Capacitance (Condensor)

→ storage device ($\vec{E}$)

$$\boxed{C = \frac{Q}{V}}$$ ✓

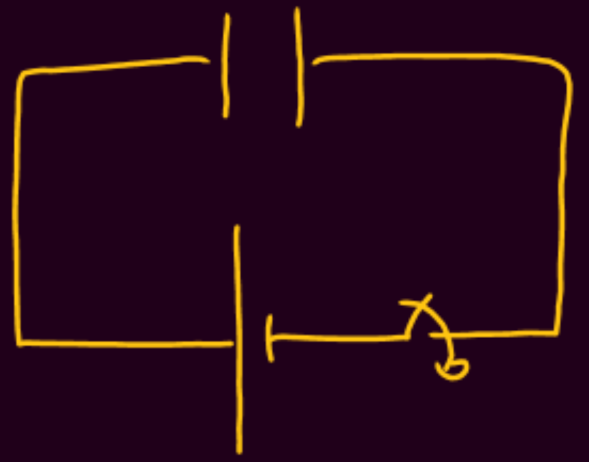
Unit Farad (F)

$\mu F = 10^{-6}\ F$

$nF = 10^{-9}\ F$

$pF = 10^{-12}\ F$

Parallel Plate Capacitor



σ = Surface charge density

= charge per unit area

$$\boxed{\sigma = \frac{q}{A}}$$

ϵ_0 = Permittivity of free Space (vacuum) ✓

ϵ = ———— " ———— Medium

$\epsilon = \epsilon_0 \epsilon_r = \boxed{\epsilon_0 K}$

$\boxed{\epsilon_r = K = \frac{\epsilon}{\epsilon_0}}$ → Relative permittivity of medium

→ Dielectric constant

→ unitless

→ Dimensionless

$\boxed{K \geq 1}$

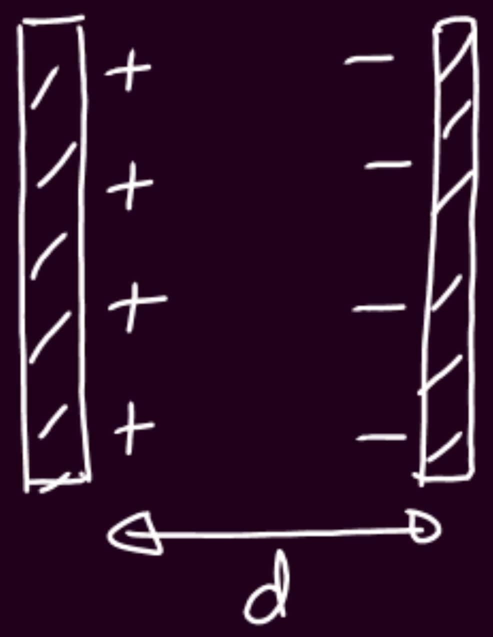
$K = 1$ vacuum

$K \simeq 1$ Air

$K \to \infty$ metals

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ \frac{N\text{-}m^2}{C^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \ \frac{C^2}{N\text{-}m^2}$$



$E_{out} = 0$

$$\boxed{E_{in} = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}}$$

Air → ϵ_0

$E_m = \frac{E_0}{K}$

$$E = \frac{q}{\epsilon_0 A} = \frac{V}{d}$$

$$\frac{q}{V} = \frac{\epsilon_0 A}{d}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Air capacitor

$$\boxed{C = \frac{\epsilon_0 K A}{d}}$$

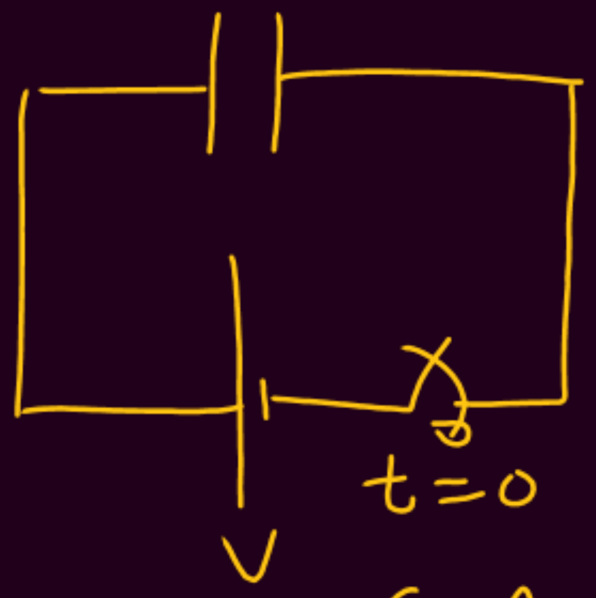
→ Area

→ Gap / Sep.

→ med

A = Area of plates

d = Sep. / Gap betⁿ plates



(1) $C = \dfrac{\epsilon_0 A}{d}$

(2) $q = CV$

(3) W.D. by battery $= qV = CV^2 = \dfrac{q^2}{C}$

(4) $\boxed{U = \dfrac{1}{2}CV^2 = \dfrac{q^2}{2C} = \dfrac{1}{2}qV}$

(5) $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{q}{\epsilon_0 A} = \dfrac{V}{d}$

(6) $E_{one} = \dfrac{\sigma}{2\epsilon_0}$

(7) $P = \dfrac{\sigma^2}{2\epsilon_0} = \dfrac{1}{2}\epsilon_0 E^2$

(8) Energy Density $= \dfrac{\text{Energy}}{\text{Volume}} = \dfrac{1}{2}\epsilon_0 E^2 = \dfrac{\sigma^2}{2\epsilon_0}$

$= \dfrac{\frac{1}{2}CV^2}{Ad}$

(9) $F = \dfrac{\sigma^2 A}{2\epsilon_0} = \dfrac{q^2}{2\epsilon_0 A}$

Q $C = \frac{\epsilon_0 K A}{d}$

$$C = \frac{(8.85 \times 10^{-12})(4)(1\cancel{6} \times \cancel{8} \times \cancel{10^{-4}})}{\cancel{8} \times \cancel{10^{-3}}}$$

$$C = 35.4 \times 10^{-12}\ F$$

Q 200 μF

20V

$$Q = CV = 4000\ \mu C$$

$$U = \frac{1}{2} C V^2 = \frac{1}{2}(200\mu)(400)$$

$$= 40{,}000\ \mu J$$

$$= 0.04\ J$$

Case Battery connected $(d \to 2d)$

① $C_0 = \frac{\epsilon_0 A}{d}$

$C = \frac{\epsilon_0 A}{2d} = \frac{C_0}{2}$

② V_0

$V = V_0$

③ $q_0 = C_0 V_0$

$q = \frac{C_0}{2} V_0 = \frac{q_0}{2}$

④ $U_0 = \frac{1}{2} C_0 V_0^2$

$U = \frac{1}{2}\left(\frac{C_0}{2}\right) V_0^2 = \frac{U_0}{2}$

⑤ $E_0 = \frac{V_0}{d}$

$E = \frac{V_0}{2d} = \frac{E_0}{2}$

⑥ $F_0 = \frac{q_0^2}{2\epsilon_0 A}$

$F = \frac{F_0}{4}$

① $C = \frac{C_0}{2}$

② $q = q_0$

③ $V = \frac{q_0}{(C_0/2)} = 2V_0$

④ $U = \frac{q_0^2}{2(C_0/2)} = 2U_0$

$U_0 = \frac{q_0^2}{2C_0}$

⑤ $E = \frac{\sigma}{\epsilon_0} = E_0$

⑥ $F = F_0$

Case Battery connected

① $C_0 = \frac{\epsilon_0 A}{d}$ $\qquad$ $C = \frac{\epsilon_0 K A}{d} = K C_0$

② V_0 $\qquad$ $V = V_0$

③ q_0 $\qquad$ $q_0 = (KC_0)V_0 = K q_0$

④ U_0 $\qquad$ $U = \frac{1}{2}(KC_0)V_0^2 = K U_0$

⑤ $E_0 = \frac{V_0}{d}$ $\qquad$ $E = E_0$

---

① $C = \frac{\epsilon_0 K A}{d} = K C_0$

② $q = q_0$

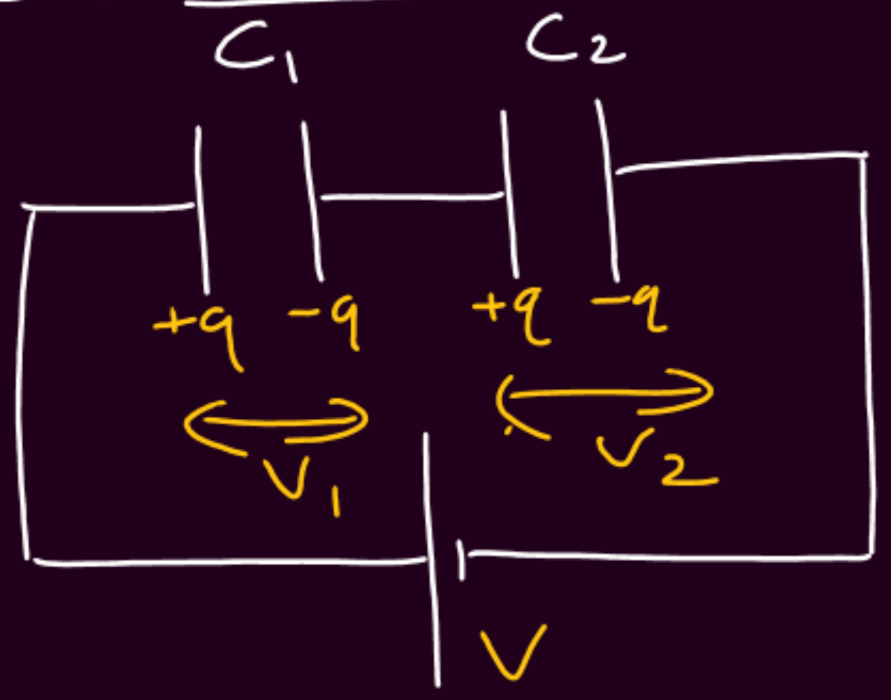
③ $V = \frac{q_0}{K C_0} = \frac{V_0}{K}$

④ $U = \frac{q_0^2}{2(KC_0)} = \frac{U_0}{K}$

⑤ $E = \frac{\sigma}{\epsilon_0 K} = \frac{E_0}{K}$ $\qquad$ $\left(\frac{V_0/K}{d}\right)$

Combination of Capacitance

① Series Combination (Same charge)



$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

$C_{eq} < C_1$

$C_{eq} < C_2$

n-identical

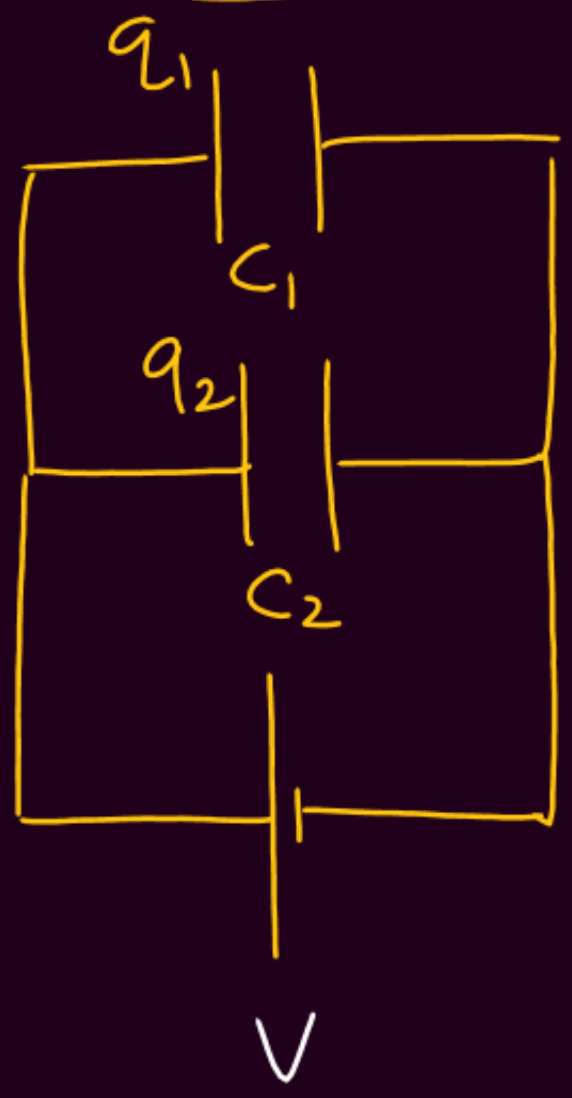
$$C_{eq} = \frac{C_0}{n}$$

$$V = \frac{q}{C}$$

$$\boxed{V_1 = \frac{V C_2}{C_1 + C_2}}$$

$$V_2 = \frac{V C_1}{C_1 + C_2}$$

(2). Parallel Combination (P. D. is same) (Voltage)



$$\boxed{C_{eq} = C_1 + C_2}$$

n - identical $\boxed{C_{eq} = nC_0}$

$q = CV$

$$\boxed{q_1 = \frac{q\,C_1}{C_1 + C_2}}$$

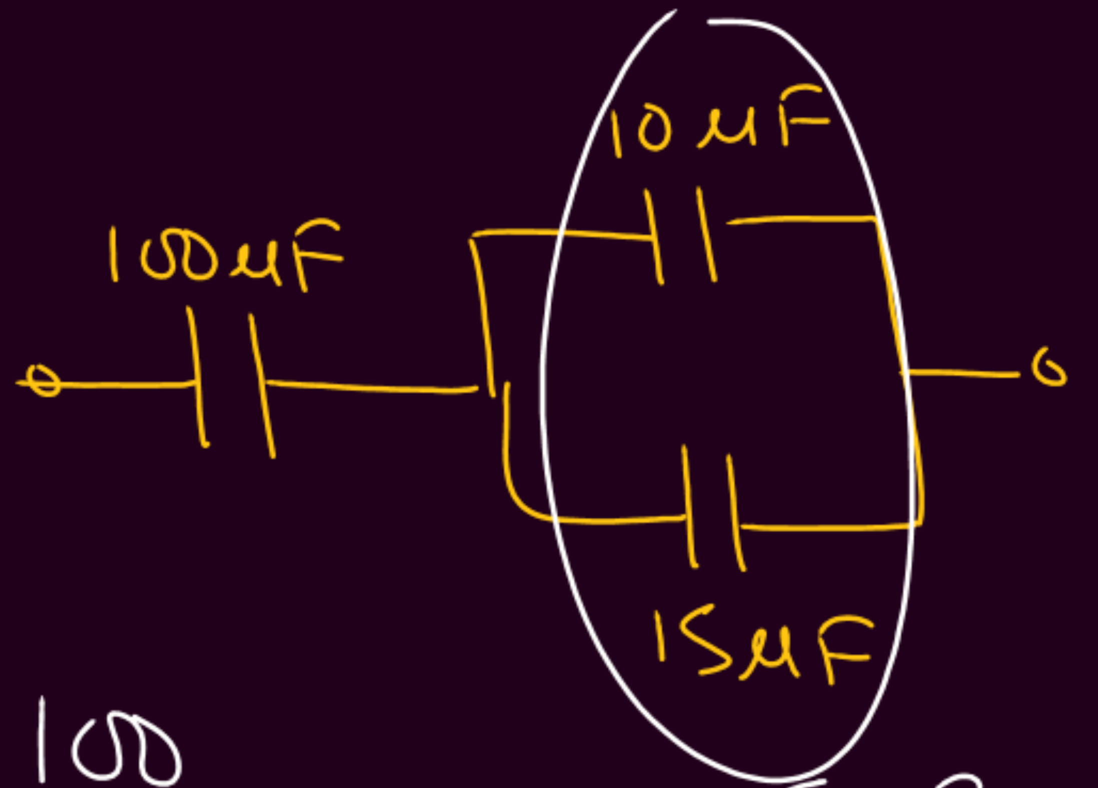
$$\boxed{q_2 = \frac{q\,C_2}{C_1 + C_2}}$$

Q.

$\frac{600}{3} = 200$

$\boxed{100\mu F}$

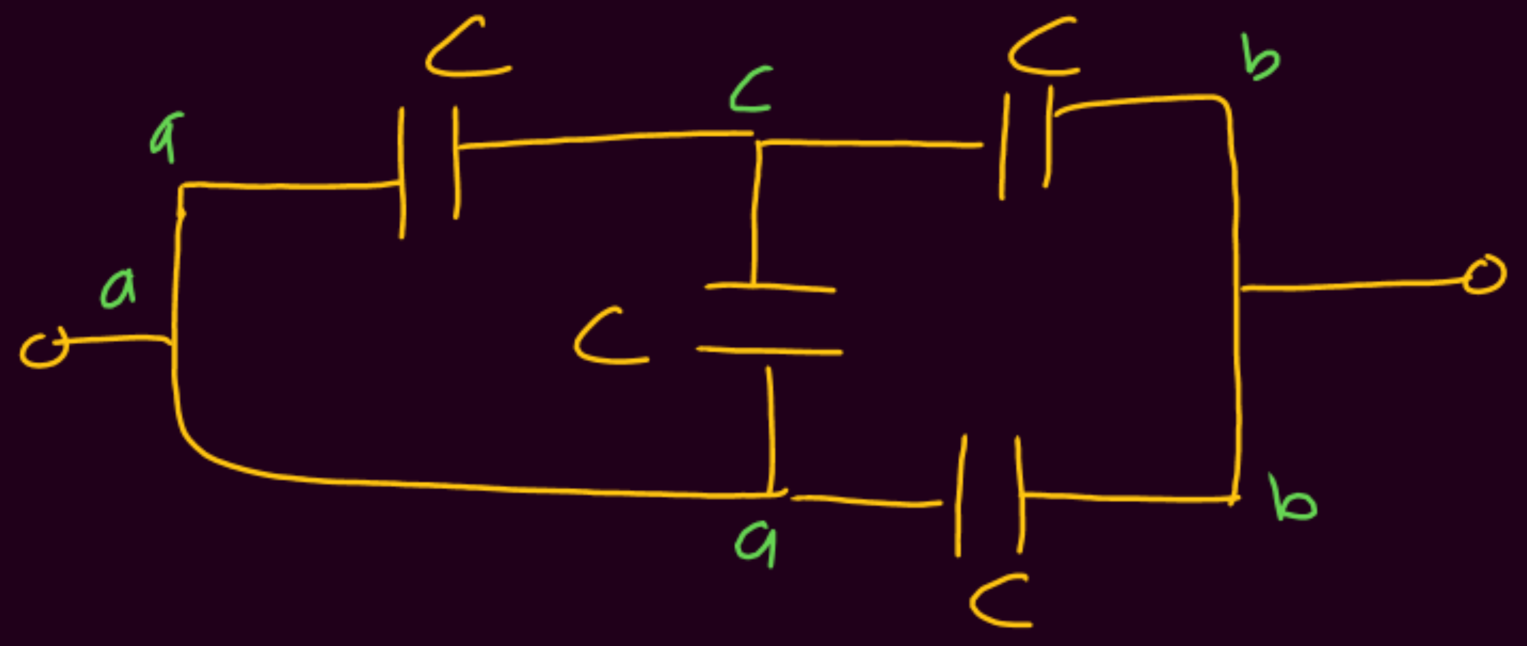
Q.

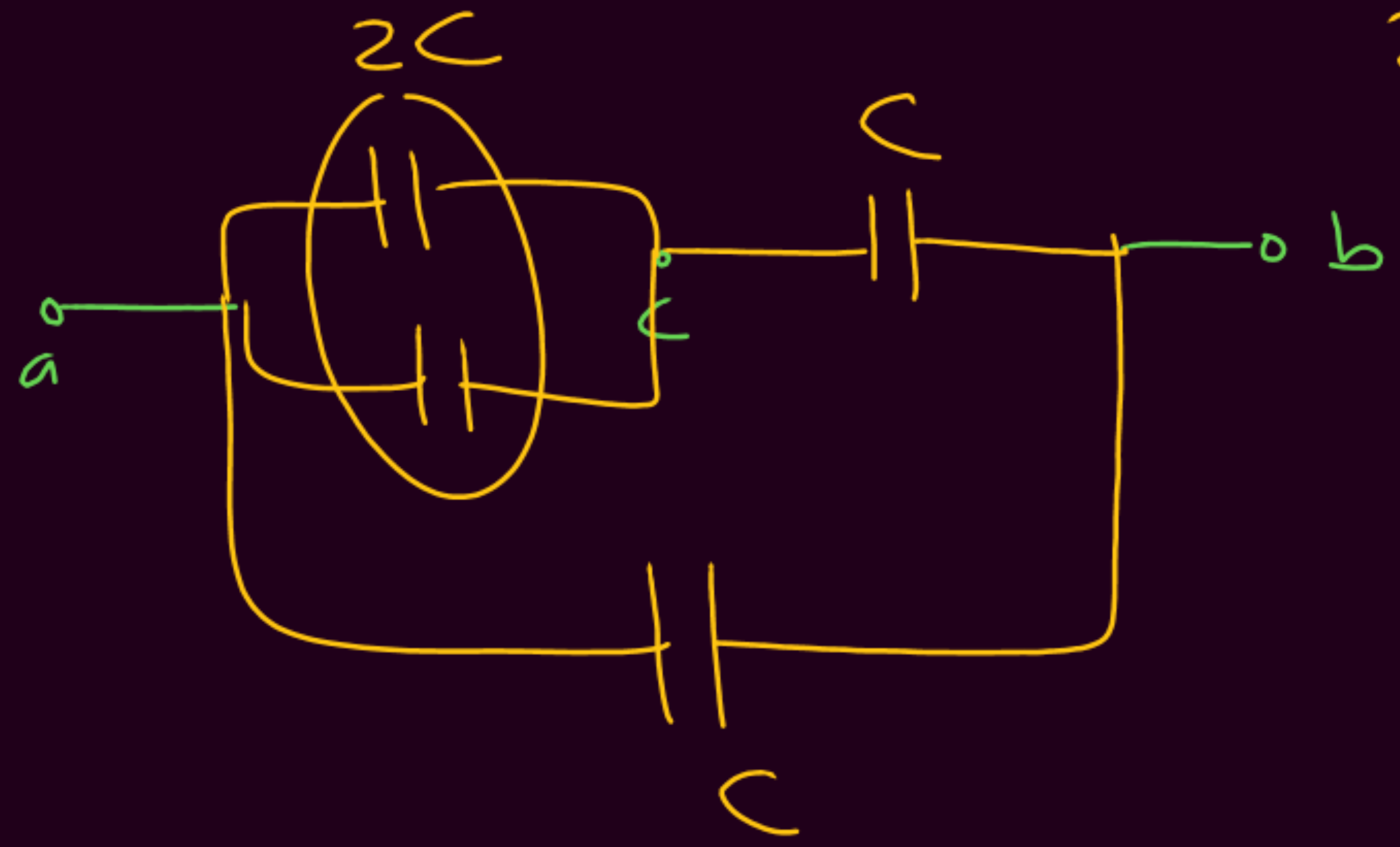


25μF

$\frac{100}{5}$

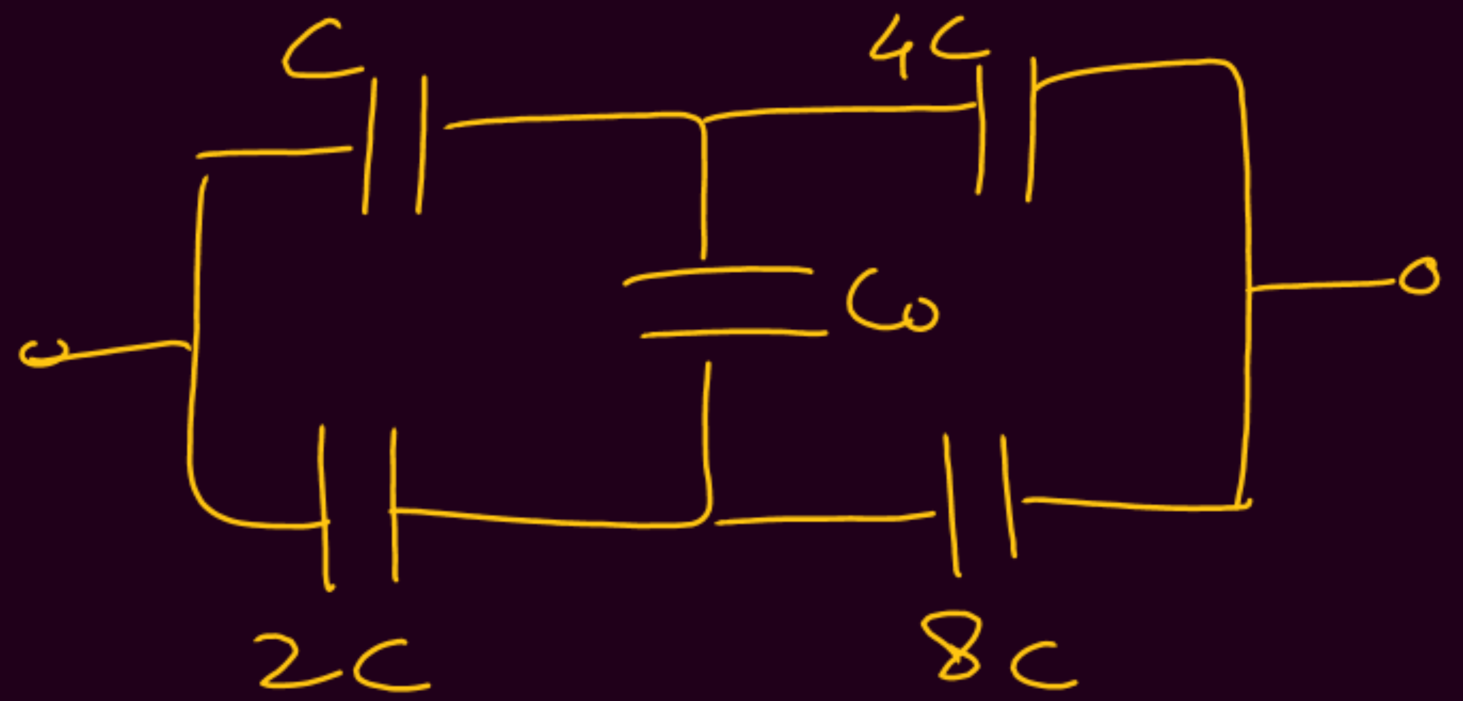
$= 20\mu F$



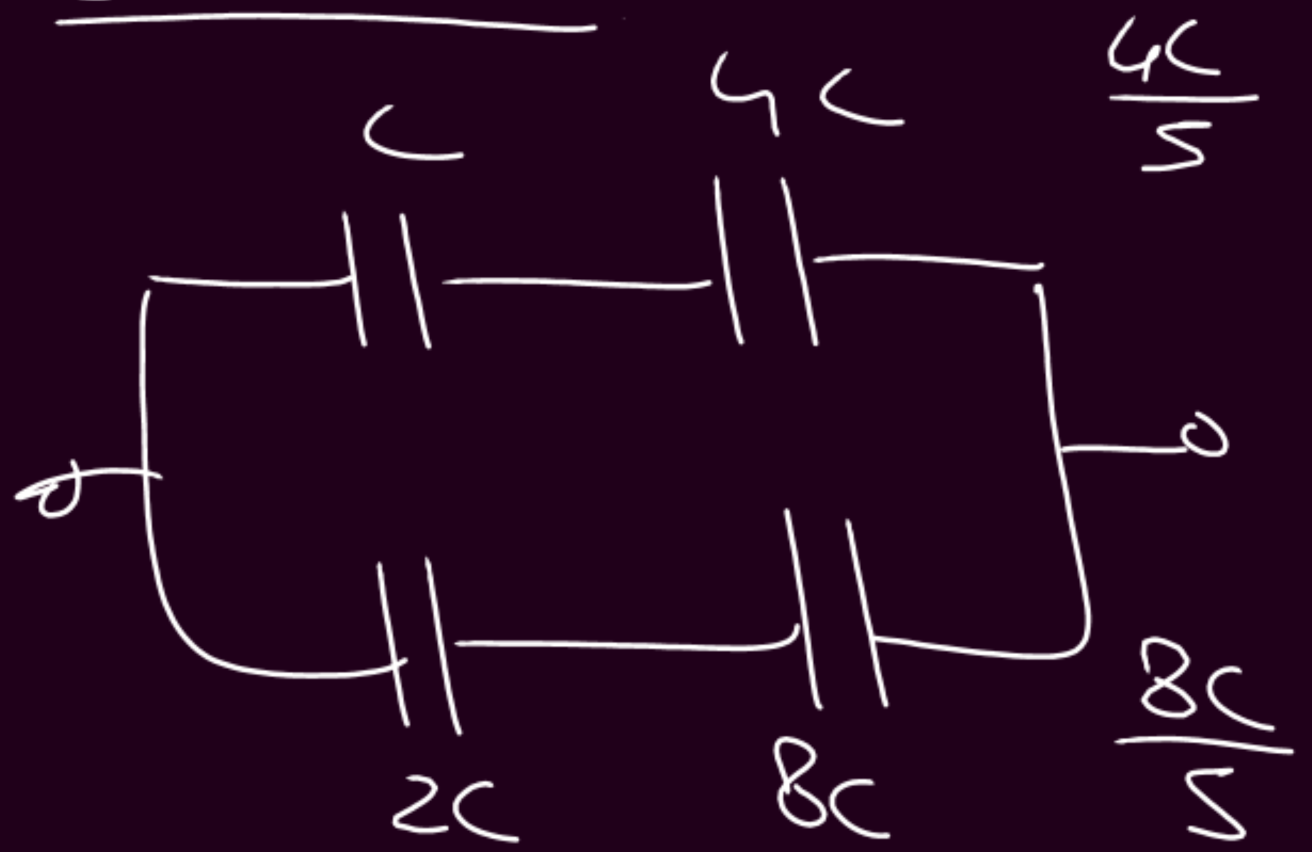


$\frac{2C}{3}$

$C_{eq} = \frac{2C}{3} + C = \frac{5C}{3}$



B - WSB



$$C_{eq} = \frac{4C}{5} + \frac{8C}{5} = \frac{12C}{5}$$

Q

$100\mu F$, $100V$

$\boxed{200\mu F \ , \ 100V}$

Unlike polarities together

1. Common P.D.
2. Final charges
3. Loss of Energy
4. Δq

① $V_c = \dfrac{(200\mu)(100) - (100\mu)(100)}{200\mu + 100\mu} = \dfrac{100}{3} V$

② $q_{1f} = C_1 V_c = (100\mu)\dfrac{100}{3} = \dfrac{10000}{3} \mu C$

$q_{2f} = C_2 V_c = (200\mu)\dfrac{100}{3} = \dfrac{20000}{3} \mu C$

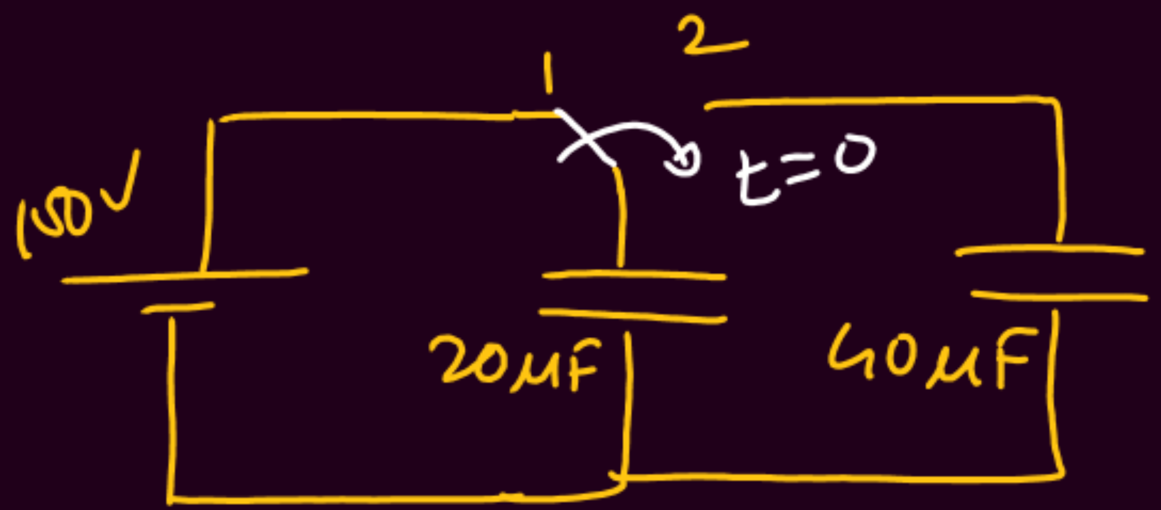
③ $\text{Loss} = \dfrac{1}{\cancel{2}}\left(\dfrac{\cancel{200\mu}}{3}\right)(2\cancel{00})(2\cancel{00}) = \dfrac{4}{3} J$

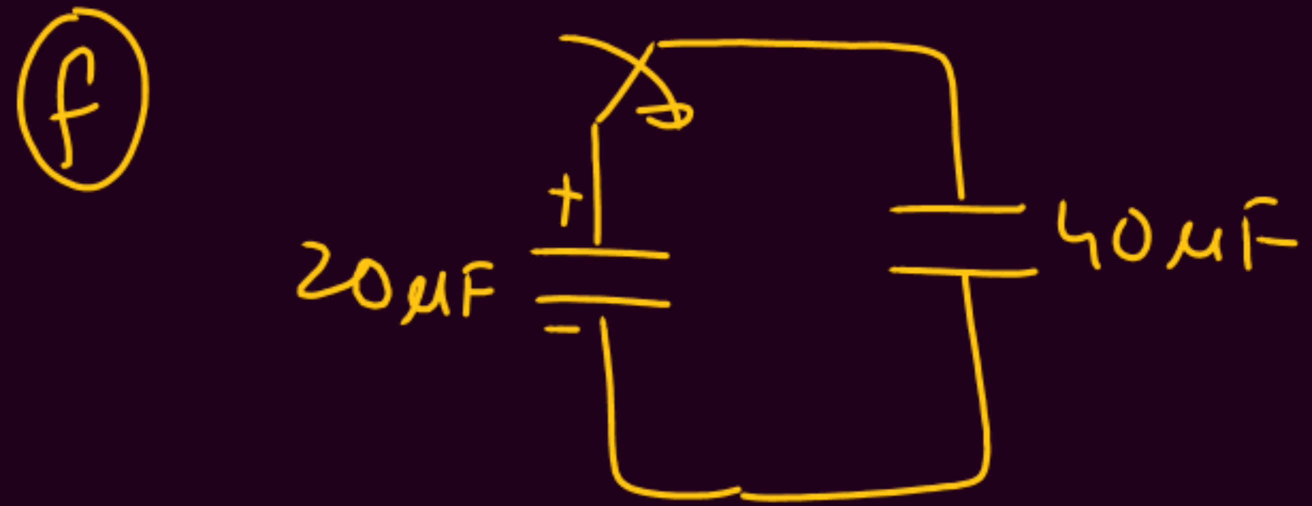
④ $200\mu F$

$q_i = (200\mu)(100) = 20000 \mu C$

$q_f = (200\mu)\left(\dfrac{100}{3}\right) = \dfrac{20000}{3} \mu C$

$$\Delta q = \frac{40000}{3} \mu C$$



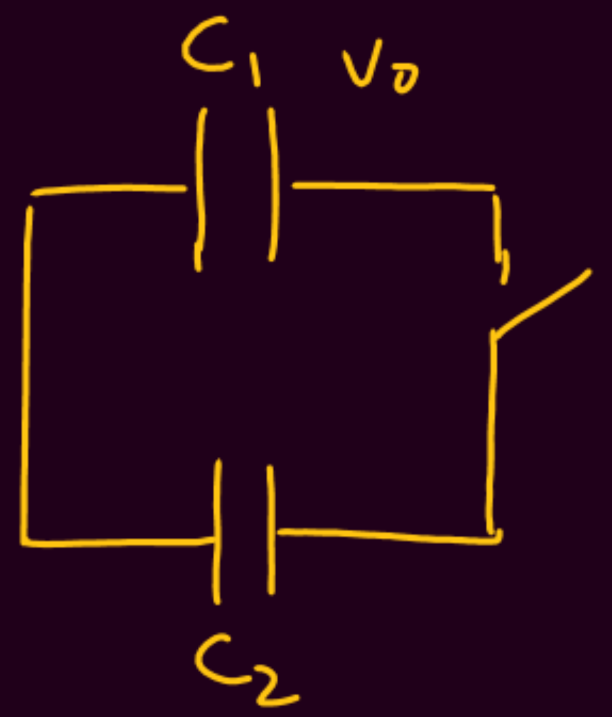


$$V_c = \frac{(20\mu)(100) + 0}{20\mu + 40\mu} = \frac{100}{3}\,V$$

$$q_{40} = (40\mu)\left(\frac{100}{3}\right) = \frac{4000}{3}\,\mu C$$

$$\text{fractional loss} = \frac{40\mu}{20\mu + 40\mu} = \frac{2}{3} = 0.67$$

$$= \underline{\underline{67\%}}$$



$$\text{fractional loss} = \frac{\text{Loss}}{\text{Initial}} = \frac{\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_0-0)^2}{\frac{1}{2}C_1V_0^2}$$

$$\boxed{\text{fractional loss} = \frac{C_2}{C_1+C_2}}$$

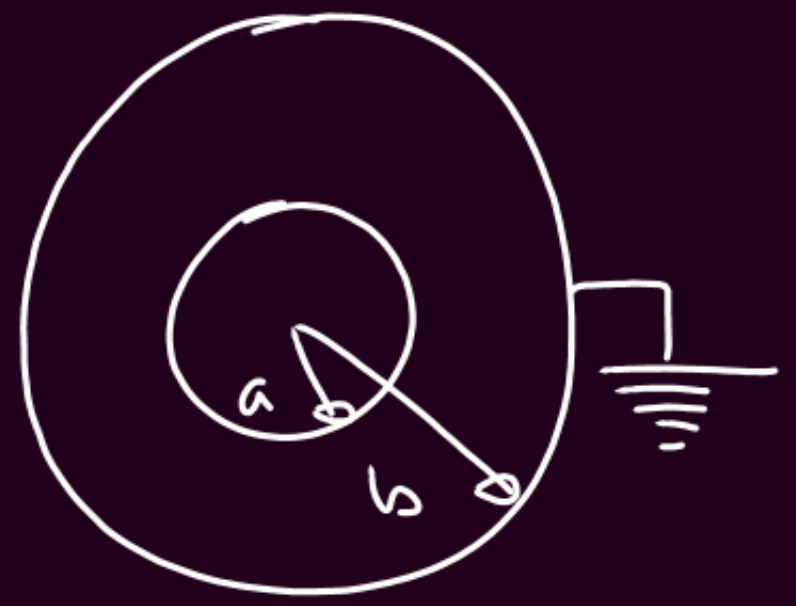
Spherical Capacitor

① $V = \dfrac{Q}{4\pi\epsilon_0 R}$

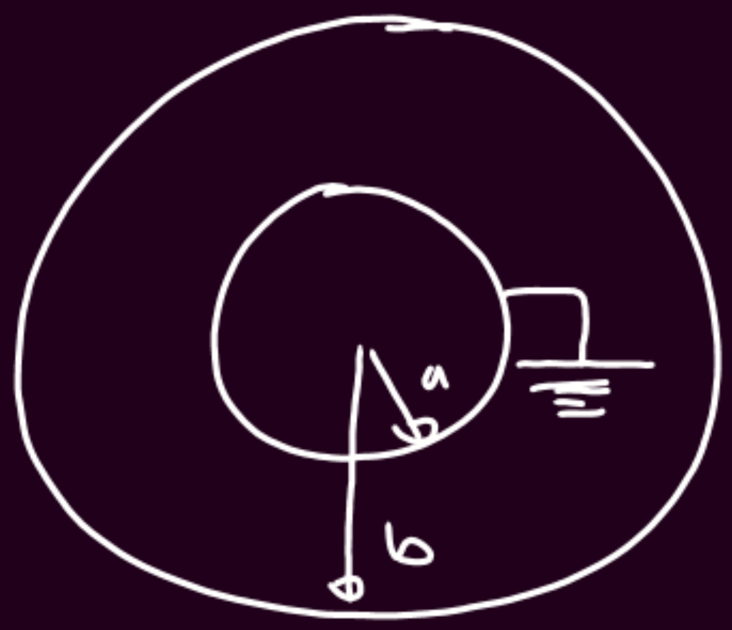
② $C = \dfrac{Q}{V}$

$$\boxed{C = 4\pi\epsilon_0 R}$$

Extra



$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$



$$C = \frac{4\pi\epsilon_0 b^2}{b-a}$$

$$C = 4\pi\epsilon_0 (a+b)$$

Capacitance of Earth

$$\frac{1}{4\pi\epsilon_0} = 9\times10^{9}$$

$R = 6400$ km

$$C = 4\pi\epsilon_0 R$$

$$C = \frac{6400\times10^{3}}{9\times10^{9}}$$

$$C = 7.11\times10^{-4}\ F$$

$$C = 711\ \mu F$$

ce to form a bigger drop

$C_0 = 4\pi\epsilon_0 r$

$C = 4\pi\epsilon_0 R = 4\pi\epsilon_0 (n^{1/3}) r$

$V_0 = \dfrac{q_0}{4\pi\epsilon_0 r}$

$V = \dfrac{n q_0}{4\pi\epsilon_0 (n^{1/3} r)}$

$(27)^{1/3} = 3$

$(64)^{1/3} = 4$

$(1000)^{1/3} = 10$

$(27)^{2/3} = (3)^2 = 9$

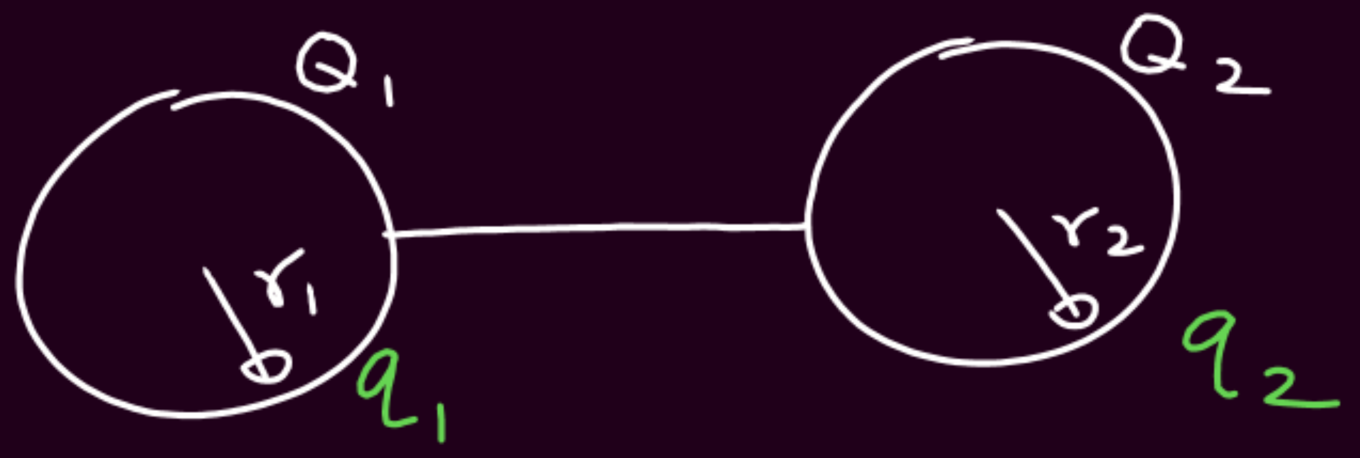
1000 drops (identical) each $10\mu F$ & $20V$ } Coalesce

$C_{new} = (1000)^{1/3} (10\mu F) = 100\mu F$

$V_{new} = (1000)^{2/3} (20) = \underline{2000V}$

	A	B	C
	20	−8	+4
1) A & C are touched & separated	12	−8	12
2) B & C —— " ——	12	2	2
3) A & B —— " ——	7	7	2

(2)



$$\sigma_1 = \frac{q_1}{4\pi r_1^2}$$

$$\frac{\sigma_1 r_1^2}{r_1} = \frac{\sigma_2 r_2^2}{r_2}$$

(1) Finally $$\frac{q_1}{4\pi\epsilon_0 r_1} = \frac{q_2}{4\pi\epsilon_0 r_2}$$

Same Potential

$$\boxed{\frac{q_1}{r_1} = \frac{q_2}{r_2}}$$

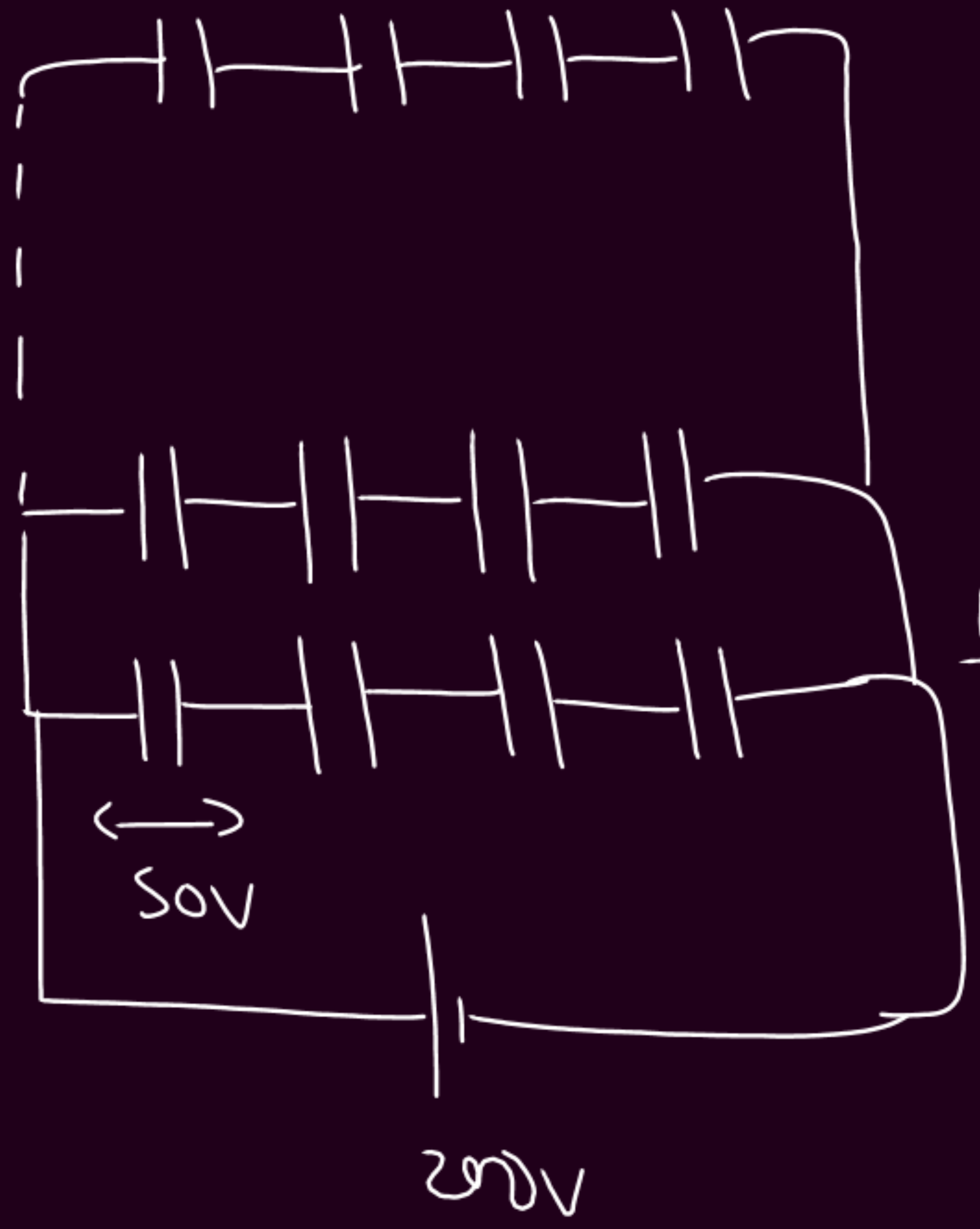
$$\boxed{\sigma_1 r_1 = \sigma_2 r_2}$$

(2) Conservation of Charge

$$\boxed{q_1 + q_2 = Q_1 + Q_2}$$

Q [100 μF, 50V] withstand

[200 μF, 200V] req



$n(25) = 200\,\mu F$

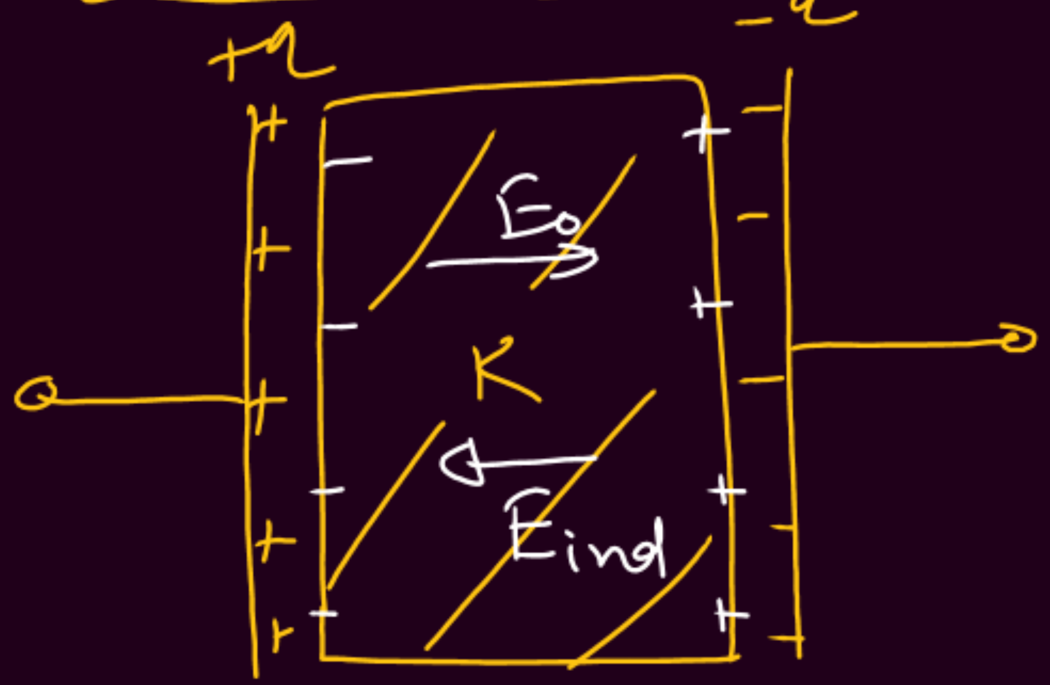
$\boxed{n = 8 \text{ rows}}$

4 in a row

Total $\boxed{32}$

$\frac{100\mu}{4} = 25\,\mu F$

Dielectric



① Field in Air $= E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$

② Field in med. $= \boxed{E_{net} = \frac{E_0}{K}}$

$$\vec{E}_{net} = \vec{E}_0 + \vec{E}_{ind}$$

$$\frac{E_0}{K} = E_0 + E_{ind}$$

$$E_{ind} = -E_0 + \frac{E_0}{K}$$

$$\boxed{\vec{E}_{ind} = -\vec{E}_0\left(1 - \frac{1}{K}\right)}$$

$$\boxed{q_{ind} = -q\left(1 - \frac{1}{K}\right)}$$

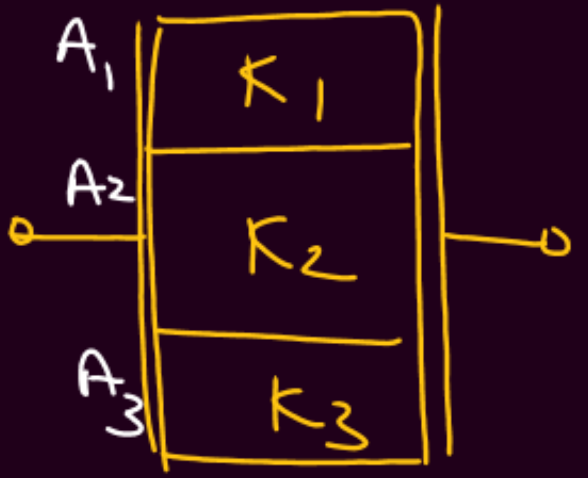
(1)

$$C = \frac{\epsilon_0 A}{d}$$

(2)

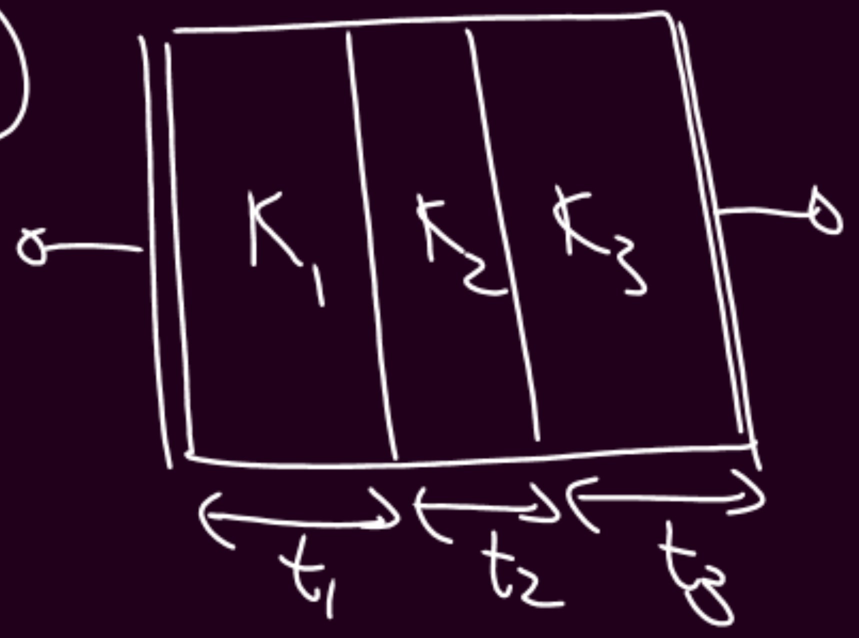
$$C = \frac{\epsilon_0 K A}{d}$$

(3)



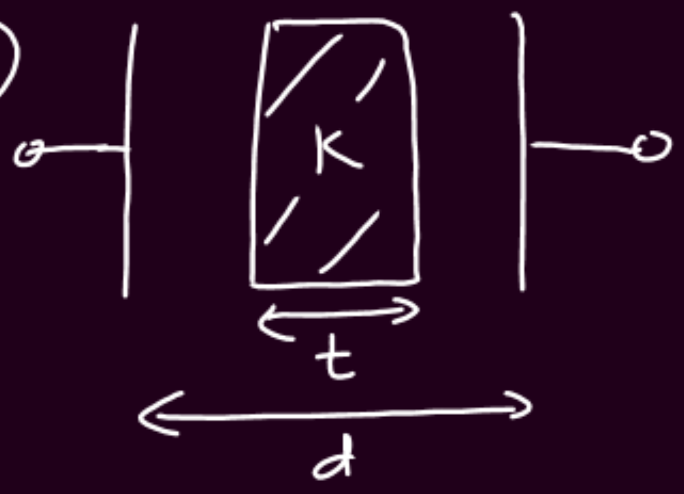
$$C = \frac{\epsilon_0 (K_1 A_1 + K_2 A_2 + K_3 A_3)}{d}$$

(4)



$$\boxed{C = \frac{\epsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}\right)}}$$

(5)



$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

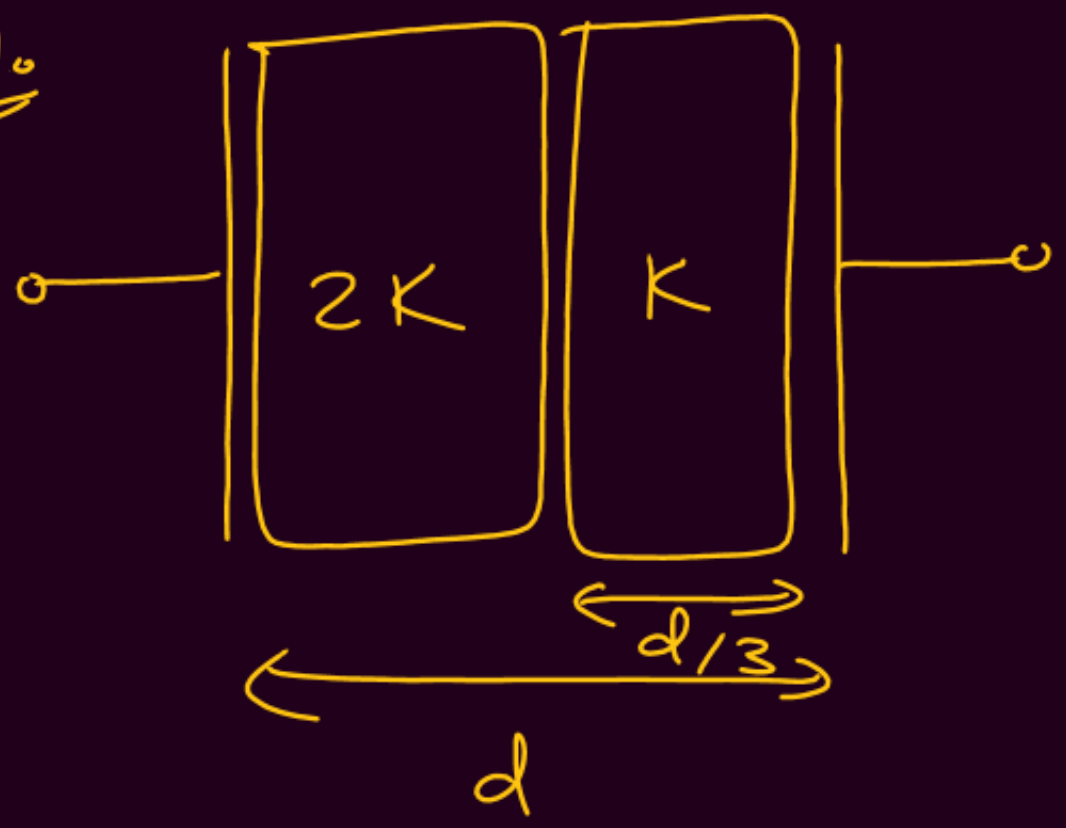
(6) Thick metal Plate

$$C = \frac{\epsilon_0 A}{d-t}$$

(7) Thin metal plate

$$C = \frac{\epsilon_0 A}{d}$$

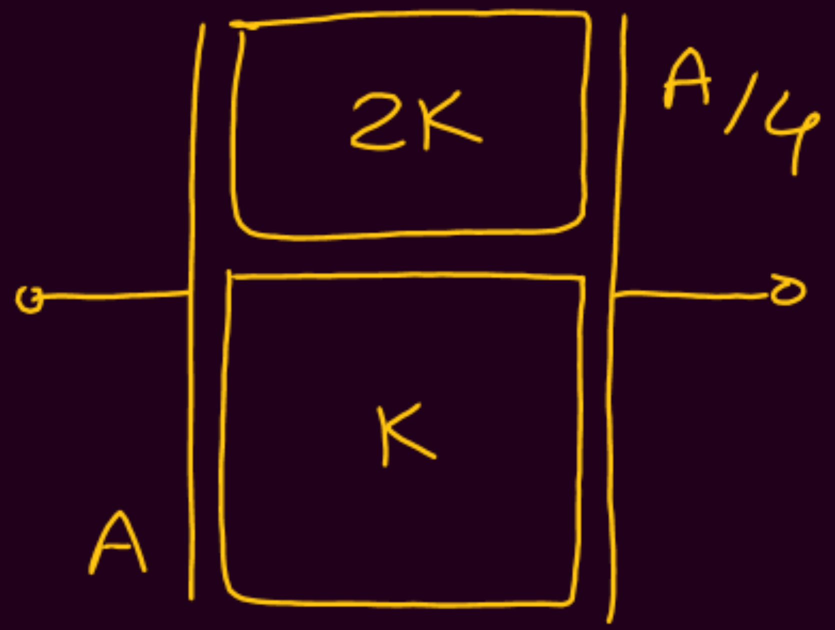
Q.



$$C = \frac{\epsilon_0 A}{\frac{\cancel{2}d}{3(\cancel{2}K)} + \frac{d}{3(K)}}$$

$$C = \frac{3\epsilon_0 K A}{2d}$$

Q.



$$C = \frac{\epsilon_0\left[2K\left(\frac{A}{4}\right) + K\left(\frac{3A}{4}\right)\right]}{d}$$

$$C = \frac{5\epsilon_0 K A}{4d}$$

Q:

$$4\mu F = \frac{\epsilon_0 A}{5}$$

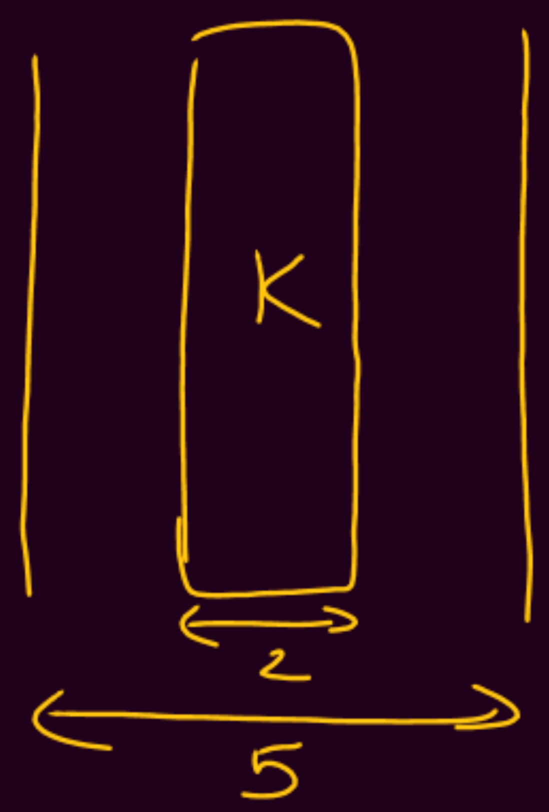
$$6\mu F = \frac{\epsilon_0 A}{(5-2) + \frac{2}{K}}$$

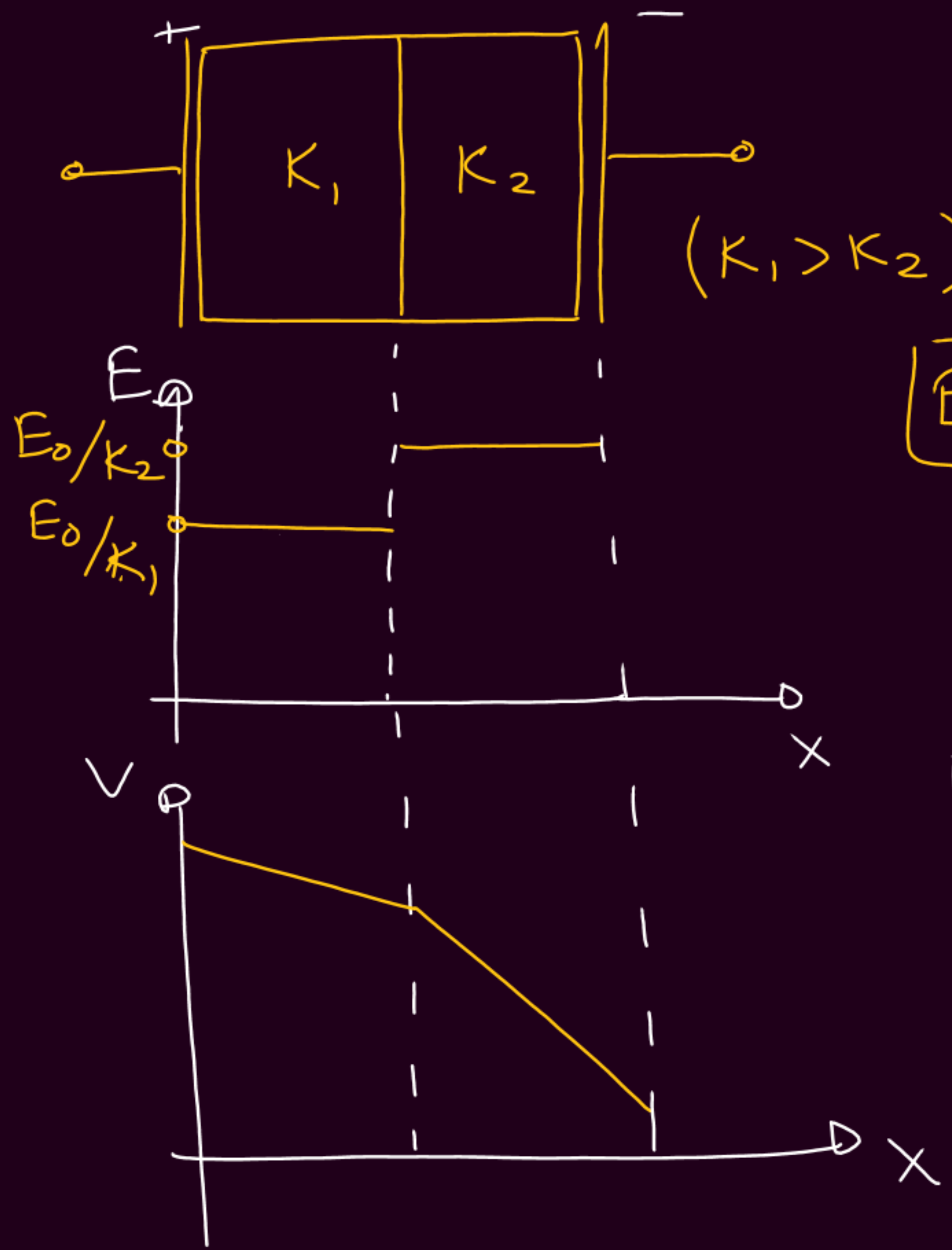
$$6 = \frac{20}{3 + \frac{2}{K}}$$

$$18 + \frac{12}{K} = 20$$

$$\frac{12}{K} = 2$$

$$\boxed{K = 6}$$



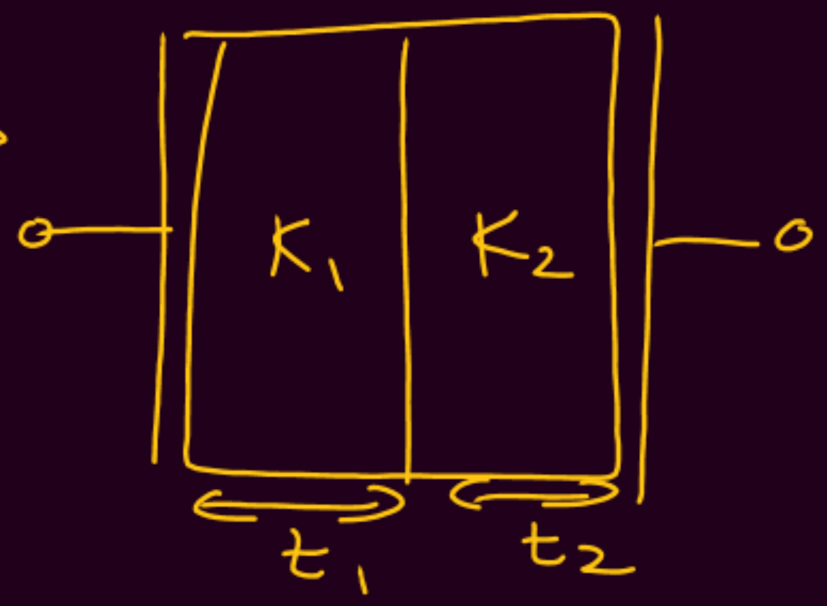


$$\boxed{E = \frac{E_0}{K}}$$

$$\left[E = -\frac{dV}{dx}\right]$$

Keq

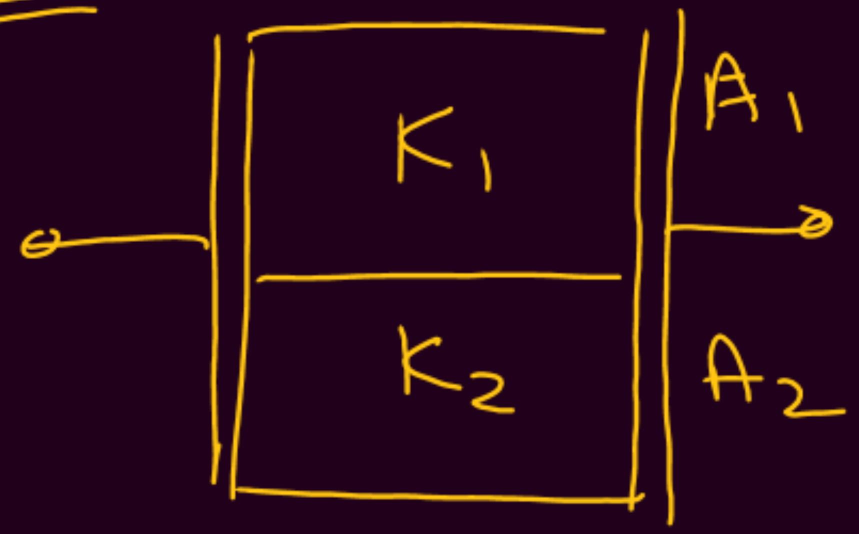
① Series



$$\frac{\epsilon_0 K_{eq} A}{t_1 + t_2} = \frac{\epsilon_0 A}{\frac{t_1}{K_1} + \frac{t_2}{K_2}}$$

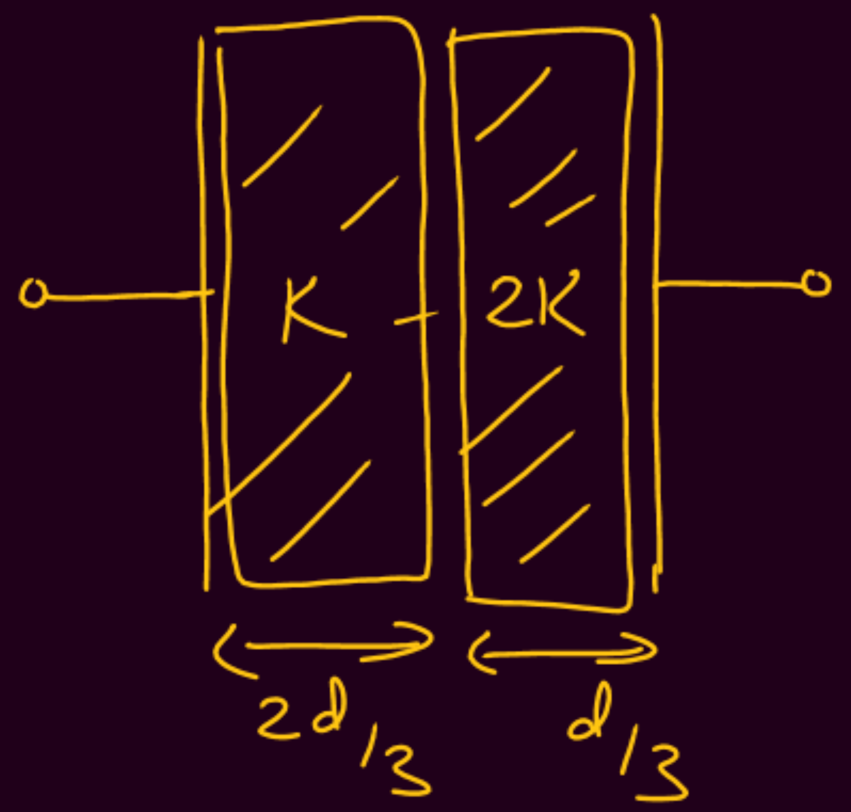
$$\boxed{K_{eq} = \frac{t_1 + t_2}{\frac{t_1}{K_1} + \frac{t_2}{K_2}}}$$

② Parallel



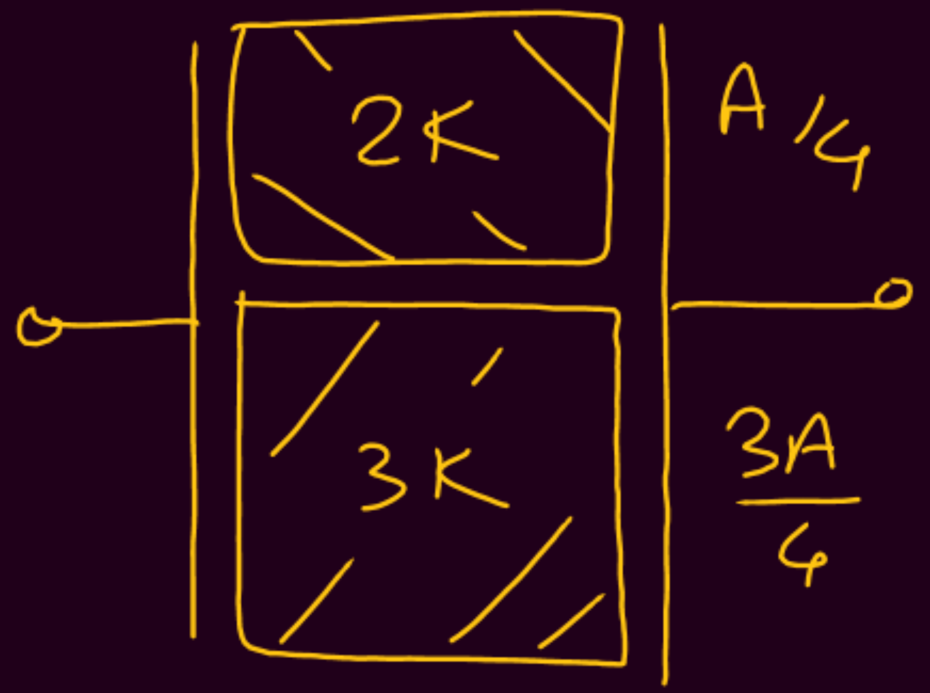
$$\frac{\epsilon_0 K_{eq}(A_1 + A_2)}{d} = \frac{\epsilon_0 (K_1 A_1 + K_2 A_2)}{d}$$

$$\boxed{K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}}$$



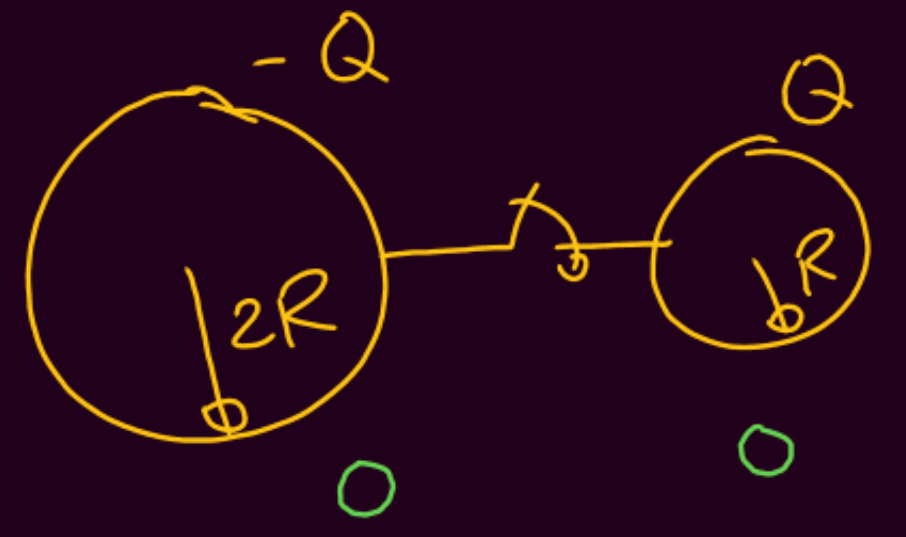
$$\frac{\epsilon_0 K_{eq} A}{d} = \frac{\epsilon_0 A}{\frac{2d/3}{K} + \frac{d/3}{2K}}$$

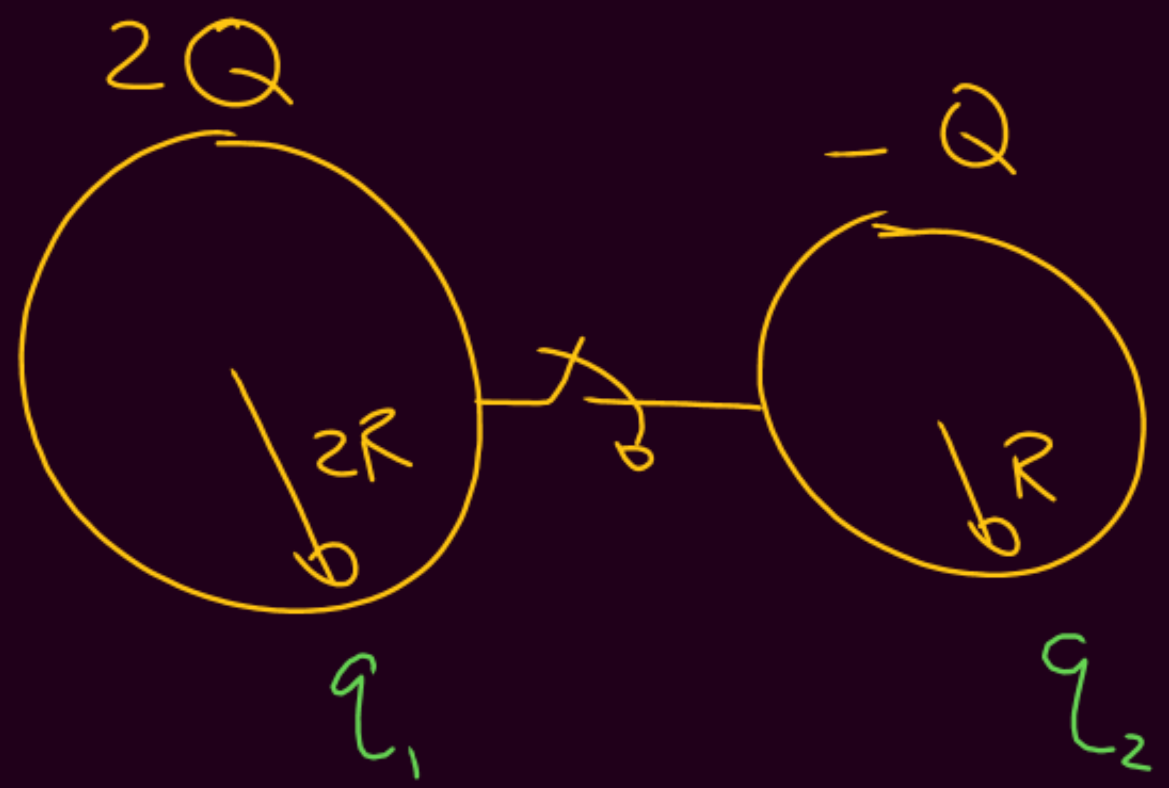
$$K_{eq} = \frac{K}{\frac{2}{3} + \frac{1}{6}} = \frac{6K}{5}$$



$$\frac{\epsilon_0 K_{eq} A}{d} = \frac{\epsilon_0 \left(2K \frac{A}{4} + 3K \frac{3A}{4}\right)}{d}$$

$$\boxed{K_{eq} = \frac{11}{4} K}$$





① $$\frac{q_1}{2R} = \frac{q_2}{R}$$

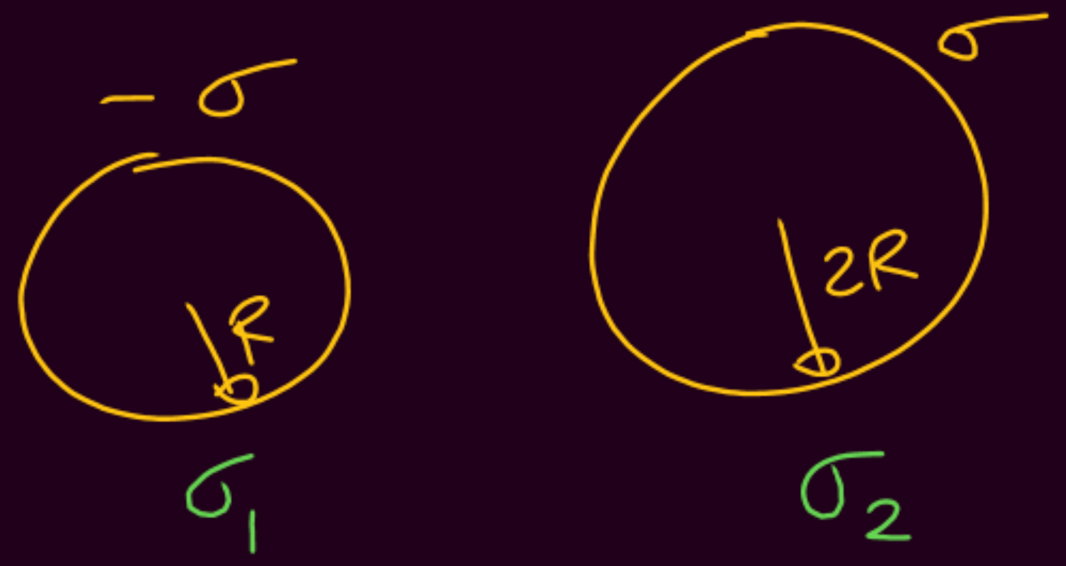
$$\boxed{q_1 = 2q_2}$$

② $$q_1 + q_2 = 2Q + (-Q)$$

$$2q_2 + q_2 = Q$$

$$\boxed{q_2 = \frac{Q}{3}}$$

$$\boxed{q_1 = \frac{2Q}{3}}$$



(1) $$\sigma_1 R = \sigma_2 (2R)$$

$$\boxed{\sigma_1 = 2\sigma_2}$$

(2) $$\sigma_1 (4\pi R^2) + \sigma_2 \, 4\pi (2R)^2 = -\sigma (4\pi R^2) + \sigma \, 4\pi (2R)^2$$

$$\sigma_1 + 4\sigma_2 = -\sigma + 4\sigma$$

$$2\sigma_2 + 4\sigma_2 = 3\sigma$$

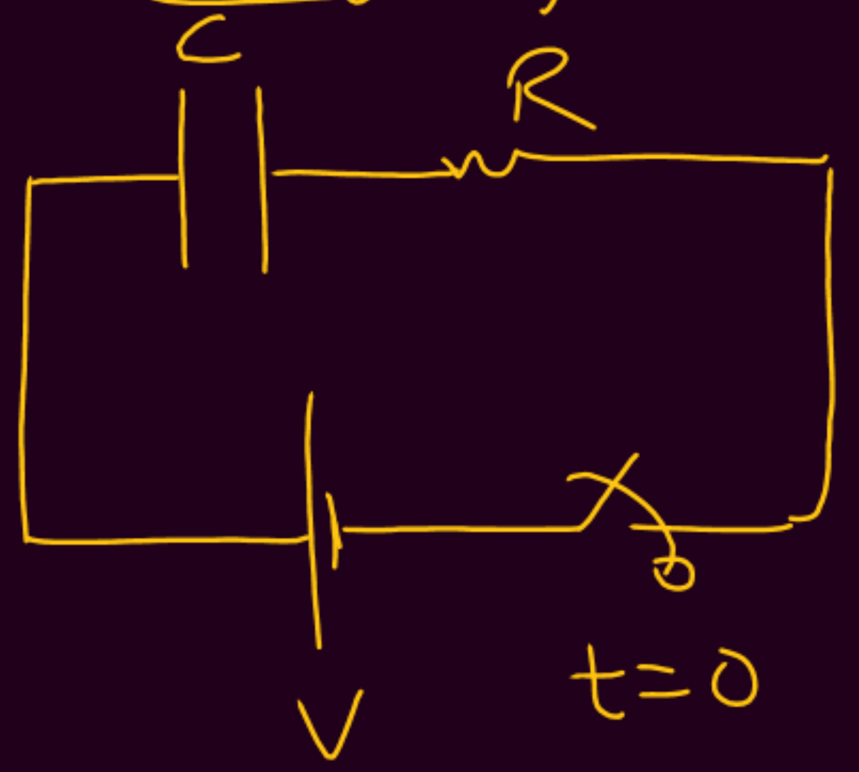
$$6\sigma_2 = 3\sigma$$

$$\boxed{\sigma_2 = \frac{\sigma}{2}}$$

$$\boxed{\sigma_1 = \sigma}$$

R-C Circuit (Extra)

(I) Charging of Capacitor

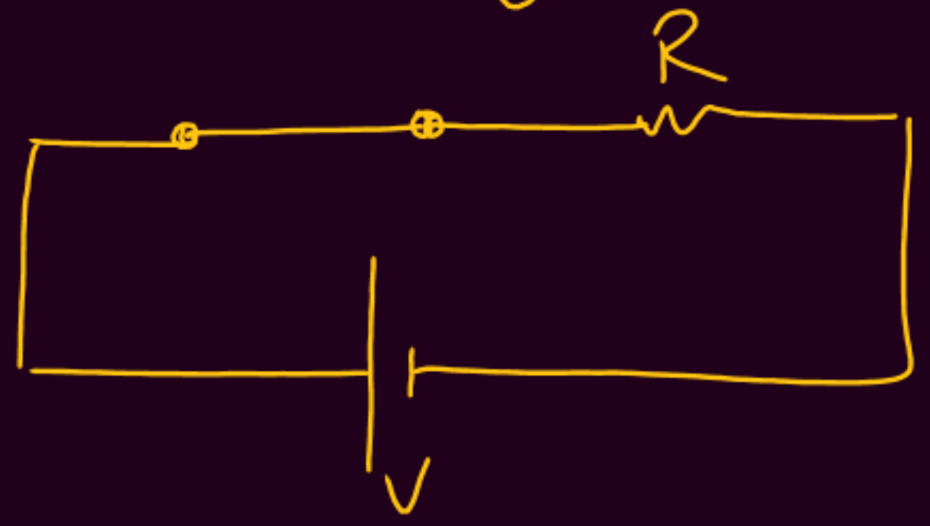


(1) $V = \left(\frac{q}{C}\right) + iR$

(2) Time Constant $\boxed{\tau = RC}$

(5) $\frac{1}{2}CV^2 = \text{Heat}$

(3) Initially $(t=0)$ $\boxed{\text{Cap.} \rightarrow SC}$



$q = 0$

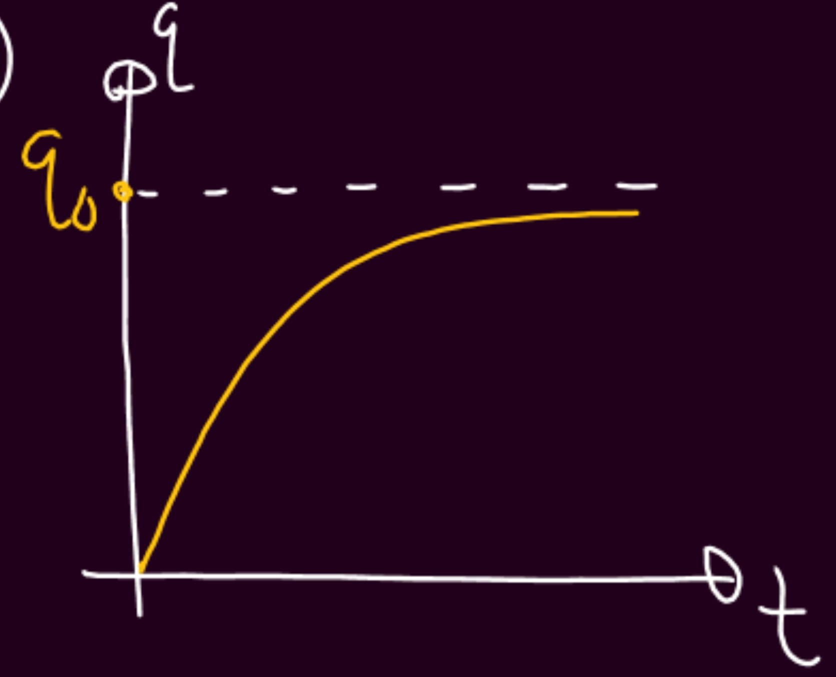
$i_{max} = \frac{V}{R}$

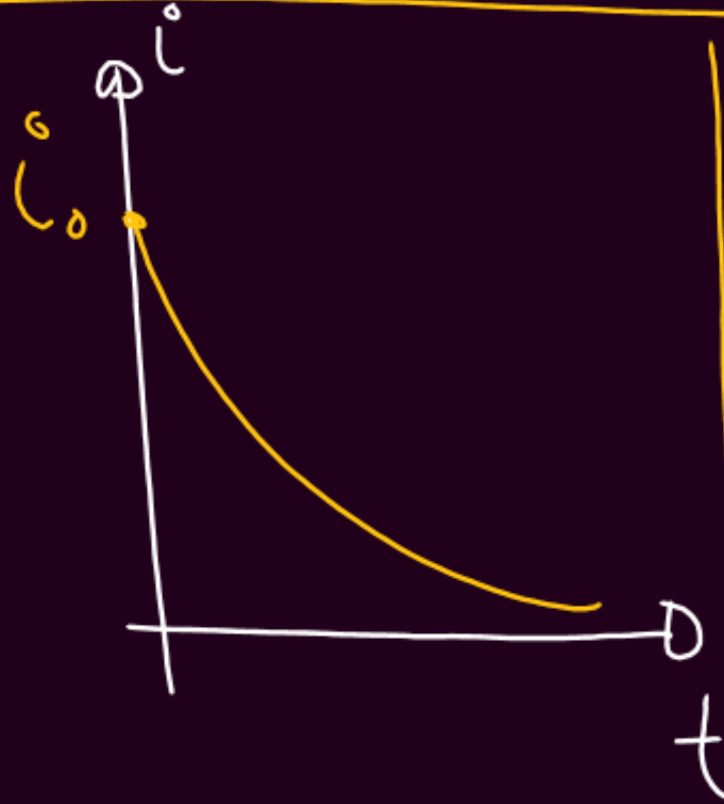
(4) Finally $(t \rightarrow \infty)$ $\boxed{\text{Cap.} \rightarrow OC}$

$q_{max} = CV$

$i = 0$

(6)



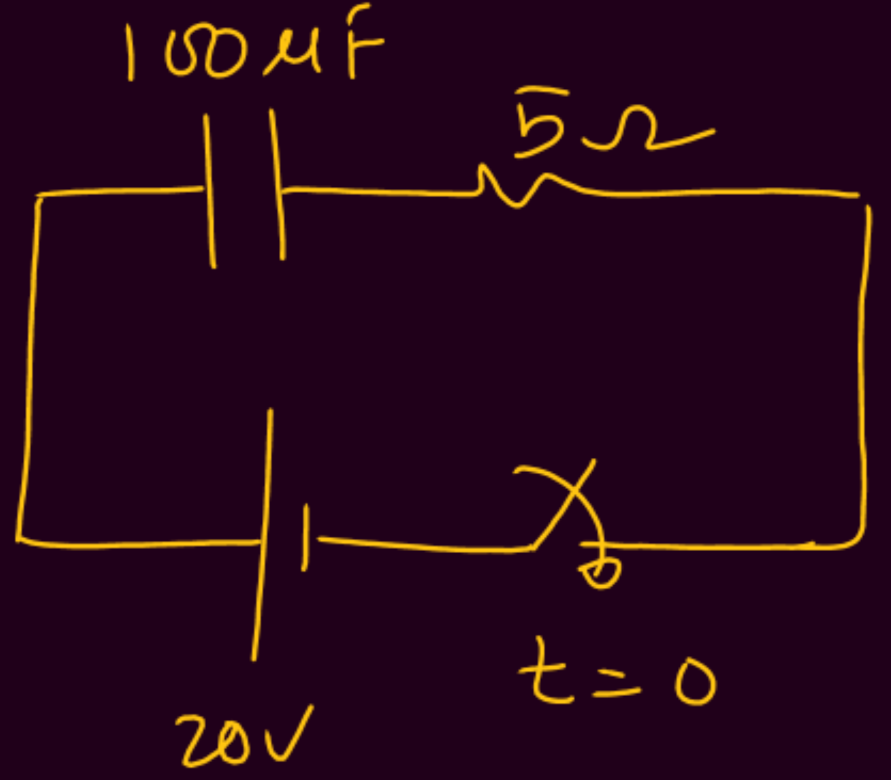


(7) Half Life

$t_{1/2} = \tau \ln(2)$

$\boxed{t_{1/2} = (0.7)\tau}$

Q.



① $\tau = 5(100\mu) = 500\,\mu s$
$= 0.5\ ms$

② $i_{max} = \frac{V}{R} = \frac{20}{5} = 4A$

③ $q_{max} = CV = (100\mu)(20)$
$= 2000\,\mu C$

④ $q = ?$ when $i = 3A$

KVL
$V = \frac{q}{C} + iR$

$20 = \frac{q}{100\mu} + (3)(5)$

$5 = \frac{q}{100\mu}$

$\boxed{q = 500\,\mu C}$

⑤ $i = ?$ when $q = 1200\,\mu C$

$20 = \frac{12\cancel{00}\cancel{\mu}}{1\cancel{00}\cancel{\mu}} + i(5)$

$8 = i(5)$

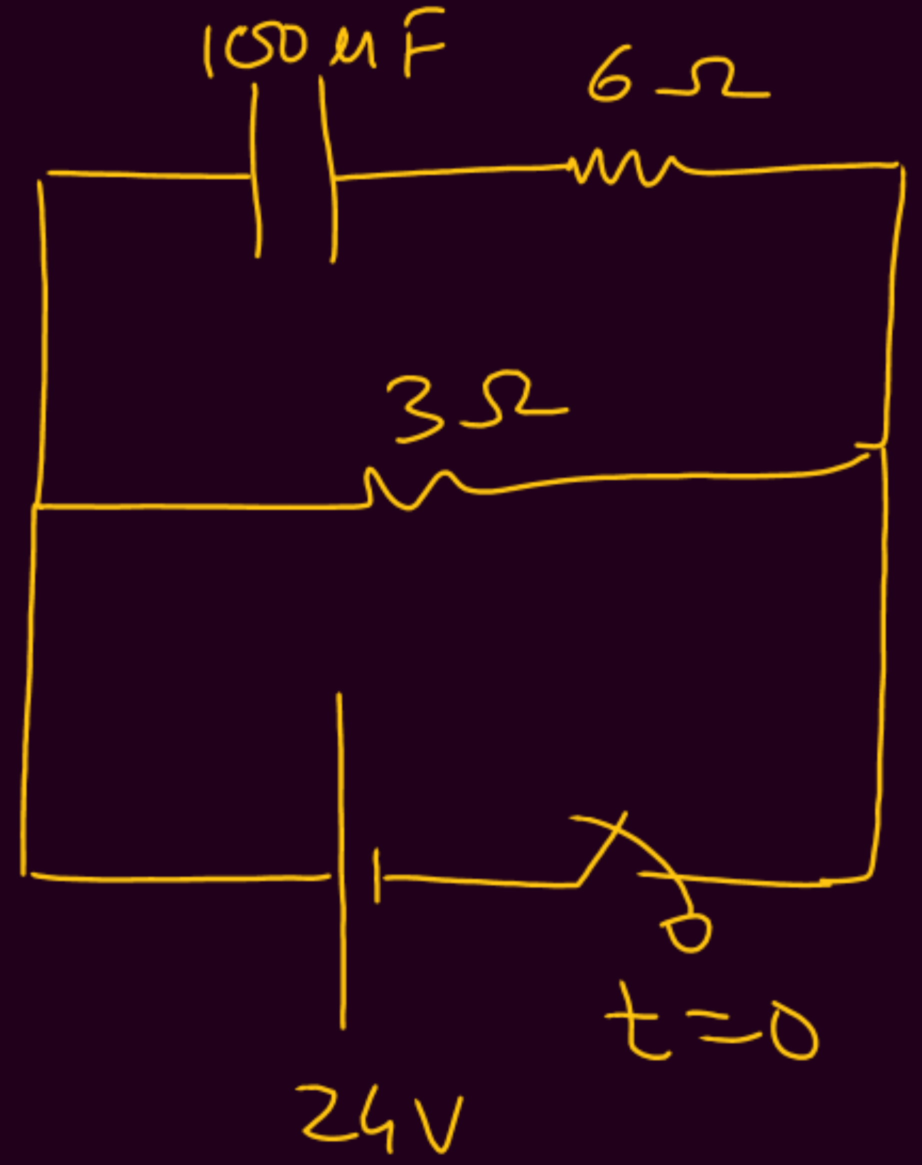
$i = \frac{8}{5} = 1.6A$

⑥ $\text{Heat} = \frac{1}{2}CV^2$

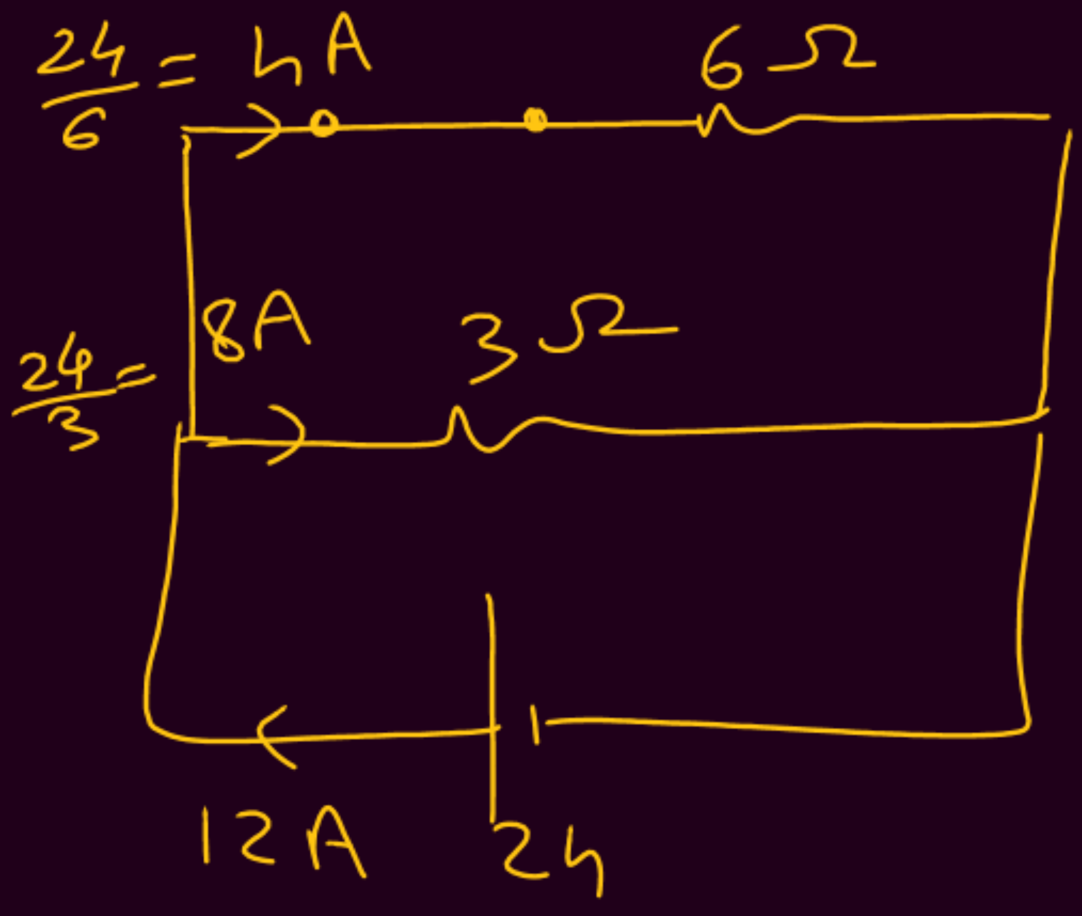
$= \frac{1}{2}(100\mu)(20)^2$

$= 2 \times 10^{-2} J$

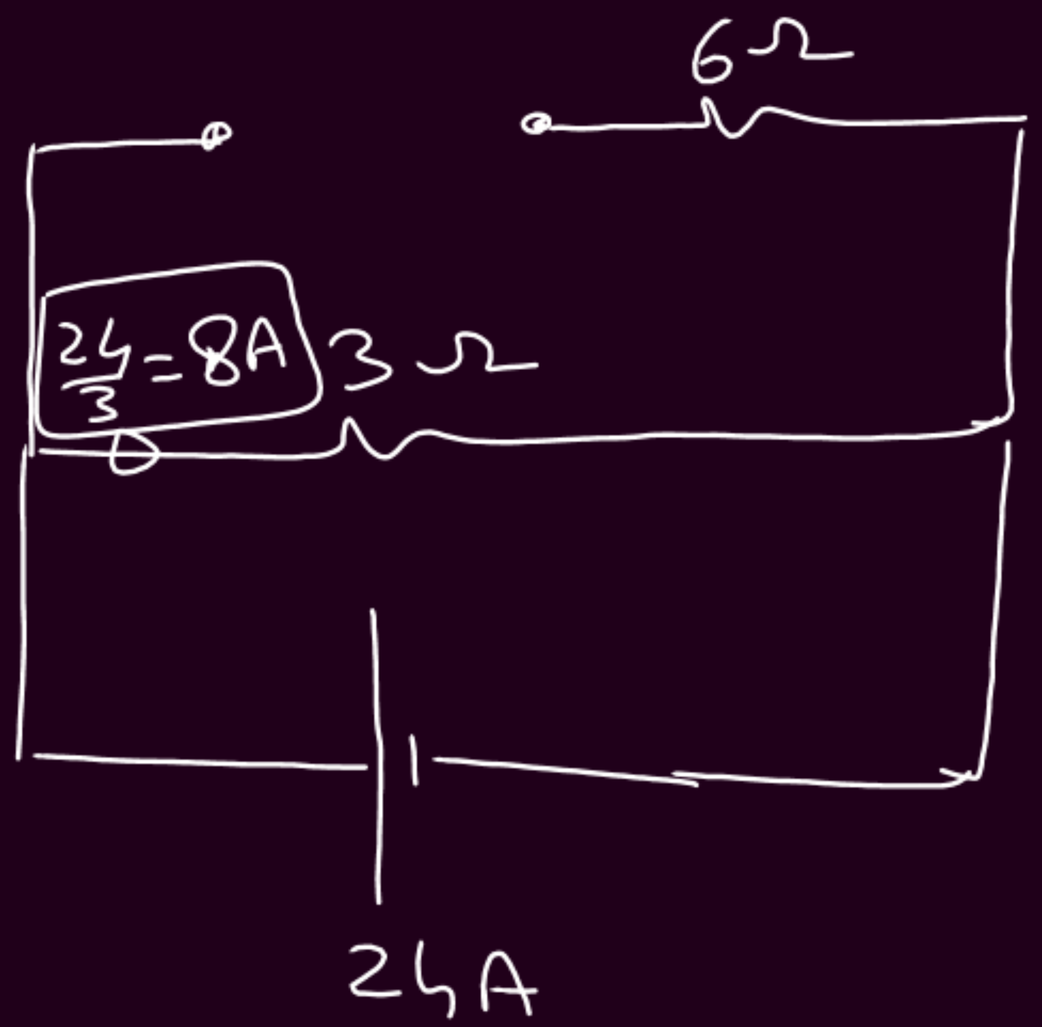
$= 0.02 J$

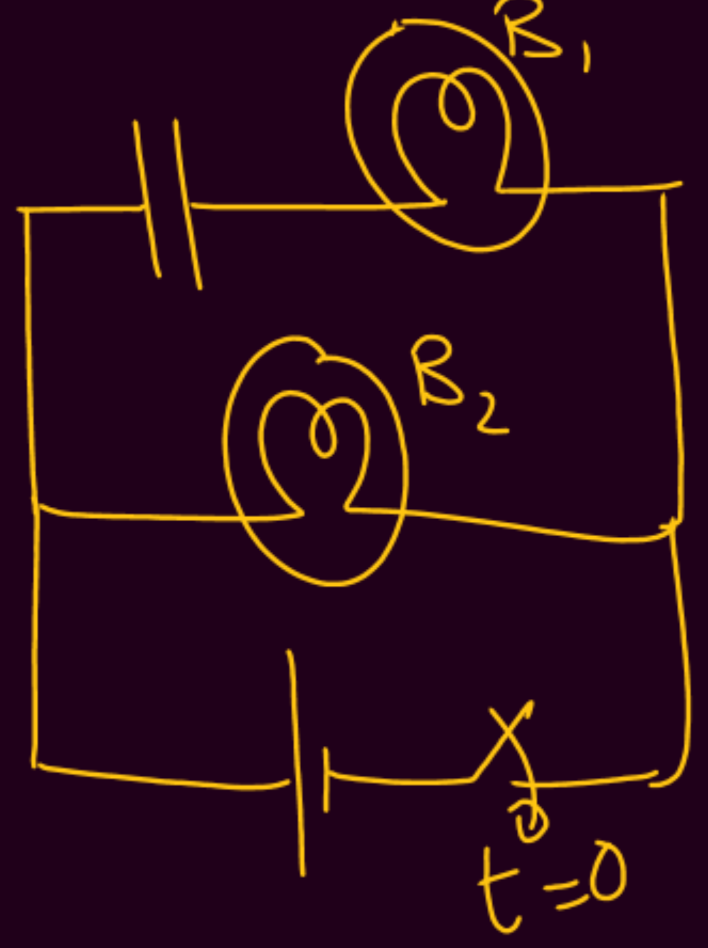


Initial $(t = 0)$



Final $(t \to \infty)$

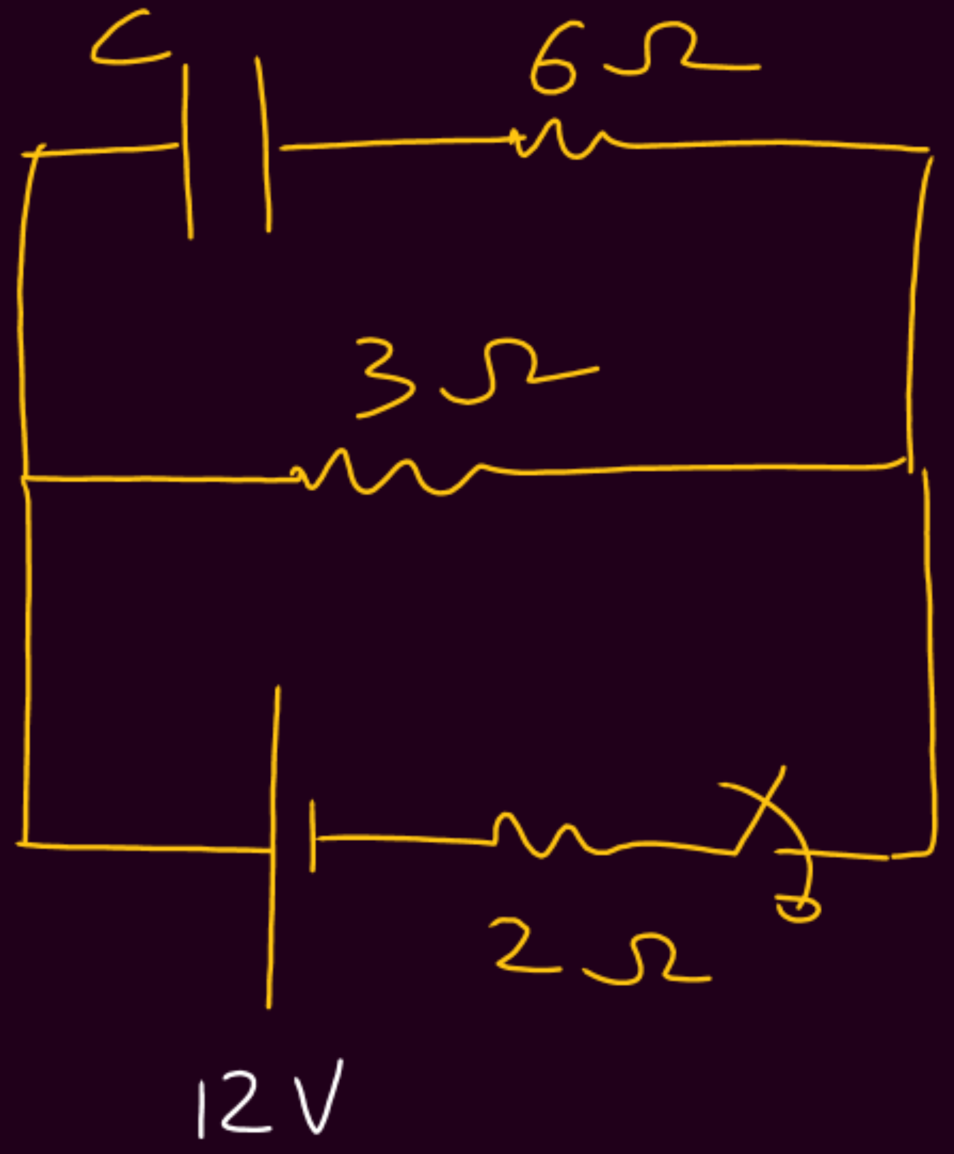




identical Bulbs

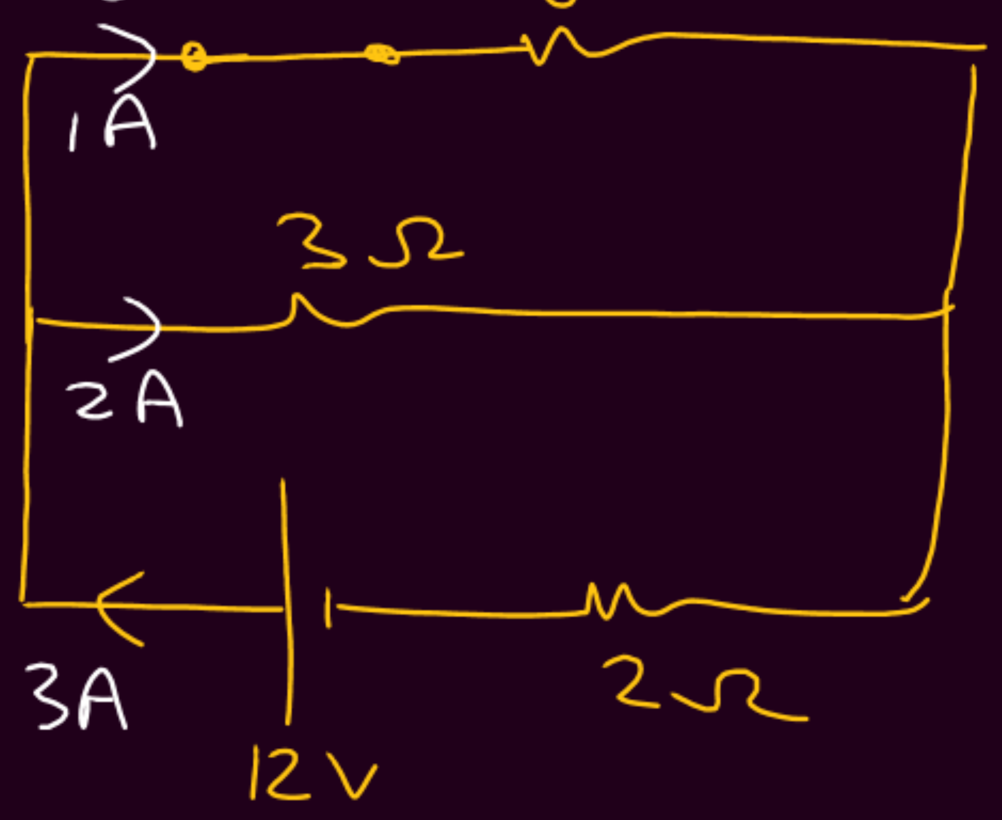
B_2 = const.

$B_1 \to \downarrow$ cont. & 0



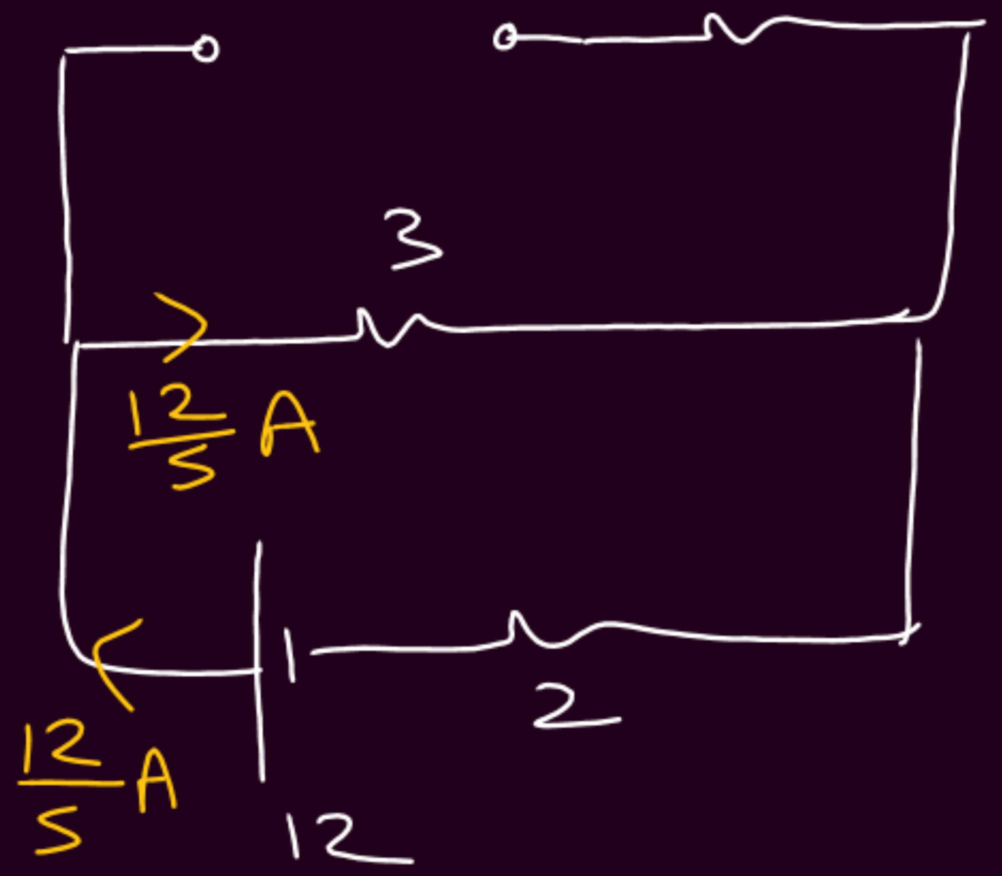
Initially ($t=0$)

$$3 \times \frac{3}{6+3}$$

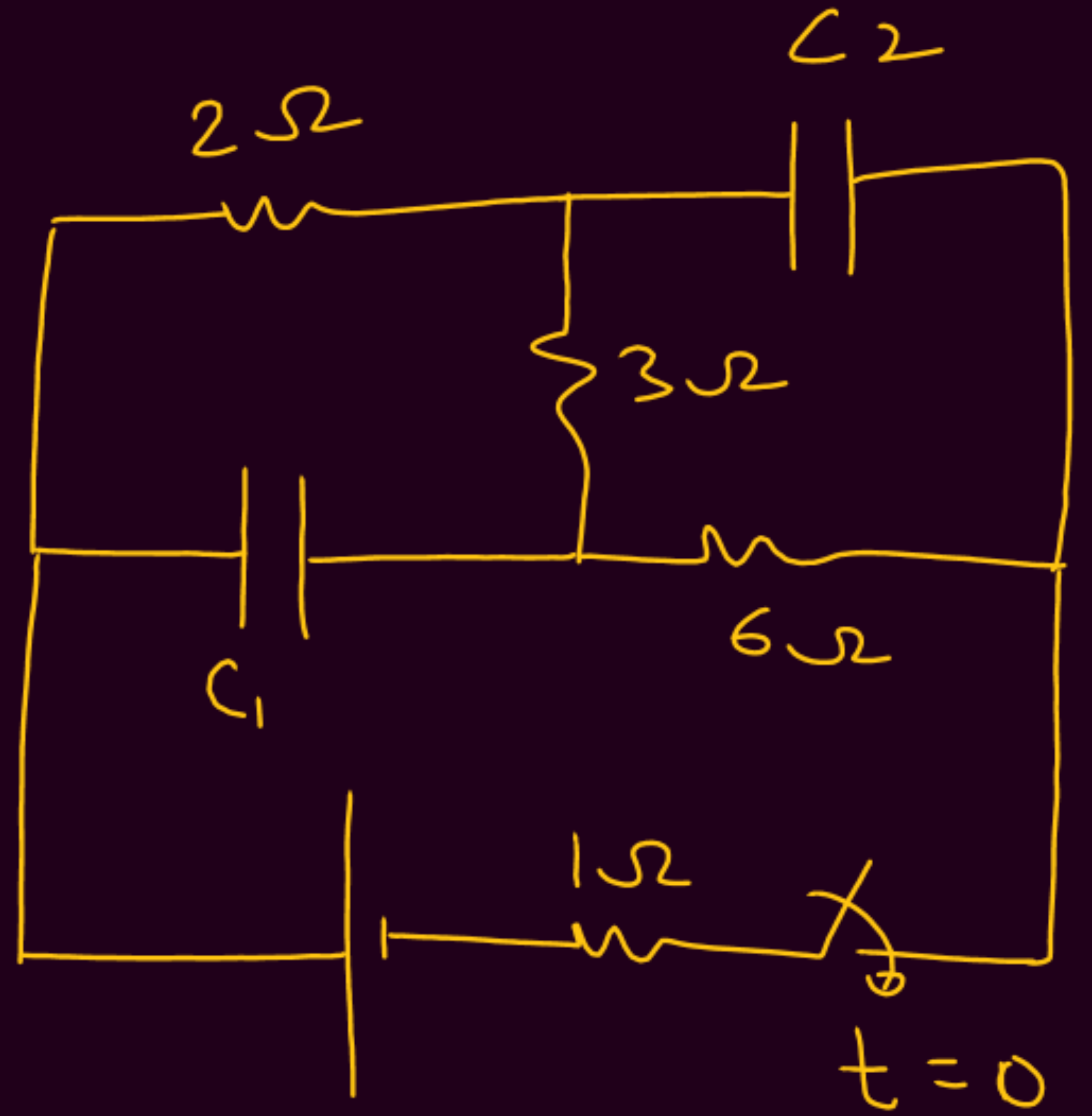


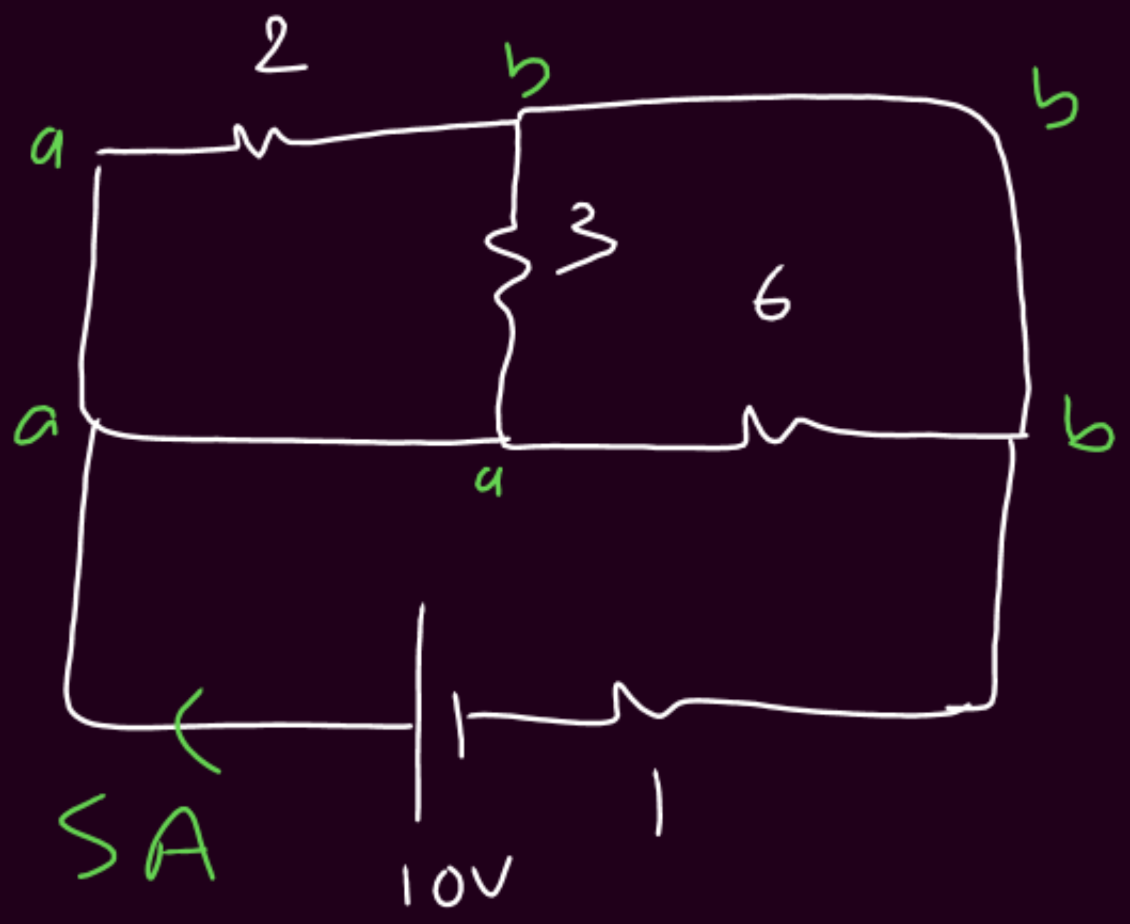
$$R_{eq} = 4\Omega$$

Finally ($t \to \infty$)

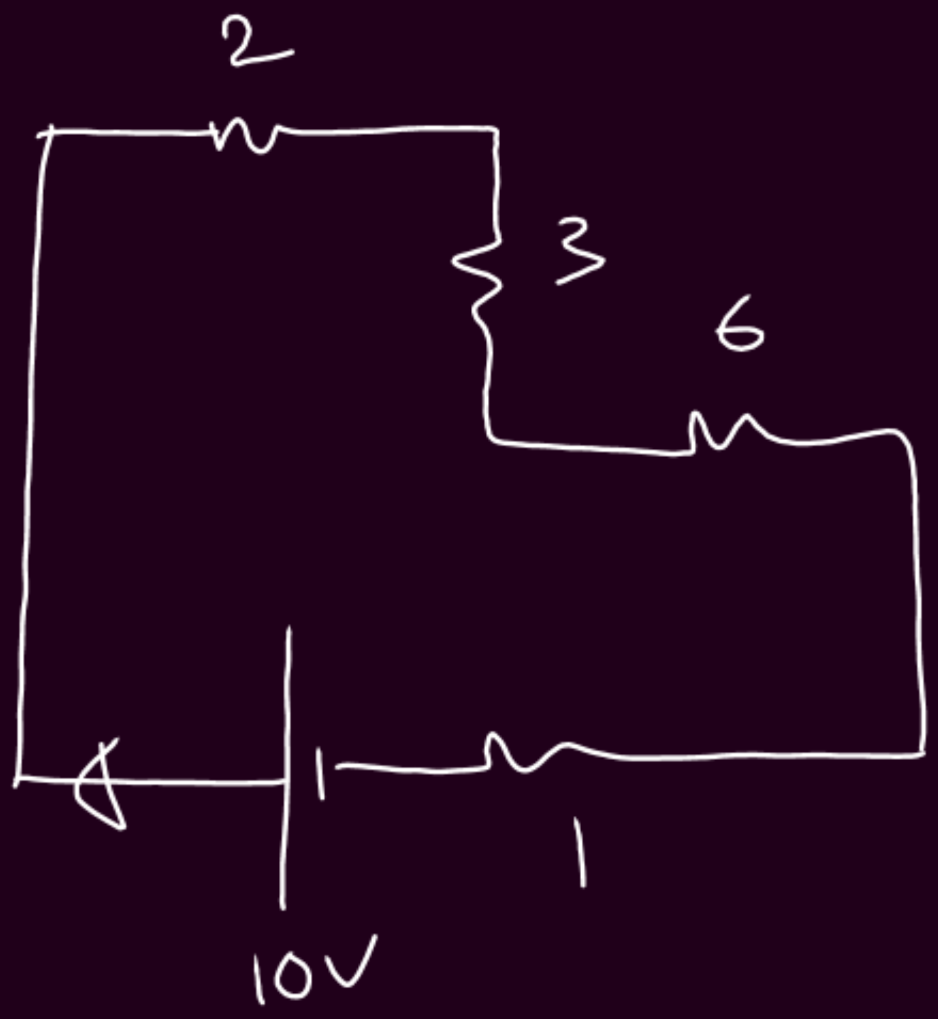


Q.



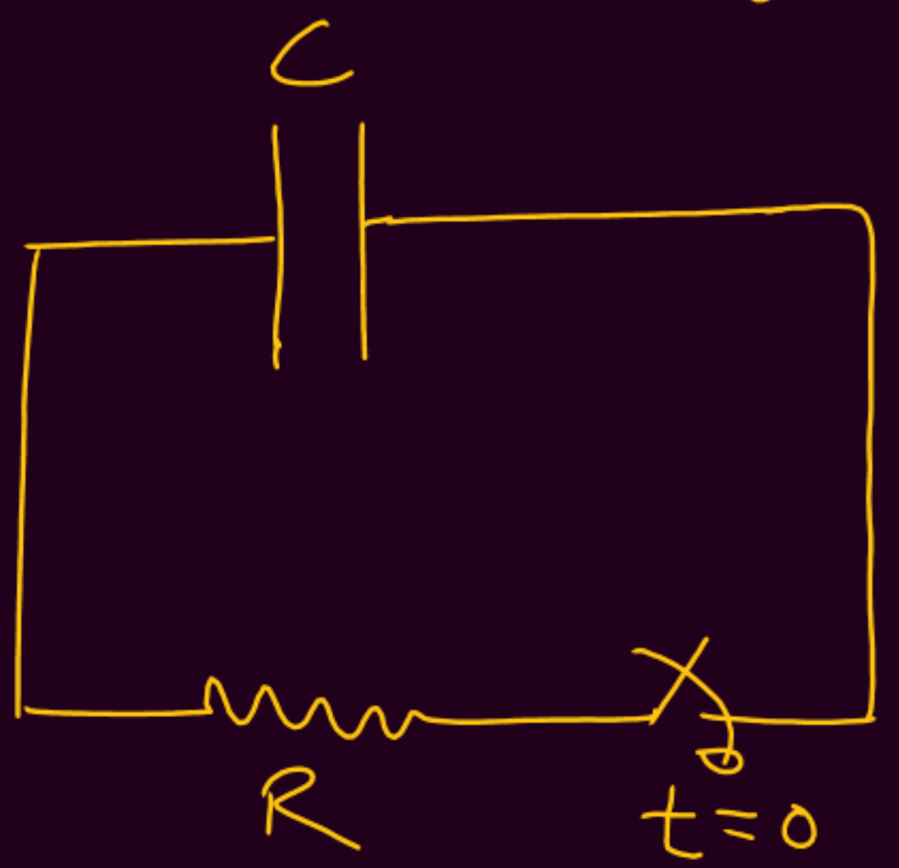


$R_{eq} = 2\Omega$



$i = \frac{10}{2+3+6+1} = \frac{10}{12} A$

(II) Discharging

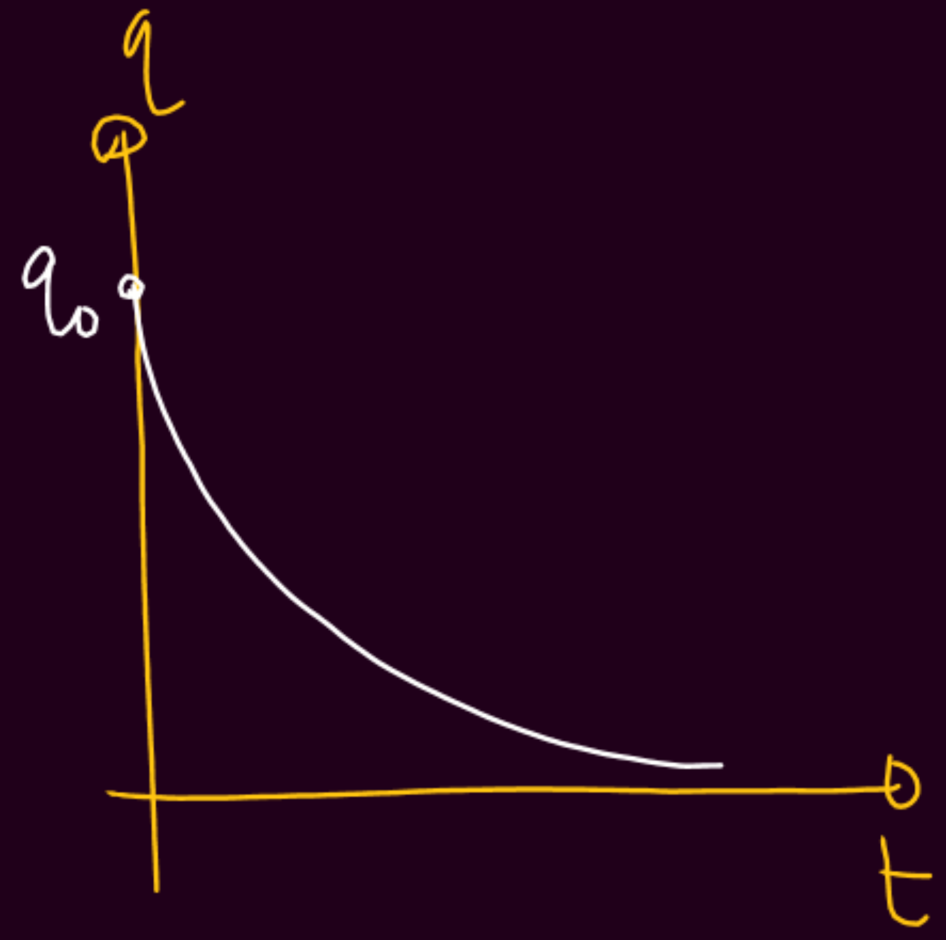


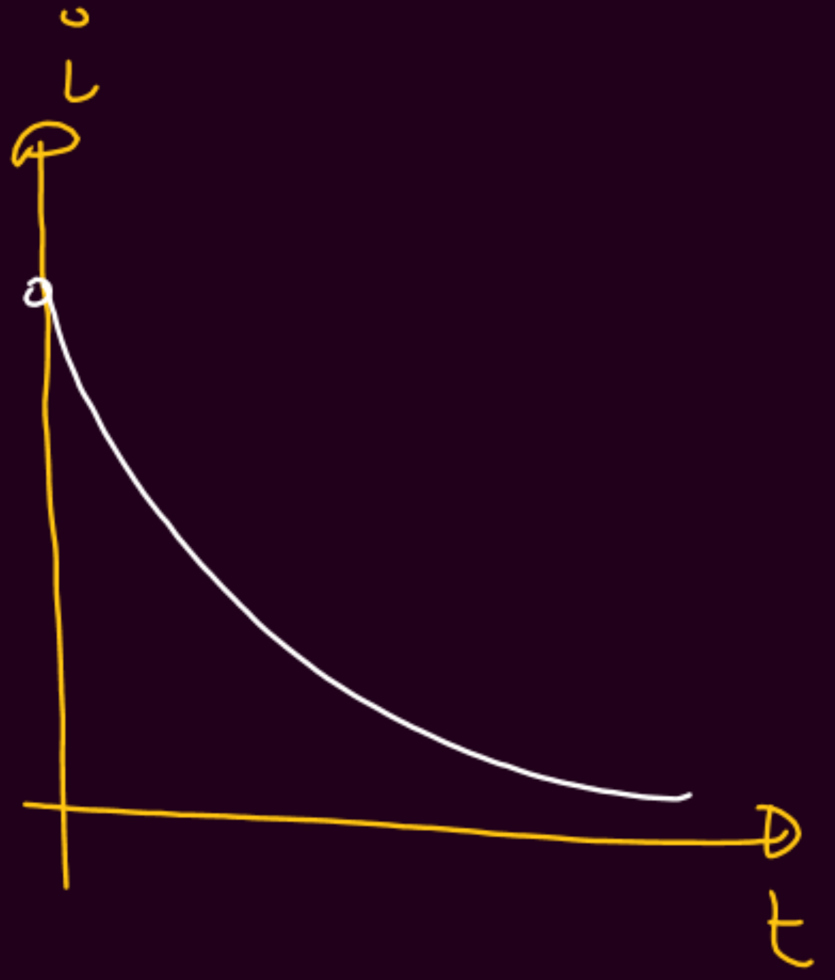
(1) $\frac{1}{2}CV_0^2 = \frac{q_0^2}{2C} = \text{Heat}$

(2) Time const. $\boxed{\tau = RC}$

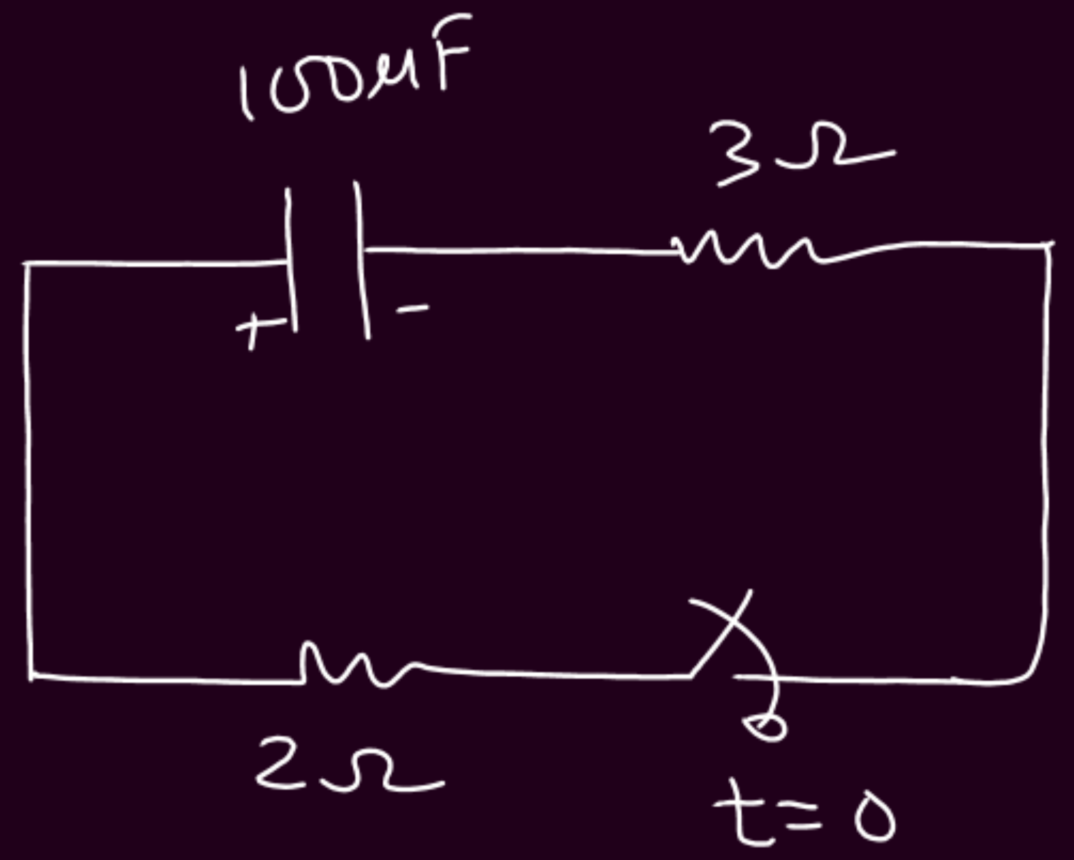
(3) KVL $\boxed{\frac{q}{C} = iR}$

(4)





Q



$$i_1 R_1 = i_2 R_2$$

$$i_1 = i \frac{R_2}{R_1 + R_2}$$

① $\frac{H_1}{H_2} = \frac{2}{3}$

② $\text{Heat} = \frac{1}{2} CV^2 = \frac{1}{2}(100\mu)(200)^2$

$= \underline{\underline{2J}}$

$$H_{2\Omega} = (2) \times \frac{2}{2+3}$$

Dielectric strength

→ E_{max}

$\boxed{\text{Lightening}}$ ✓