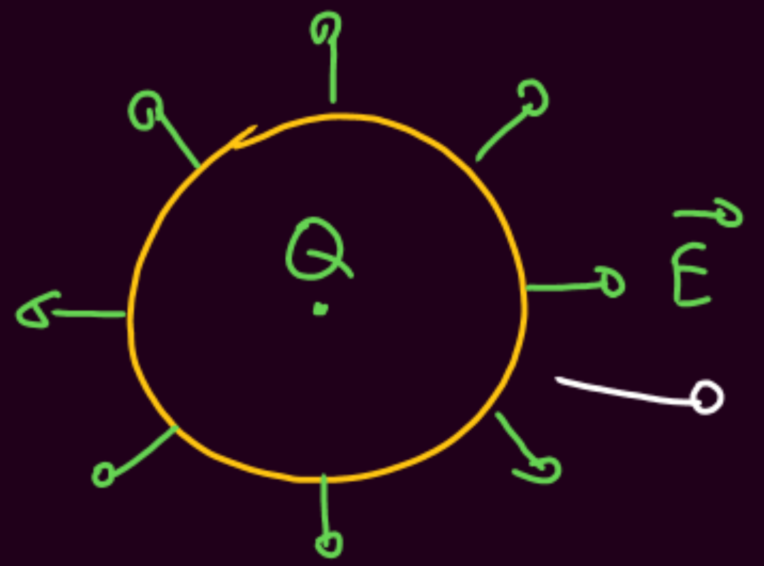
$W = 0$

(conservative $\vec{E}$)

④ If $\vec{E} \perp d\vec{r}$

$\Delta V = 0$

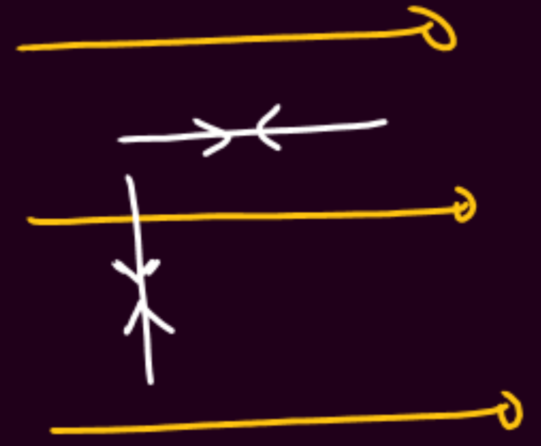
$W = 0$



Equipotential Surface

⑤ Uniform $\vec{E}$

$$\boxed{\Delta V = -\vec{E} \cdot \Delta\vec{r}}$$



$$\boxed{\Delta V_{\perp} = 0}$$

$$\boxed{\Delta V_{\parallel} = \pm E d}$$

In the dir of $\vec{E}$ ⊖

Against —— " —— ⊕

Q. $\vec{E} = (2\hat{i} + 3\hat{j})$ V/m

$q = -2C$ is displaced from $(1, 2, -1)$ to $(2, 0, 1)$

W.D. by field ?

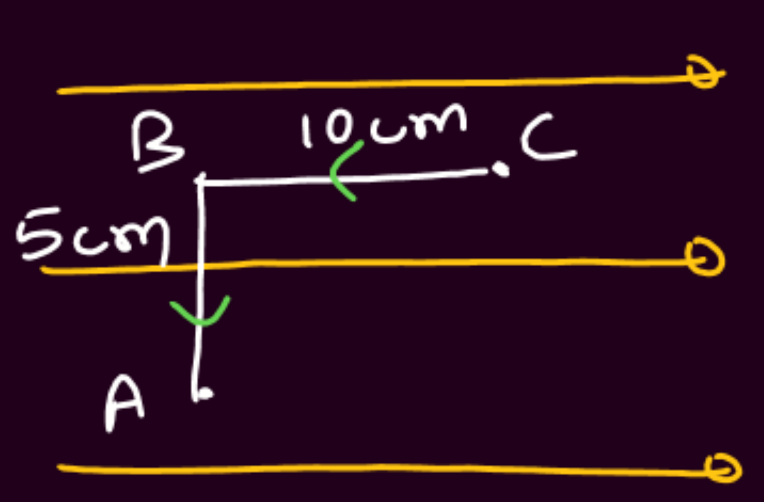
$$\Delta\vec{r} = (\hat{i} - 2\hat{j} + 2\hat{k})$$

$$W = \vec{F}\cdot\Delta\vec{r}$$

$$W = (-4\hat{i} - 6\hat{j})\cdot(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$= -4 + 12 = 8J$$

Q.



$\vec{E} = 10$ N/C

P.D. bet^n A & C ?

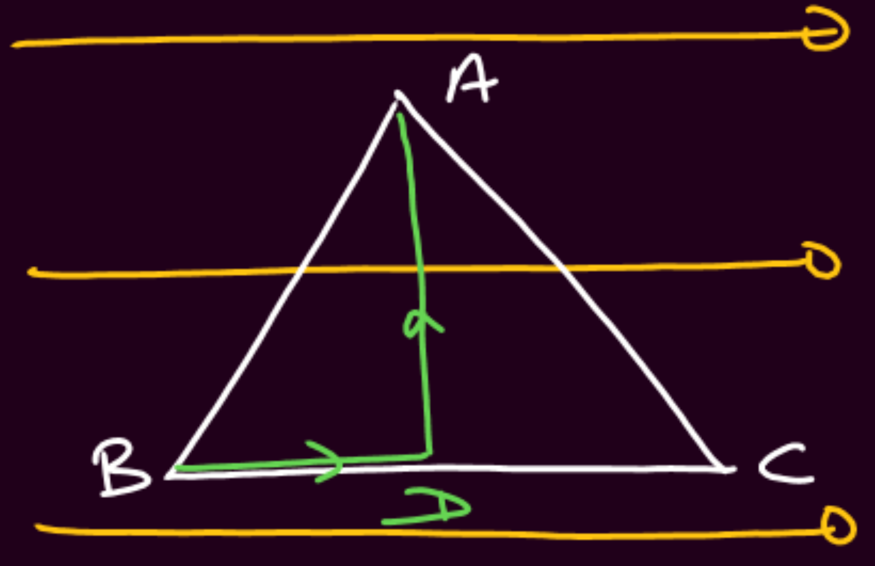
$$V_{AC} = V_A - V_C \qquad [C \text{ to } A]$$

$$V_{AC} = C \to B + B \to A$$

$$= +10\left(\frac{10}{100}\right) + 0$$

$$= 1V$$

Q.



$l = 10\,cm \qquad E = 20\ N/C$

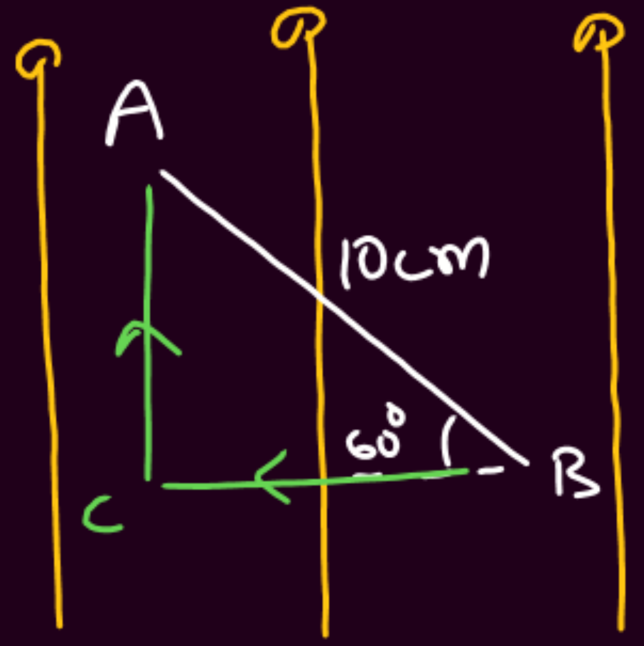
$V_{AB} = ?$

B → D D → A

$V_{AB} = -20\left(\frac{5}{100}\right) + 0$

$= \underline{\underline{-1V}}$

Q.



$E = 10\ N/C$

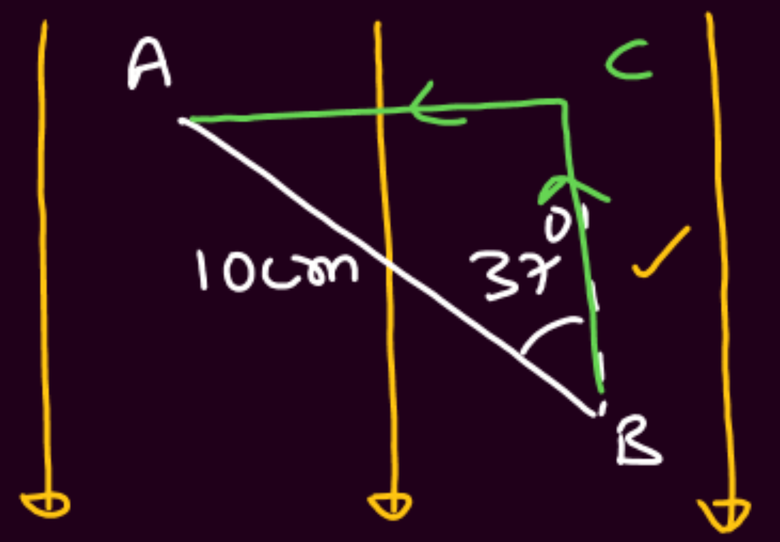
$V_{AB} = ?$

B → C C → A

$V_{AB} = 0 - 10\left(\frac{5\sqrt{3}}{100}\right)$

$= -\frac{\sqrt{3}}{2}\ V$

Q.



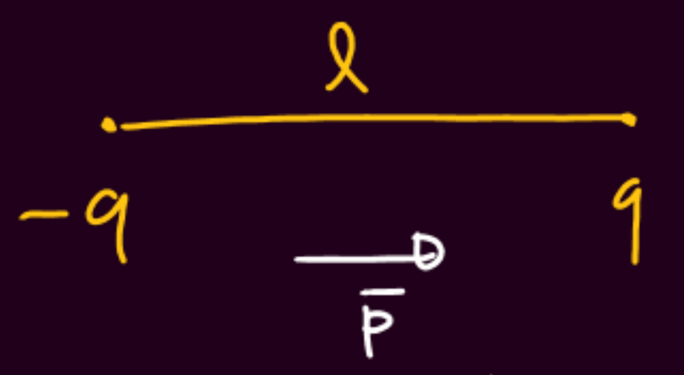
$V_{AB} = ?$ $E = 10\ N/C$

B → C C → A

$V_{AB} = +10\left(\frac{8}{100}\right) + 0$

$= \underline{\underline{0.8V}}$

Electric Dipole



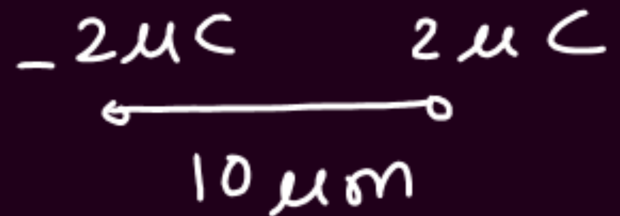
Electric Dipole Moment ($\vec{p}$)

$$\vec{p} = q\vec{l}$$

Vector Dir ⊖ to ⊕

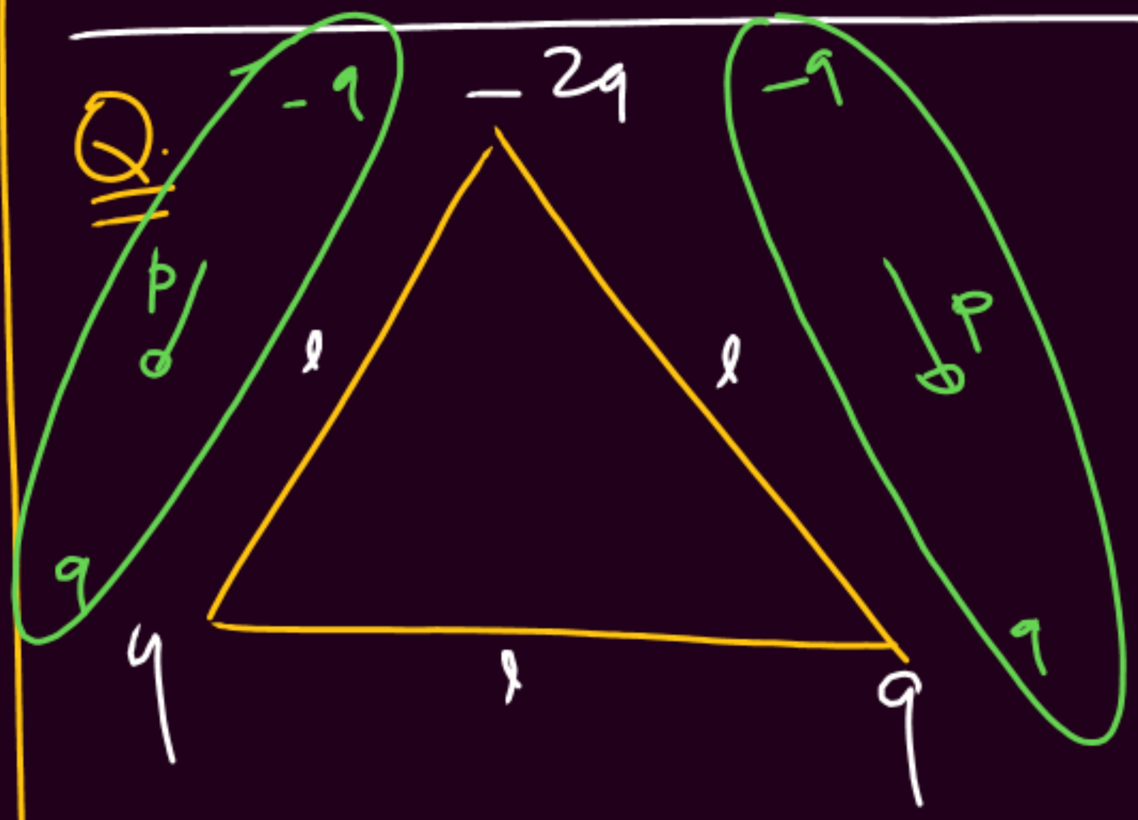
Unit C-m

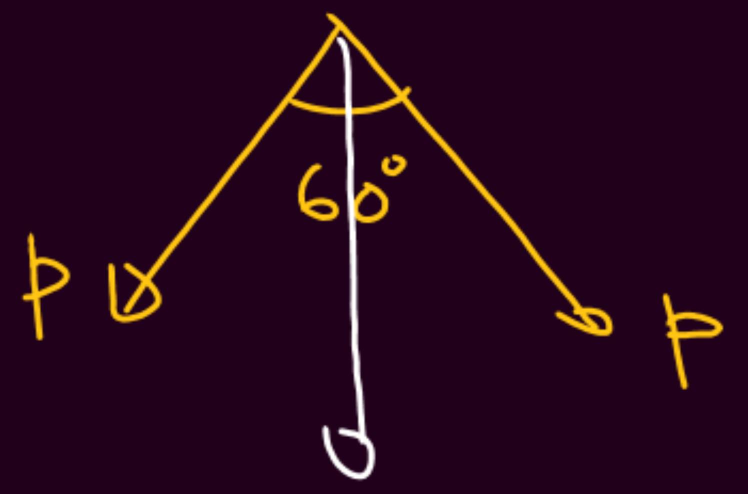
Q.



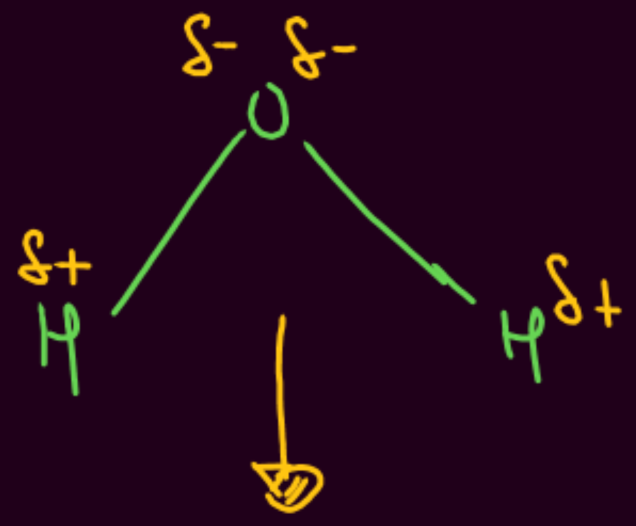
$$p = (2\times10^{-6})(10\times10^{-6}) = 2\times10^{-11}\ \text{C-m}$$

Q.

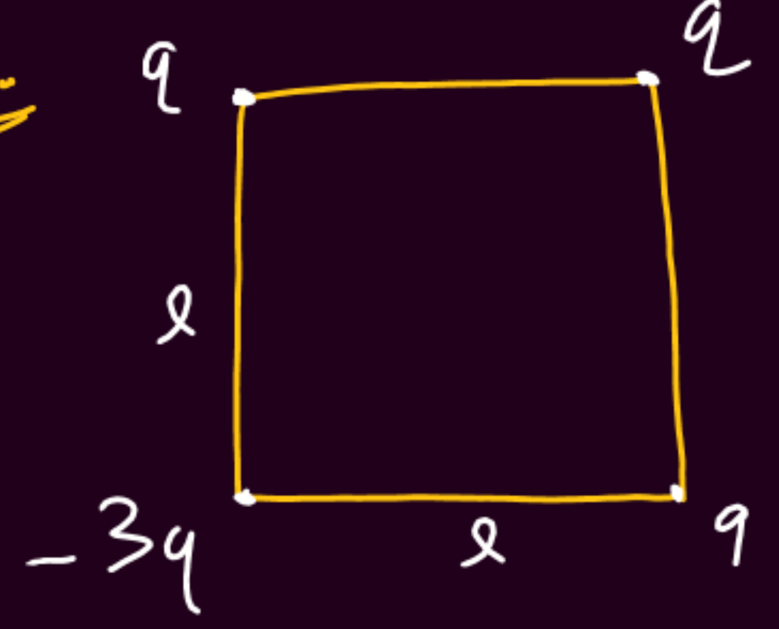




$$P_{net} = 2p\cos 30^\circ$$
$$= p\sqrt{3}$$
$$= ql\sqrt{3}$$



Q.



p_1

$p_2 = q(l\sqrt{2})$

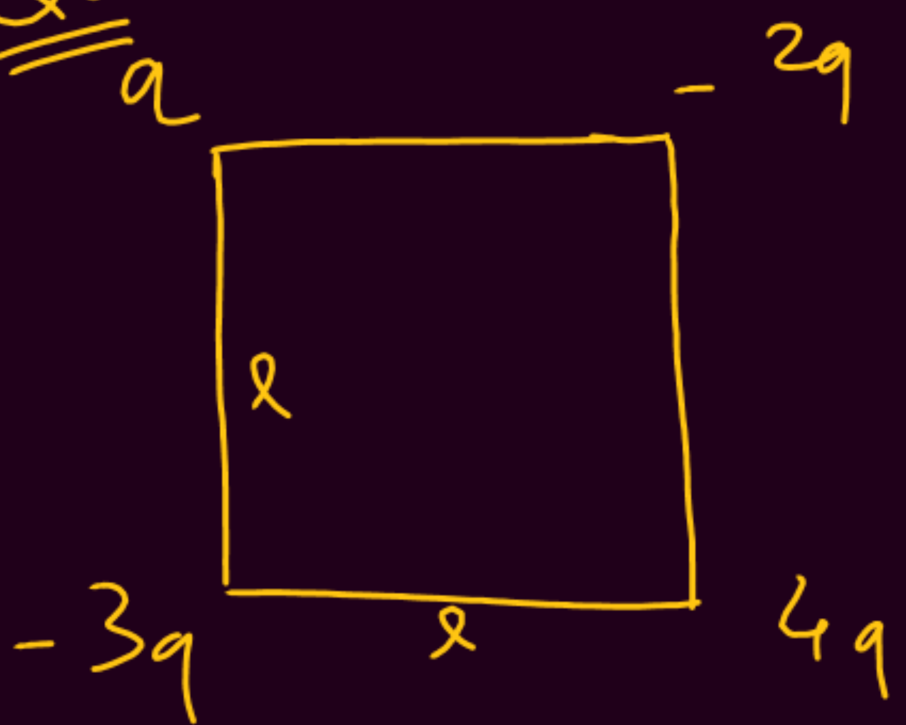
$p_1 = ql$

$$p_{net} = p_1\sqrt{2} + p_2$$

$$= (ql)\sqrt{2} + (ql\sqrt{2})$$

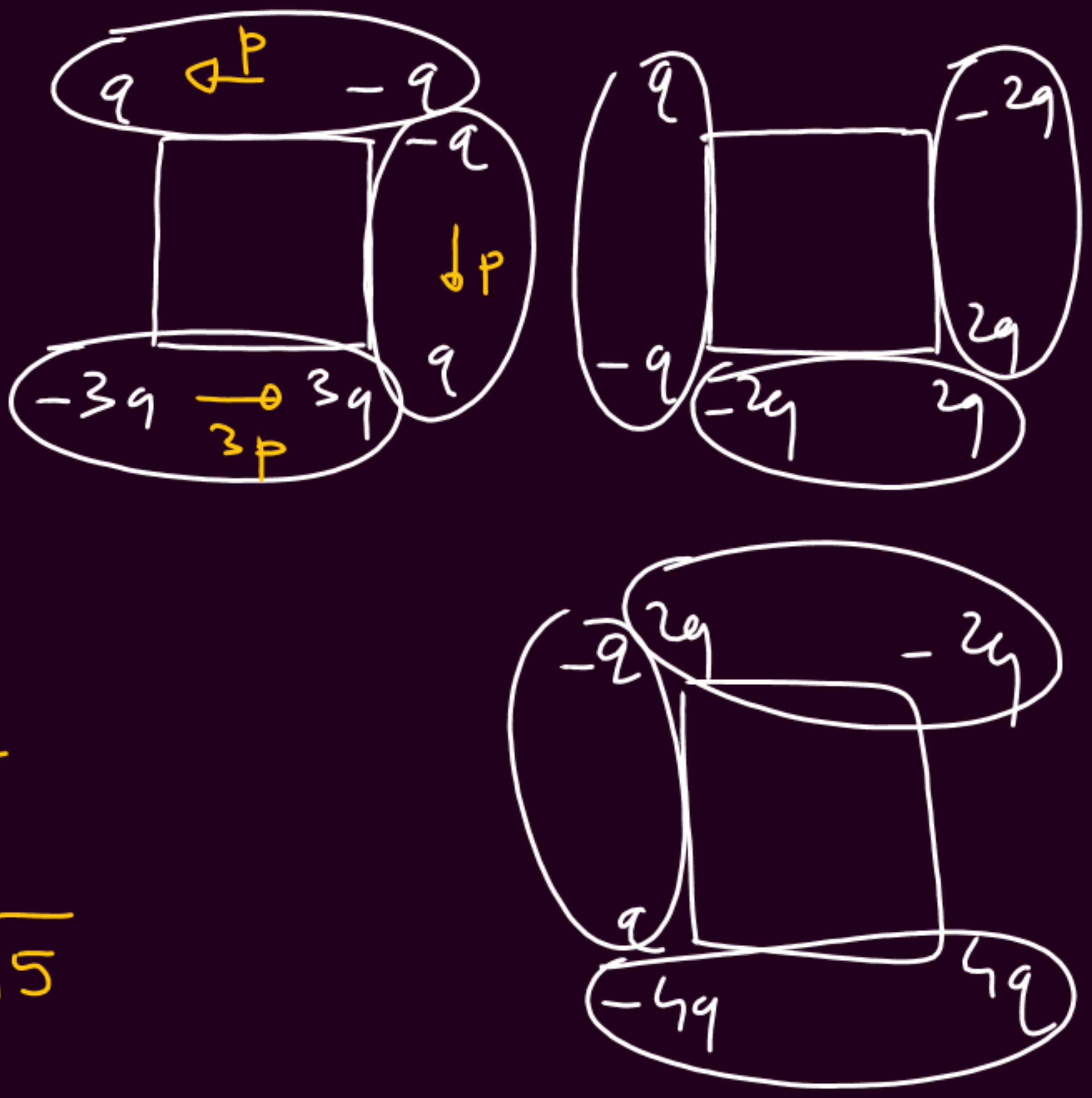
$$= 2\sqrt{2}\,ql$$

Q.



$$p_{net} = p\sqrt{5}$$

$$= ql\sqrt{5}$$



Field & Potential Due to Tiny dipole ($r >> l$)

Equatorial / Broadside-on Position

$$E_{eq} = -\frac{kp}{r^3}$$

$$V_{eq} = 0$$

Extra

$$E = \frac{kp}{r^3}\sqrt{1 + 3\cos^2\theta}$$

$$V = \frac{kp\cos\theta}{r^2} = \frac{k\vec{p}\cdot\vec{r}}{r^3} = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

Axial / End-on Position

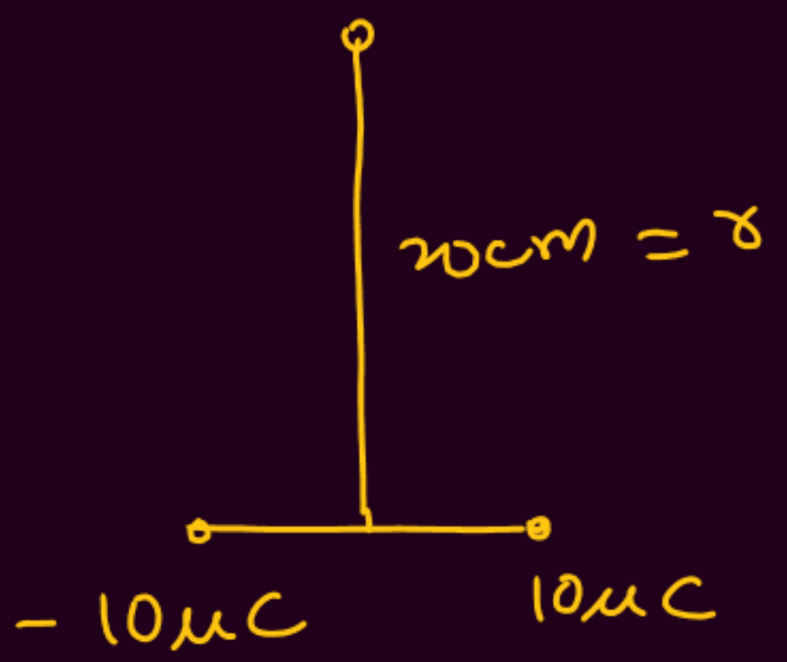
$$E_a = \frac{2kp}{r^3}$$

$$V_a = \frac{kp}{r^2}$$

① $r \to 2r$
$E \to E/8$

② $E_a = 2E_{eq}$

Q.



$$E = \frac{kp}{r^3} = \frac{k\,ql}{r^3}$$

$$= \frac{(9\times10^9)(10\times10^{-6})(2\times10^{-2})}{(0.2)^3}$$

$$= \frac{9}{4}\times10^5 \text{ N/C}$$

Q

$$E_0 = \frac{2kp}{r^3}$$

$$E = \frac{kp}{(2r)^3} = \frac{kp}{8r^3} = \frac{E_0/2}{8} = \frac{E_0}{16}$$

Q.

$$E_0 = \frac{2kp}{r^3}$$

$$E = \frac{kp}{r^3} = \frac{E_0}{2}$$

Q.

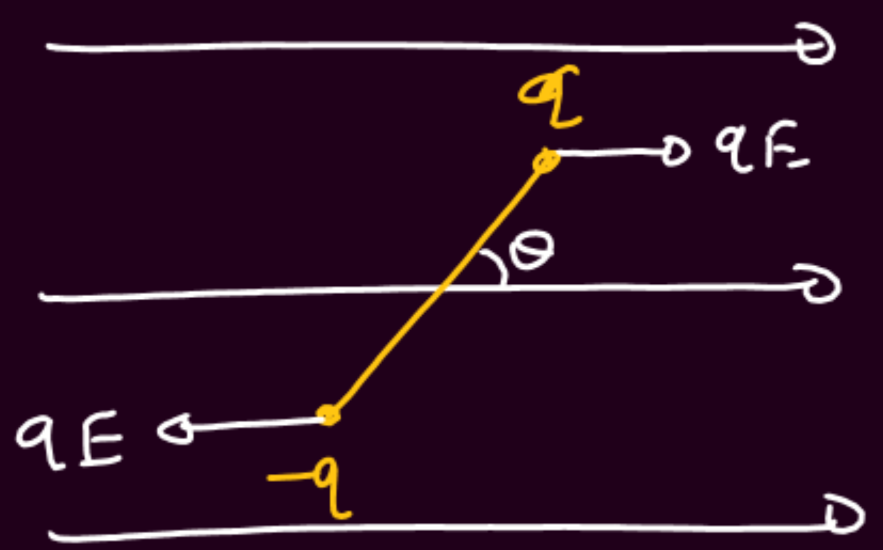
$$E_a = E_{eq}$$

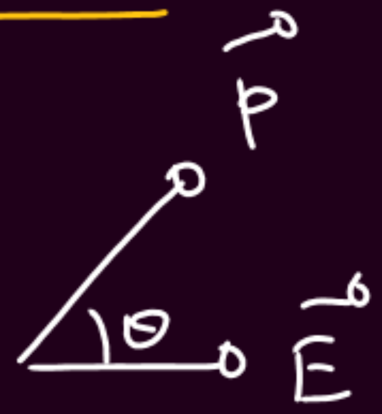
$$\frac{2\cancel{kp}}{r^3} = \frac{\cancel{kp}}{(10)^3}$$

$$r^3 = 2\,(10)^3$$

$$\boxed{r = 2^{1/3}(10)\text{ cm}}$$

Dipole in Uniform $\vec{E}$





① $\vec{F}_{net} = 0$

② Torque (Couple)

$$\boxed{\begin{aligned}\vec{\tau} &= \vec{p} \times \vec{E} \\ \tau &= pE\sin\theta\end{aligned}}$$

RHTR

Thumb ACW

③ $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

④ $W = \Delta U = -pE(\cos\theta_2 - \cos\theta_1)$

Stable Eq.	Unstable Eq.
	
$\theta = 0°$	$\theta = 180°$
$F = 0$	$F = 0$
$\tau = 0$	$\tau = 0$
$U_{min} = -pE$	$U_{max} = pE$

Disturb → Oscillations
↓
SHM for small θ ($<5°$)

$$T = 2\pi\sqrt{\frac{I}{pE}}$$

I = moment of Inertia

Q. $\theta = 60^\circ \qquad U_0$

$$U_0 = -pE\cos 60^\circ = -\frac{pE}{2}$$

$$\tau = pE\sin 60^\circ = \frac{pE\sqrt{3}}{2}$$

$$\tau = -U_0\sqrt{3}$$

Q. $W = \Delta U = (pE) - (-pE)$

$$= 2pE$$

$$W = -pE(\cos 180^\circ - \cos 0^\circ)$$
$$= -pE(-1-1)$$
$$= 2pE$$

Q. $W_0 = -pE(\cos 60^\circ - \cos 0^\circ)$

$$= -pE\left(\frac{1}{2} - 1\right) = \frac{pE}{2}$$

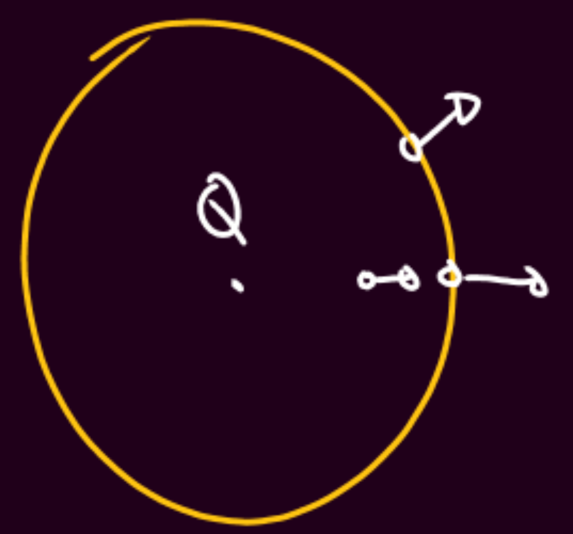
$$\tau = pE\sin 60^\circ = pE\frac{\sqrt{3}}{2} = W_0\sqrt{3}$$

Q.

$$F = \left(\frac{2kp}{r^3}\right)q$$

	F	τ
① Dipole in uniform $\vec{E}$	Must be 0	may/may not be 0
② Dipole in non uniform $\vec{E}$	may/may not be 0	may/may not be 0
③ Dipole in $\vec{E}$ of pt. charge	can not be zero	—— " ——

$qE_1 - qE_2$

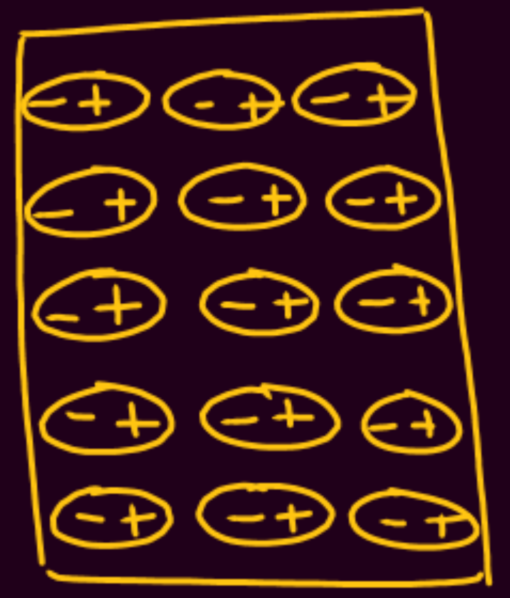


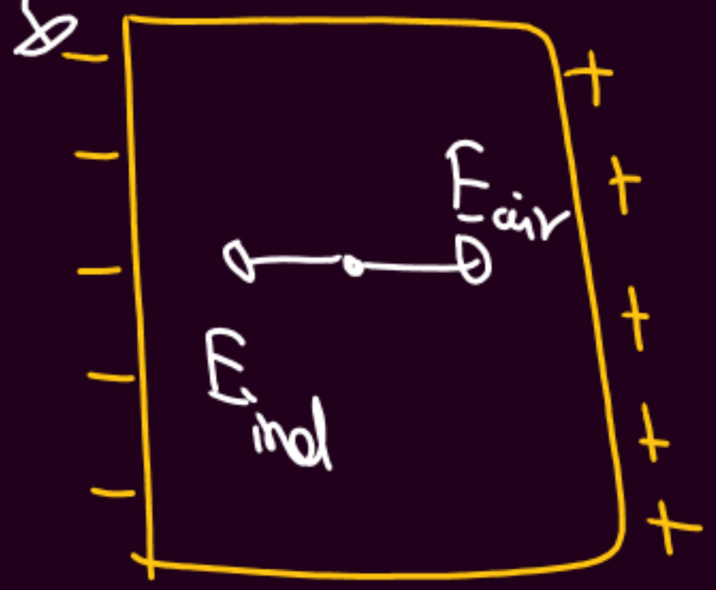
Dielectrics

<u>Polar</u> (H_2O, NH_3)

<u>Non-polar</u> (C_6H_6)

$\vec{E}_{air}$





$$\vec{E}_{net} = \vec{E}_{air} + \vec{E}_{ind}$$

$$\frac{\vec{E}_{air}}{K} = \vec{E}_{air} + \vec{E}_{ind}$$

$$\vec{E}_{ind} = - \vec{E}_{air}\left(1 - \frac{1}{K}\right)$$

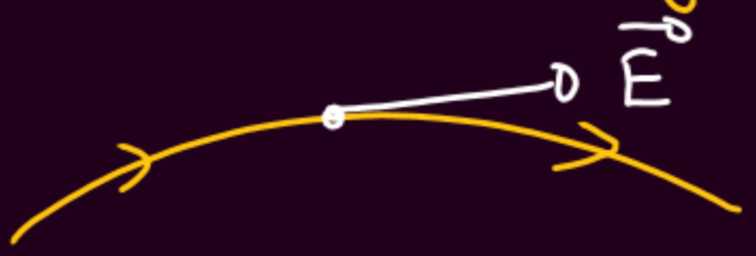
$K \geq 1$

$$q_{in} = -q\left(1 - \frac{1}{K}\right)$$

Electric Field Lines

→ Faraday

→ Imaginary



→ Never

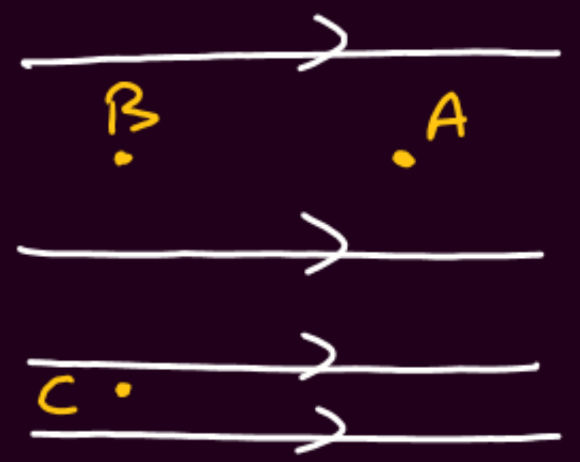
→ Closer / far

$E_A > E_B$

$V_A > V_B$

$E_A < E_B$

$V_A > V_B$



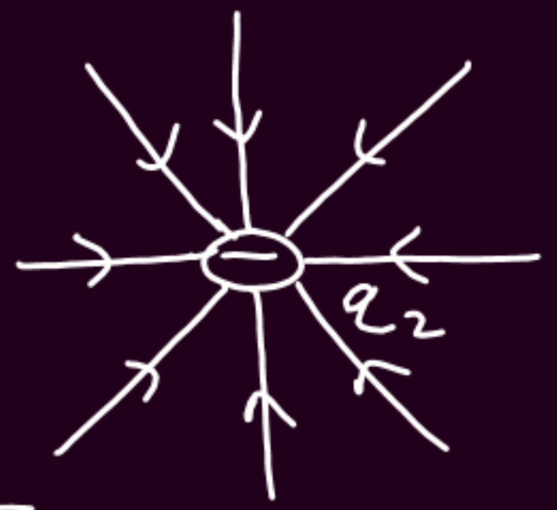
① $E_A = E_B < E_C$

② $V_B = V_C > V_A$

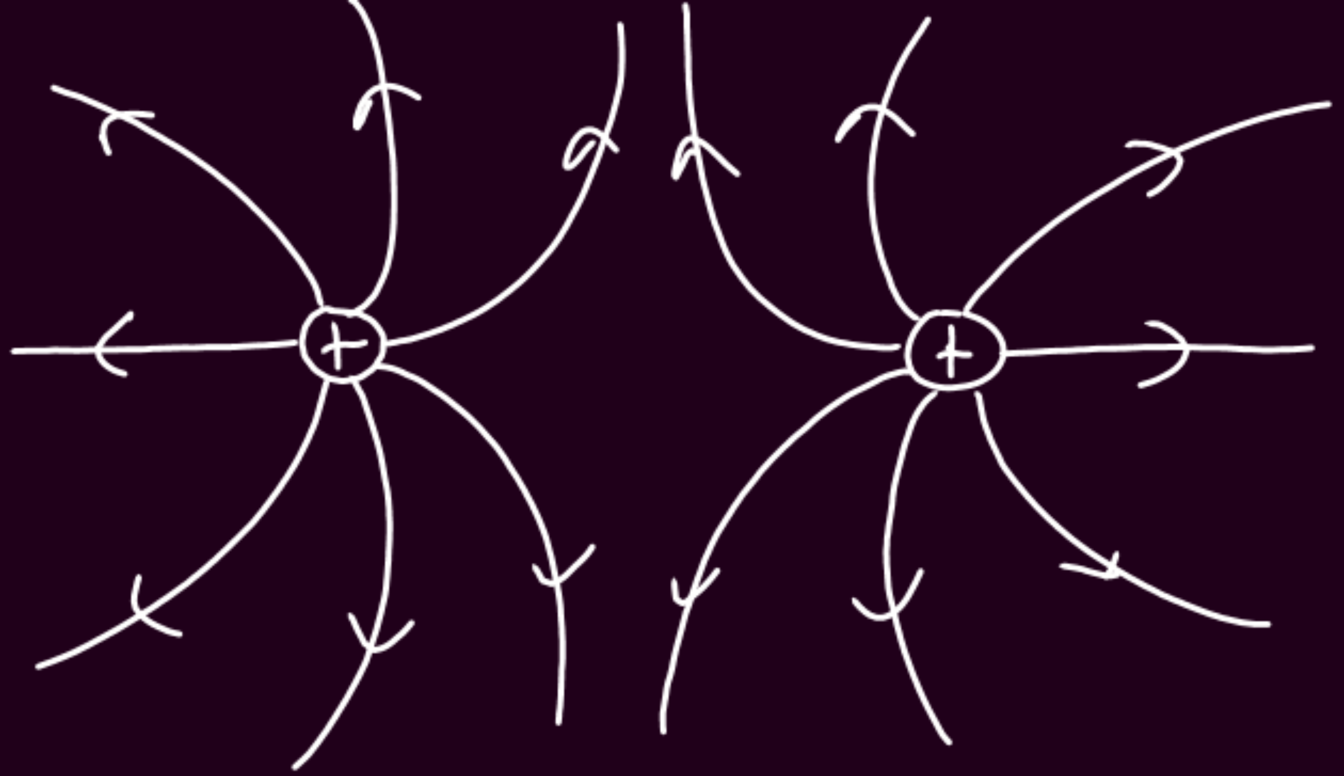
$V_A > V_B$

→  → Induced EFL

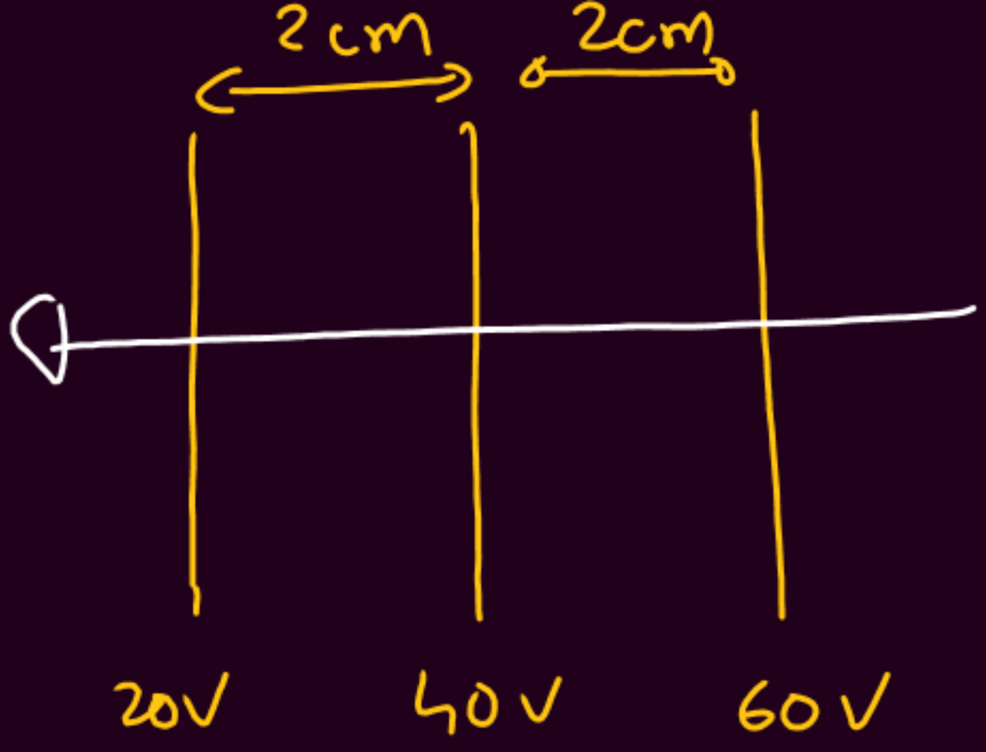
$\frac{W}{q} = \mathcal{E}$



$\frac{q_1}{q_2} = -\frac{1}{2}$



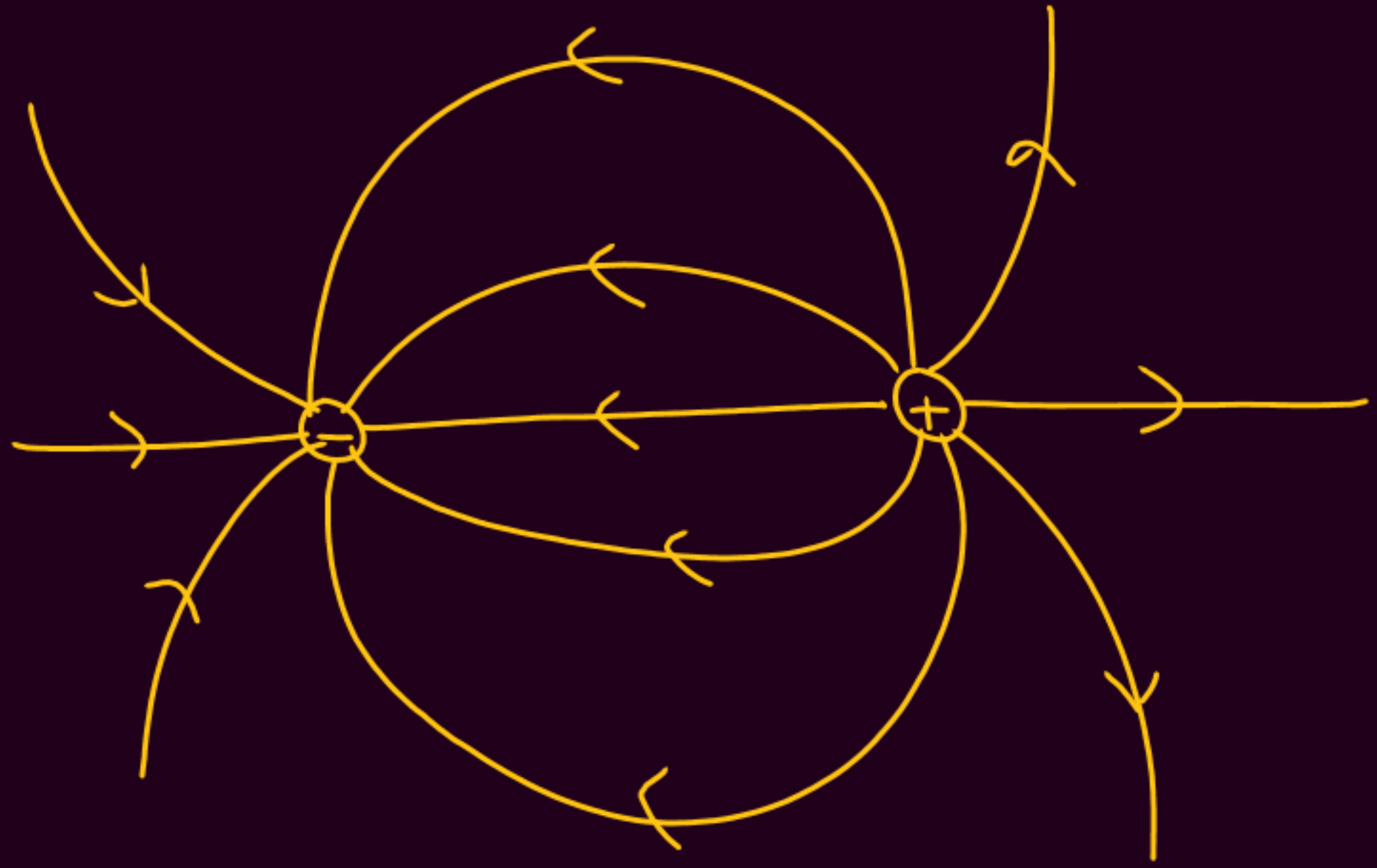
Q.



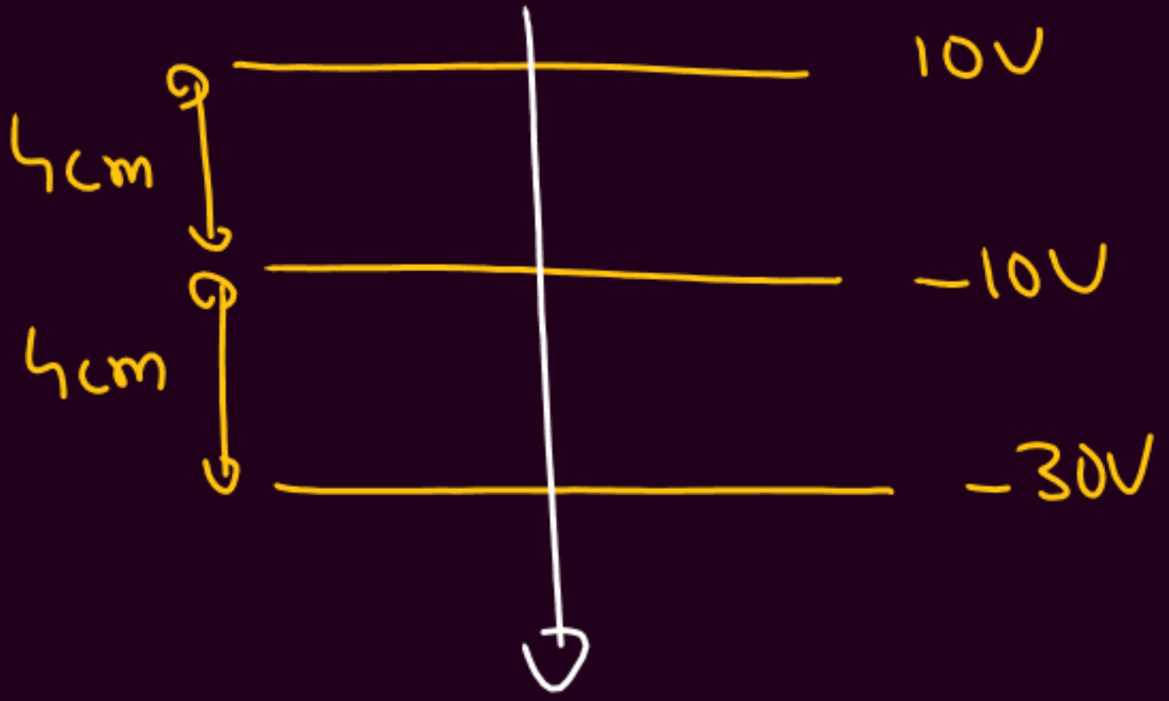
$$E = \frac{\Delta V}{\Delta r}$$

$$E = \frac{40-20}{2\times10^{-2}}$$

$$E = 1000\ V/m$$



Q.



$$E = \frac{10-(-10)}{4\times10^{-2}} = 500\ V/m$$

Q. If $\vec{E} = (3\hat{i} - 4\hat{j})$ N/C then find P.D. between P(1, 2) & Q (−1, 1)

$$\Delta V = -\vec{E} \cdot \Delta\vec{r}$$

$$\Delta V = -(3\hat{i} - 4\hat{j}) \cdot (-2\hat{i} - \hat{j})$$

$$= -[-6 + 4]$$

$$= +2V$$

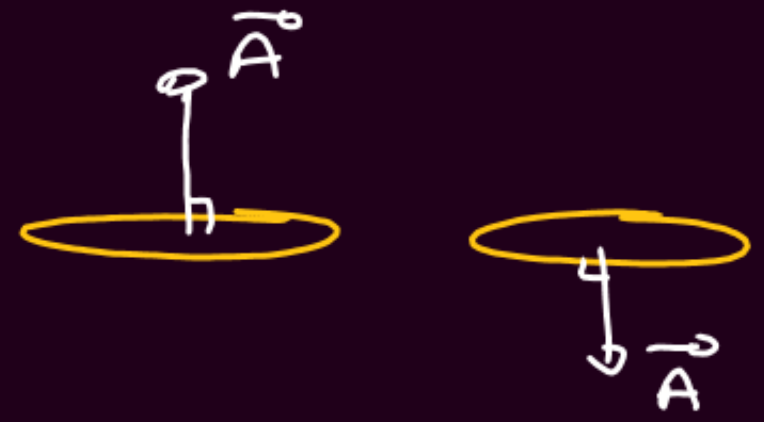
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$= (-\hat{i} + \hat{j}) - (\hat{i} + 2\hat{j})$$

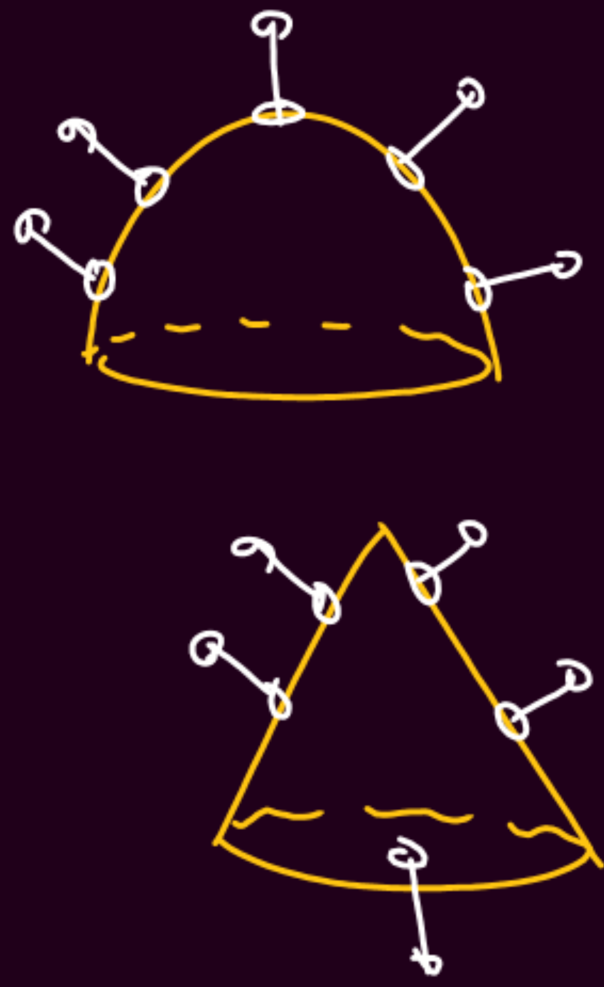
$$\Delta\vec{r} = -2\hat{i} - \hat{j}$$

Area Vector

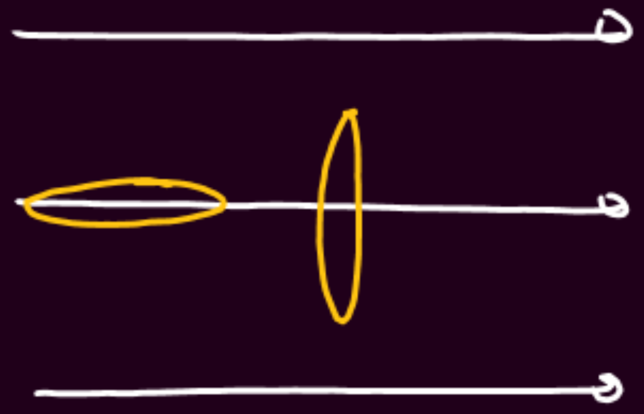
① Plane Surface



② 3D



Electric Flux



$\phi = \int \vec{E} \cdot d\vec{A}$ $\qquad \dfrac{N\text{-}m^2}{C} = V\text{-}m$

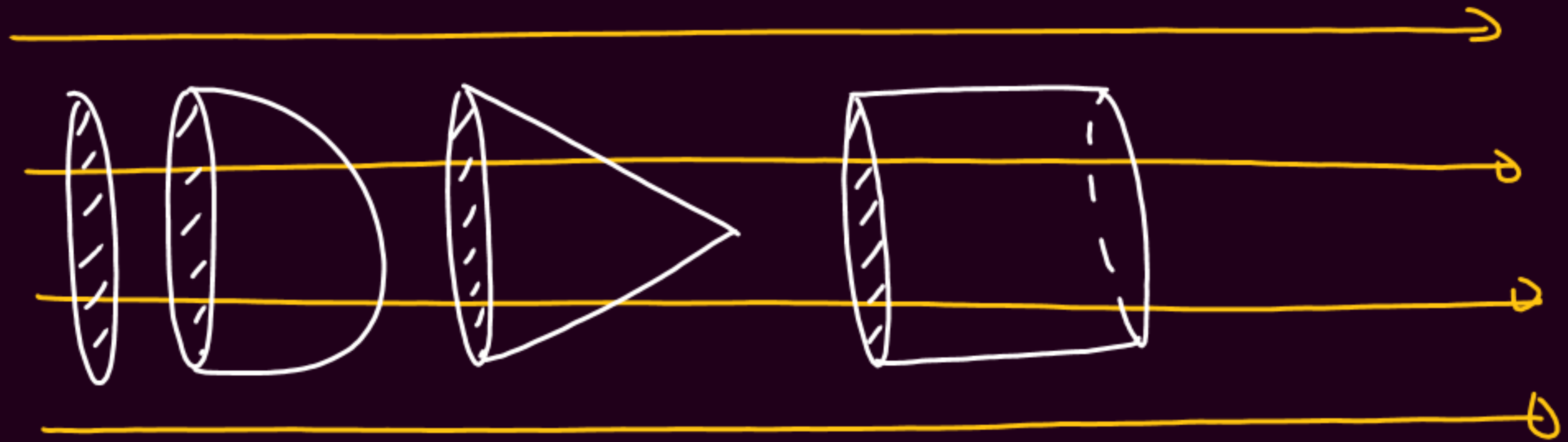
if EFL are in the plane $\phi = 0$

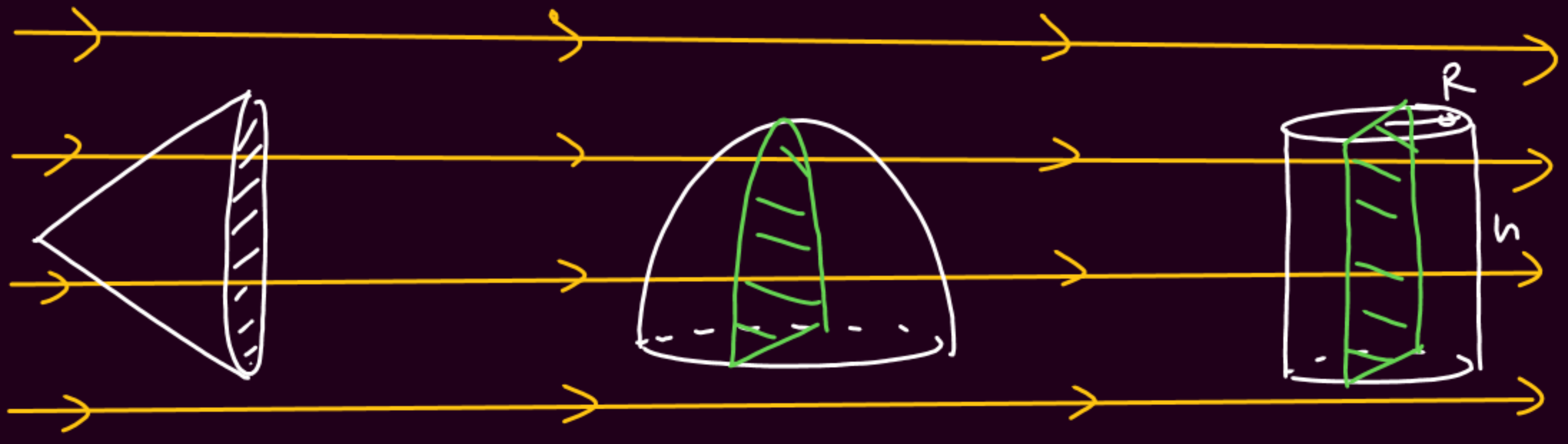
— " — $\perp$ — " — ϕ_{max}

Uniform $\vec{E}$

$\phi = \vec{E} \cdot \vec{A} = EA\cos\theta = EA\sin\alpha$

Projected Area $\phi = E(\pi R^2)$





$$\phi_{curved} = -E(\pi R^2)$$

$$\phi_{flat} = E(\pi R^2)$$

$$\phi_{net} = 0$$

$$\phi_{flat} = 0$$

$$\phi_{curved} = 0$$

$$\phi_{in} = -E\left(\frac{\pi R^2}{2}\right)$$

$$\phi_{flat} = 0$$

$$\phi_{curved} = 0$$

$$\phi_{in} = -E(2Rh)$$

Convention

EFL going out $\phi = +$

EFL going in $\phi = -$

Q. $A = 20 cm^2$

$E = 10 \ V/m$

60° with plate

$\phi = ?$

$\phi = E A \sin\alpha$

$= 10 \left(\frac{20}{100}\right) \frac{\sqrt{3}}{2}$

$= \sqrt{3} \ V\text{-}m$

Q. $\vec{E} = (2\hat{i} + \hat{j} + 3\hat{k}) \ V/m$

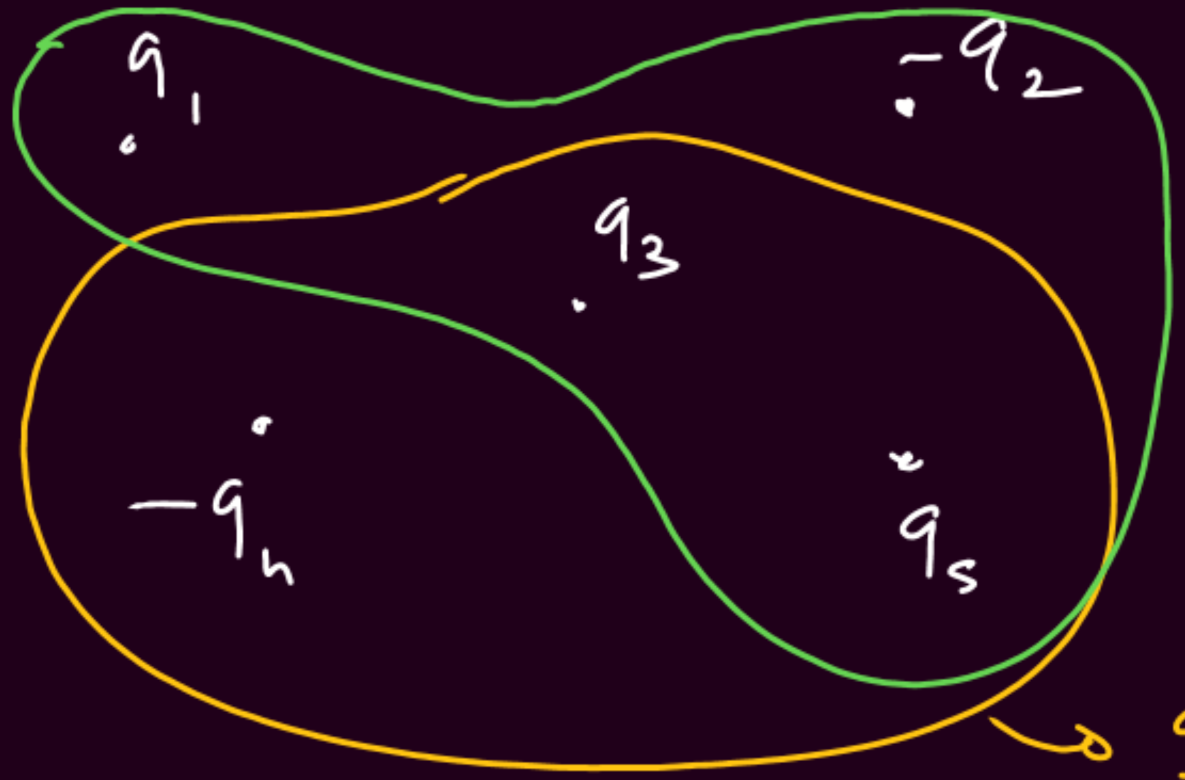
$\phi = ?$ through plate $A = 2 \ m^2$ placed in y-z plane.

$\vec{A} = 2\hat{i}$

$\phi = (2\hat{i} + \hat{j} + 3\hat{k}) \cdot (2\hat{i}) = 4 \ V\text{-}m$

Gauss Law

$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\epsilon_0}}$$



→ gaussian surface

$$\phi = \frac{q_3 + q_5 - q_4}{\epsilon_0}$$

$$\phi = \frac{q_1 - q_2 + q_3 + q_5}{\epsilon_0}$$

→ only for closed Surface

→ q_{en}

→ $\vec{E}$ → net

→ valid $\left(E \propto \frac{1}{r^2}\right)$

→

① $\vec{E} = 0$ at all pts. ⇒ $\phi = 0$ $\quad q_{en} = 0$

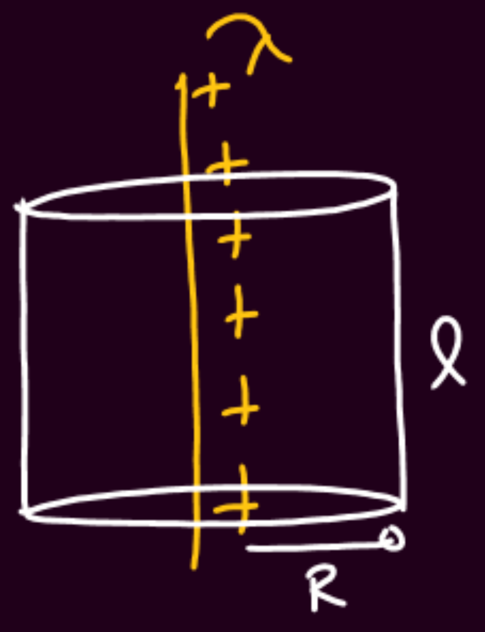
② $q_{en} = 0$ ⇒ $\phi = 0$ $\quad E$ may / may not be 0

③ $\phi = 0$ ⇒ $q_{en} = 0$ $\quad$ —— " ——

④ $\phi = 0$ if $\vec{E}$ is uniform

⑤ ϕ_{flat} containing the charge is zero.

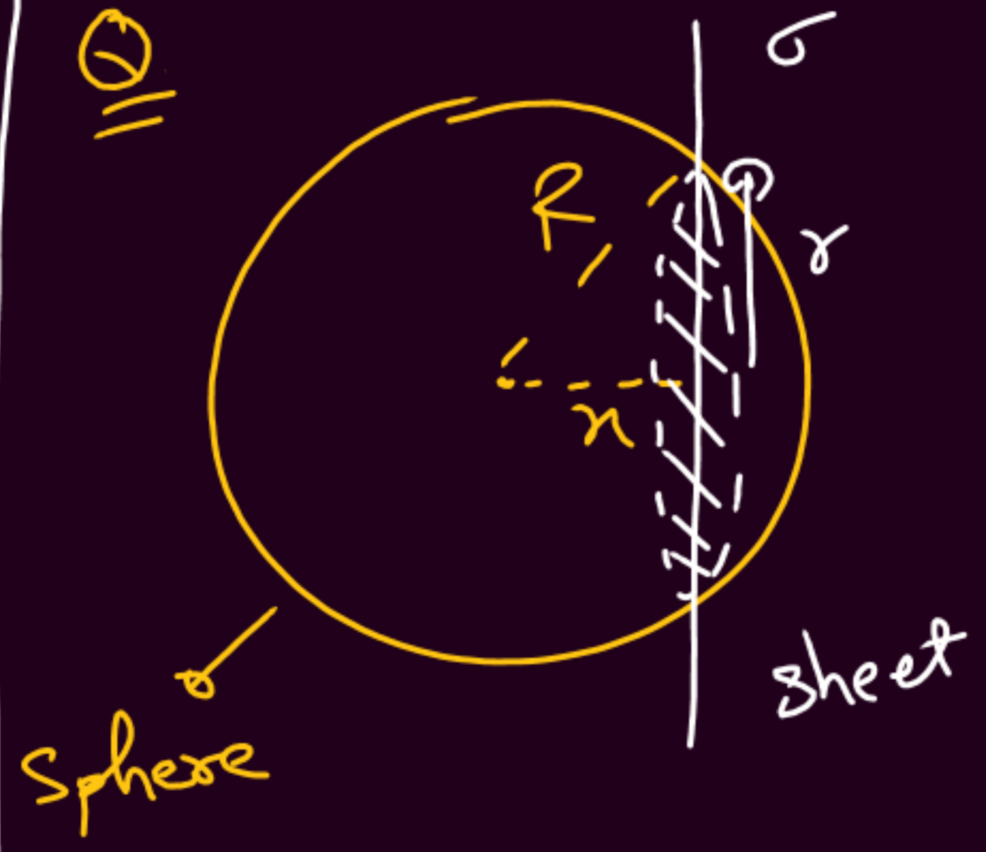
Q.



$$\phi_{cylinder} = \frac{q_{en}}{\epsilon_0}$$

$$= \frac{(\lambda l)}{\epsilon_0}$$

Q.



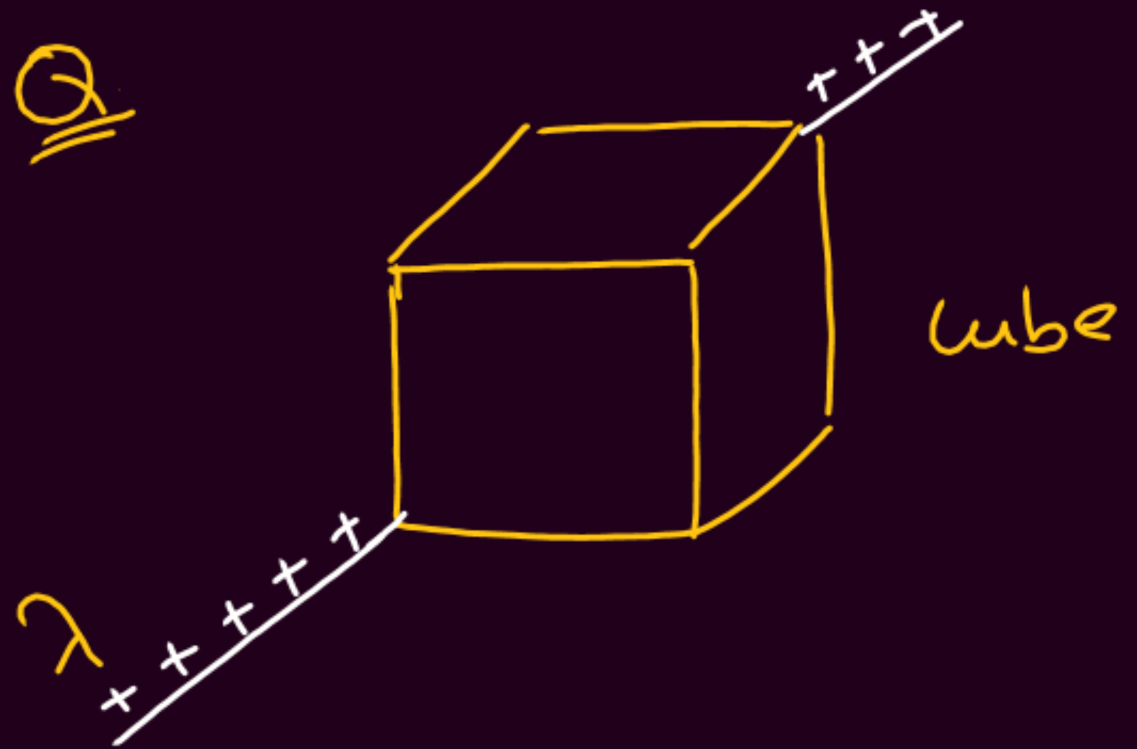
$$\phi_{sphere} = \frac{\sigma A}{\epsilon_0}$$

$$= \frac{\sigma \pi r^2}{\epsilon_0}$$

$$= \frac{\sigma \pi (R^2 - x^2)}{\epsilon_0}$$

$$R^2 = x^2 + r^2$$

Q.



$$\phi_{cube} = \frac{\lambda (l\sqrt{3})}{\epsilon_0}$$

Q.

$$\phi_{flat} = 0$$

Q.

given

ϕ_0 = flux through curved Surface

$\phi_{flat} = ?$

$$\phi_{cylinder} = \phi_{curved} + 2\,\phi_{flat}$$

$$\frac{q}{\epsilon_0} = \phi_0 + 2\,\phi_{flat}$$

$$\left(\frac{q}{\epsilon_0} - \phi\right) = 2\,\phi_{flat}$$

$$\phi_{flat} = \frac{1}{2}\left(\frac{q}{\epsilon_0} - \phi_0\right)$$

Symmetry

①

$$\phi_{flat} = 0$$

$$\phi_{curved} = \frac{q}{2\epsilon_0}$$

②

$$\phi_{cylinder} = \frac{q}{2\epsilon_0}$$

③ Cube

Cube

① Body Center

$$\phi = \frac{q}{\epsilon_0}$$

$$\phi_{face} = \frac{q}{6\epsilon_0}$$

② Face Center

$$\phi = \frac{q}{2\epsilon_0}$$

③ Edge Center

$$\phi = \frac{q}{4\epsilon_0}$$

④ Corner / Vertex

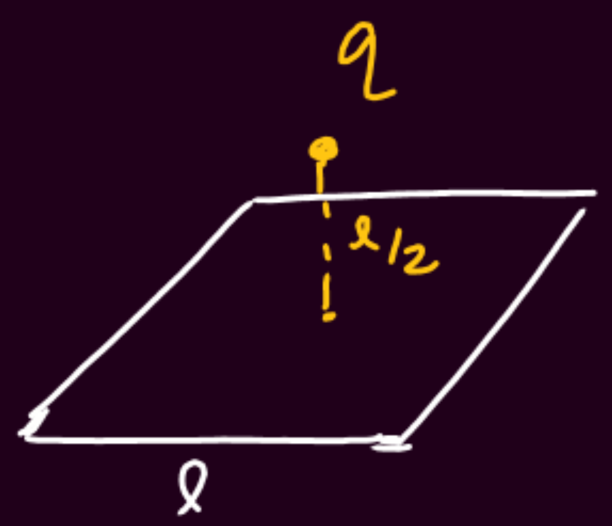
$$\phi = \frac{q}{8\epsilon_0}$$

$$\phi_{Top} = \phi_{Right} = \phi_{Back} = 0$$

$$\phi_{Bottom} = \phi_{Left} = \phi_{front} = \frac{q}{24\epsilon_0}$$

Q.

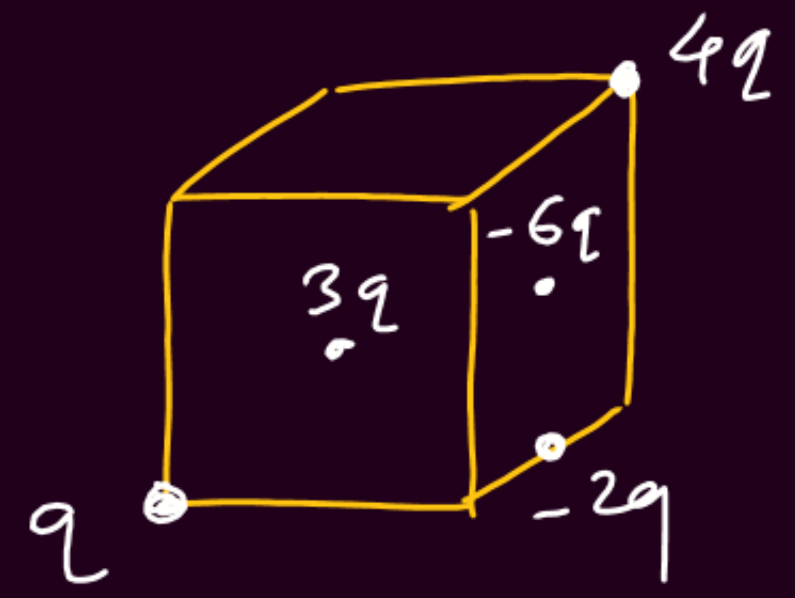
Symmetry



$$\phi = \frac{1}{6}\left(\phi_{cube}\right)$$

$$\phi = \frac{q}{6\epsilon_0}$$

Q

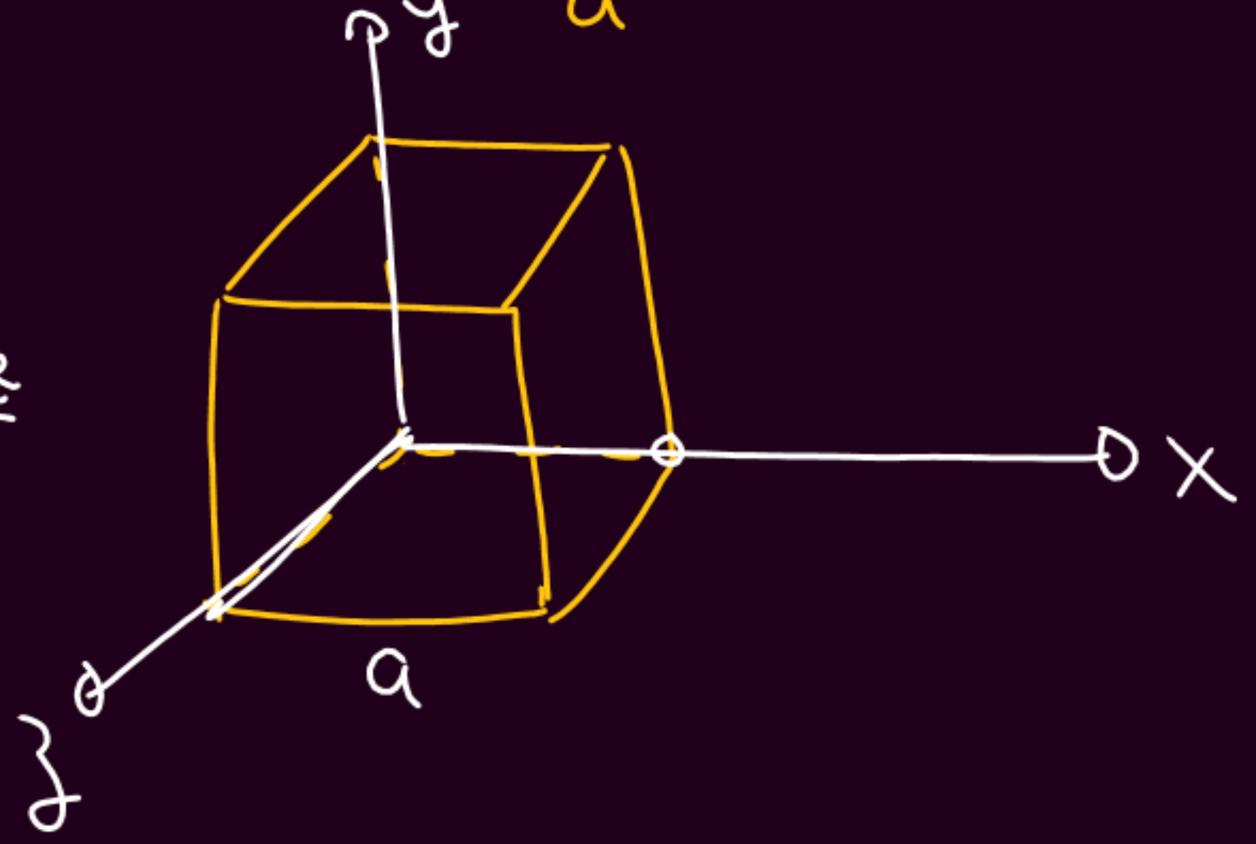


$$\phi_{cube} = \frac{q}{8\epsilon_0} - \frac{2q}{4\epsilon_0} + \frac{3q}{\epsilon_0} + \frac{4q}{8\epsilon_0} - \frac{6q}{2\epsilon_0}$$

Q. $E = E_0 \frac{x^2}{a^2} \hat{i}$

Top / Bottom / front / Back $\boxed{\phi = 0}$

cube



$\phi_{Left} = 0$ $\quad [x = 0 \ , \ E = 0]$

$\phi_{Right} = E_0 a^2$ $\quad [x = a \ , \ E = E_0 \hat{i}]$

$\phi_{net} = \frac{q_{en}}{\epsilon_0} = E_0 a^2$

$q_{en} = \epsilon_0 E_0 a^2$

Spherical Symmetry

Point charge

$$\phi = \frac{q_{en}}{\epsilon_0}$$

$$\boxed{E(4\pi R^2) = \frac{Q}{\epsilon_0}}$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2}$$

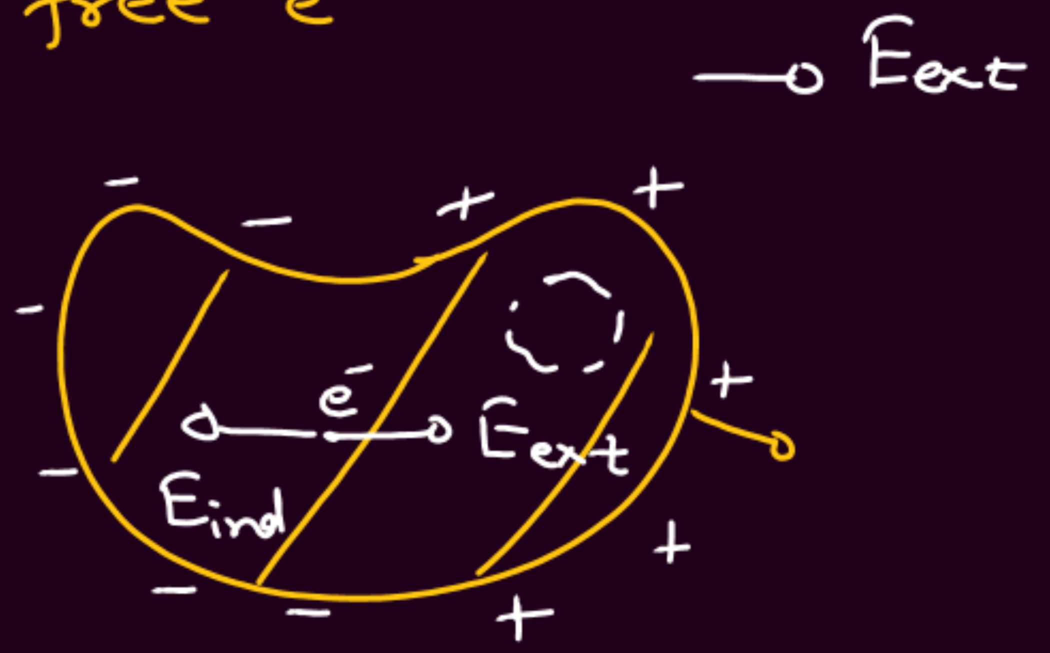
Q. $E = c r^2$ in a sphere of Radius 'a' find q_{en}

$$(c a^2)(4\pi a^2) = \frac{q_{en}}{\epsilon_0}$$

$$\boxed{q_{en} = 4\pi\epsilon_0 c a^4}$$

Conductors (Electrostatic condition)

→ free e^-



→ $E_{net} = 0$

→ q only on Surface

→ $\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{ind} = 0$

$$\vec{E}_{ind} = -\vec{E}_{ext}$$

→ EFL

→ $V_{inside} = V_{surface}$ = constant

→ $\frac{\sigma}{\epsilon_0}$ → $\frac{\sigma^2}{2\epsilon_0}$ → Electrostatic sheilding

→ σr = Const.

→ sparking → corona discharge

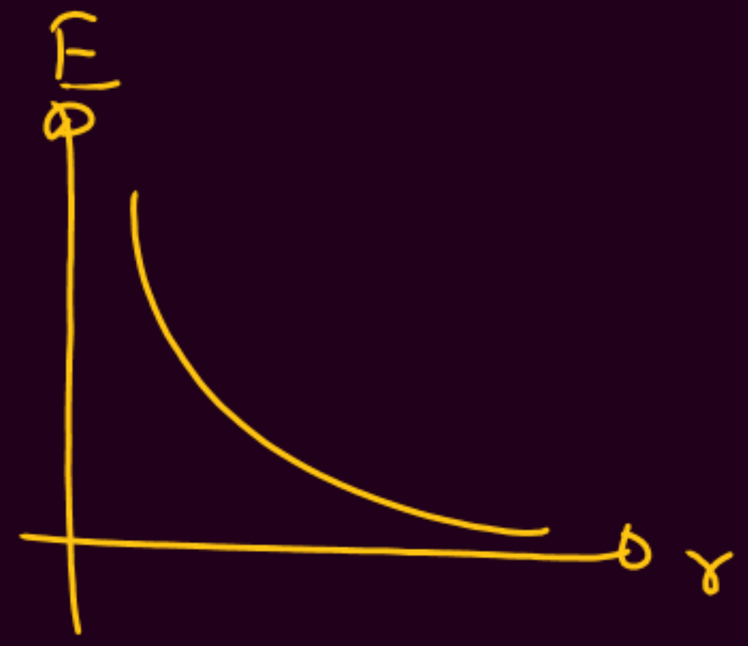
↳ "Dielectric Breakdown"

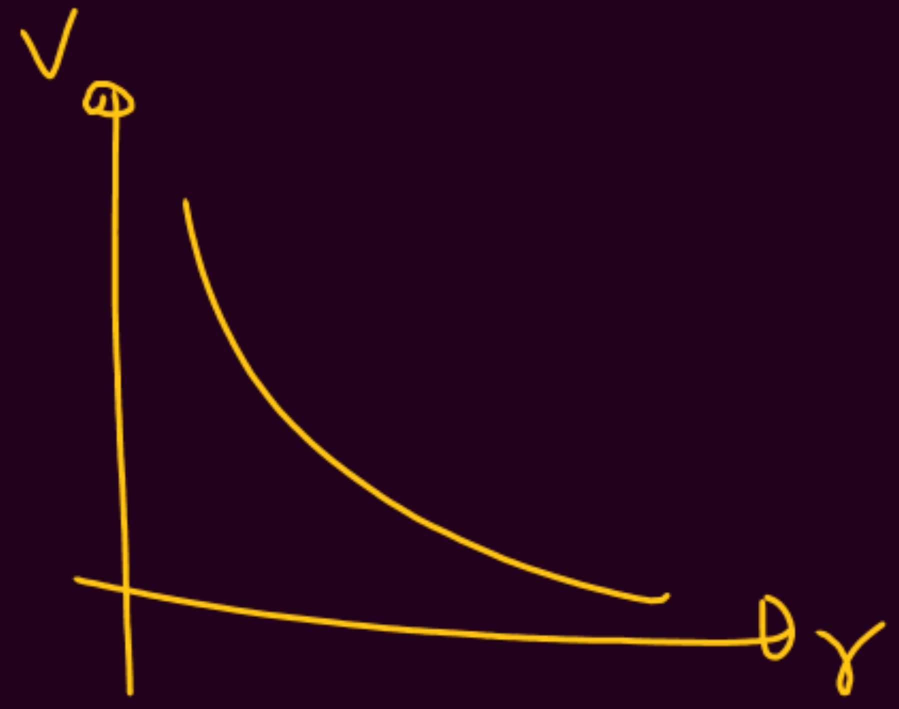
① Point charge

Q

$$E = \frac{kQ}{r^2}$$

$$V = \frac{kQ}{r}$$





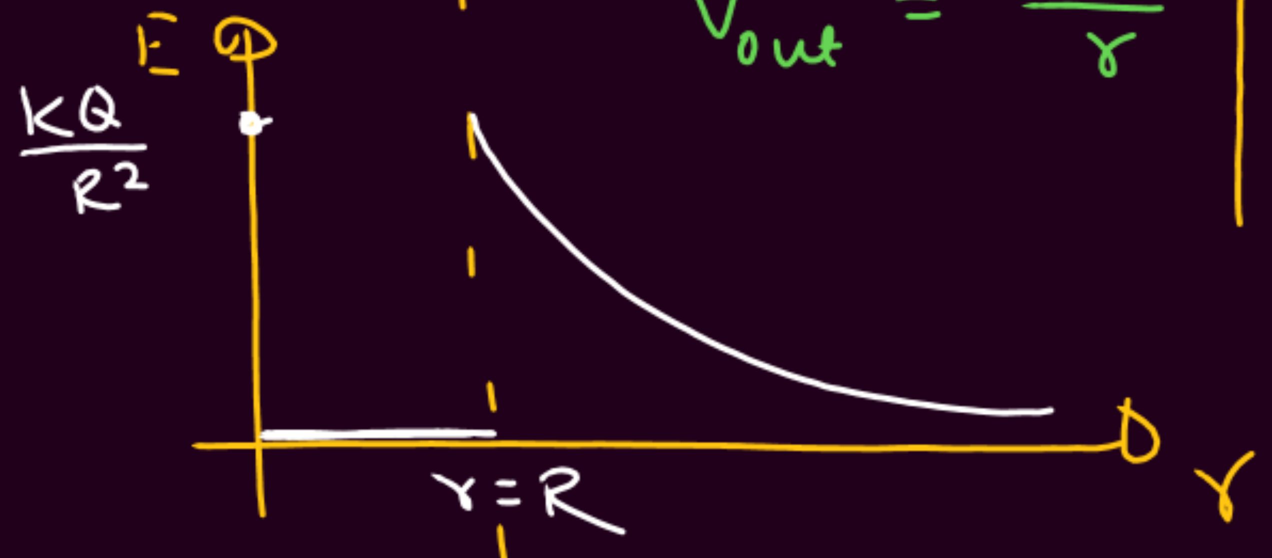
② Sphere (Surface distribution) → Hollow sphere

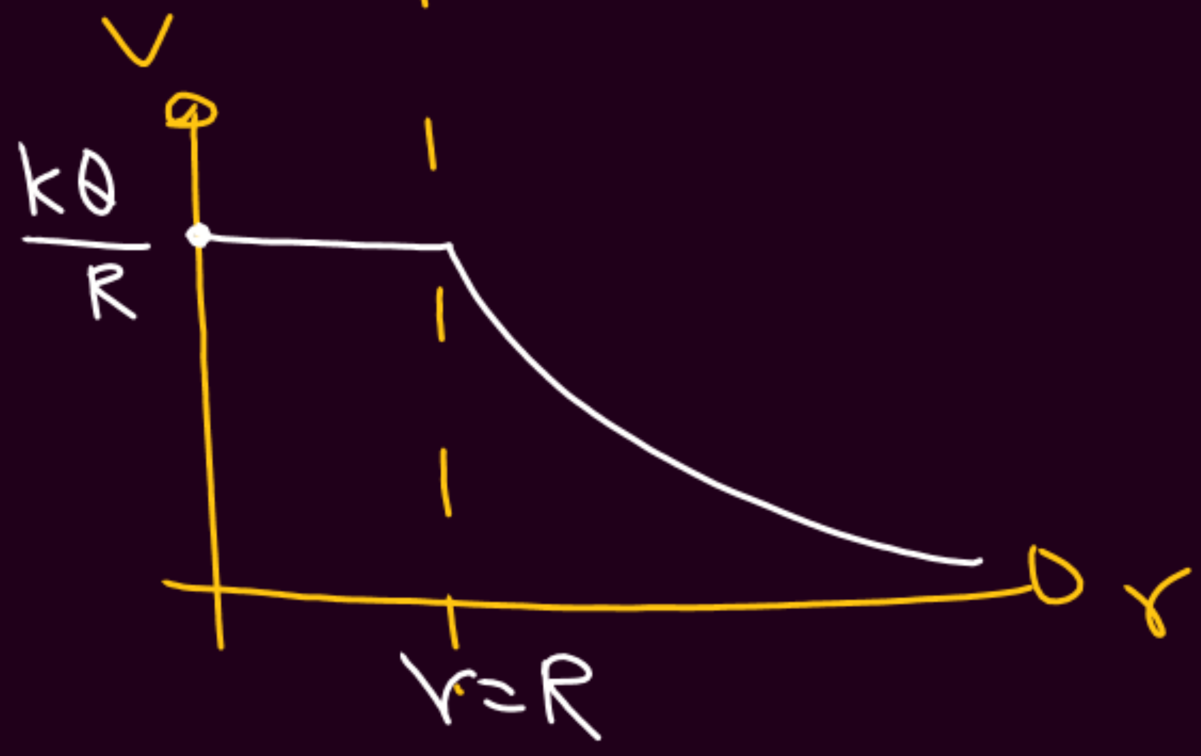
→ Metallic → SS

$$E_{out} = \frac{kQ}{r^2} \quad \Big| \quad E_{surface} = \frac{kQ}{R^2} \quad \Big| \quad E_{in} = 0$$

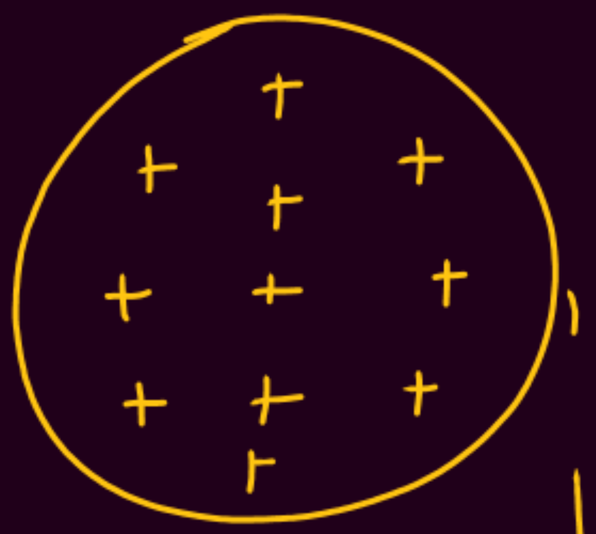
$$V_{out} = \frac{kQ}{r}$$

$$V_{in} = V_{surface} = \frac{kQ}{R}$$





③ Sphere (Volume Distribution)

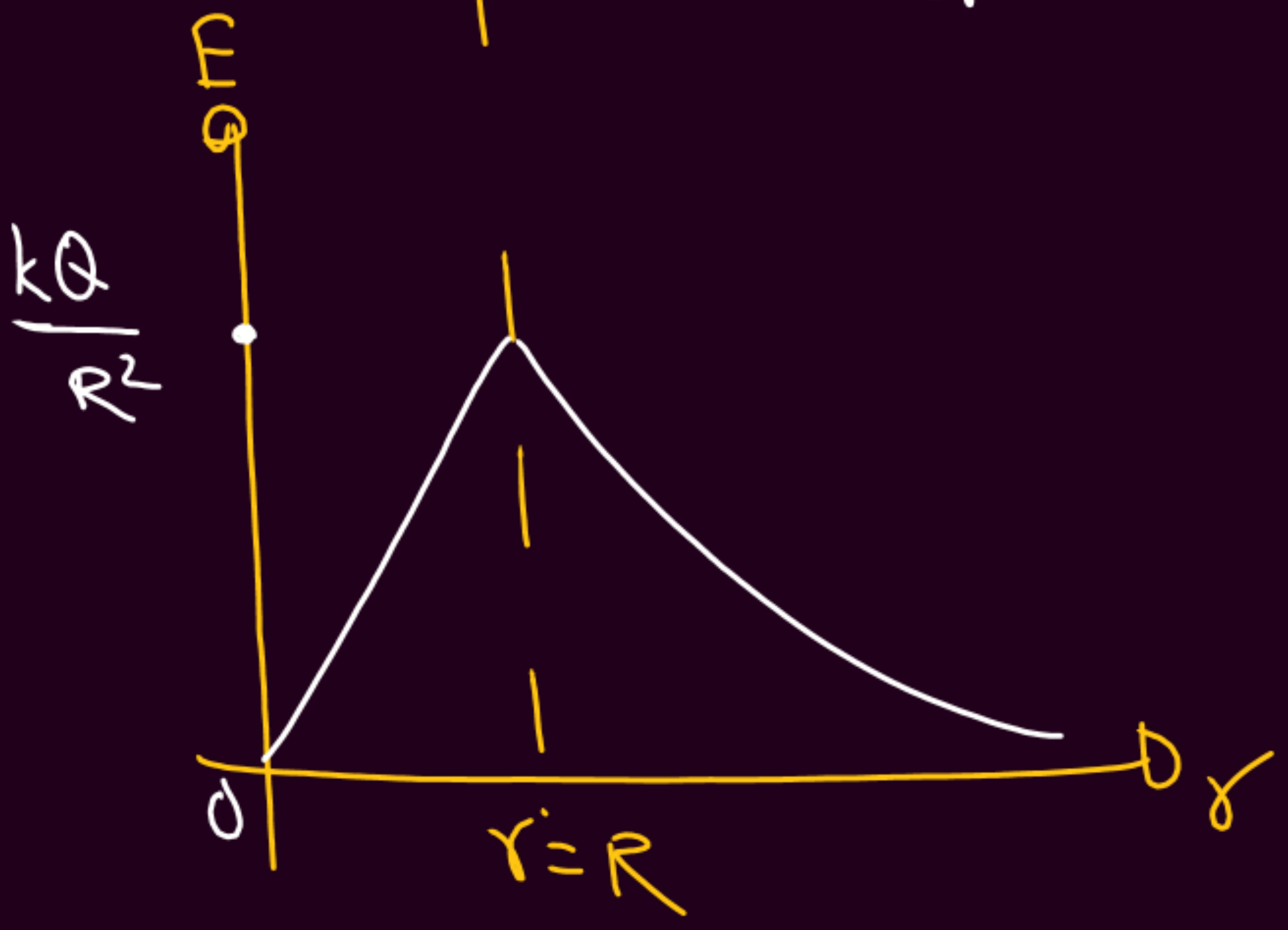


$$E_{out} = \frac{kQ}{r^2}$$

$$E_{surface} = \frac{kQ}{R^2}$$

$$E_{in} = \frac{kQr}{R^3} = \frac{\rho r}{3\epsilon_0}$$

$$E_{center} = 0$$

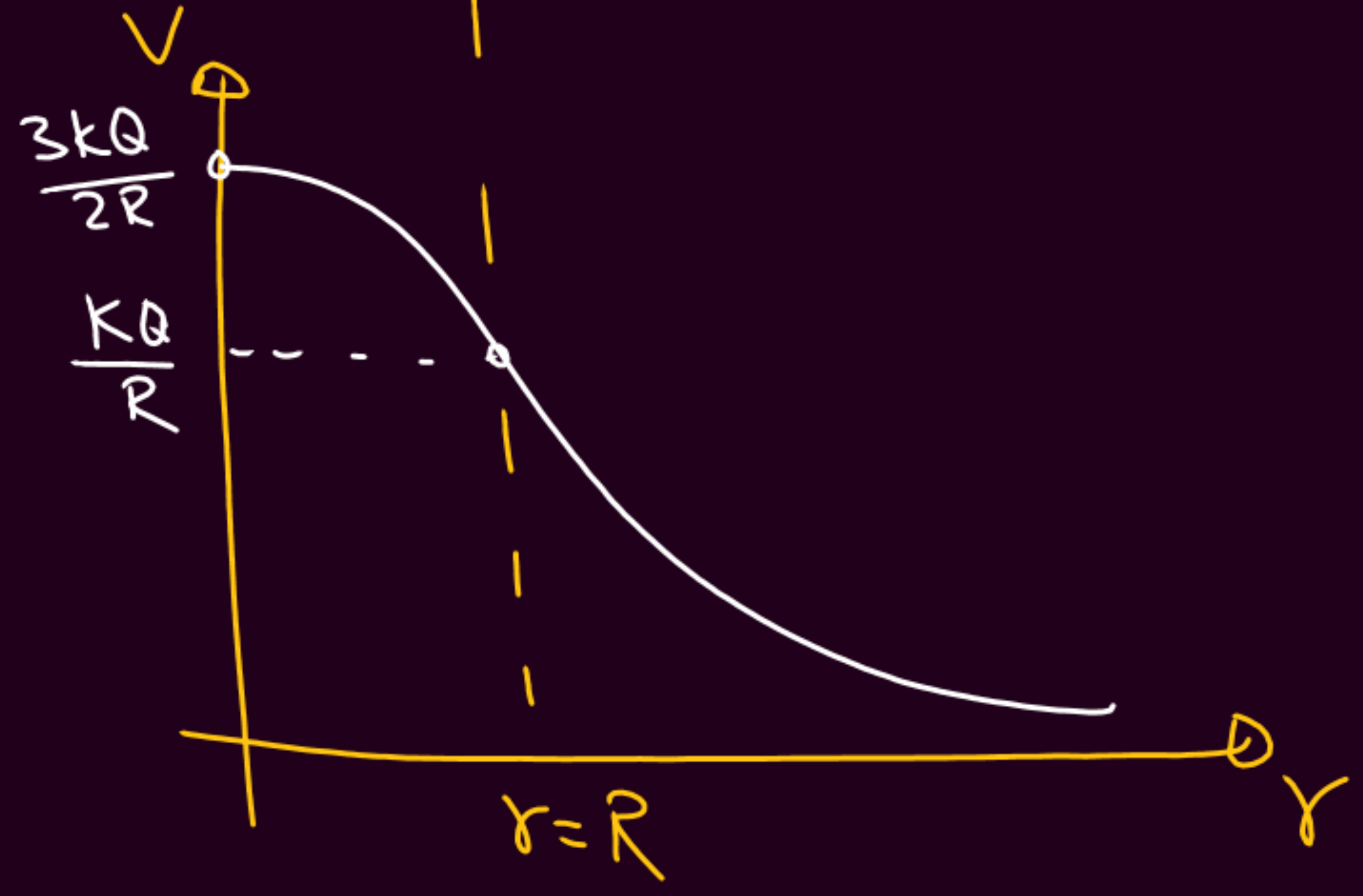


Insulator

$$\begin{cases} V_{out} = \dfrac{kQ}{r} \\ V_{surface} = \dfrac{kQ}{R} \end{cases}$$

$$V_{in} = \frac{kQ}{2R^3}\left(3R^2 - r^2\right)$$

$$V_{center} = \frac{3kQ}{2R}$$



$r = 2R \qquad E = E_0$

$r = R \qquad V = ?$

$$E_0 = \frac{kQ}{4R^2}$$

$$V = \frac{kQ}{R} = \frac{E_0(4R^2)}{R}$$

$$V = 4RE_0$$

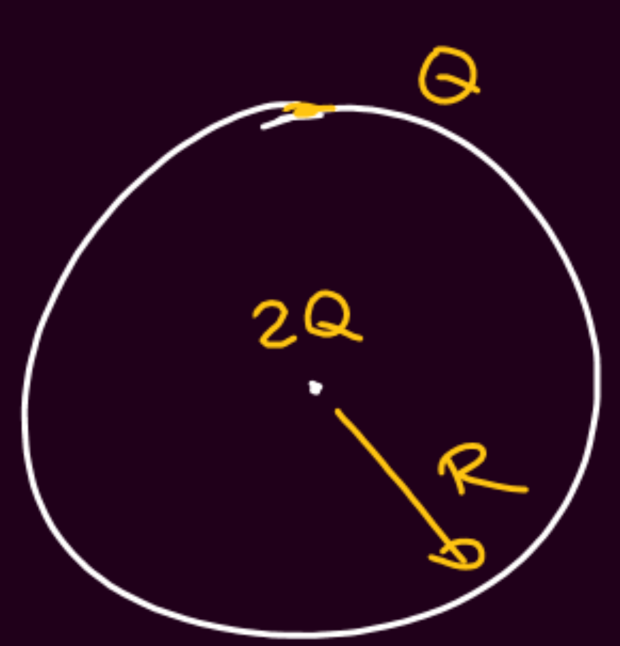
$r = 2R \qquad E_0$

$r = \frac{R}{2} \qquad ?$

$$E_0 = \frac{kQ}{4R^2}$$

$$E = \frac{kQ(R/2)}{R^3} = \frac{kQ}{2R^2} = 2E_0$$

Q.



$\boxed{r = 2R}$ outside

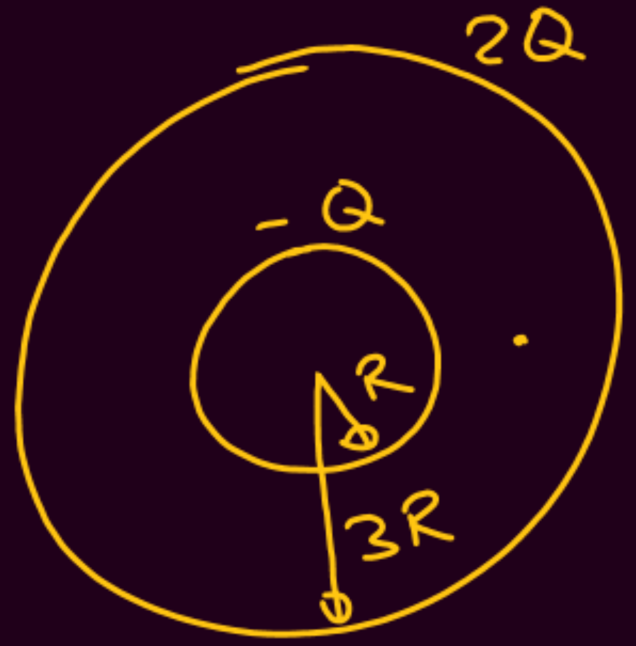
$$E = \frac{k(2Q)}{(2R)^2} + \frac{kQ}{(2R)^2}$$

$$V = \frac{k(2Q)}{(2R)} + \frac{kQ}{(2R)}$$

$\boxed{r = R/2}$ inside

$$E = \frac{k(2Q)}{(R/2)^2} + 0$$

$$V = \frac{k(2Q)}{(R/2)} + \frac{kQ}{R}$$



	E	V
$r = 4R$	$\frac{k(-Q)}{(4R)^2} + \frac{k(2Q)}{(4R)^2}$	$\frac{k(-Q)}{4R} + \frac{k(2Q)}{4R}$
$r = 2R$	$\frac{k(-Q)}{(2R)^2} + 0$	$\frac{k(-Q)}{2R} + \frac{k(2Q)}{3R}$
center	$0 + 0$	$\frac{k(-Q)}{R} + \frac{k(2Q)}{3R}$

Sharing of charges

Potential

① $\dfrac{kq_1}{R_1} = \dfrac{kq_2}{R_2}$

$$\sigma_1 R_1 = \sigma_2 R_2$$

---

② $q_1 + q_2 = Q_1 + Q_2$

Spherical conductor

$$E_{out} = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\sigma(4\pi R^2)}{4\pi\epsilon_0 r^2} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

$$\boxed{E_{surface} = \frac{\sigma}{\epsilon_0}} \qquad E_{in} = 0$$

$$V_{out} = \frac{Q}{4\pi\epsilon_0 r} = \frac{\sigma 4\pi R^2}{4\pi\epsilon_0 r} = \frac{\sigma R^2}{\epsilon_0 r}$$

$$V_{in} = V_{surface} = \frac{\sigma R}{\epsilon_0}$$

Same charge density (σ)

① $\dfrac{E_1}{E_2} = \dfrac{1}{1}$ $\left[E_{sur} = \dfrac{\sigma}{\epsilon_0}\right]$

② $\dfrac{V_1}{V_2} = \dfrac{R_1}{R_2}$ $\left[V_{sur} = \dfrac{\sigma R}{\epsilon_0}\right]$

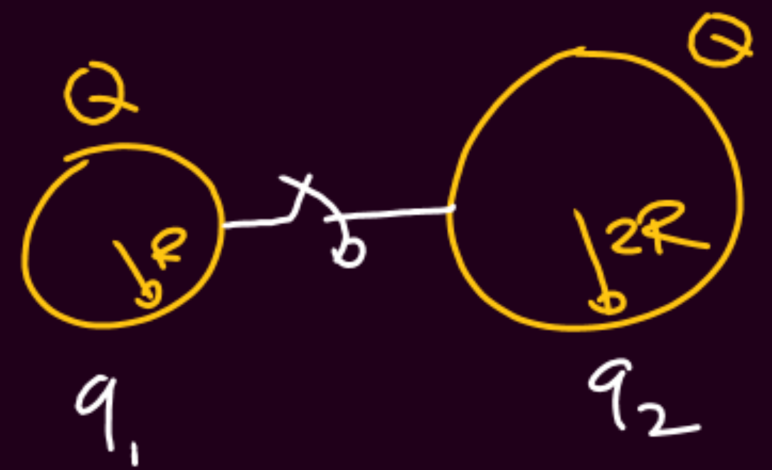
③ $\dfrac{q_1}{q_2} = \dfrac{R_1^2}{R_2^2}$ $\left[q = \sigma 4\pi R^2\right]$

Both at same potential $\left[\sigma_1 R_1 = \sigma_2 R_2\right]$

① $\dfrac{E_1}{E_2} = \dfrac{\sigma_1}{\sigma_2} = \dfrac{R_2}{R_1}$ $E_{sur} = \dfrac{\sigma}{\epsilon_0}$

② $\dfrac{V_1}{V_2} = \dfrac{1}{1}$

③ $\dfrac{q_1}{q_2} = \dfrac{R_1}{R_2}$ $\dfrac{kq_1}{R_1} = \dfrac{kq_2}{R_2}$



Potential

① $$\frac{k q_1}{R} = \frac{k q_2}{2R}$$

$$\boxed{2q_1 = q_2}$$

② $$q_1 + q_2 = Q + Q$$

$$q_1 + 2q_1 = 2Q$$

$$\boxed{q_1 = \frac{2Q}{3}}$$

$$\boxed{q_2 = \frac{4Q}{3}}$$

① $$\frac{k q_1}{R} = \frac{k q_2}{3R}$$

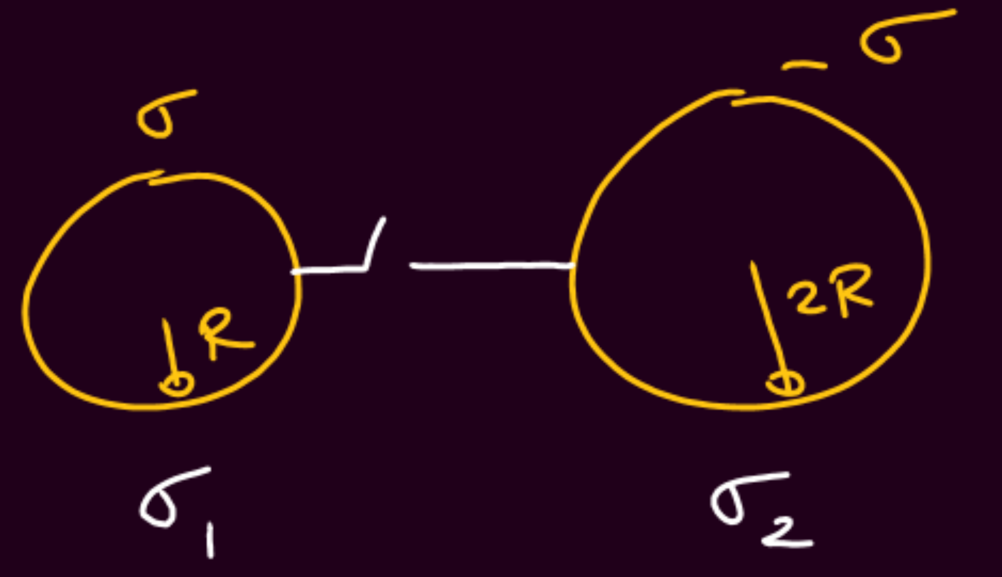
$$\boxed{q_2 = 3q_1}$$

② $$q_1 + q_2 = -2Q + 3Q$$

$$4q_1 = Q$$

$$\boxed{q_1 = \frac{Q}{4}}$$

$$\boxed{q_2 = \frac{3Q}{4}}$$



① $$\sigma_1 R = \sigma_2 \, 2R$$

$$\boxed{\sigma_1 = 2\sigma_2}$$

② $$\sigma_1 (4\pi R^2) + \sigma_2 \, 4\pi (2R)^2 = \sigma \, 4\pi R^2 - \sigma \, 4\pi (2R)^2$$

$$\sigma_1 + 4\sigma_2 = \sigma - 4\sigma$$

$$2\sigma_2 + 4\sigma_2 = -3\sigma$$

$$\sigma_2 = -\frac{3\sigma}{6} = -\frac{\sigma}{2}$$

$$\sigma_2 = -\frac{\sigma}{2}$$

$$\sigma_1 = -\sigma$$

(n) - identical drops → Coalesce

Each
q
r
V_0
C_0

R

① $R^3 = n r^3$

② $Q = n q$

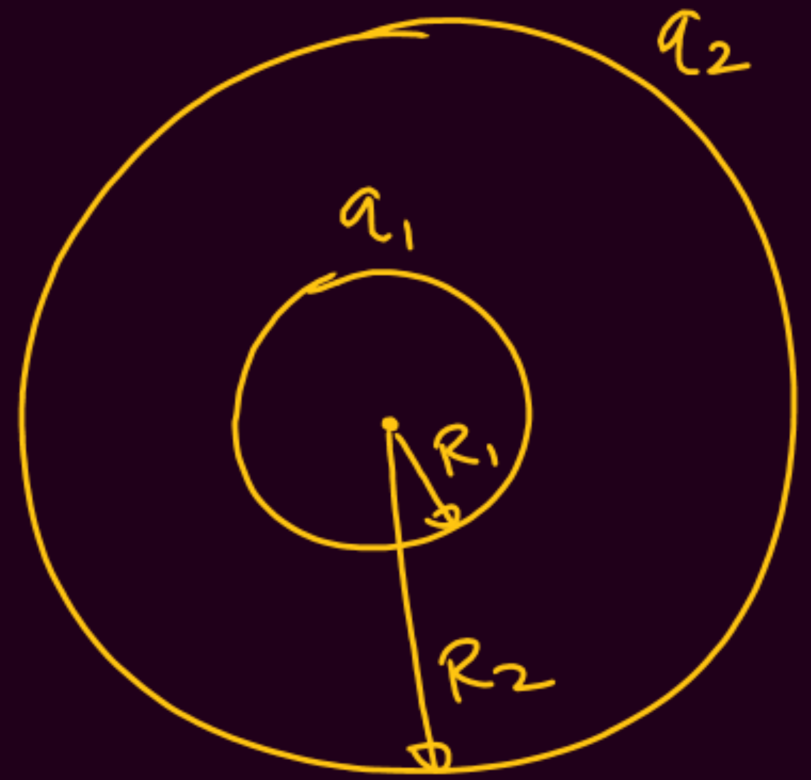
③ $\boxed{V = n^{2/3} V_0}$

④ $\boxed{C = n^{1/3} C_0}$

1000 drops

$V = (1000)^{2/3} V_0$
$= 100 V_0$

$C = (1000)^{1/3} C_0$
$= 10 C_0$



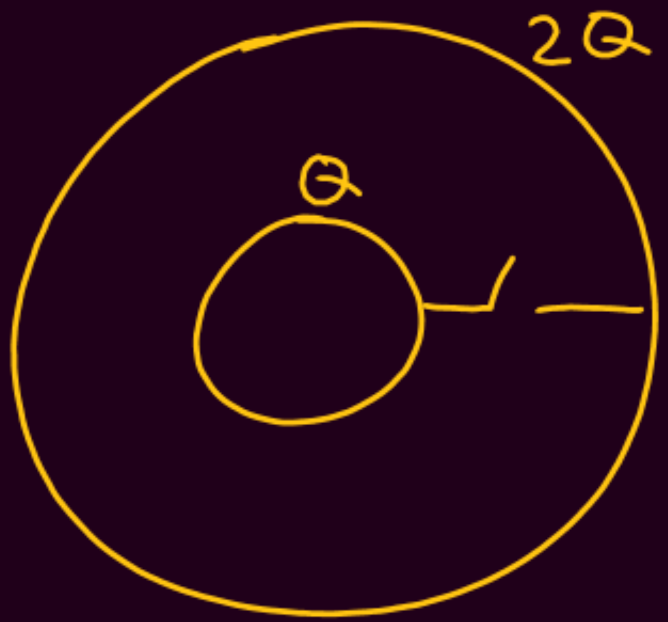
① Potential of Bigger Sphere $= \frac{Kq_1}{R_2} + \frac{Kq_2}{R_2}$

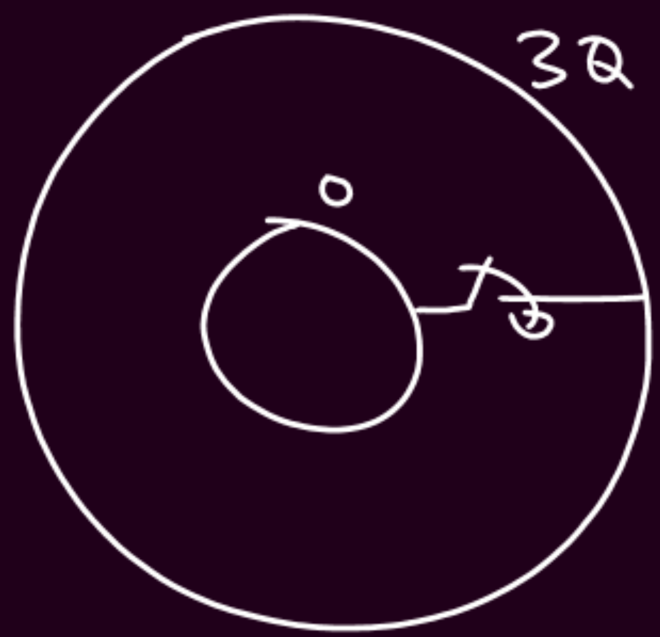
② —— " —— Smaller — " — $= \frac{Kq_1}{R_1} + \frac{Kq_2}{R_2}$

③ P.D. bet^n spheres $= Kq_1\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

① P.D. independent of q_2

② If connected then





① Charged Sphere → Vol.

② Hollow metallic sphre → Sur

③ Solid metallic — " ——— → Sur

④ Hollow sphere → Sur.

⑤ Solid Non-conducting sphree → volume
has charge Q

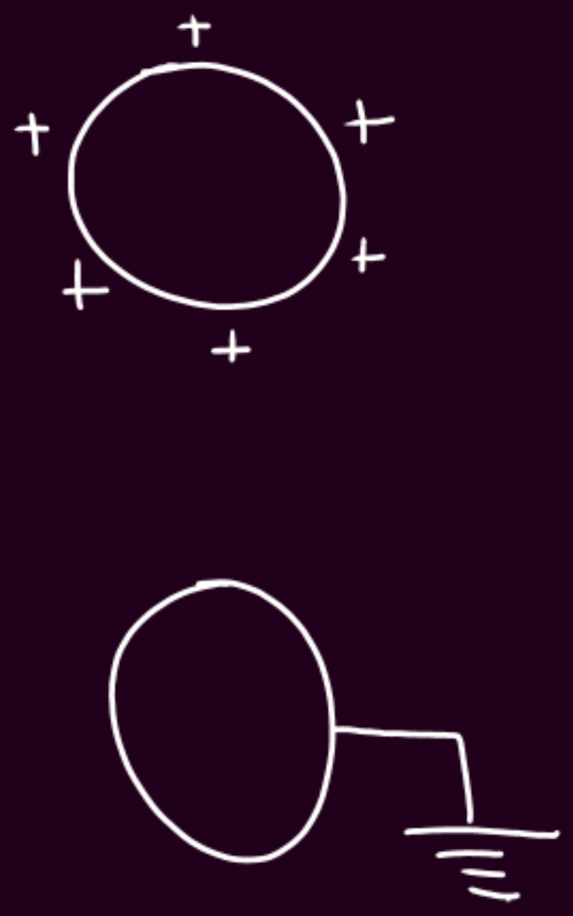
⑥ Solid Non-conducting sphere → Sur.
has charge Q on Sur.

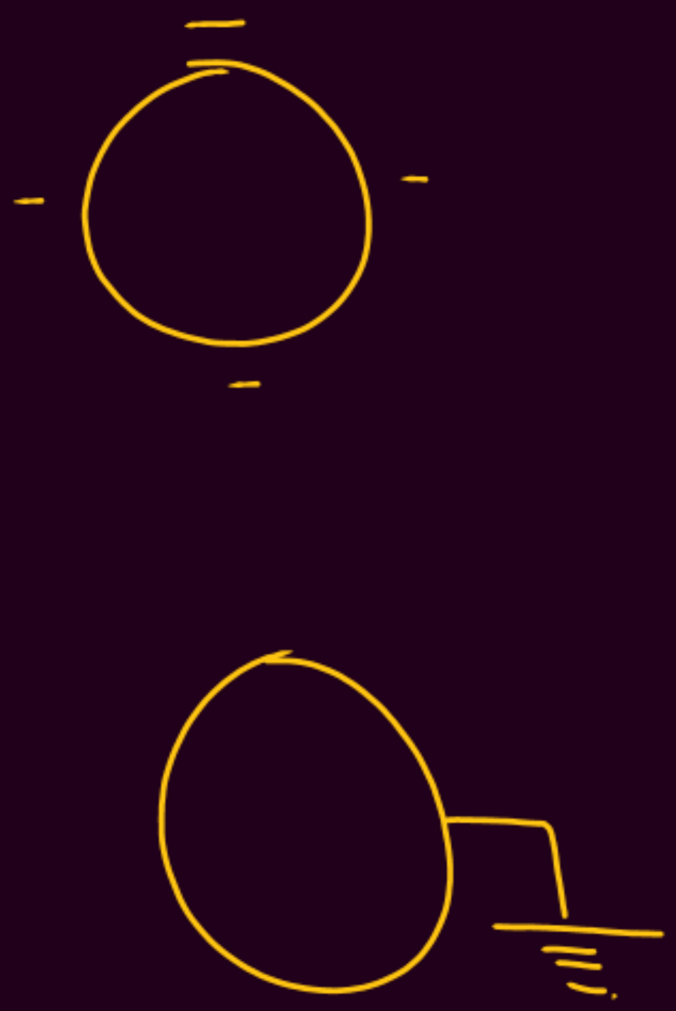
Earthing

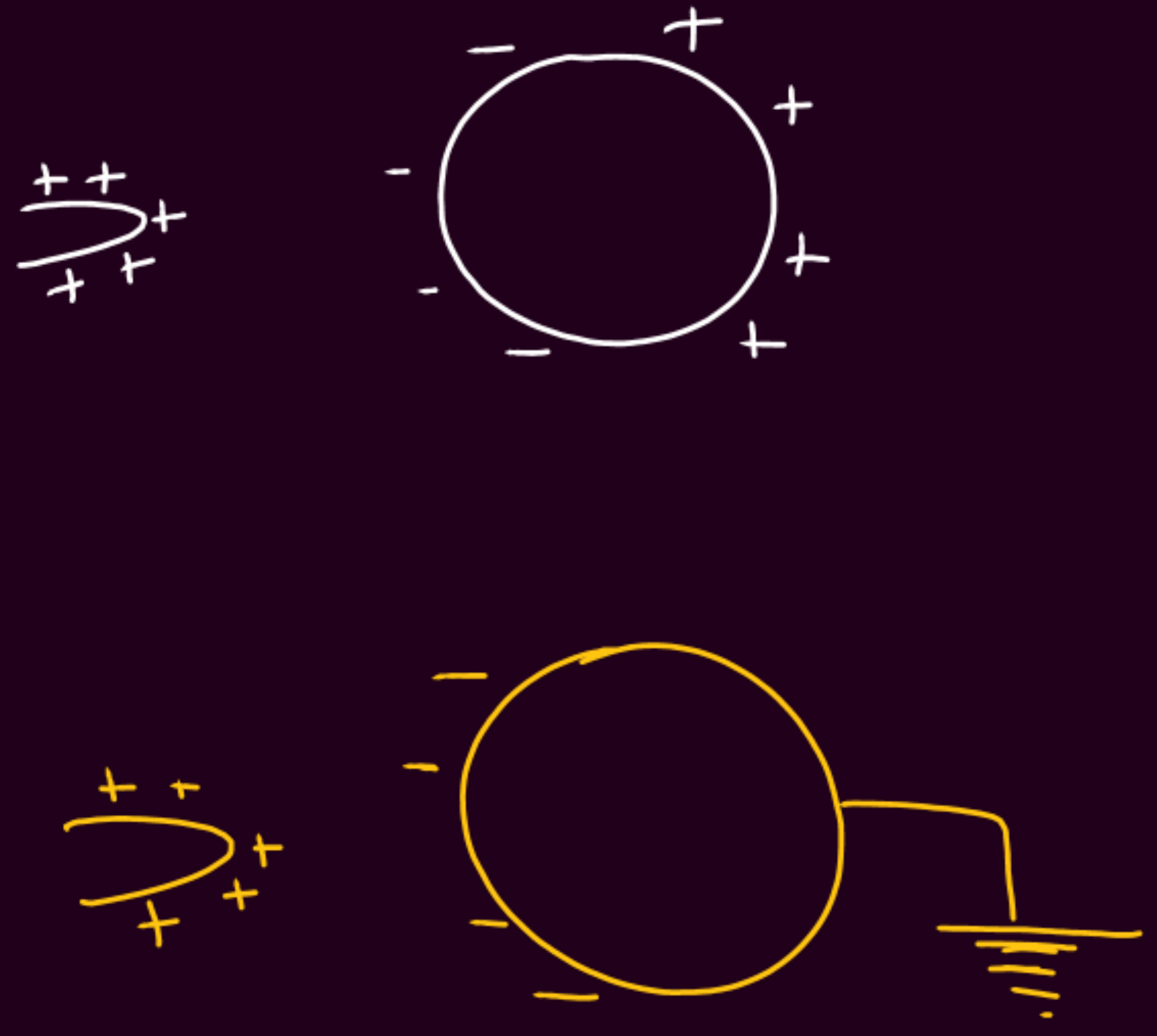
→ $V_{earth} = 0$

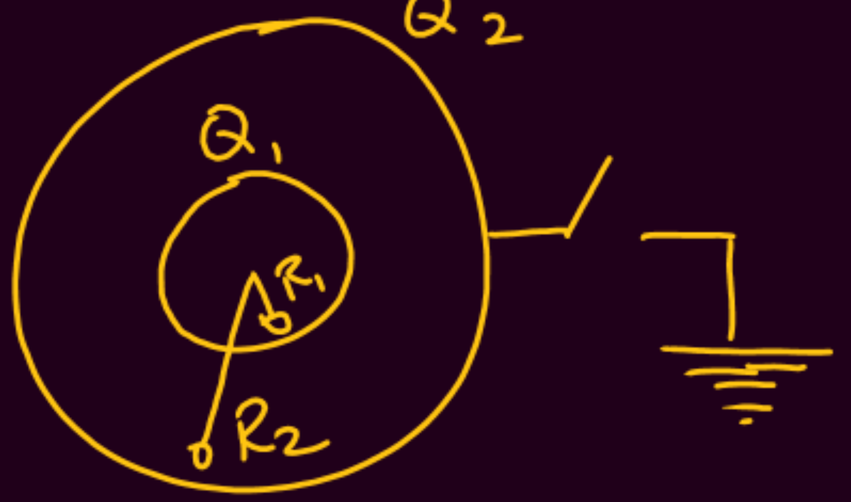
→ ⊖

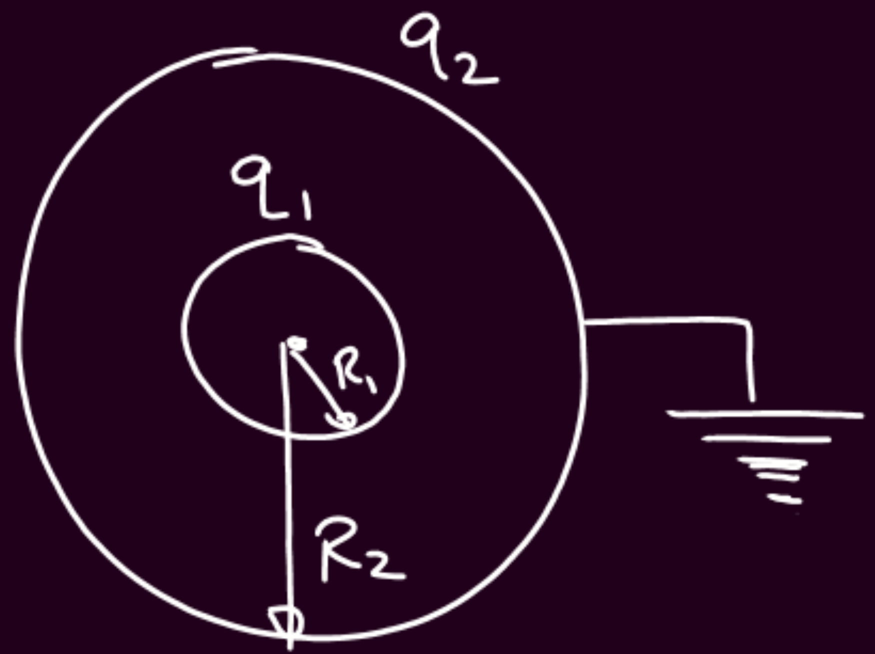
→











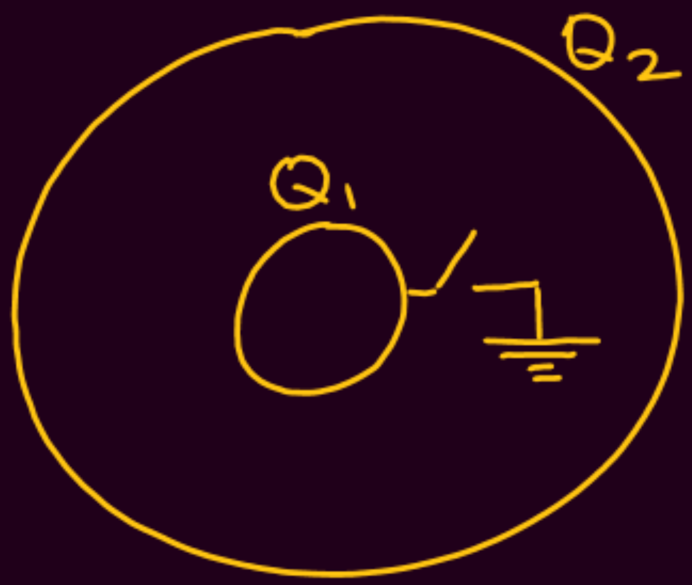
$$\frac{kq_1}{R_2} + \frac{kq_2}{R_2} = 0$$

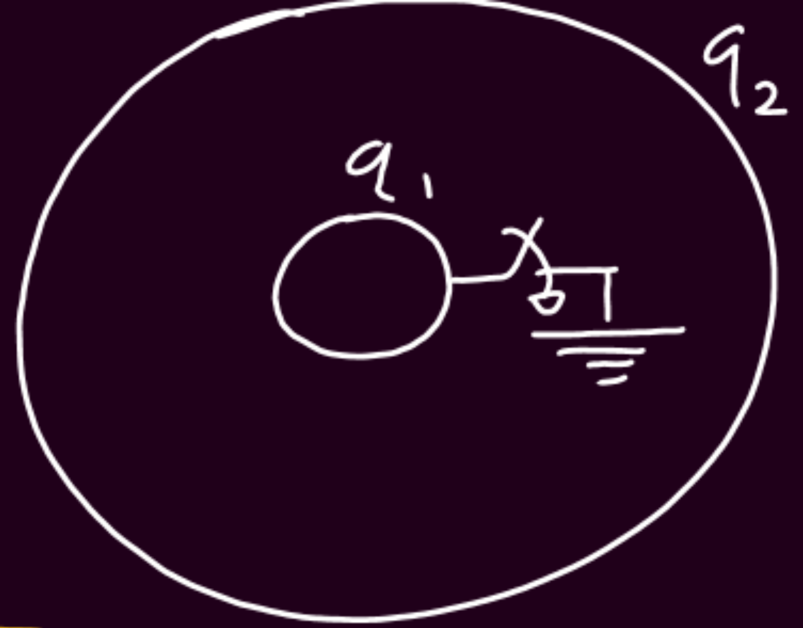
$$\boxed{q_2 = -q_1}$$

$$\boxed{q_1 = Q_1}$$

isolated

$$\boxed{q_2 = -Q_1}$$





$$\boxed{q_2 = Q_2}$$

$$\boxed{q_1 = -\frac{R_1}{R_2} Q_2}$$

$$\frac{kq_1}{R_1} + \frac{kq_2}{R_2} = 0$$

$$q_1 = -\frac{R_1}{R_2} q_2$$

Infinitely large sheet of charge

→ Single Surface

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

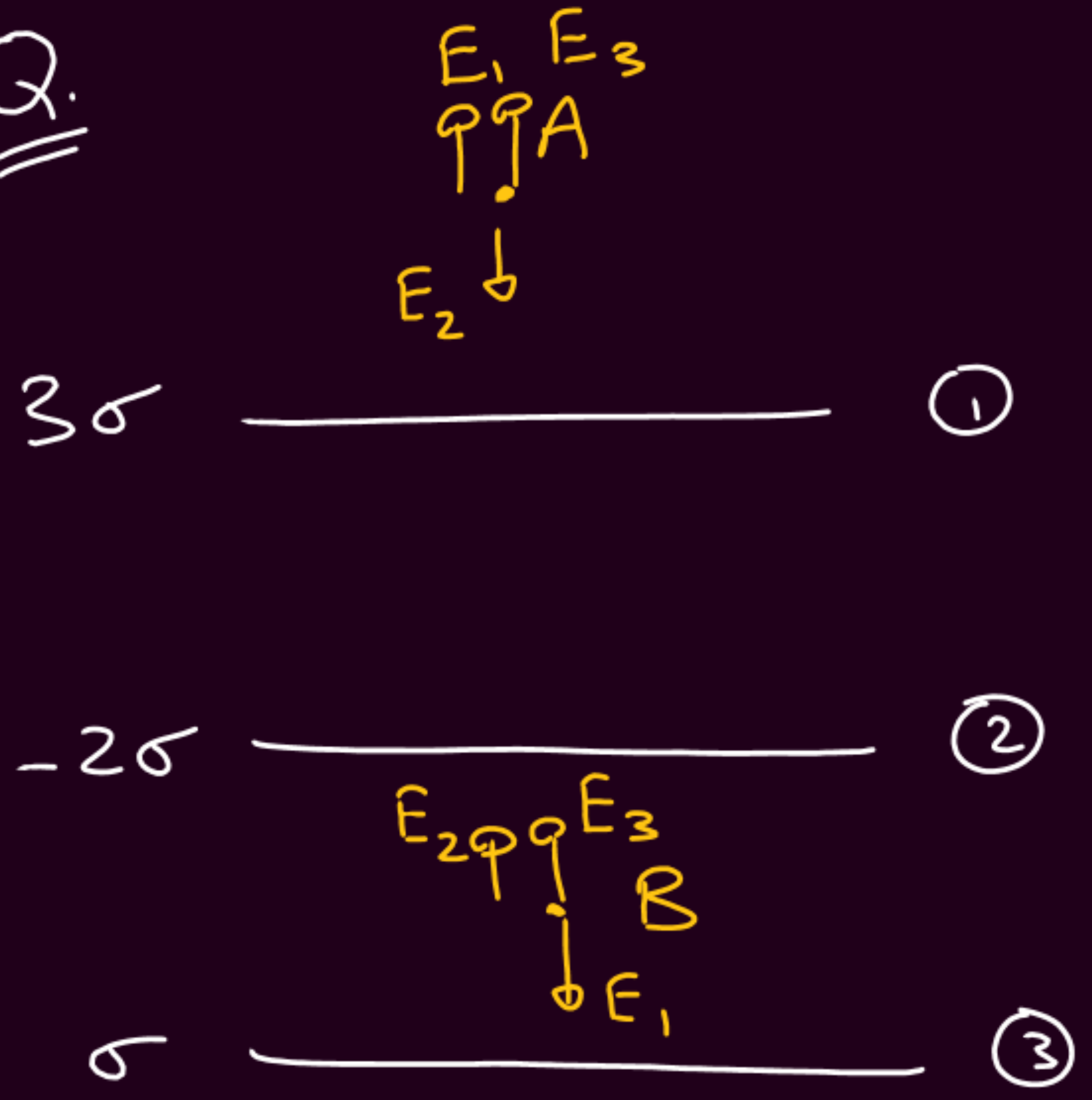
$$V_{AB} = \left(\frac{\sigma}{2\epsilon_0}\right) d$$

Uniform field

Metallic / Conducting Sheet

→ $\boxed{E = \frac{\sigma}{\epsilon_0}}$

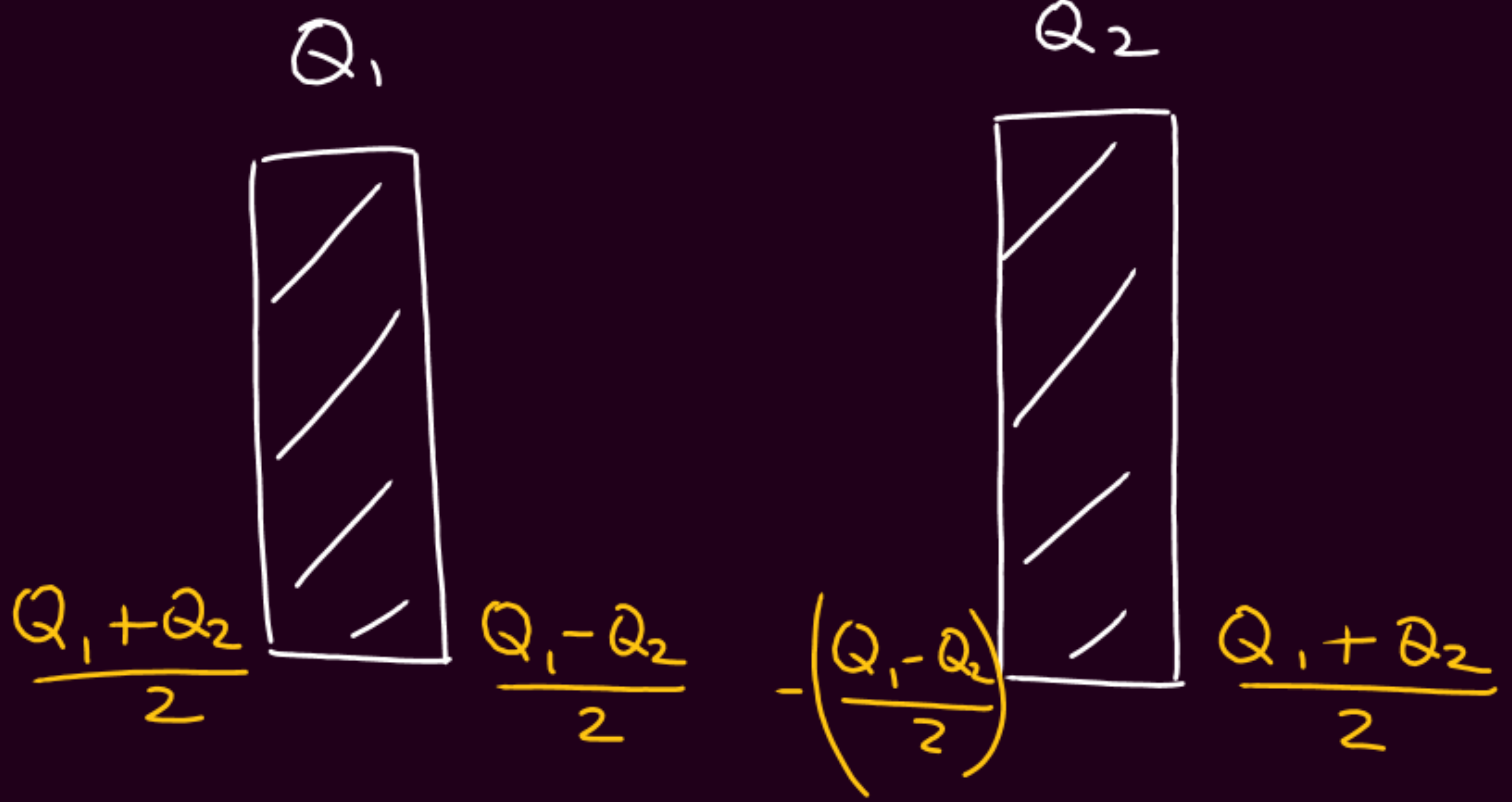
Q.



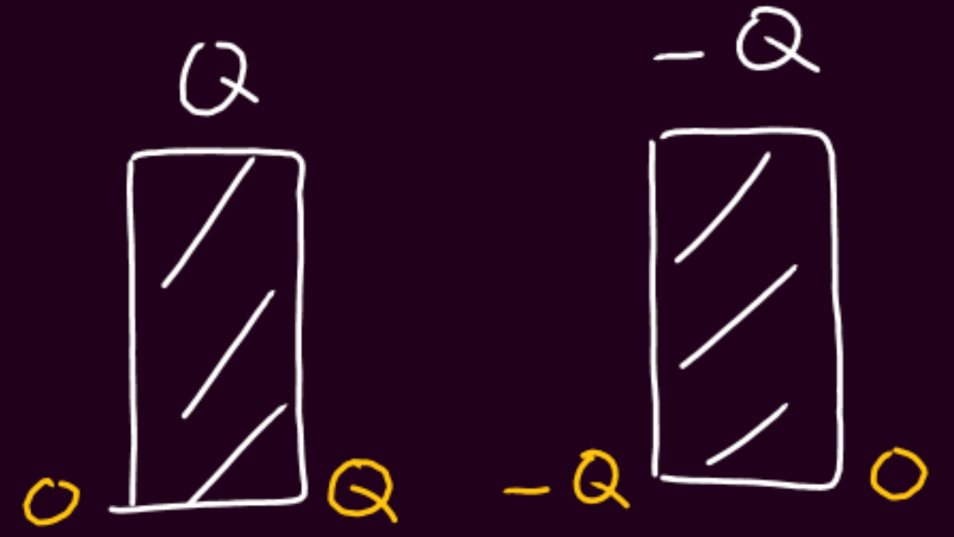
$$E_A = \frac{3\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} - \frac{2\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \uparrow$$

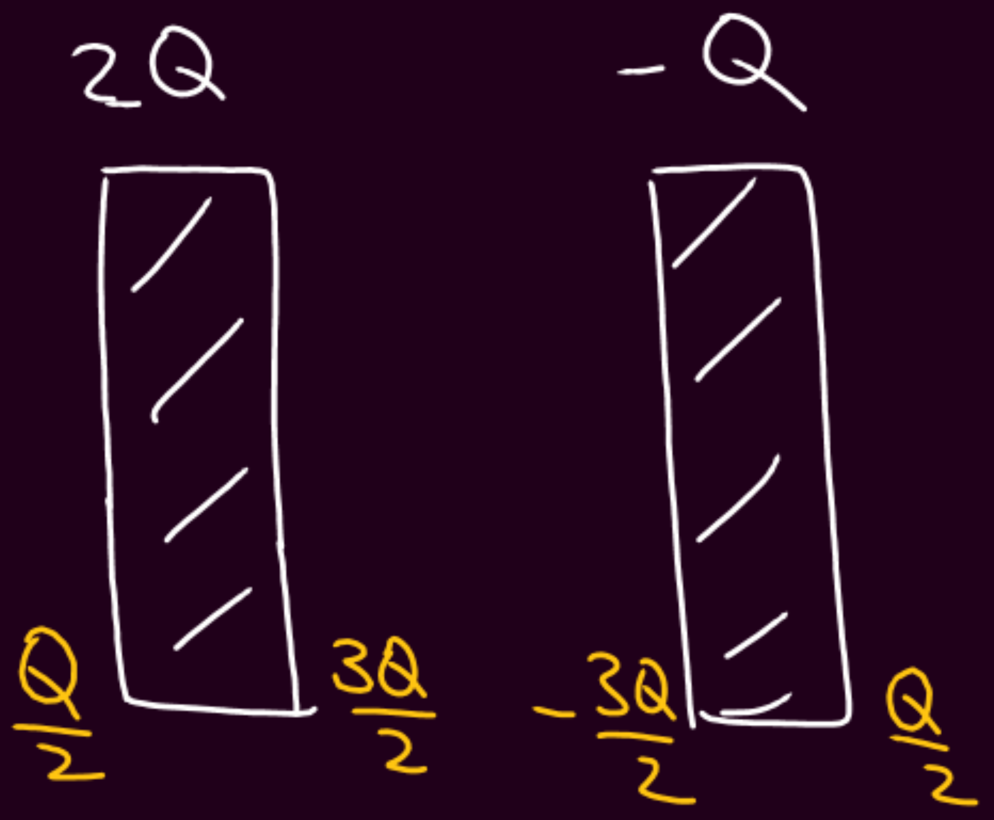
$$E_B = \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} - \frac{3\sigma}{2\epsilon_0} = 0$$

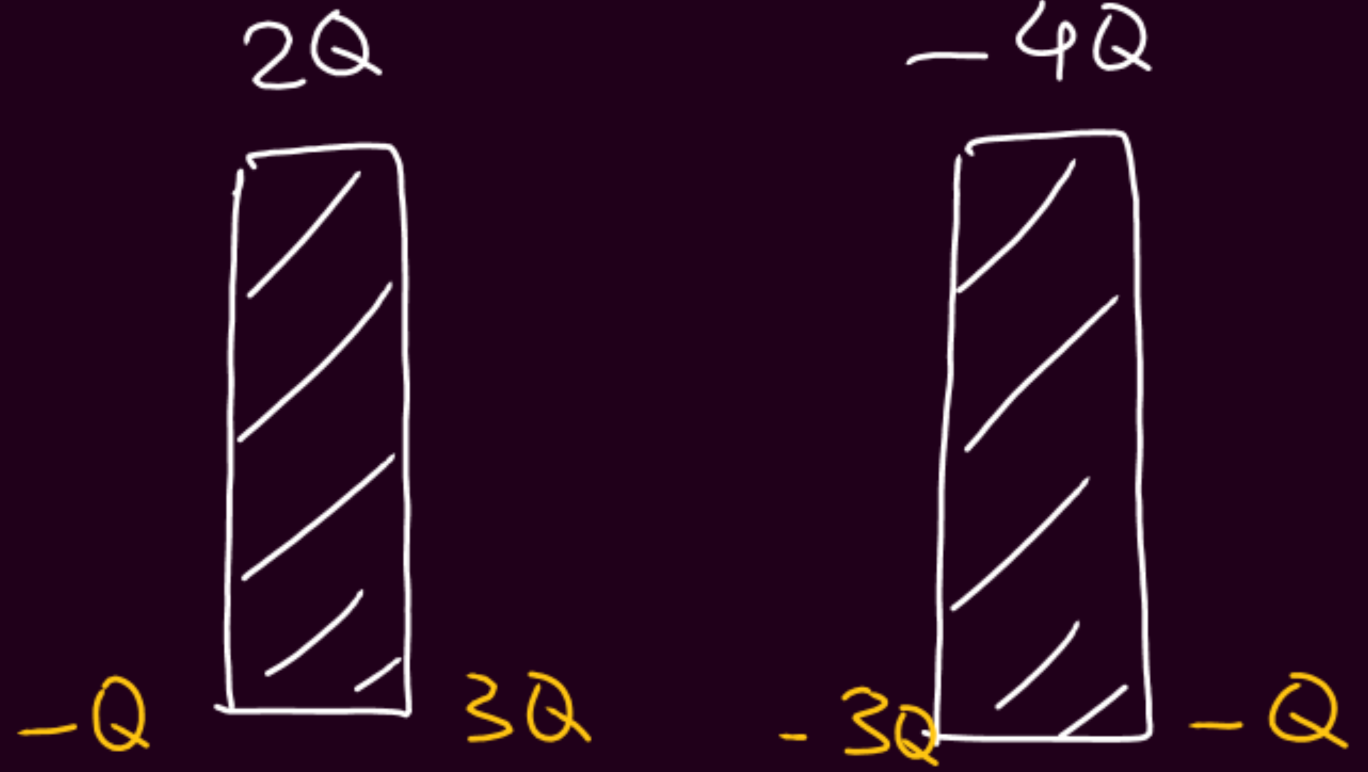
Parallel Conducting Sheets (Extra)

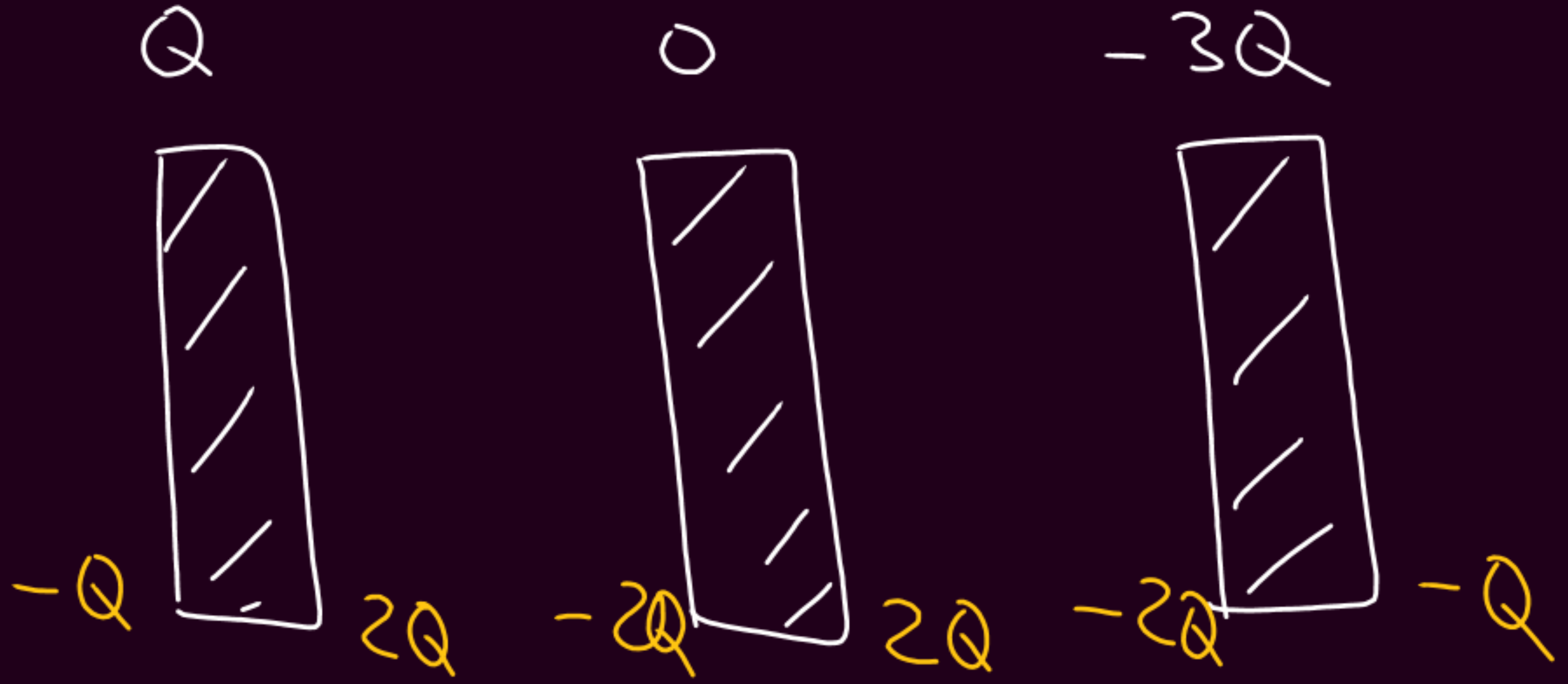


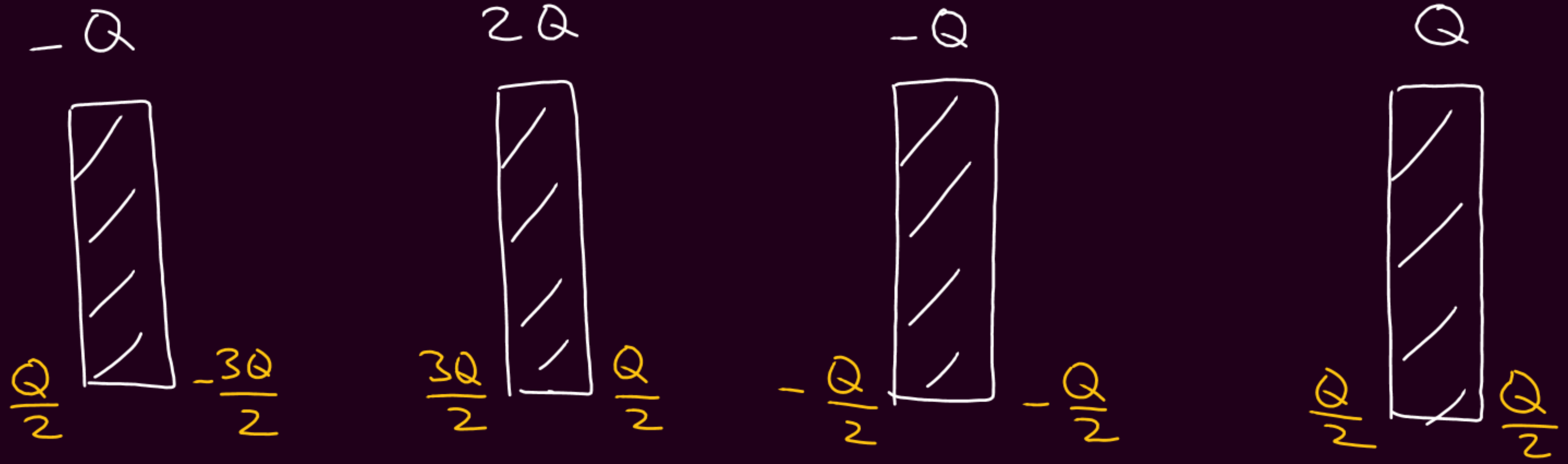
1. Facing $\rightarrow$ Equal & opp.
2. outermost $\rightarrow$ Equal $= \frac{\text{Total}}{2}$

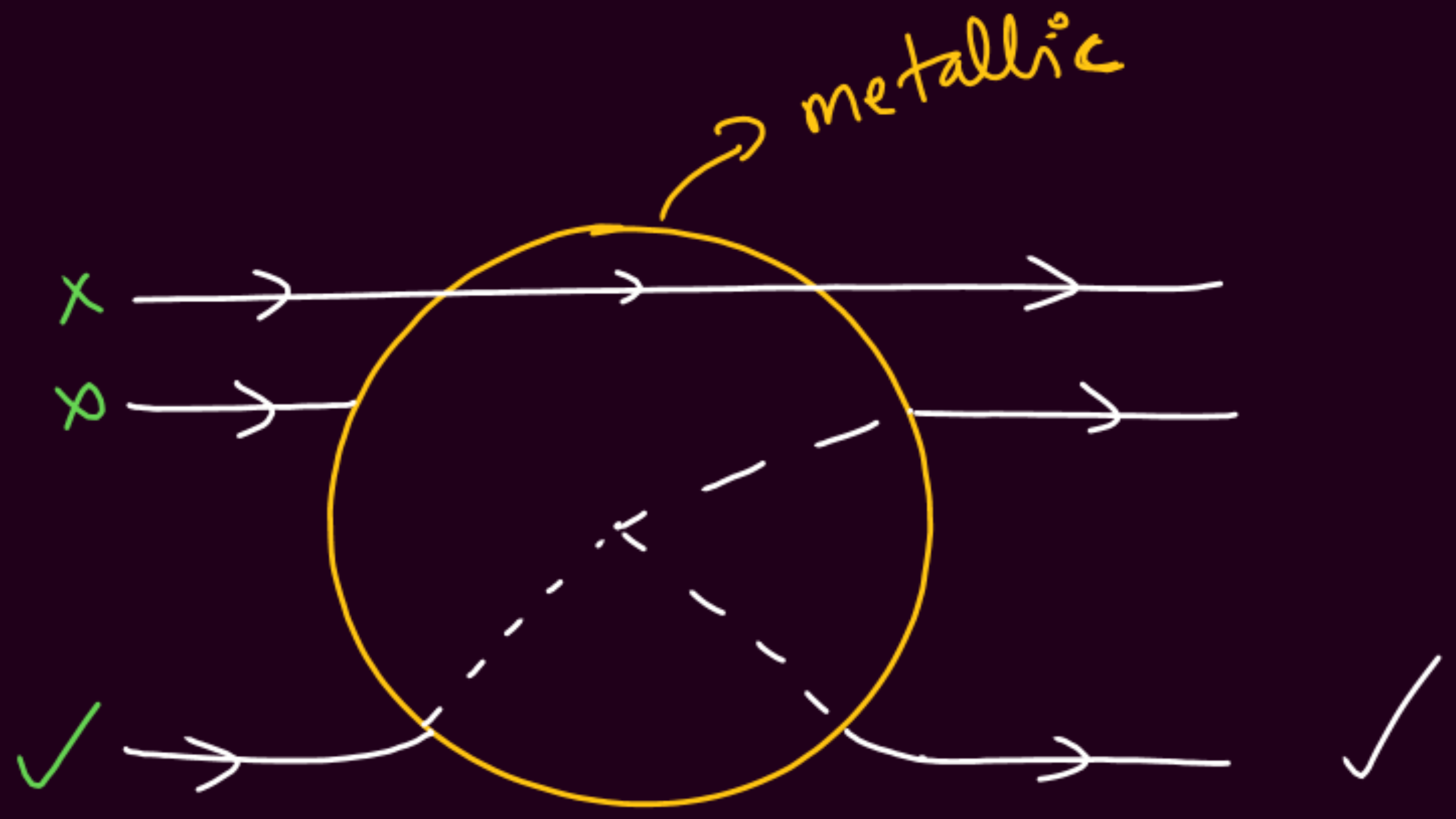

Q
-Q
0
Q
-Q
0

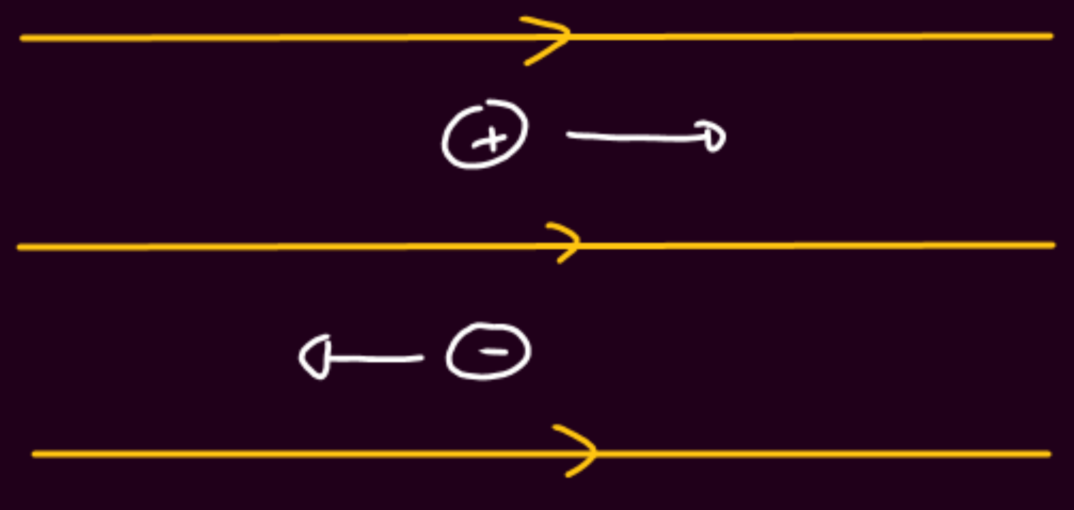

2Q
-Q
Q/2
3Q/2
-3Q/2
Q/2


2Q
-4Q
-Q
3Q
-3Q
-Q


Q
0
-3Q
-Q
2Q
-2Q
2Q
-2Q
-Q


-Q
2Q
-Q
Q
Q/2
-3Q/2
3Q/2
Q/2
-Q/2
-Q/2
Q/2
Q/2





$$\Delta U = -W_{by}$$