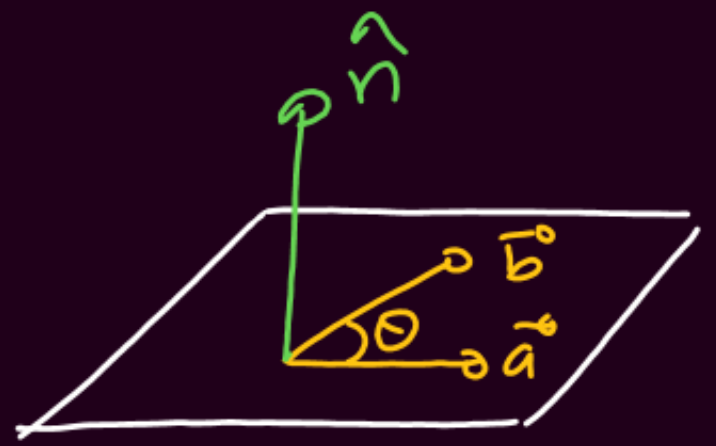
Cross Product (vector)

$$\vec{a} \times \vec{b} = (ab\sin\theta)\,\hat{n}$$



Direction $\rightarrow$ RHTR

$\odot$ out of the plane / Thumb / ACW / North

$\otimes$ into the plane / / CW / South

---

① $\vec{a} \times \vec{a} = 0$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

② If $\vec{a} \parallel \vec{b}$ then $\vec{a} \times \vec{b} = 0$

$\theta = 0^\circ / 180^\circ$

③ If $\vec{a} \parallel \vec{b}$ then

$$\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$$

④

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\overset{\oplus}{\longrightarrow}$$

$$\hat{i} \quad \hat{j} \quad \hat{k} \quad \hat{i} \quad \hat{j} \quad \hat{k}$$

$$\underset{\ominus}{\longleftarrow}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

⑤

$$(\vec{a} \times \vec{b}) \perp \vec{a}$$

$$(\vec{a} \times \vec{b}) \perp \vec{b}$$

$$\begin{cases} \vec{a} \cdot (\vec{a} \times \vec{b}) = 0 \\ \vec{b} \cdot (\vec{a} \times \vec{b}) = 0 \end{cases}$$

⑥

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$$

⑦

$$\vec{a} = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

$$\vec{b} = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1 z_2 - z_1 y_2)\hat{i} - (x_1 z_2 - z_1 x_2)\hat{j} + (x_1 y_2 - y_1 x_2)\hat{k}$$

Q. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & 2 \end{vmatrix} = [4-(-3)]\hat{i} - (2-6)\hat{j} + (-1-4)\hat{k}$$

$$= 7\hat{i} + 4\hat{j} - 5\hat{k}$$

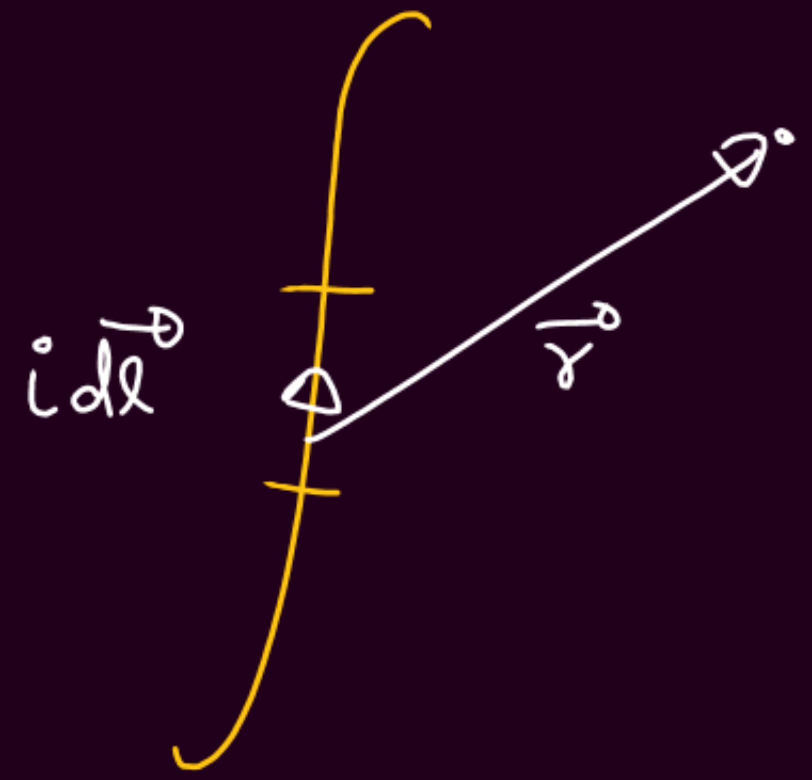
Q. $\vec{a} = 3\hat{i} - 4\hat{j}$

$\vec{b} = 3\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ 0 & 3 & 1 \end{vmatrix} = (-4)\hat{i} - 3\hat{j} + 9\hat{k}$$

Magnetic Effects of Current

Biot Savart Law



$$\boxed{\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,d\vec{l} \times \hat{r}}{r^2}}$$

unit 1 Tesla $= \frac{Wb}{m^2} = 10^4$ Gauss
(T)

$\vec{B}$ = Intensity of mag. field
= Magnetic flux density

μ_0 = permeability of free space

$$\frac{\mu_0}{4\pi} = 10^{-7} \quad \frac{T\text{-}m}{A}$$

R Palm Rule

Thumb $\rightarrow$ i

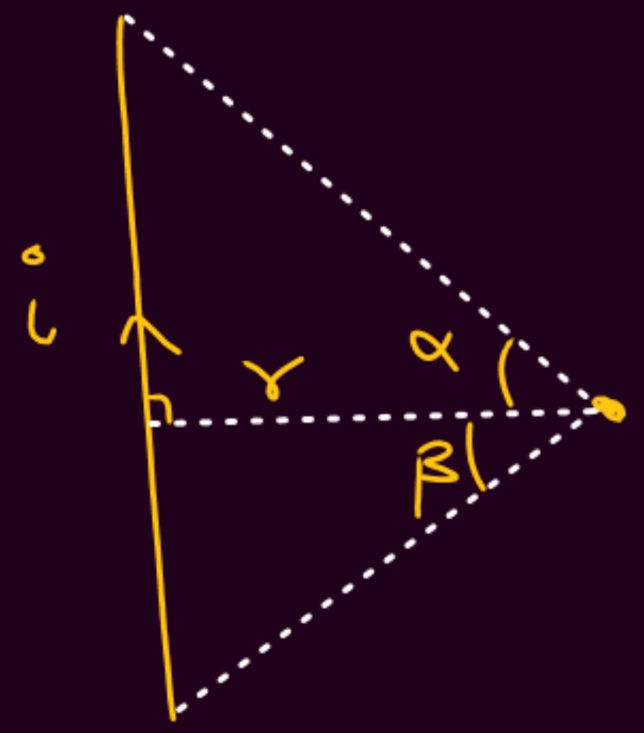
fingers – r

Palm $\rightarrow$ B

Straight wire

① 

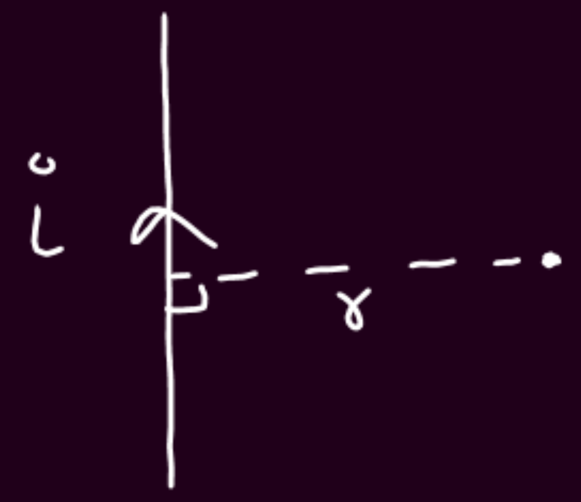
$\bullet\ B = 0$

② 

$$B = \frac{\mu_0 i}{4\pi r}(\sin\alpha + \sin\beta)$$

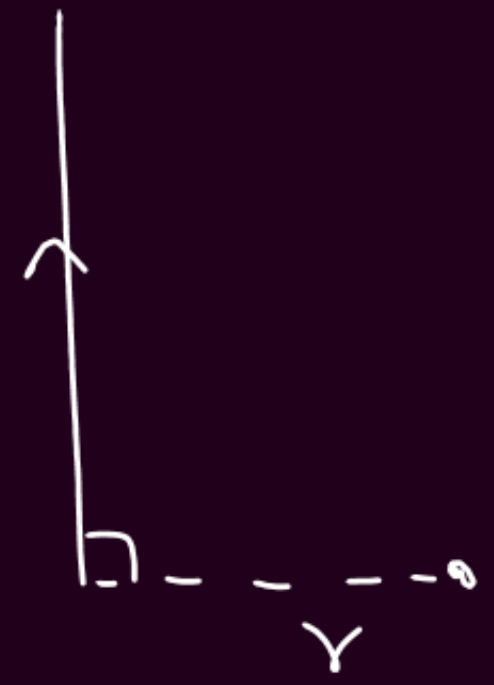
③ Infinitely long thin st. wire

$(\alpha = \beta = 90^\circ)$



$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

④ Semi infinite

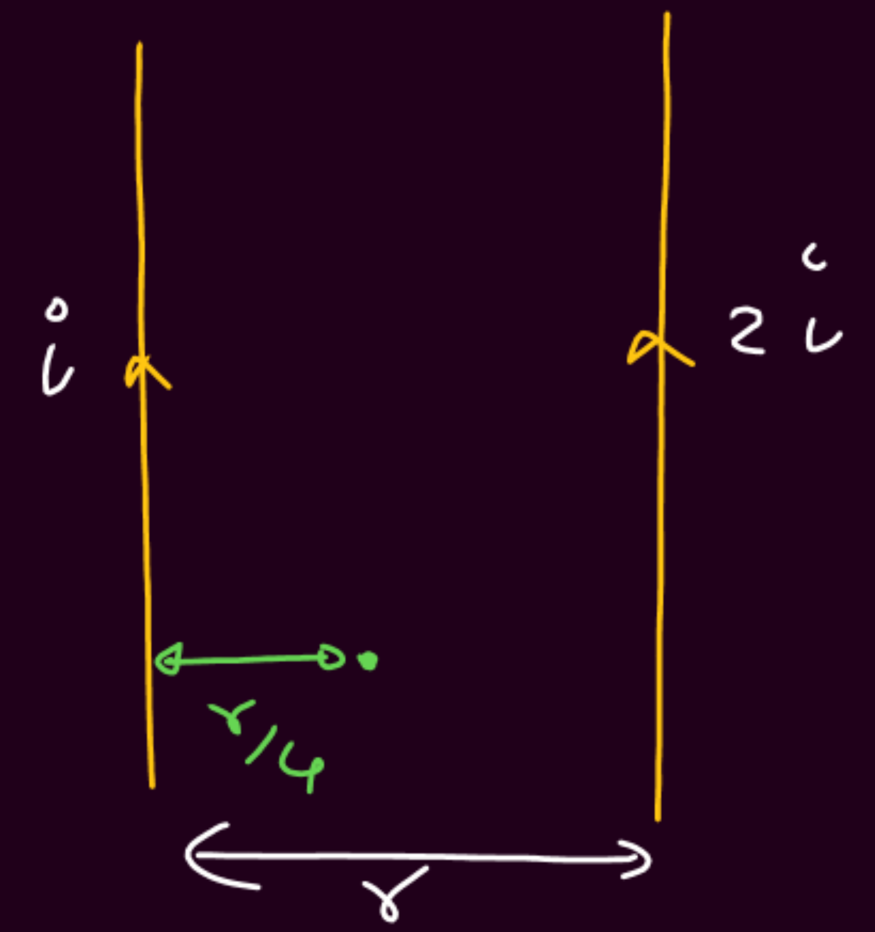


$\alpha = 90^\circ$

$\beta = 0^\circ$

$$B = \frac{\mu_0 i}{4\pi r}$$

Q.



$$\otimes \quad B_1 = \frac{\mu_0 i}{2\pi (r/4)} = \frac{2\mu_0 i}{\pi r}$$

$$\odot \quad B_2 = \frac{\mu_0 (2i)}{2\pi \left(\frac{3r}{4}\right)} = \frac{4\mu_0 i}{3r}$$

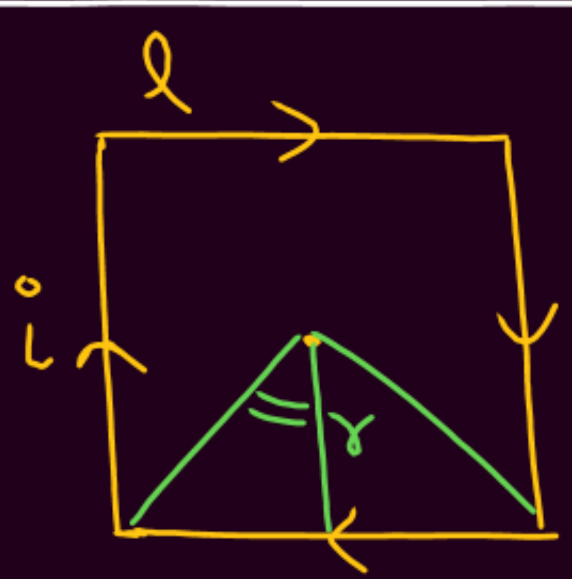
$$B_{net} = B_1 - B_2 = \left(2 - 4/3\right)\frac{\mu_0 i}{r} \quad \otimes$$

Q.

$$B = \frac{\mu_0 i}{4\pi r}\left(\sin 60^\circ + \sin 30^\circ\right)$$

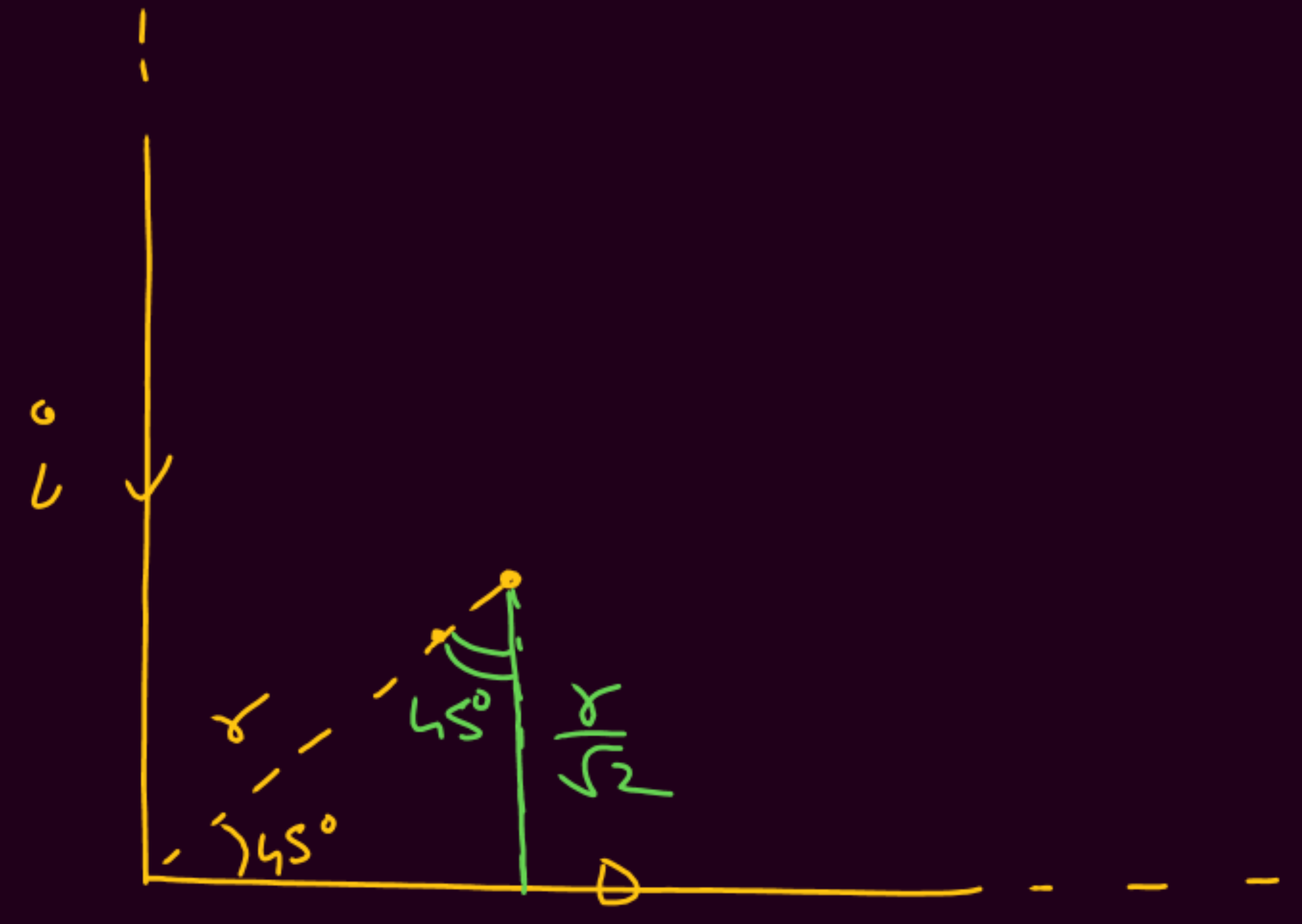
$$B = \frac{\mu_0 i}{4\pi r}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \quad \otimes$$

Q



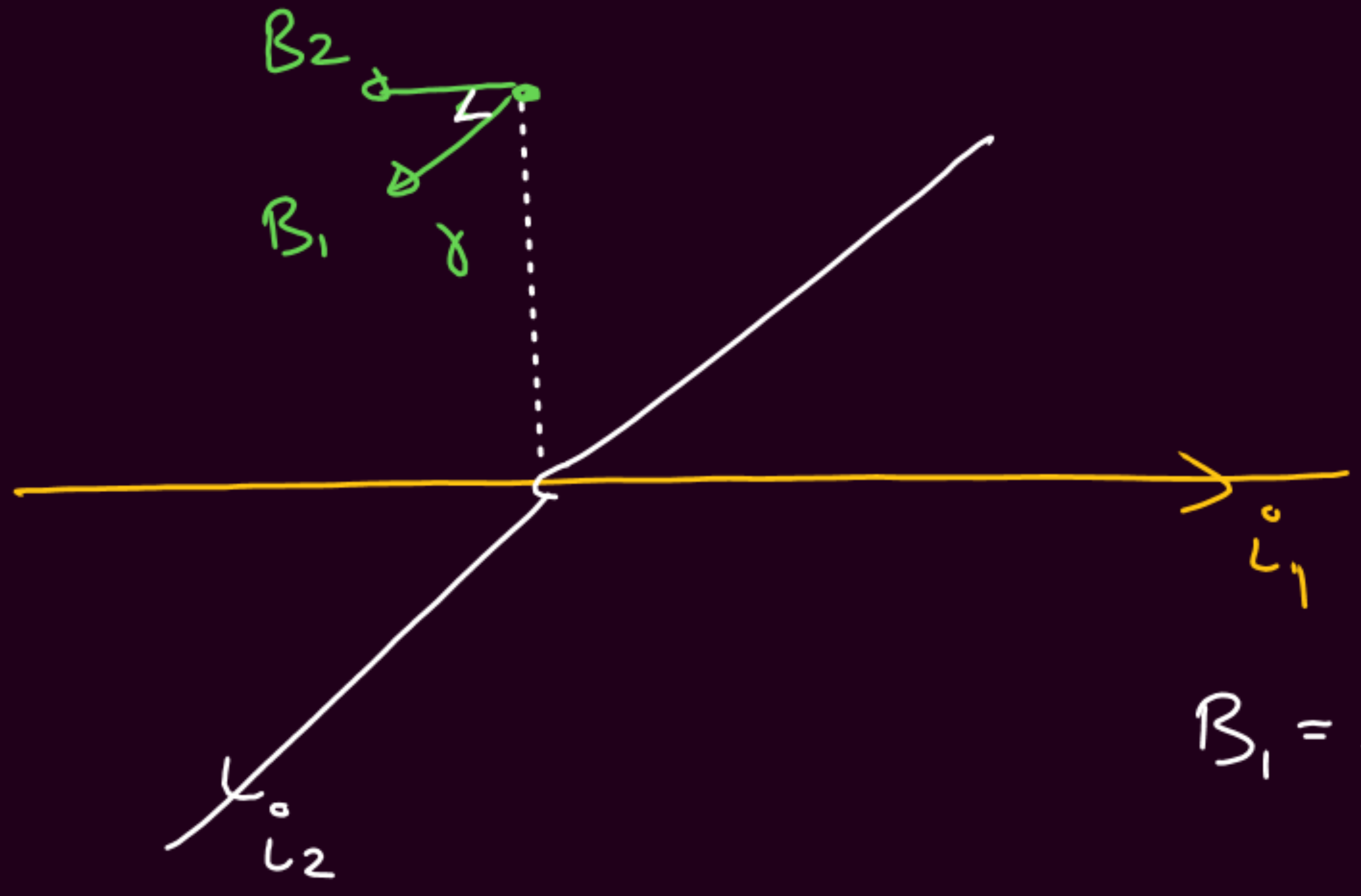
$$B = 4\left(\frac{\mu_0 i}{4\pi\left(\frac{l}{2}\right)}\right)\left(\sin 45^\circ + \sin 45^\circ\right)$$

Q.



$$\odot \ \vec{B} = 2\left(\frac{\mu_0 i}{4\pi\left(\frac{r}{\sqrt{2}}\right)}\right)\left(\sin 90^\circ + \sin 45^\circ\right)$$

Q.



$$B_1 = \frac{\mu_0 i_1}{2\pi r}$$

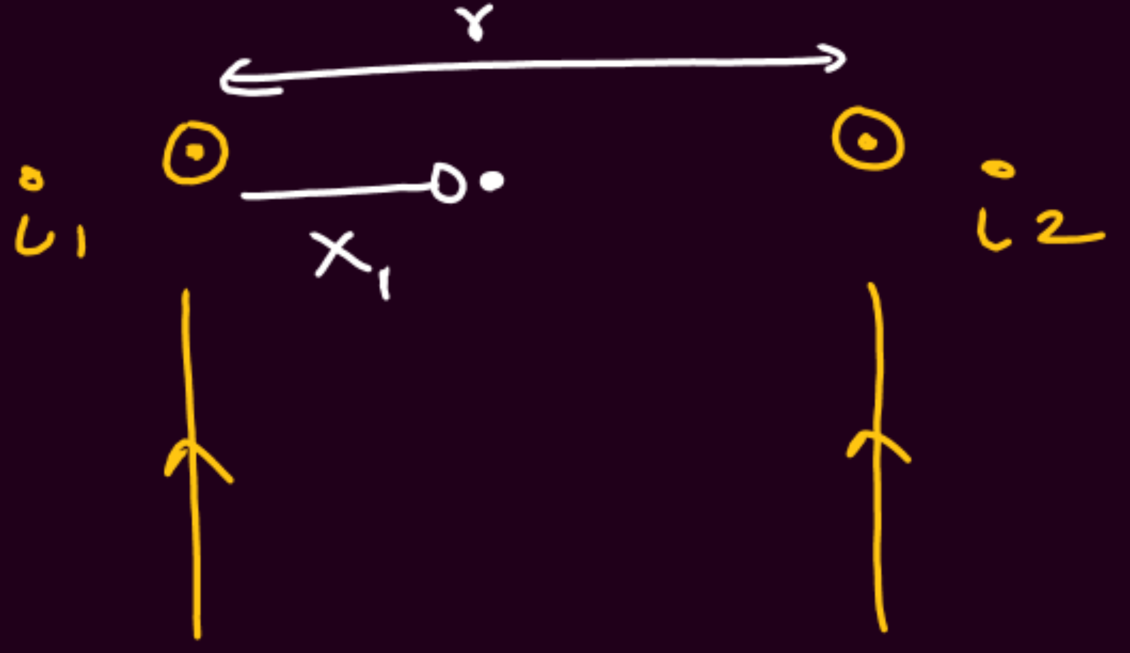
$$B_2 = \frac{\mu_0 i_2}{2\pi r}$$

$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$B_{net} = \frac{\mu_0}{2\pi r}\sqrt{i_1^2 + i_2^2}$$

Null Point

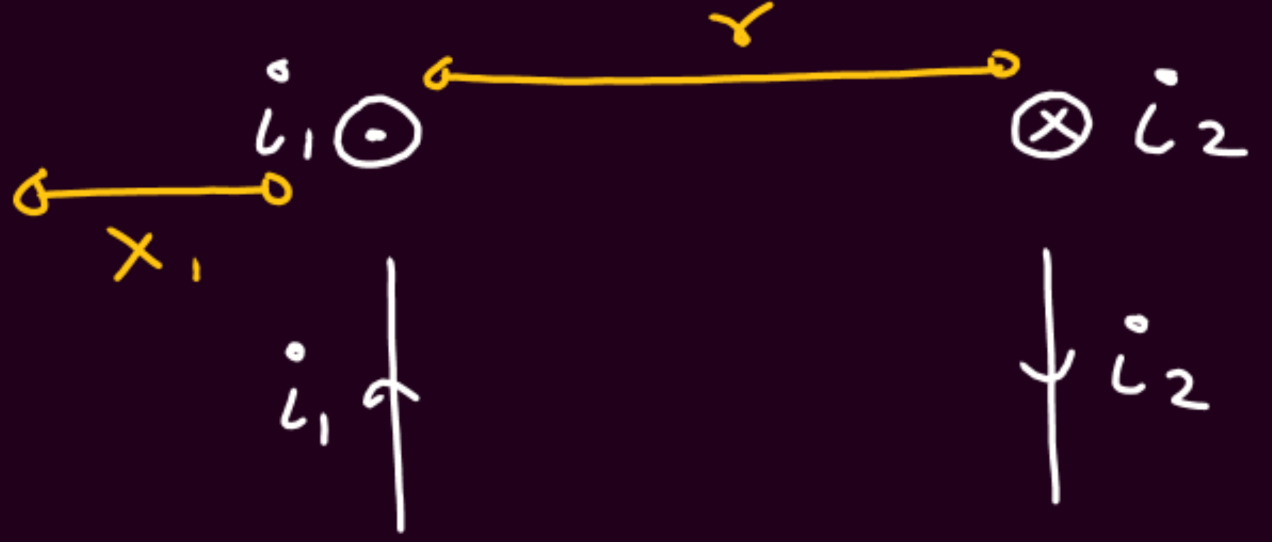
① Like Currents



$$\boxed{x_1 = \frac{r}{\frac{i_2}{i_1}+1}}$$

② Equal & opposite (No null point)

③ Unlike Currents (Unequal)



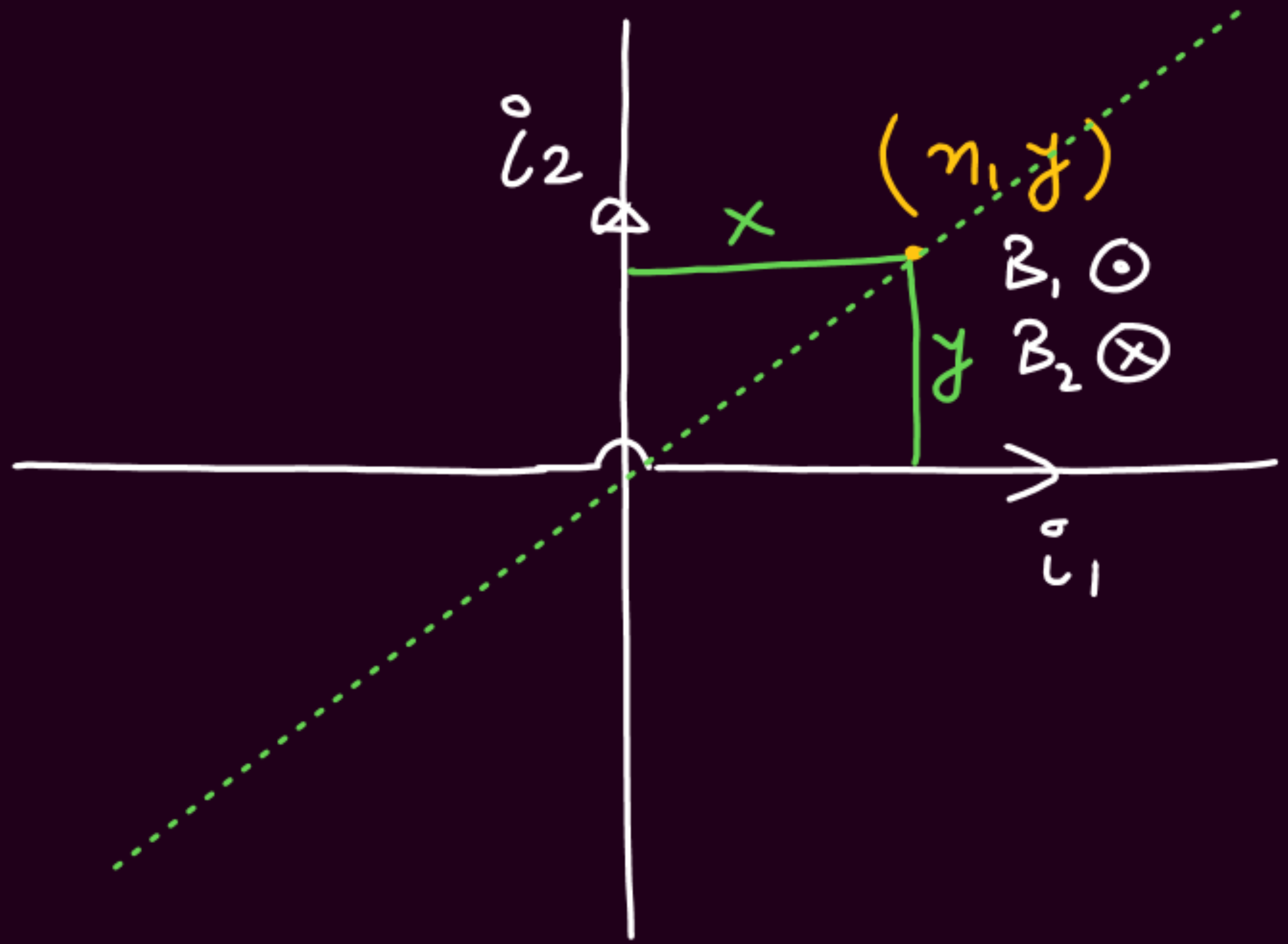
$$\boxed{x_1 = \frac{r}{\frac{i_2}{i_1}-1}}$$

Q.

$$x_1 = \frac{20}{\frac{6}{2}+1} = 5cm$$

Q.

$$x_1 = \frac{20}{\frac{10}{2}-1} = 5cm$$



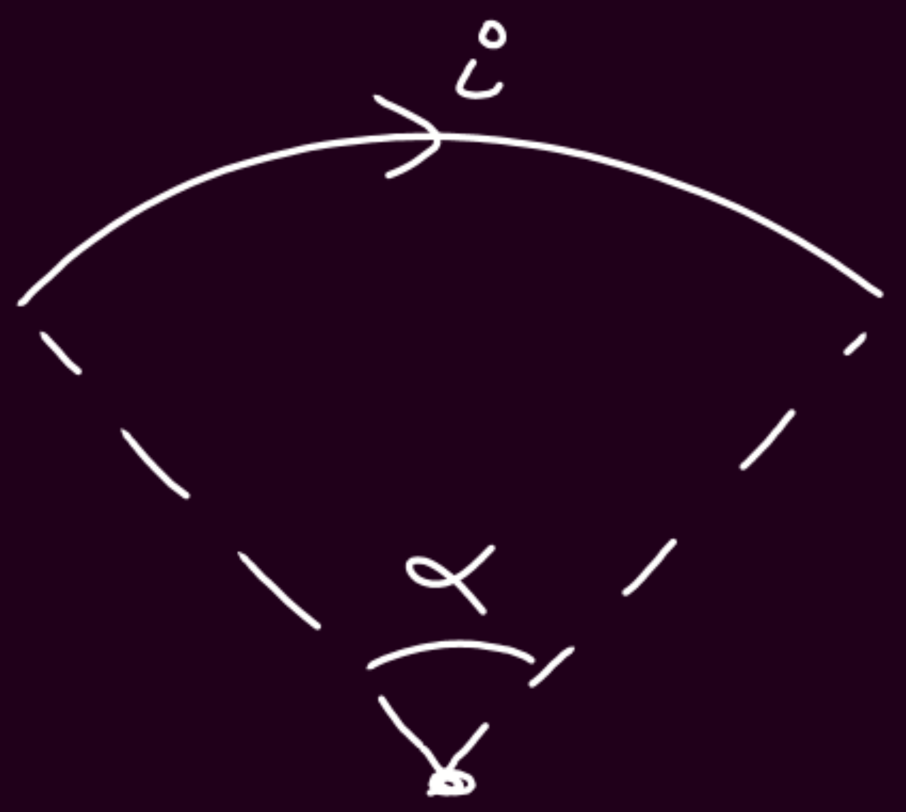
For Nullpoint

$$B_1 = B_2$$

$$\frac{\mu_0 i_1}{2\pi y} = \frac{\mu_0 i_2}{2\pi x}$$

$$\boxed{i_1 x = i_2 y}$$

② Circular Arc



$$\boxed{B = \frac{\mu_0 i \alpha}{4\pi R}}$$

RHTR → Curl finger → i

Thumb → B

③ Circular Coil

$360^\circ \rightarrow 2\pi$

$180^\circ \rightarrow \pi$

$30^\circ \rightarrow \pi/6$

$60^\circ \rightarrow \pi/3$

$90^\circ \rightarrow \pi/2$

$45^\circ \rightarrow \pi/4$

$120^\circ \rightarrow 2\pi/3$

$135^\circ \rightarrow 3\pi/4$

$150^\circ \rightarrow \frac{5\pi}{6}$

$$\boxed{B_{center} = \frac{\mu_0 N i}{2R}}$$

N = no. of turns

If pt. charge in UCM

$$\boxed{i = qf = \frac{q}{T} = \frac{q\omega}{2\pi} = \frac{qv}{2\pi r}}$$

Case

B_0

N

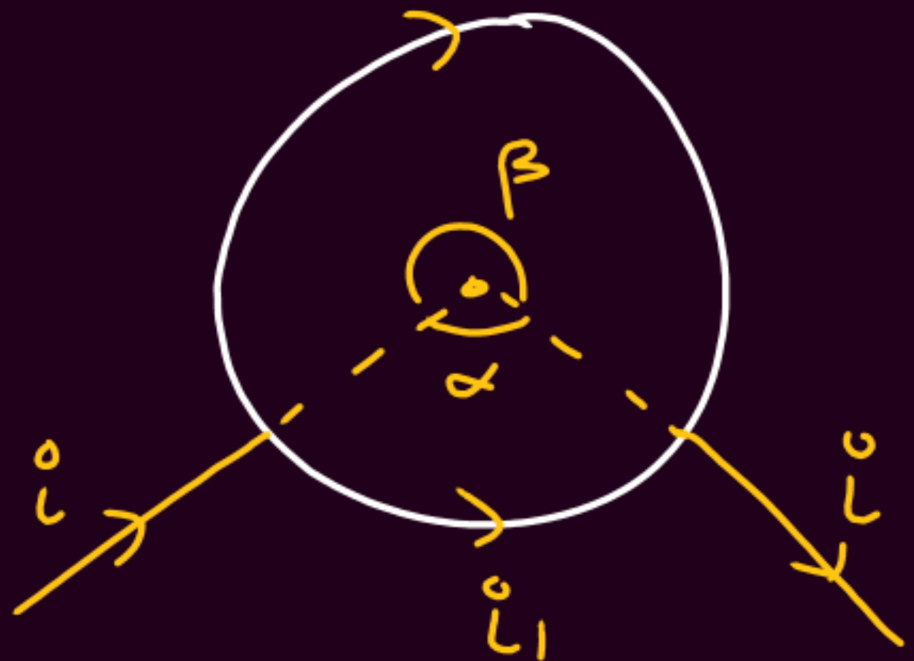
$$\boxed{B = N^2 B_0}$$

$$B_0 = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 N i}{2\pi \left(\frac{R}{N}\right)} = N^2 B_0$$

$$2\pi R = N(2\pi r)$$

Symmetry



$$B_1 = \frac{\mu_0 i_1 \alpha}{4\pi R}$$

$$B_2 = \frac{\mu_0 i_2 \beta}{4\pi R}$$

$$\boxed{B_{center} = 0}$$

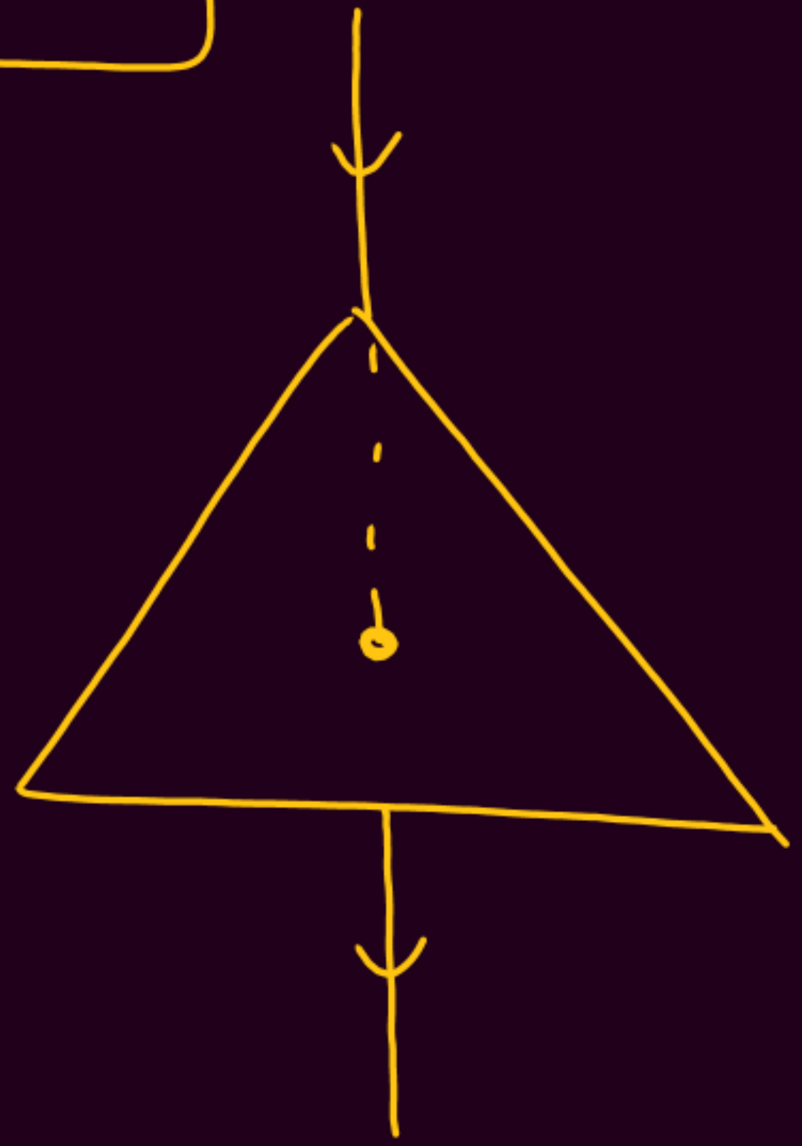
$$\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\beta}{\alpha}$$

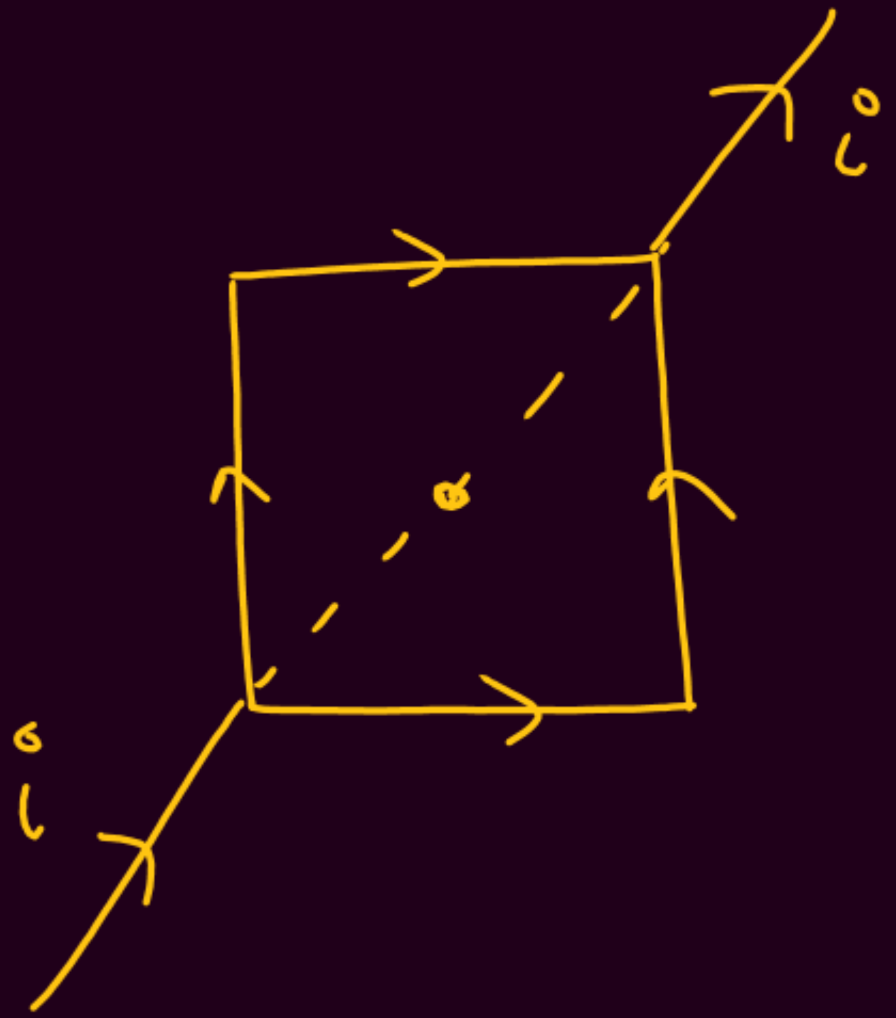
$$i_1 \alpha = i_2 \beta$$

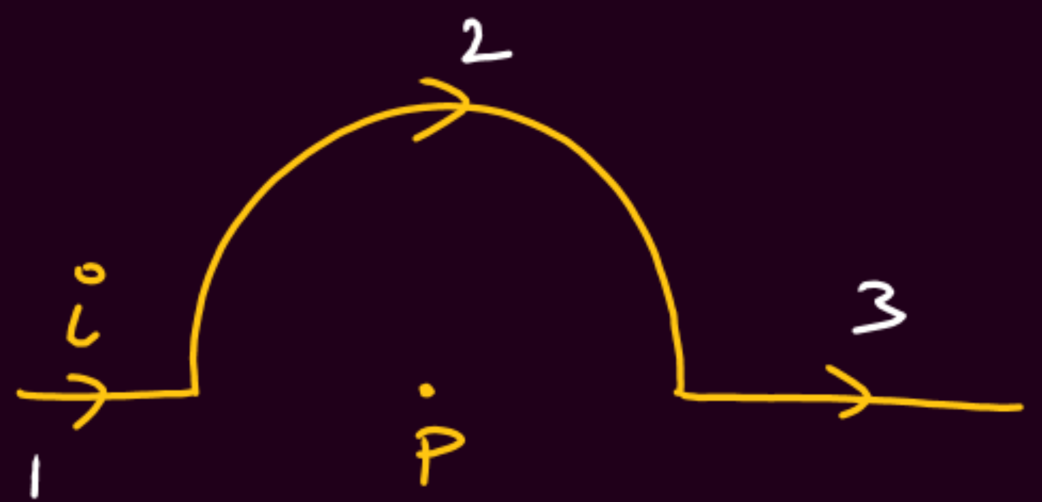
$$B_1 = B_2$$

$$\odot \qquad \otimes$$

$$\boxed{B_{center} = 0}$$

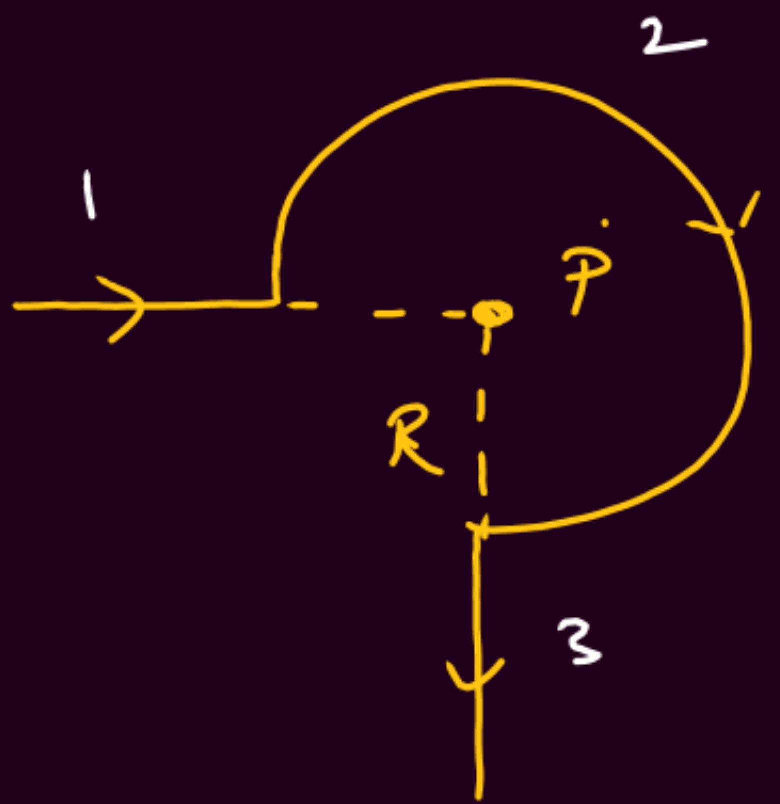






$$B_1 = B_3 = 0$$

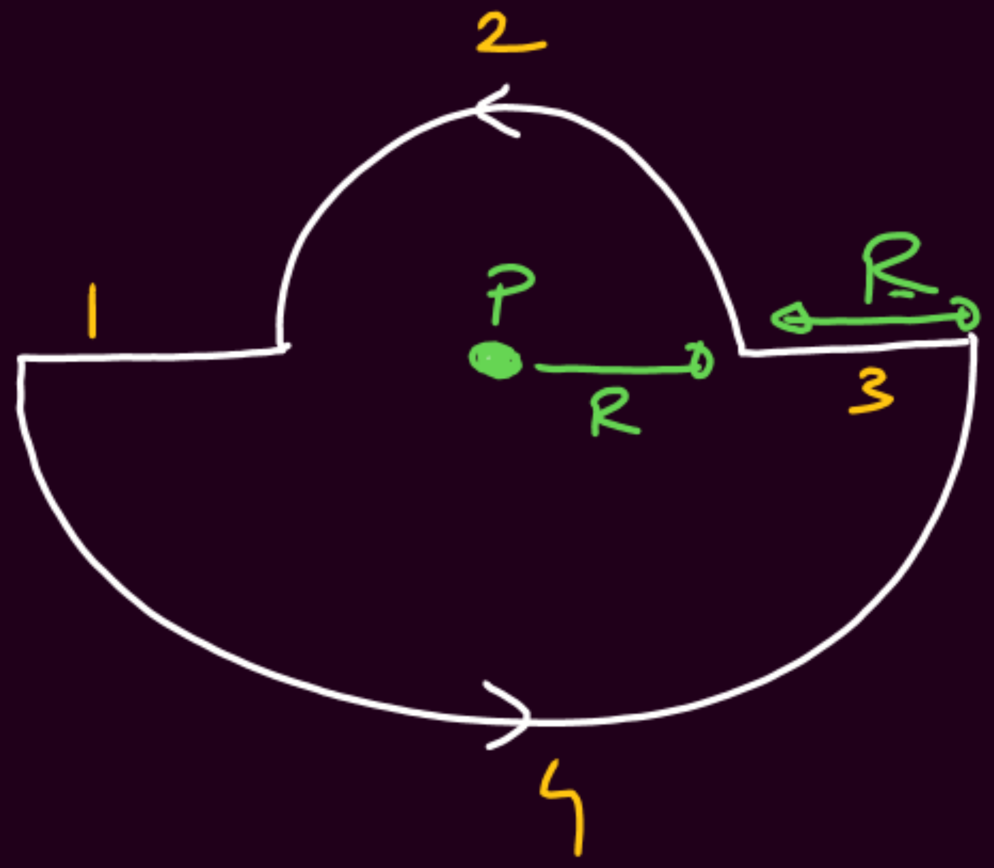
$$B = B_2 = \frac{\mu_0 i \pi}{4\pi R} \otimes$$



$$B_1 = B_3 = 0$$

$$B = B_2 = \frac{\mu_0 i \left(3\pi/2\right)}{4\pi R}$$

$$\otimes$$

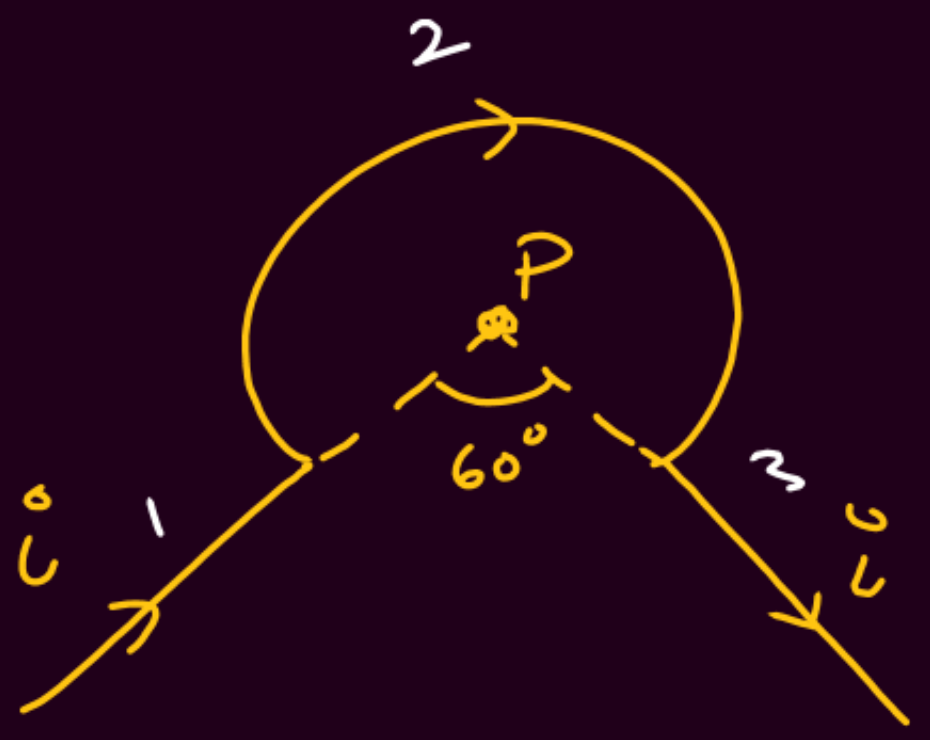


$$B_1 = B_3 = 0$$

$$B_2 = \frac{\mu_0 i \pi}{4\pi R} \odot$$

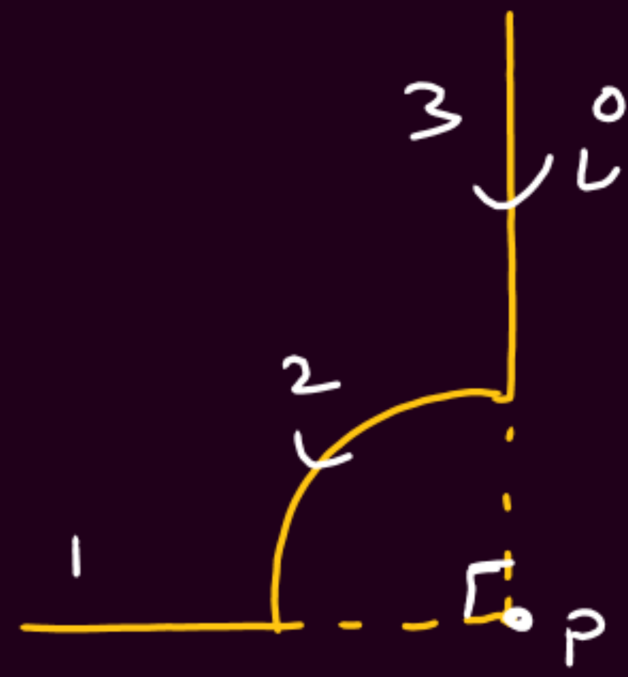
$$B_4 = \frac{\mu_0 i \pi}{4\pi (2R)} \odot$$

$$B_{net} = B_2 + B_4$$



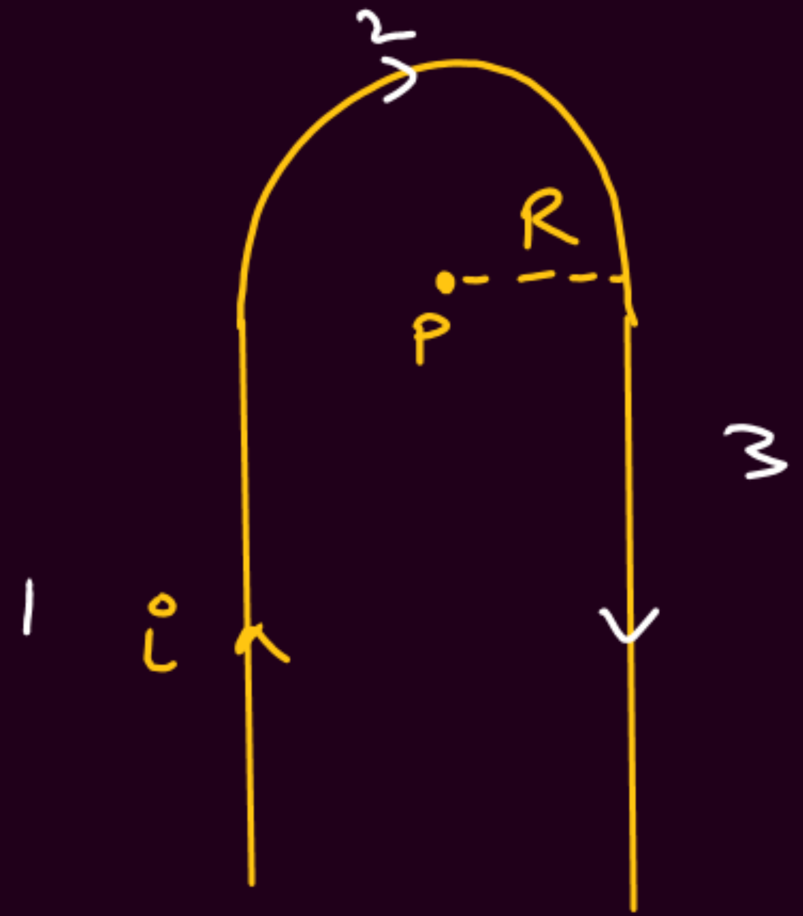
$$B_1 = B_3 = 0$$

$$B = B_2 = \frac{\mu_0 i \left(\frac{5\pi}{3}\right)}{4\pi R} \quad \otimes$$



$$B_1 = B_3 = 0$$

$$B = B_2 = \frac{\mu_0 i (\pi/2)}{4\pi R} \quad \odot$$



$$\boxed{\begin{aligned} \alpha &= 90^\circ \\ \beta &= 0^\circ \end{aligned}}$$

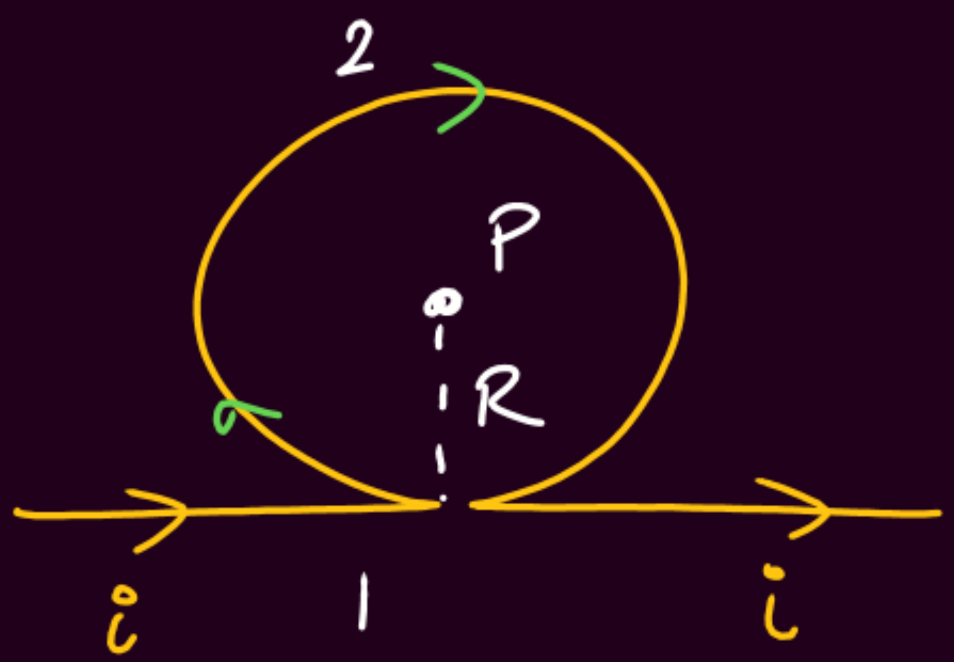
$$B_1 = B_3 = \frac{\mu_0 i}{4\pi R}$$

$\otimes \qquad \otimes$

$$\otimes\ B_2 = \frac{\mu_0 i (\pi)}{4\pi R}$$

$$B_{net} = B_1 + B_2 + B_3$$

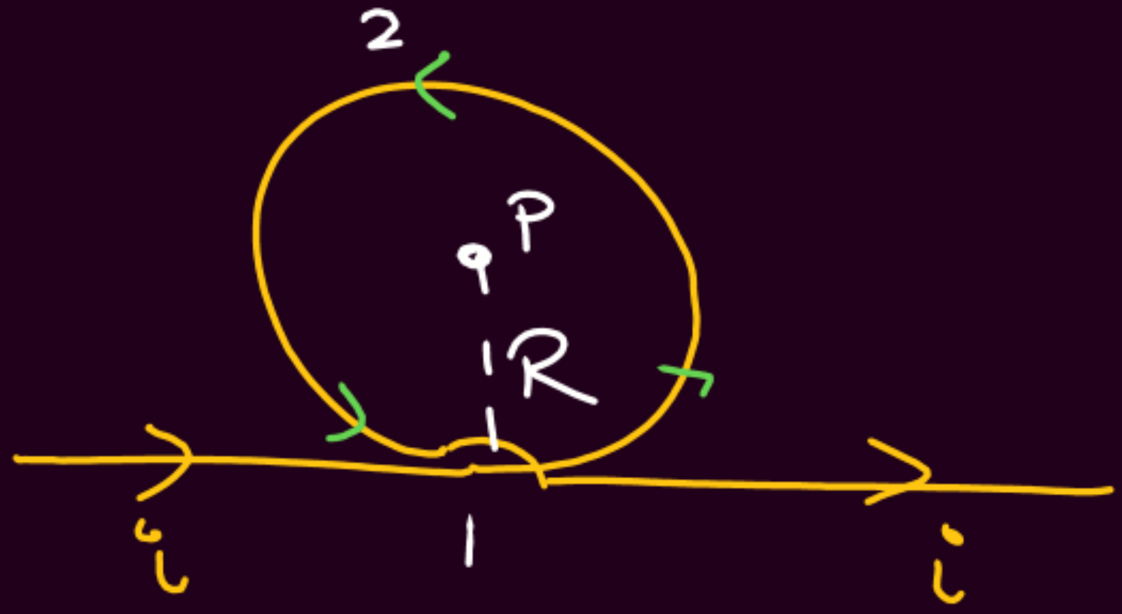
$$= \frac{\mu_0 i}{4\pi R}(1 + \pi + 1)$$



$$B_1 = \frac{\mu_0 i}{2\pi R} \quad \odot$$

$$B_2 = \frac{\mu_0 i}{2R} \quad \otimes$$

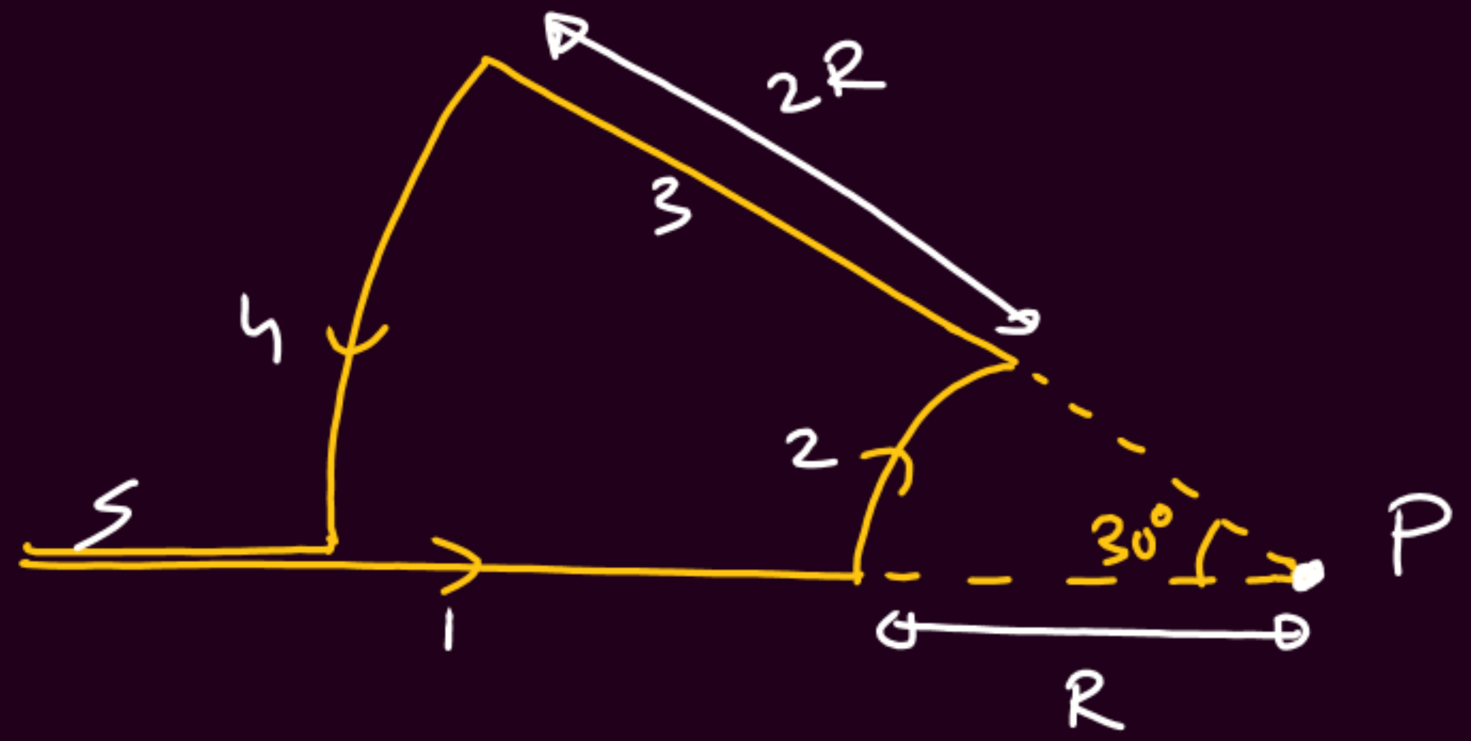
$$B_{net} = B_2 - B_1 \quad \otimes$$



$$B_1 = \frac{\mu_0 i}{2\pi R} \quad \odot$$

$$B_2 = \frac{\mu_0 i}{2R} \quad \odot$$

$$B_{net} = B_2 + B_1 \quad \odot$$



$$B_1 = B_3 = B_5 = 0$$

$$B_2 = \frac{\mu_0 i \left(\pi/6\right)}{4\pi R} \quad \otimes$$

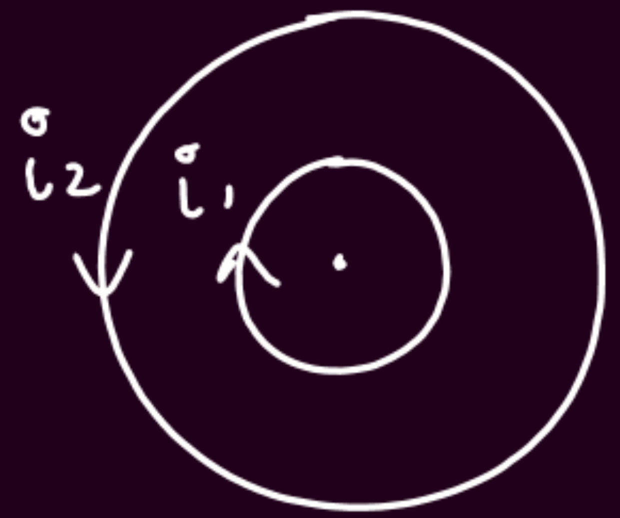
$$B_4 = \frac{\mu_0 i \left(\pi/6\right)}{4\pi (3R)} \quad \odot$$

$$B_{net} = B_2 - B_4 \quad \otimes$$

Q. Two concentric / coaxial / circular coils of R_1 & R_2 → opp. current

if $B_{center} = 0$

then $\frac{i_1}{i_2} = ?$



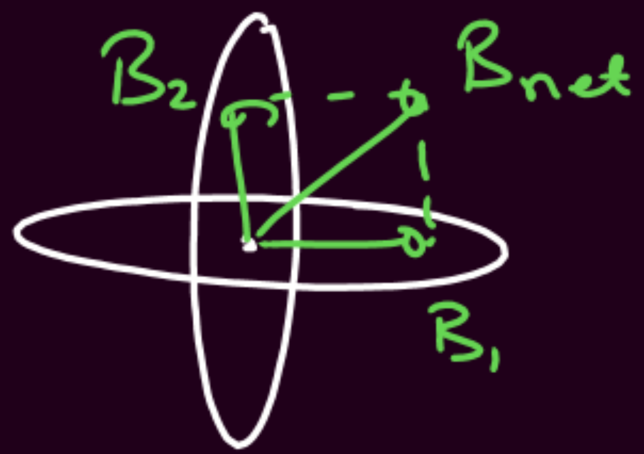
$$B_1 = B_2$$

$$\frac{\mu_0 i_1}{2R_1} = \frac{\mu_0 i_2}{2R_2}$$

$$\boxed{\frac{i_1}{i_2} = \frac{R_1}{R_2}}$$

Q. Two identical / concentric / ⊥ circular coils of R (i_1 & i_2)

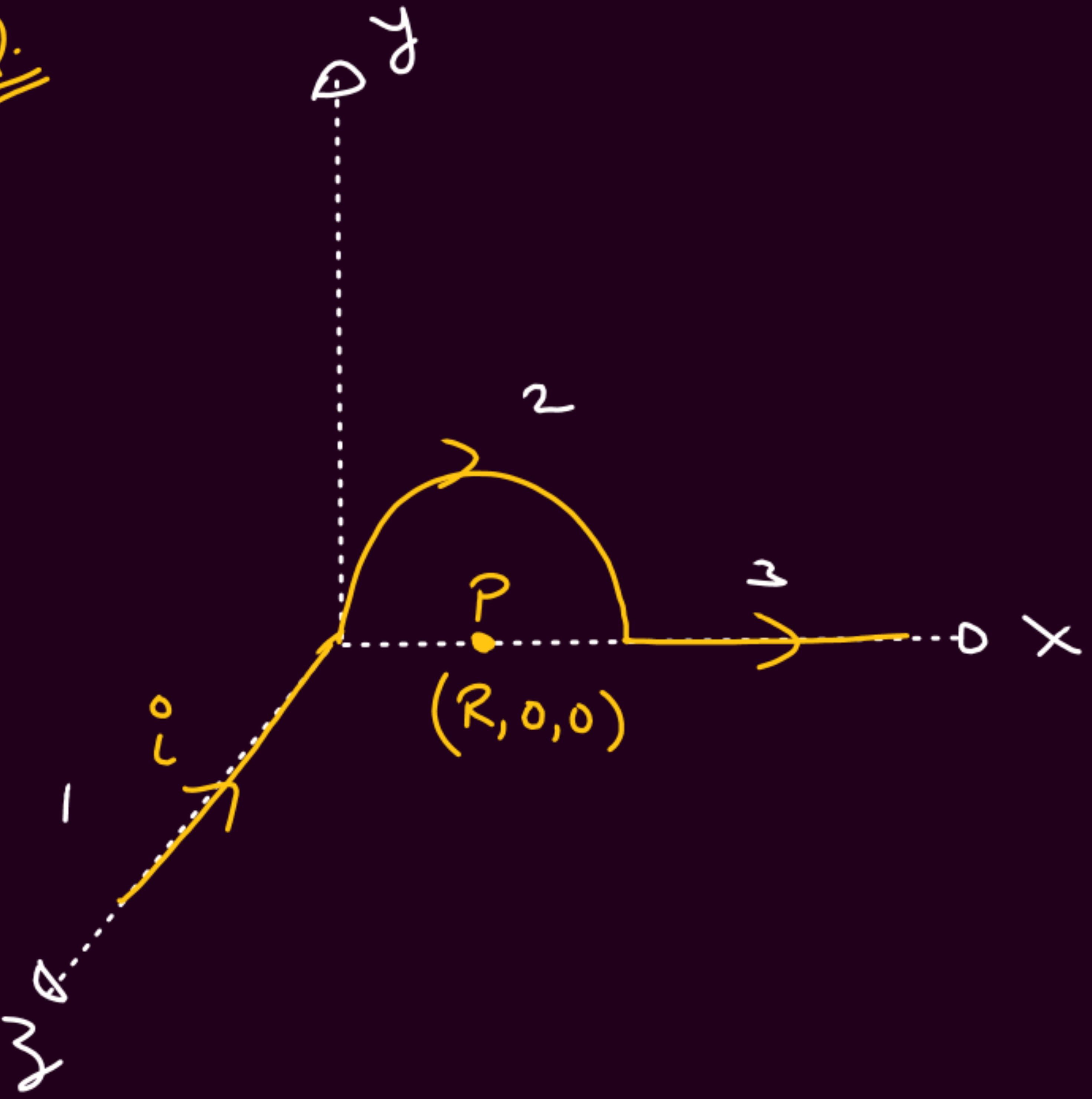
$B_{center} = ?$



$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$B_{net} = \frac{\mu_0}{2R}\sqrt{i_1^2 + i_2^2}$$

Q.



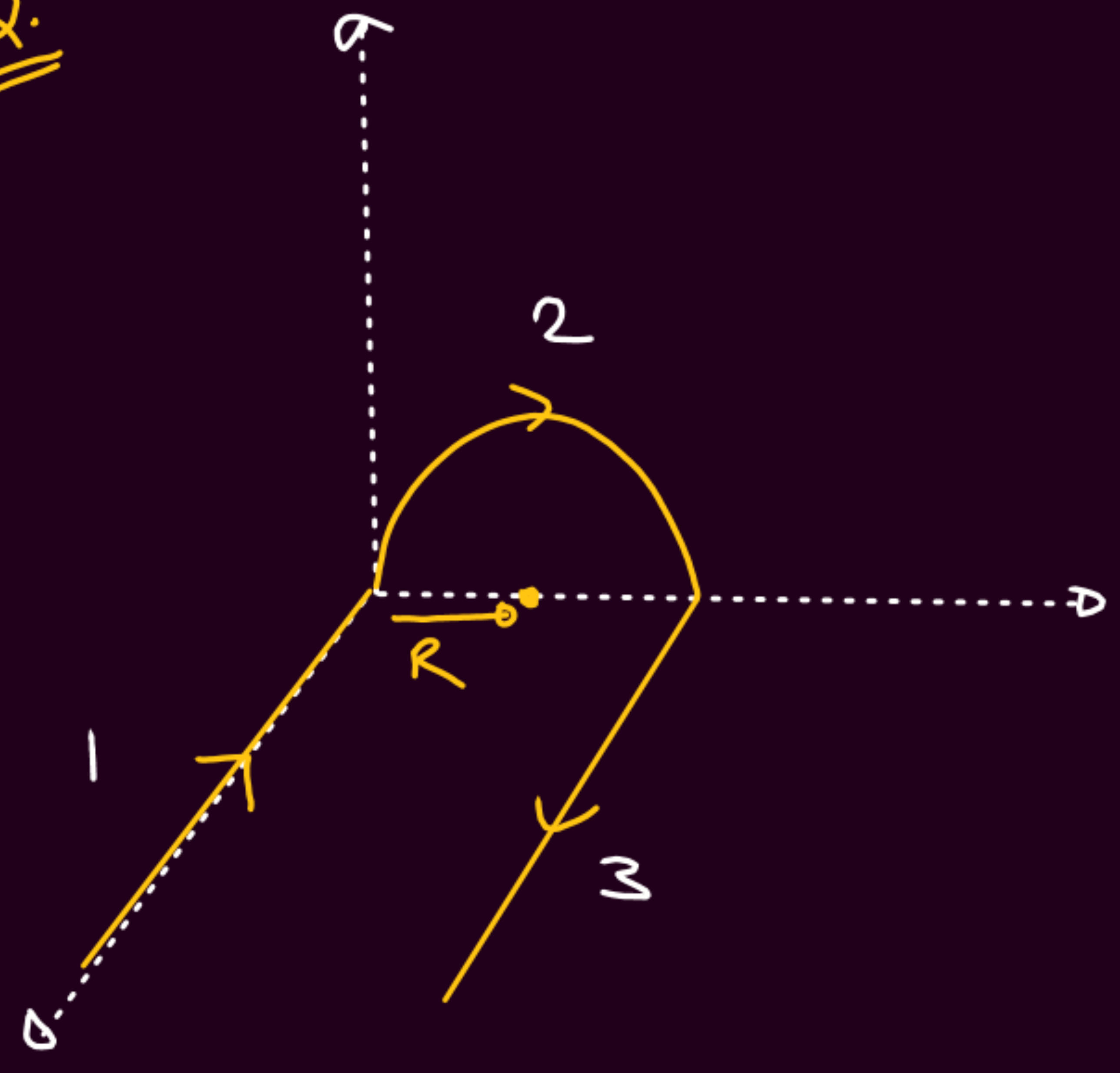
$$\vec{B}_1 = \frac{\mu_0 i}{4\pi R}(-\hat{j})$$

$$\vec{B}_2 = \frac{\mu_0 i(\pi)}{4\pi R}(-\hat{k})$$

$$\vec{B}_3 = 0$$

$$\vec{B}_{net} = -\frac{\mu_0 i}{4\pi R}\left(\hat{j} + \pi\hat{k}\right)$$

Q.



$$\vec{B}_1 = \frac{\mu_0 i}{4\pi R}(-\hat{j})$$

$$\vec{B}_2 = \frac{\mu_0 i \pi}{4\pi R}(-\hat{k})$$

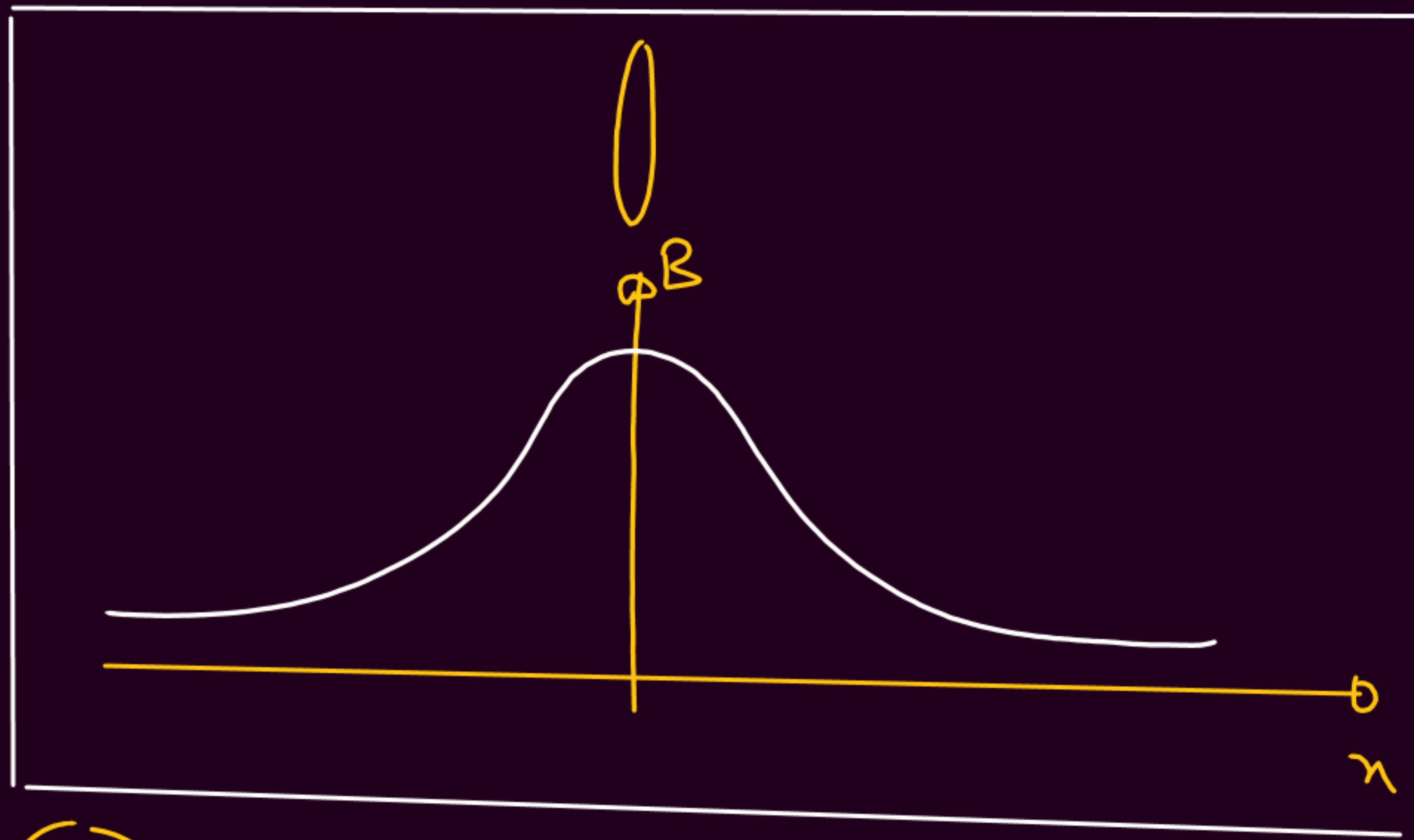
$$\vec{B}_3 = \frac{\mu_0 i}{4\pi R}(-\hat{j})$$

$$\vec{B}_{net} = -\frac{\mu_0 i}{4\pi R}\left(2\hat{j} + \pi\hat{k}\right)$$

④ $\vec{B}$ on Axis of Circular Coil

Dir → Curl the finger → i
thumb → $\vec{B}$

$$\boxed{\vec{B} = \frac{\mu_0 N i R^2}{2\left(x^2+R^2\right)^{3/2}}}$$



① $x=0$ (center) $\quad B = \frac{\mu_0 N i}{2R}$

② $x \ll R$ (very near center) $\quad B = \frac{\mu_0 N i}{2R}\left(1-\frac{3x^2}{2R^2}\right)$ extra

③ $x \gg R$ $\quad \boxed{B = \frac{\mu_0 N i R^2}{2x^3} = \frac{2k\vec{M}}{r^3}}$

$\vec{M}$ = magnetic dipole Moment $= Ni(\pi R^2)$

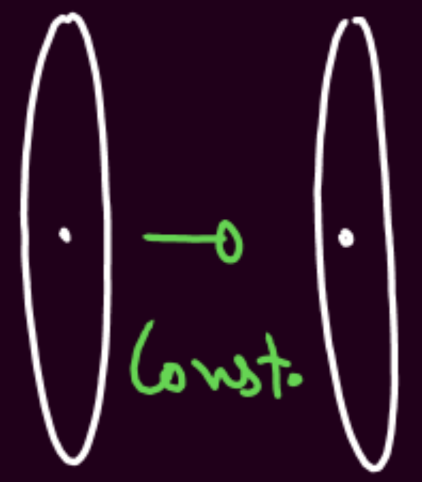
Helmholtz Coil (Extra)

→ Nearly const. $\vec{B}$

→ Two identical/coaxial coils

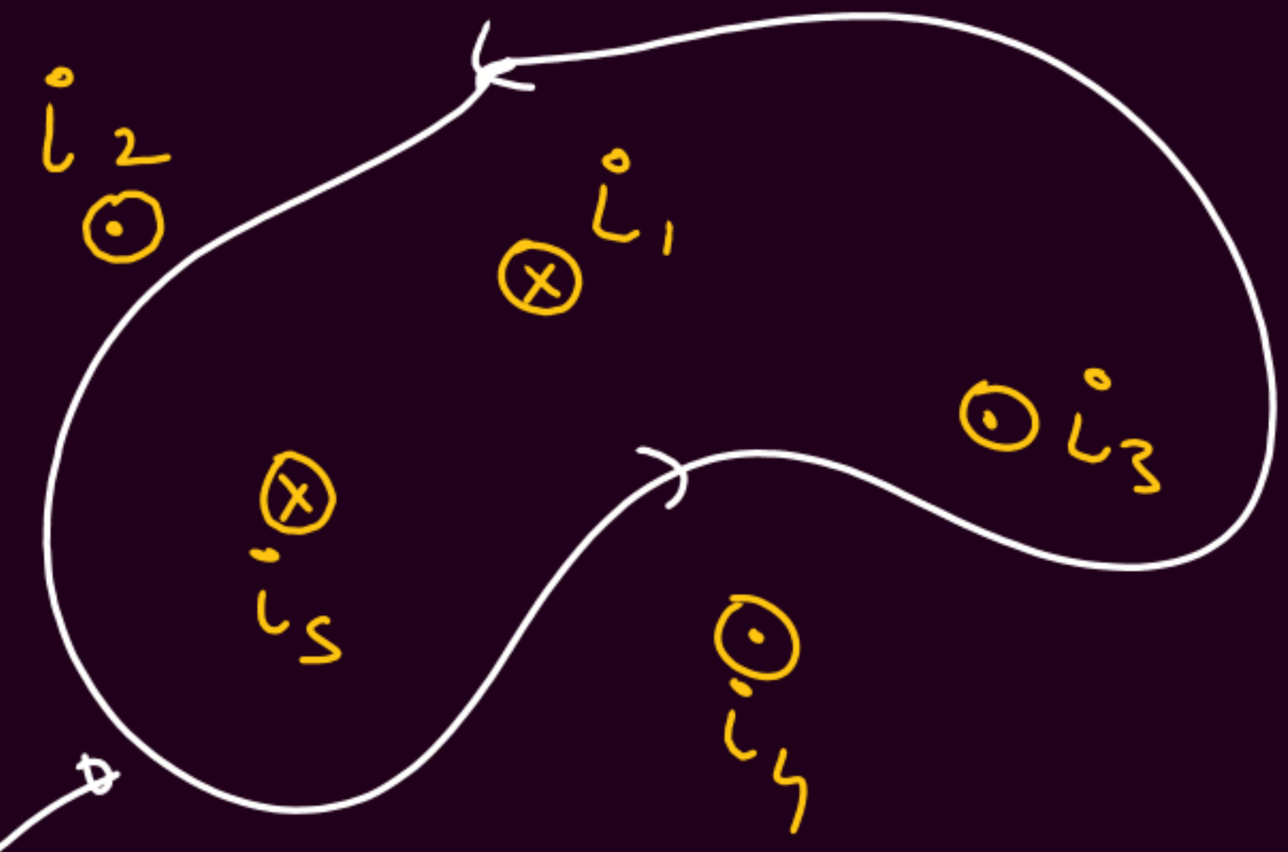
→ At sep. R

→ Same current in same sense



Ampere's Circuital law

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{en}}$$



Amperian loop

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (i_3 - i_1 - i_5)$$

→ Sign convention In the sensation of integration → i ⊕

→ only for closed loop

→ Depends on enclosed current

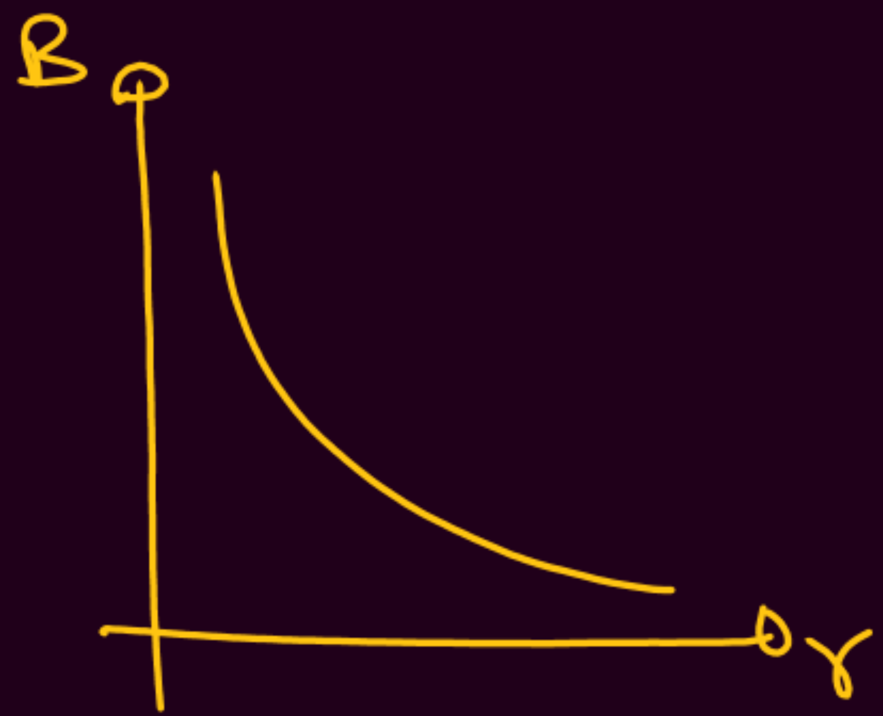
→ $\vec{B}$ due to all currents

→ used for $\vec{B}$ in symmetry

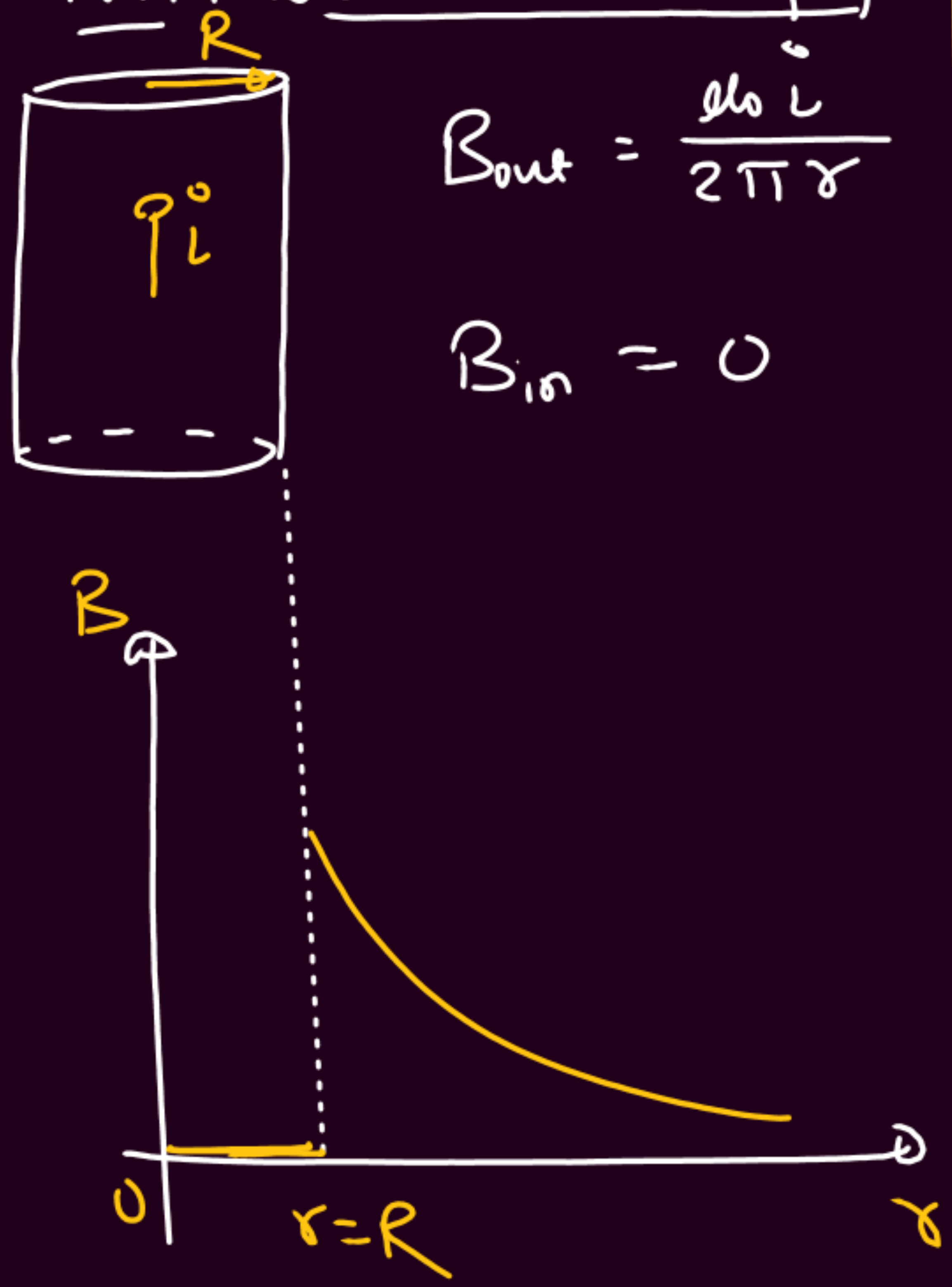
But valid in asymmetry

① Thin wire

$$\vec{B} = \frac{\mu_0 i}{2\pi r}$$



② Thin walled Hollow pipe



$$B_{out} = \frac{\mu_0 i}{2\pi r}$$

$$B_{in} = 0$$

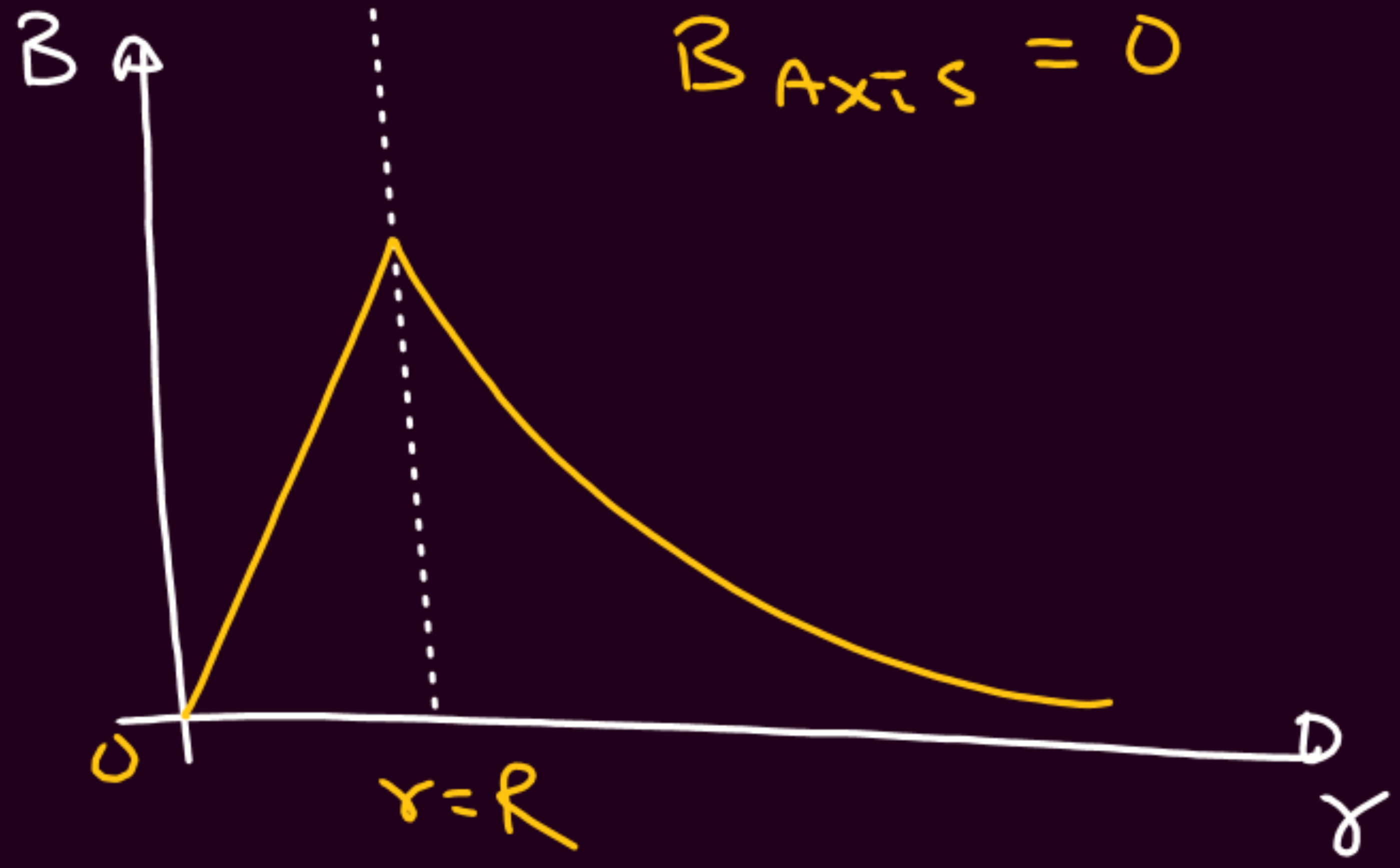
③ Solid Wire

$$B_{out} = \frac{\mu_0 i}{2\pi r}$$

$$B_{surface} = \frac{\mu_0 i}{2\pi R}$$

$$B_{in} = \frac{\mu_0 i r}{2\pi R^2}$$

$$B_{Axis} = 0$$



Q.

$r = 2R \qquad B_0$

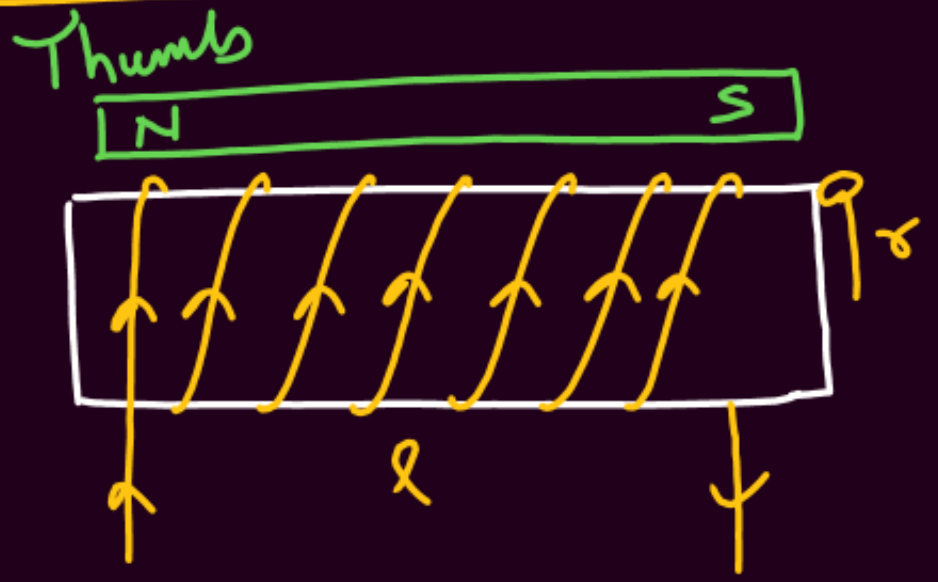
$r = R/3$

$$B_0 = \frac{\mu_0 i}{2\pi(2R)} = \frac{\mu_0 i}{4\pi R}$$

$$B = \frac{\mu_0 i (R/3)}{2\pi R^2} = \frac{\mu_0 i}{6\pi R}$$

$$B = \frac{4B_0}{6} = \frac{2B_0}{3}$$

Solenoid / Choke Coil / Inductor



N = Total no of turns

$n = \frac{N}{l}$ = turn Density

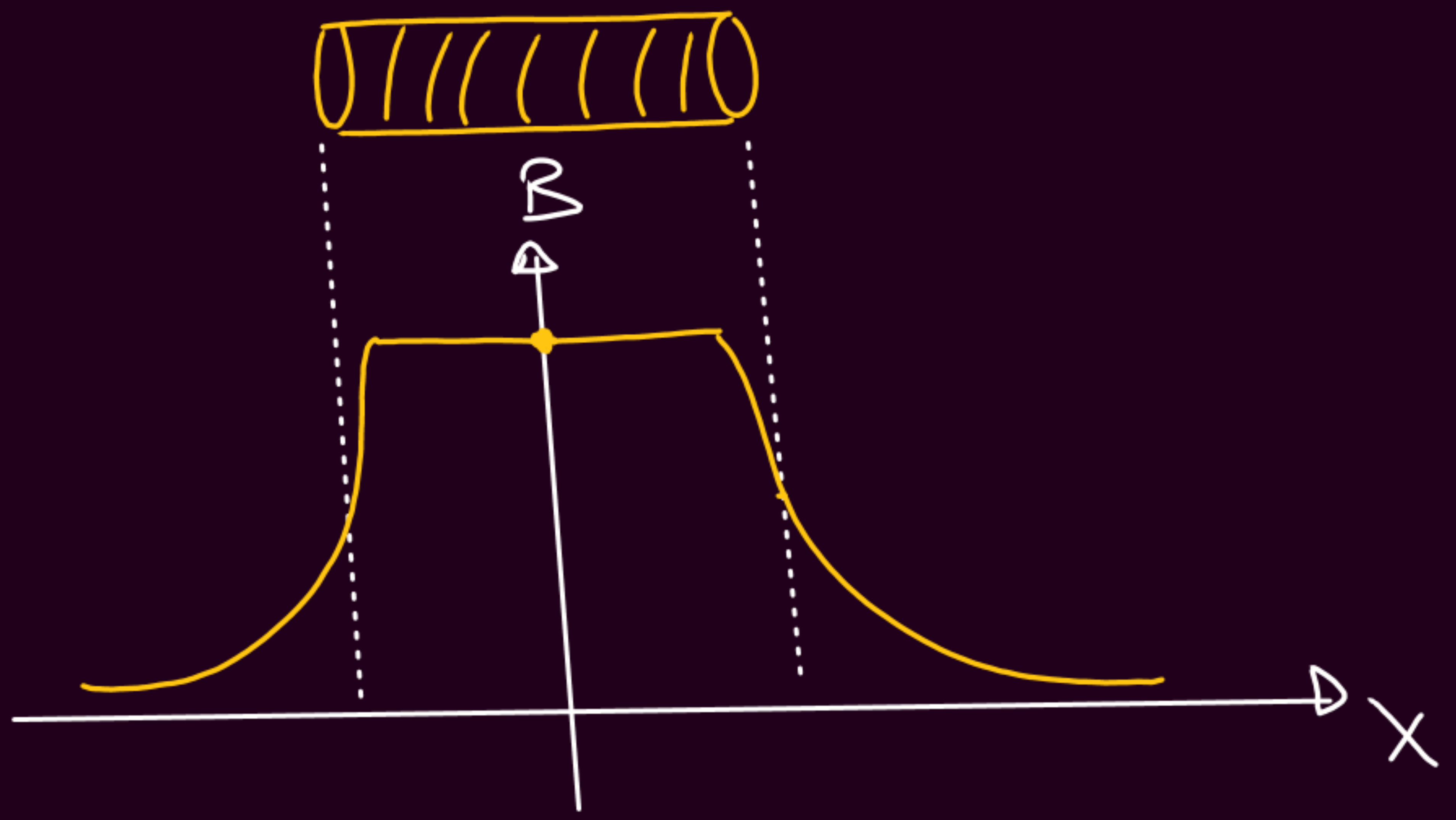
long Solenoid $(l >> r)$

↳ B_{in} is uniform

→ Act like Bar Magnet

$$B_{inside} = \mu_0 n i = \frac{\mu_0 N i}{l}$$

$$B_{end} = \frac{1}{2}(\mu_0 n i)$$



Q. Solenoid 50 turns/cm

$i = 2A$

$B_{inside} = ?$

$$B = \mu_0 n i$$

$$B = (4\pi \times 10^{-7}) \frac{50}{10^{-2}} (2)$$

$$\boxed{B = 4\pi \times 10^{-3} \text{ T}}$$

Q. Solenoid B_0

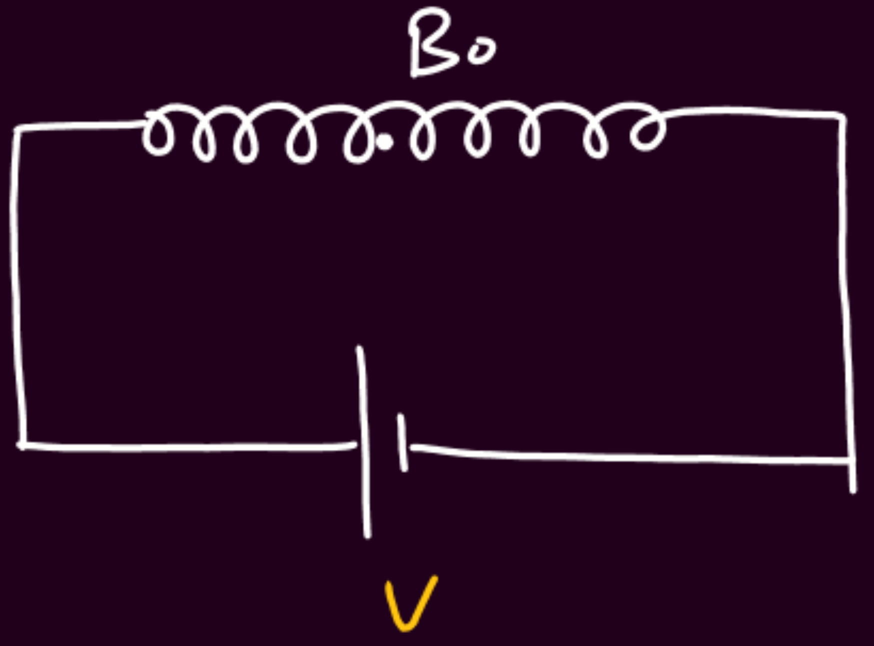
$N \rightarrow 2N$

$l \rightarrow l/2$

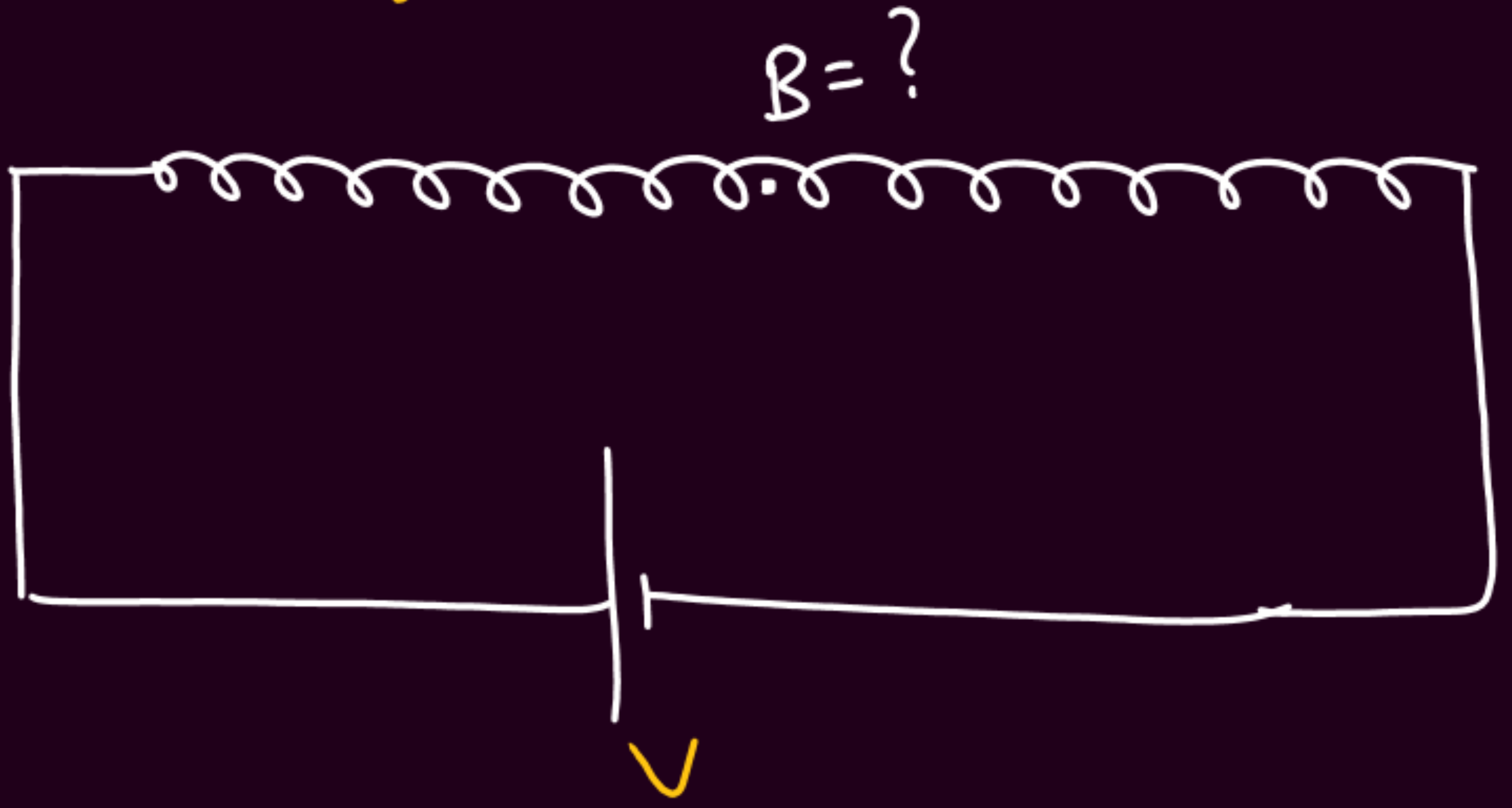
i (same)

$$B = \frac{\mu_0 N i}{l} \frac{(2)}{(1/2)} = \underline{\underline{4 B_0}}$$

Q.

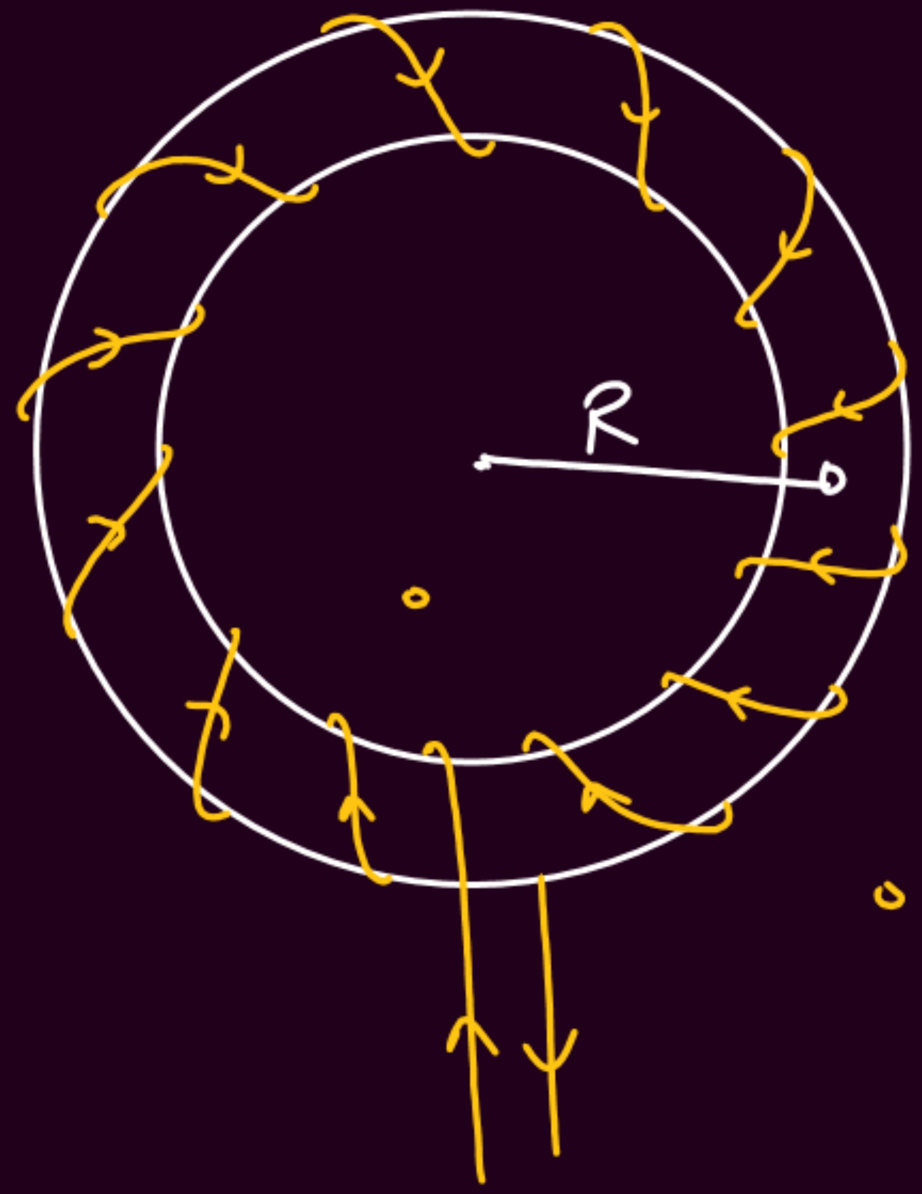


$$B_0 = \frac{\mu_0 N i}{l}$$



$$B = \frac{\mu_0 (2N)(i/2)}{(2l)} = \frac{B_0}{2}$$

Toroid



$$B_{in} = \frac{\mu_0 N i}{2\pi R} = \mu_0 n i$$

$$B_{out} = 0$$

Force on moving charge due to $\vec{B}$.

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

$F = qvB\sin\theta$

① Charge at Rest, $F_B = 0$

② If $\vec{v} \parallel \vec{B}$, $F_B = 0$

③ $\vec{F} \perp \vec{v}$ | $\vec{F} \cdot \vec{v} = 0$

$\vec{F} \perp \vec{B}$ | $\vec{F} \cdot \vec{B} = 0$

④ W.D. by $\vec{B}$ is 0
(Power)

⑤ KE = const.

Speed = const.

⑥ Velocity
- Speed (const.)
- Dir (change)

⑦ For ⊕ charge, Dir($\vec{F}$) = Dir($\vec{v} \times \vec{B}$)

For ⊖ charge, Dir($\vec{F}$) opp. of Dir($\vec{v} \times \vec{B}$)

Q $q = 2C$

$\vec{v} = (2\hat{i} - 3\hat{j})\ m/s$

$\vec{B} = (3\hat{i} + 2\hat{k})\ T$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 0 \\ 3 & 0 & 2 \end{vmatrix}$$

$$= (-6-0)\hat{i} - (4-0)\hat{j} + (0-(-9))\hat{k}$$

$$= -6\hat{i} - 4\hat{j} + 9\hat{k}$$

$$\vec{F} = 2(-6\hat{i} - 4\hat{j} + 9\hat{k})\ N$$

Q. If $\vec{B} = (2\hat{i} - 3\hat{j} + \alpha\hat{k})\ T$

Find α, $\vec{F} = (3\hat{i} - \hat{j} - 3\hat{k})\ N$

$$\vec{F} \perp \vec{B}$$

$$\vec{F} \cdot \vec{B} = 0$$

$$(2)(3) + (-3)(-1) + \alpha(-3) = 0$$

$$6 + 3 - 3\alpha = 0$$

$$9 = 3\alpha$$

$$\boxed{\alpha = 3}$$

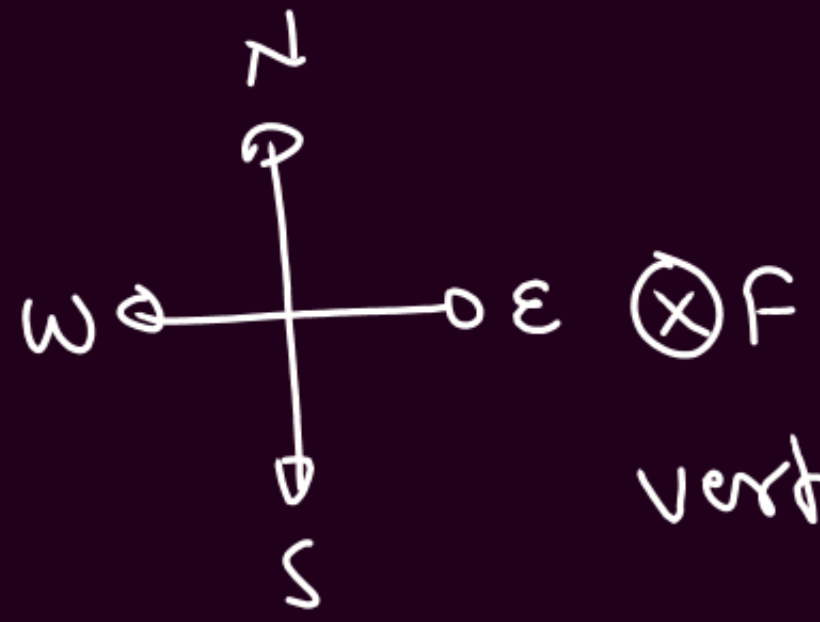
Q. $q = 2C$

$v = 10\ m/s$ East

$B = 0.5\ T$ South

$F = qvB\sin 90^\circ$

$F = (2)(10)(0.5) = 10\ N$

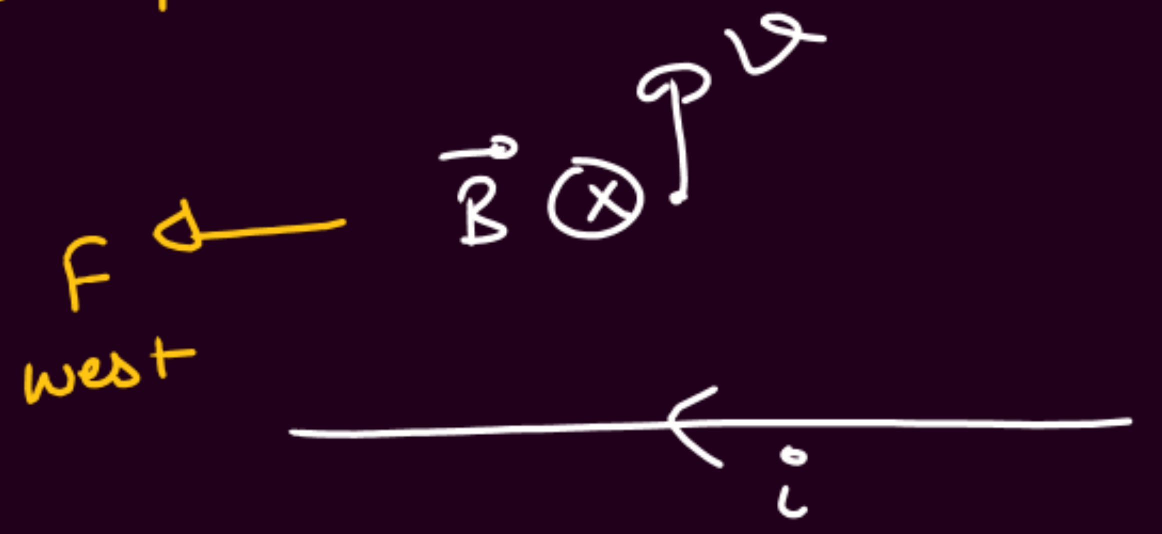


vertically Downward

$\hat{i} \times (-\hat{j}) = -\hat{k}$

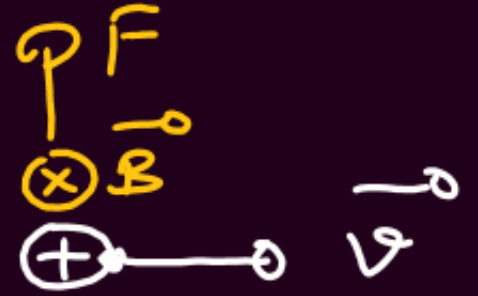
Q. Current carrying wire East to West

⊕ pt. charge above it vertically up.

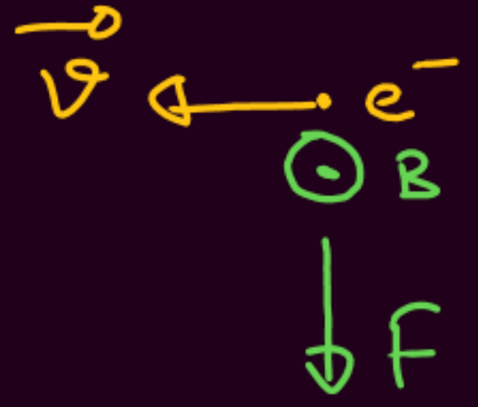


$\vec{v} \times \vec{B} = \hat{j} \times (-\hat{k}) = -\hat{i}$

Q



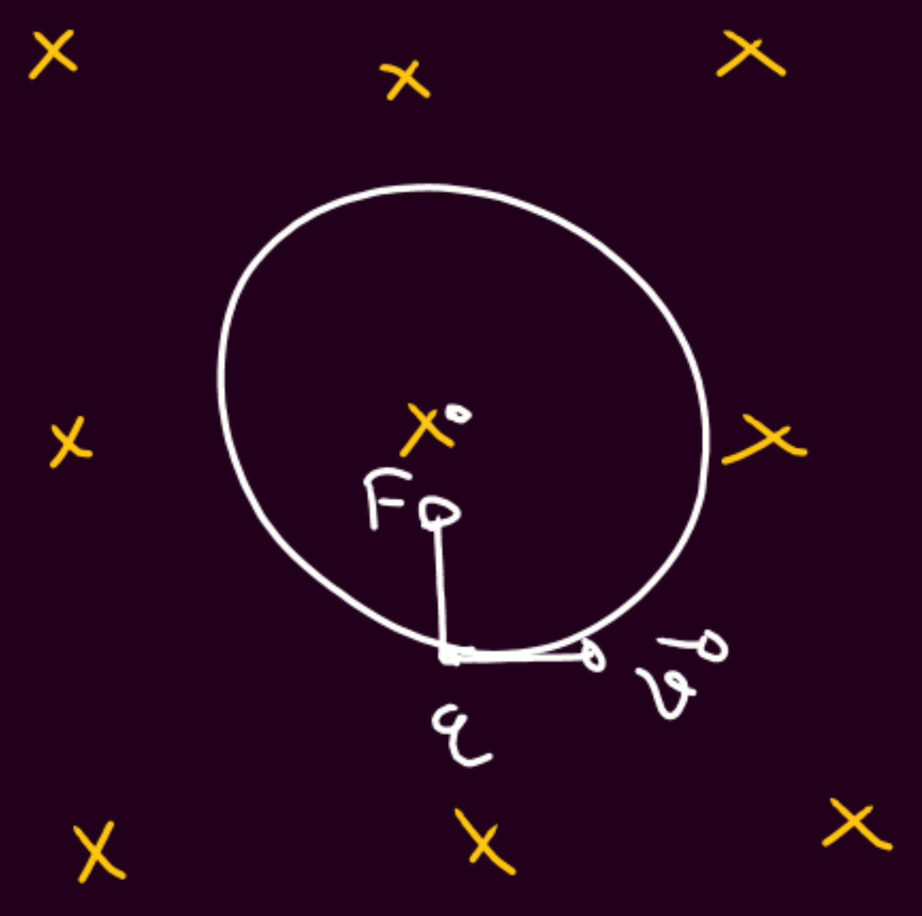
$$\hat{i} \times (-\hat{k}) = -(\hat{i} \times \hat{k})$$
$$= -(-\hat{j})$$
$$= \hat{j}$$



$$\vec{F} = q(\vec{v} \times \vec{B})$$
$$= -(-\hat{i} \times \hat{k})$$
$$= \hat{i} \times \hat{k}$$
$$= -\hat{j}$$

Motion of q in uniform $\perp$ $\vec{B}$

- Transverse $\vec{v} \perp \vec{B}$
- $\boxed{F = qvB}$



- UCM
- Centripetal

$$\frac{mv^2}{r} = qvB$$

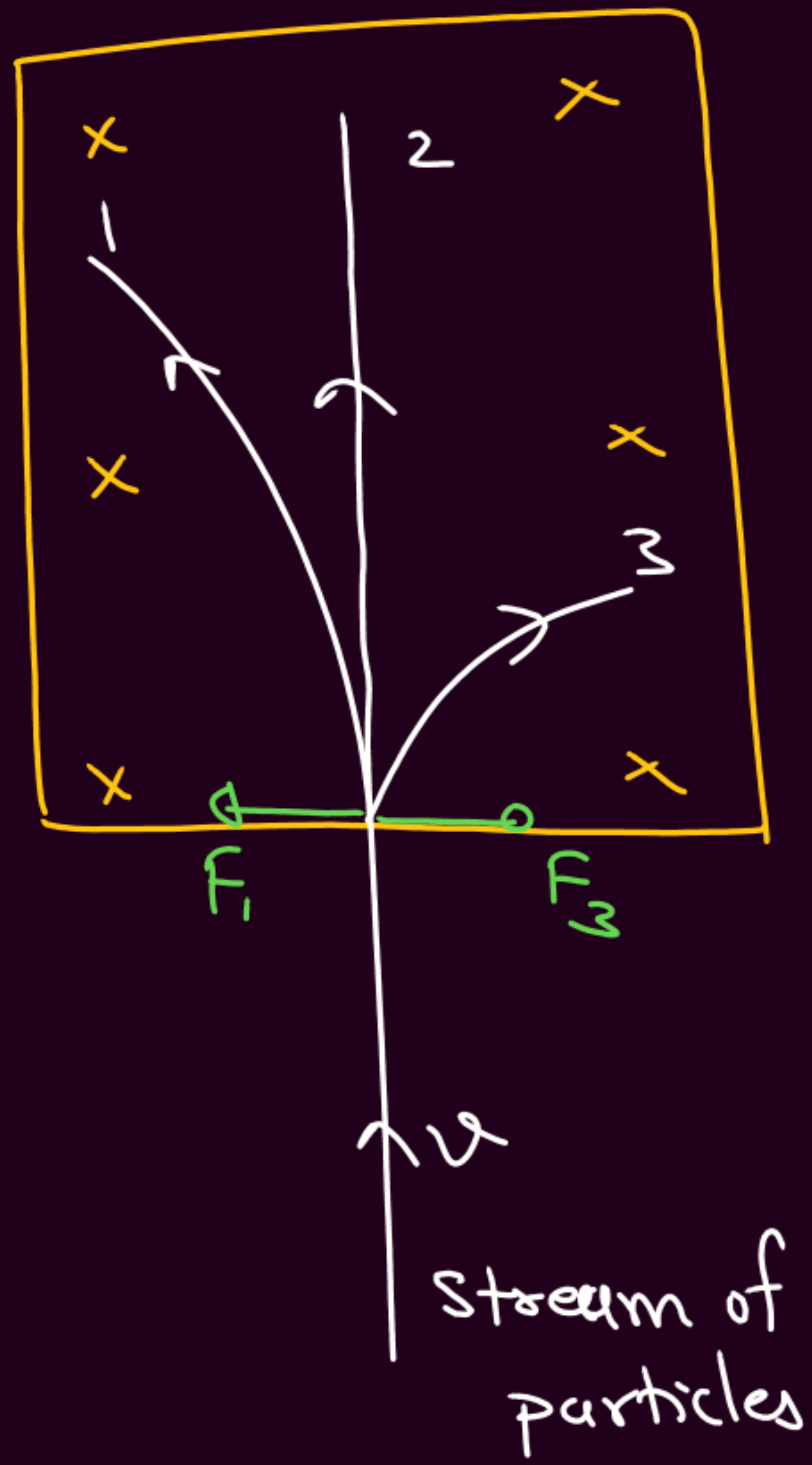
$$\boxed{K = qV_a}$$

$$\boxed{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV_a}{q}}}$$

$$T = \frac{2\pi r}{v}$$

$$\boxed{T = \frac{2\pi m}{qB}}$$

$$\boxed{f = \frac{qB}{2\pi m}}$$ cyclotron freq.



② → <u>Neutral</u>

$$\text{Dir}(\vec{v} \times \vec{B})$$

<u>west</u>

① ⊕

③ ⊖

$$r_1 > r_3$$

$$\left(\frac{mv}{qB}\right)_1 > \left(\frac{mv}{qB}\right)_3$$

$$\left(\frac{m}{q}\right)_1 > \left(\frac{m}{q}\right)_3$$

$$\left(\frac{q}{m}\right)_3 > \left(\frac{q}{m}\right)_1$$

Q. Proton & deutron in transverse B same radius

① Ratio of v

p d

$$\frac{m v_1}{eB} = \frac{2m v_2}{eB}$$

$$\boxed{\frac{v_1}{v_2} = \frac{2}{1}}$$

proton e e m

Deutron (p n) e 2m

Alpha (p n n p) 2e 4m

② Ratio of p

$$\frac{p_1}{eB} = \frac{p_2}{eB}$$

$$\frac{p_1}{p_2} = \frac{1}{1}$$

③ Ratio of KE

$$\frac{\sqrt{2mK_1}}{eB} = \frac{\sqrt{2(2m)K_2}}{eB}$$

$$K_1 = 2K_2$$

$$\frac{K_1}{K_2} = \frac{2}{1}$$

④ Ratio of V_a

$$\frac{1}{B}\sqrt{\frac{2mV_1}{e}} = \frac{1}{B}\sqrt{\frac{2(2m)V_2}{e}}$$

$$\frac{V_1}{V_2} = \frac{2}{1}$$

Q. Deutron and α in transverse B. Find ratio of radius

① Same speed

$$r = \frac{mv}{qB} \qquad v = \frac{qBr}{m}$$

$$\frac{(e)Br_1}{2m} = \frac{(2e)Br_2}{(4m)}$$

$$\boxed{\frac{r_1}{r_2} = \frac{1}{1}}$$

② Same p

$$r = \frac{p}{qB} \qquad p = qBr$$

$$eBr_1 = (2e)Br_2$$

$$\boxed{\frac{r_1}{r_2} = \frac{2}{1}}$$

③ Same KE

$$r = \frac{\sqrt{2mK}}{qB}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{2m}{4m}} \; \frac{2e}{e}$$

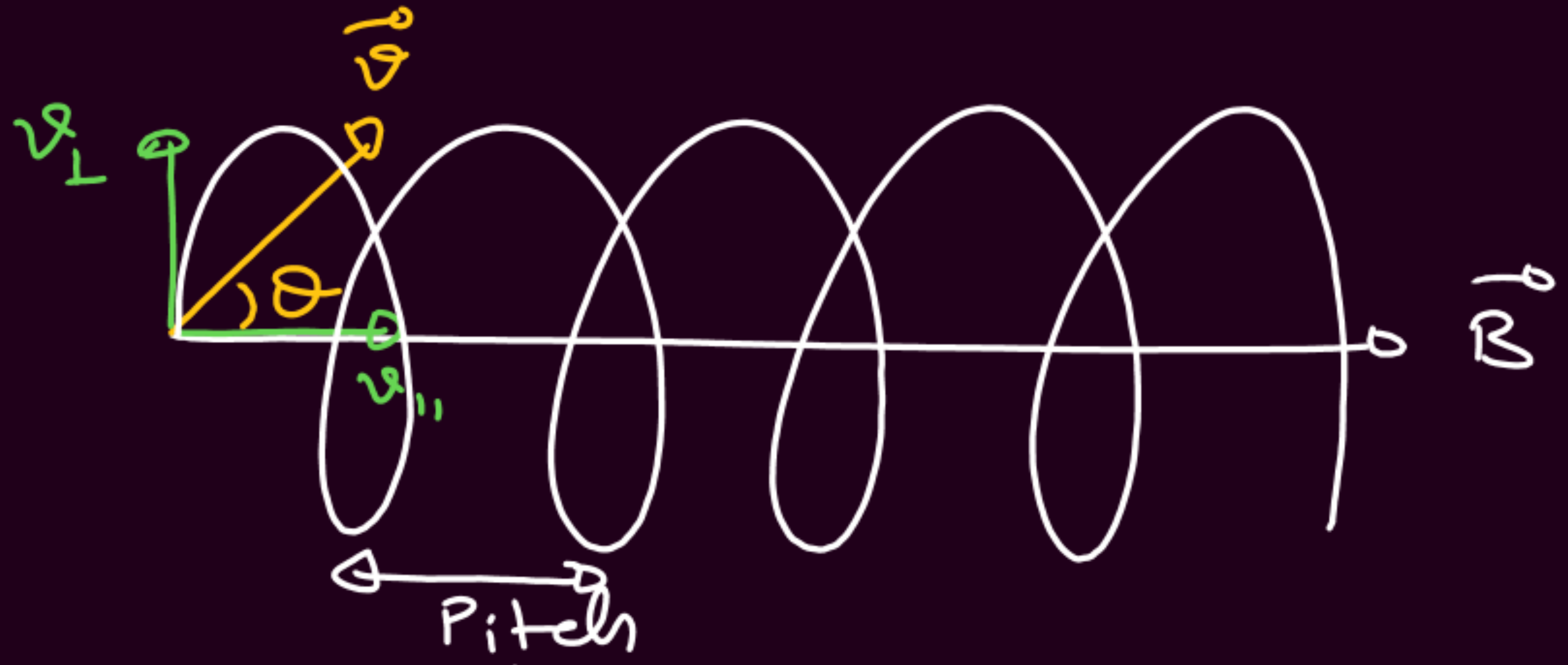
$$\boxed{\frac{r_1}{r_2} = \sqrt{2}}$$

④ Same V_a

$$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{2m(2e)}{4m(e)}} = \frac{1}{1}$$

uniform $\vec{B}$ at θ to $\vec{u}$



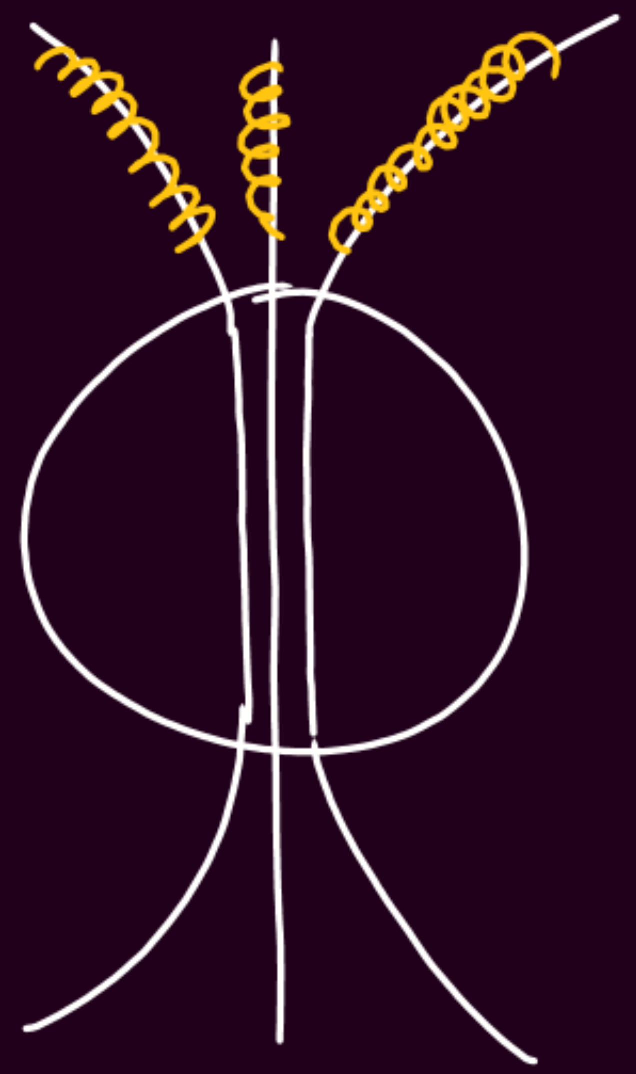
Helical Path

① $r = \dfrac{m v_\perp}{qB}$

② $T = \dfrac{2\pi m}{qB}$

③ $P = v_{\parallel} T = \dfrac{2\pi m v_{\parallel}}{qB}$

Aurora Borialis



$N_2 \rightarrow$ Pink

$O_2 \rightarrow$ Green

Lorentz Force

$$\vec{F} = \vec{F}_e + \vec{F}_B$$

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

① Only uniform $\vec{E}$ $(\vec{B} = 0)$

→ Rest or $\vec{u} \parallel \vec{E}$ — st. line

→ $\vec{u}$ not parallel $\vec{E}$ — parabolic

② Only uniform $\vec{B}$ $(\vec{E} = 0)$

→ Rest — $F = 0$ — Rest

→ $\vec{u} \parallel \vec{B}$ — $F = 0$ — (st. line with uniform $\vec{u}$)

→ $\vec{v} \perp \vec{B}$ — Circular (UCM)

→ $\vec{v}$ oblique to $\vec{B}$ — [Helical with uniform pitch]

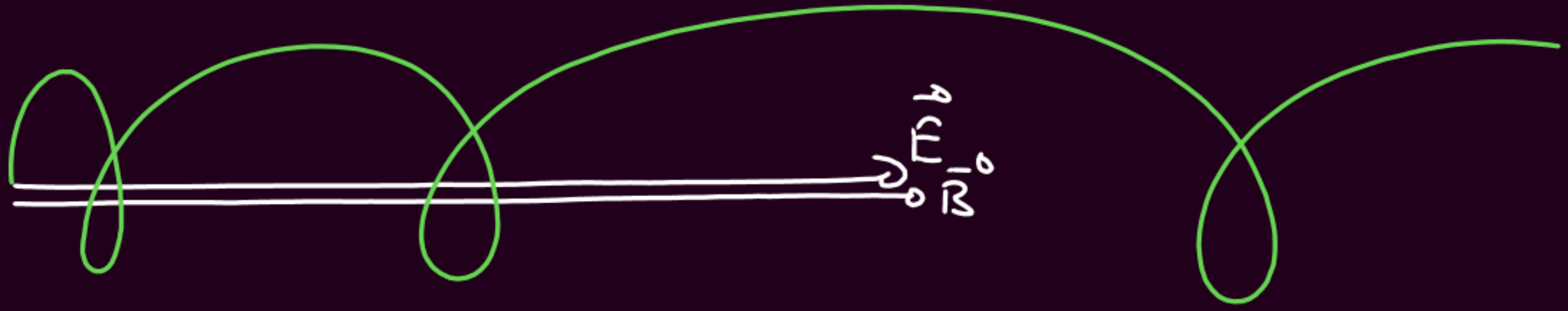
③ Uniform $\vec{E} \parallel \vec{B}$

① Rest — st. line — speed ↑ — $F_B = 0$

② $\vec{u} \parallel \vec{E} \parallel \vec{B}$ — st. line — speed ↑/↓ — $F_B = 0$

③ $\vec{u} \perp (\vec{E} \parallel \vec{B})$

$\vec{u}$ oblique $(\vec{E} \parallel \vec{B})$



Helical path with non-uniform pitch

④ – Uniform $\vec{E}$ & $\vec{B}$

– $\vec{v} \perp \vec{E} \perp \vec{B}$

– $F_{net} = 0$

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

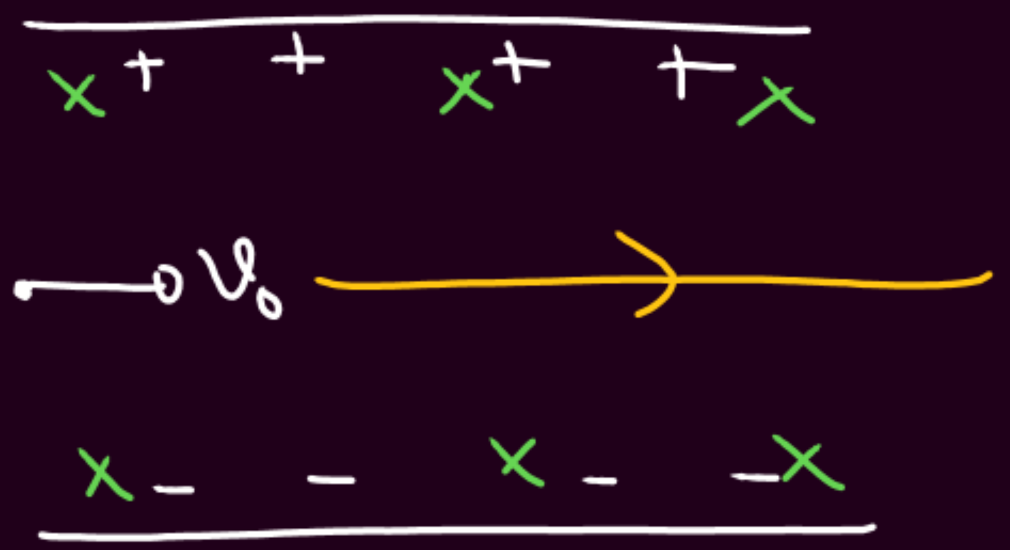
$$\vec{E} + (\vec{v} \times \vec{B}) = 0$$

① $E = vB$

$$\boxed{v_0 = \frac{E}{B}}$$

② $Dir(\vec{v}) = Dir(\vec{E} \times \vec{B})$

$$\left[v\,E\,\underbrace{B\,v\,E}\,B \right]$$



Dir

$$\vec{B} \times \vec{v} = \vec{E}$$

$$-\hat{k} \times \hat{i} = -\hat{j}$$

Q Const. $\vec{v}$

1) $E = 0$ $B = 0$ ✓

2) $E \neq 0$ $B = 0$ ✗

3) $E = 0$ $B \neq 0$ ✓ $(\vec{v} \parallel \vec{B})$

4) $E \neq 0$ $B \neq 0$ ✓

Q Undeviated (Dir same)

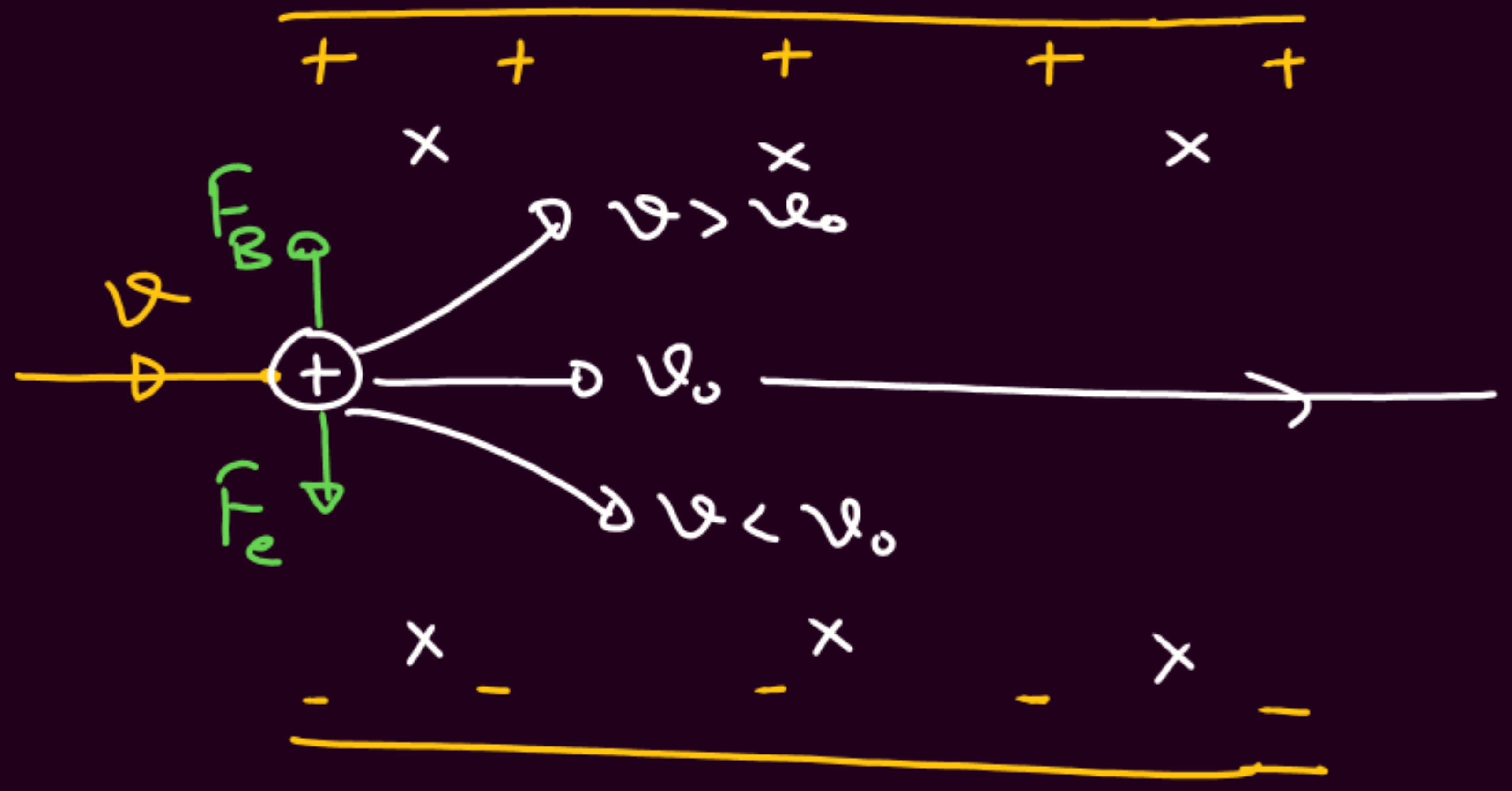
1) $E = 0$ $B = 0$ ✓

2) $E \neq 0$ $B = 0$ ✓

3) $E = 0$ $B \neq 0$ ✓

4) $E \neq 0$ $B \neq 0$ ✓

Velocity Selector



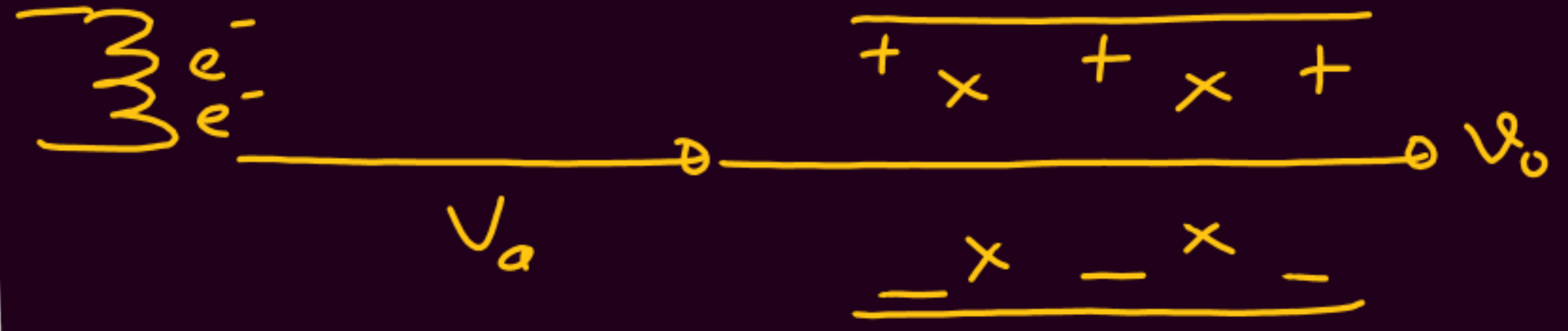
① $\boxed{v_0 = \frac{E}{B}}$ 'undeviated'

② $v > v_0$

③ $v < v_0$

Thomson's Exp.

Thermionic Emission



① $\frac{1}{2} m v^2 = eV_a$ ② $v = \frac{E}{B}$

$$\frac{1}{2} m \frac{E^2}{B^2} = eV_a$$

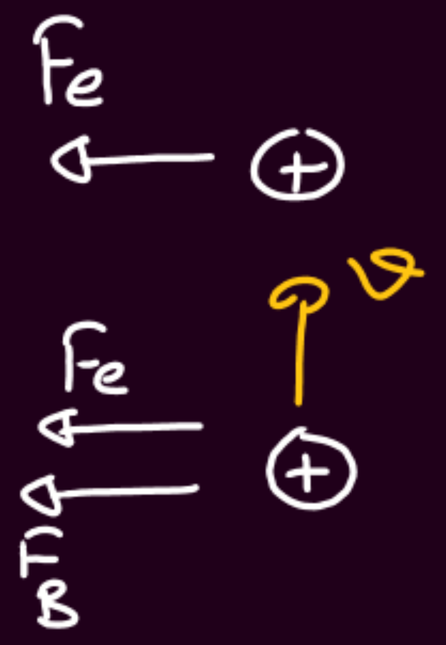
$$\boxed{\frac{e}{m} = \frac{E^2}{2B^2 V_a}}$$

NEET-2013

Proton

① Rest $\quad a_0$ west

② $v_0 \uparrow \quad 3a_0$ west



$$ma_0 = eE$$

$$\boxed{E = \frac{ma_0}{e}}$$ west

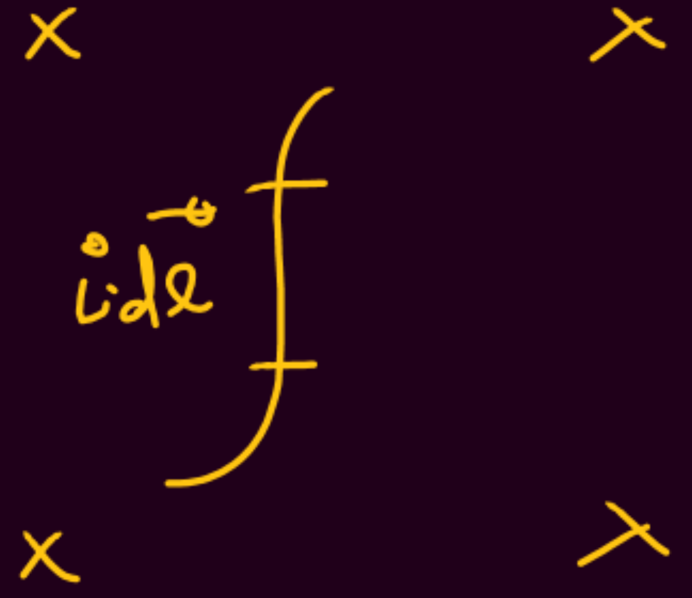
$$m(2a_0) = evB$$

$$\boxed{B = \frac{2ma_0}{ev}}$$ Down

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

$$-\hat{i} = \oplus\left(\hat{j} \times (\quad)\right)$$

$\vec{F}$ on Current Carrying Conductor in $\vec{B}$



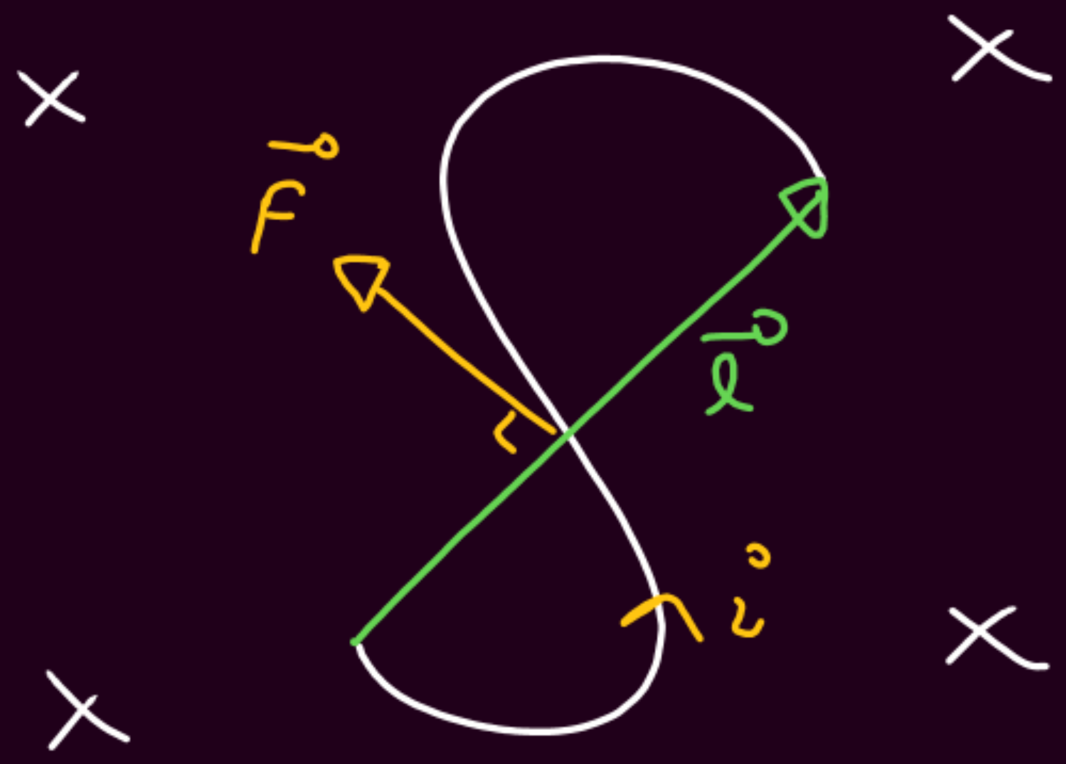
$$\vec{F} = \int \left(i\, d\vec{l} \times \vec{B} \right)$$

RHTR

Uniform $\vec{B}$

$$\boxed{\vec{F} = i\left(\vec{l} \times \vec{B}\right)}$$

$\vec{l} \rightarrow$ Displacement vector

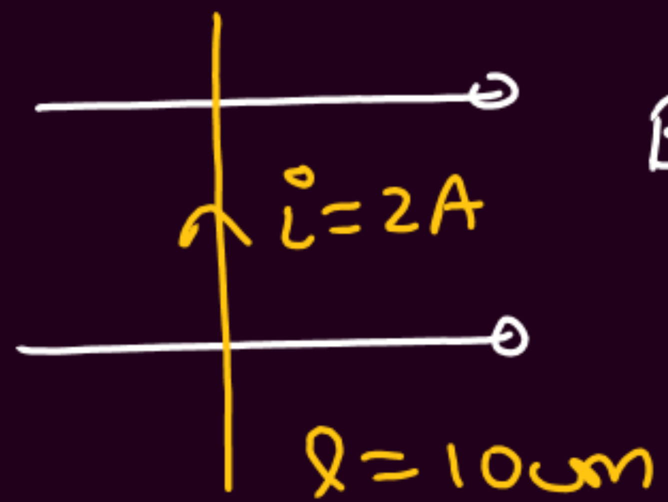


$\boxed{\vec{F}_{loop} = 0}$ if $\vec{B}$ = const.

Uniform Transverse B

$$\boxed{F = Bil}$$

Q



$B = 10^{-3}$ T

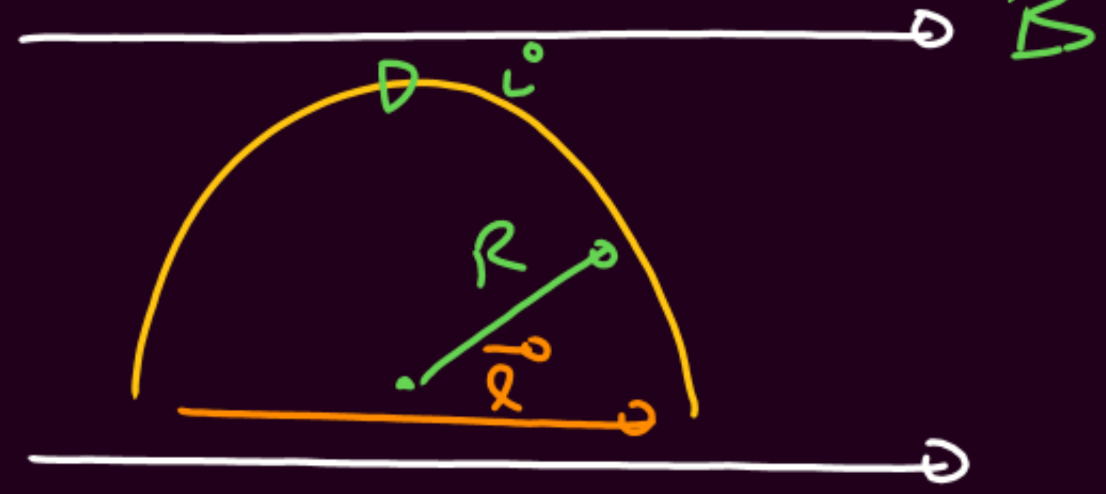
$$F = Bil = (10^{-3})(2)(0.1)$$

$$F = 2 \times 10^{-4} \text{ N} \quad \otimes$$

$$\vec{F} = i\,\vec{l} \times \vec{B}$$

$$= \hat{j} \times \hat{i} = -\hat{k}$$

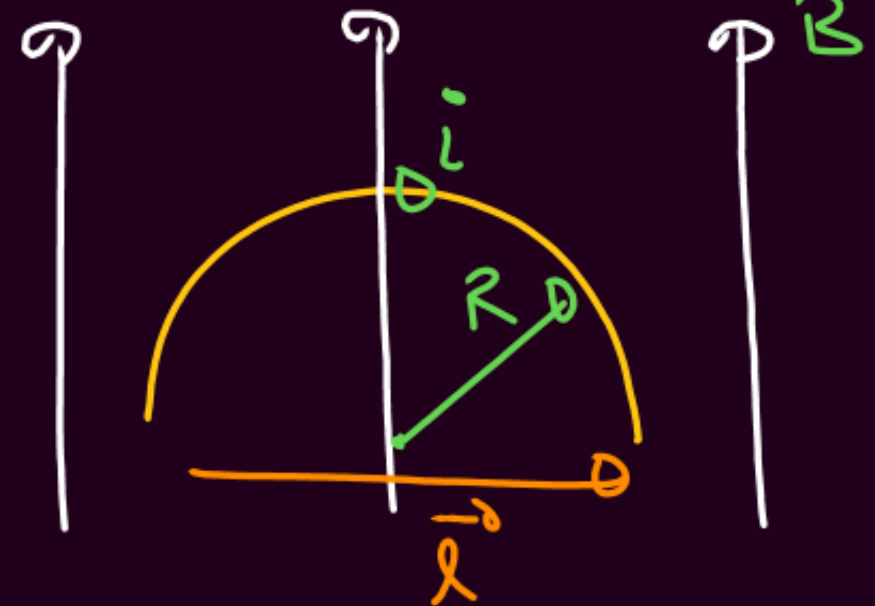
Q.



$\vec{l} \parallel \vec{B}$

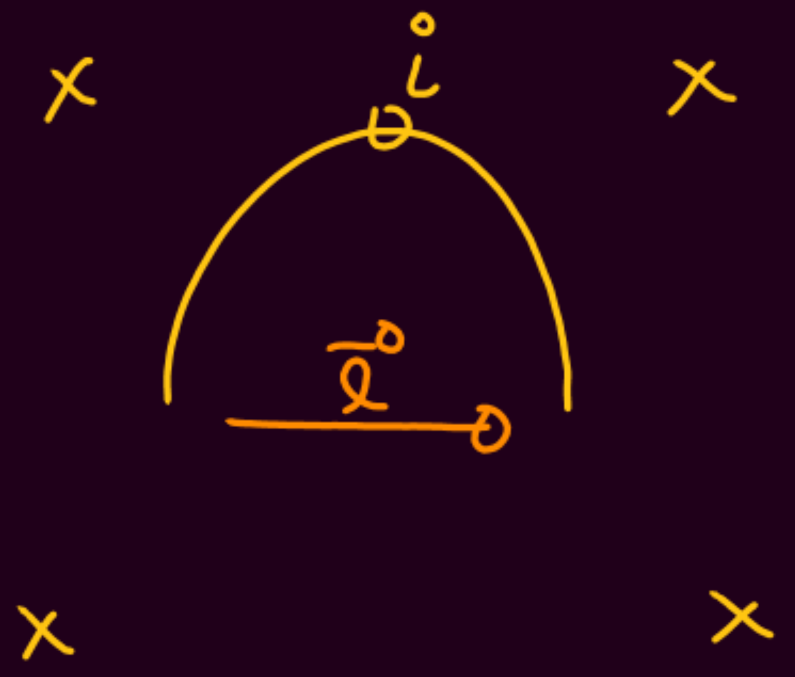
$$\boxed{F = 0}$$

Q



$$F = Bi(2R) \quad \odot$$

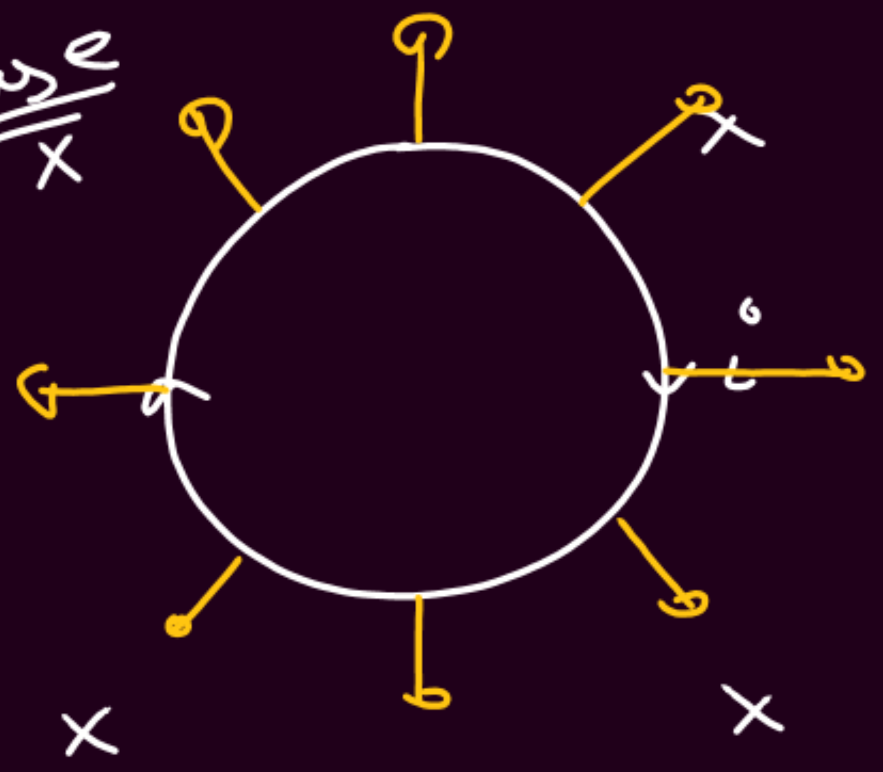
Q.



$$F = Bi(2R) \quad \hat{j}$$

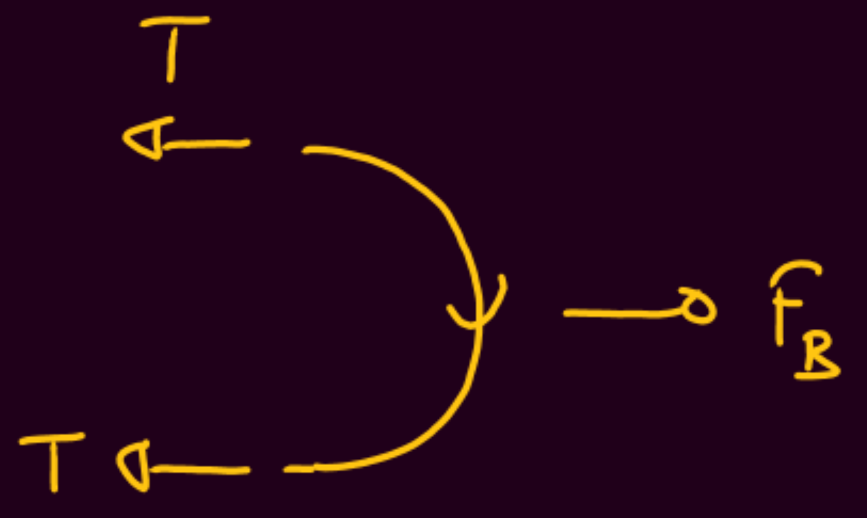
$$(\hat{i} \times -\hat{k}) = \hat{j}$$

Case ×



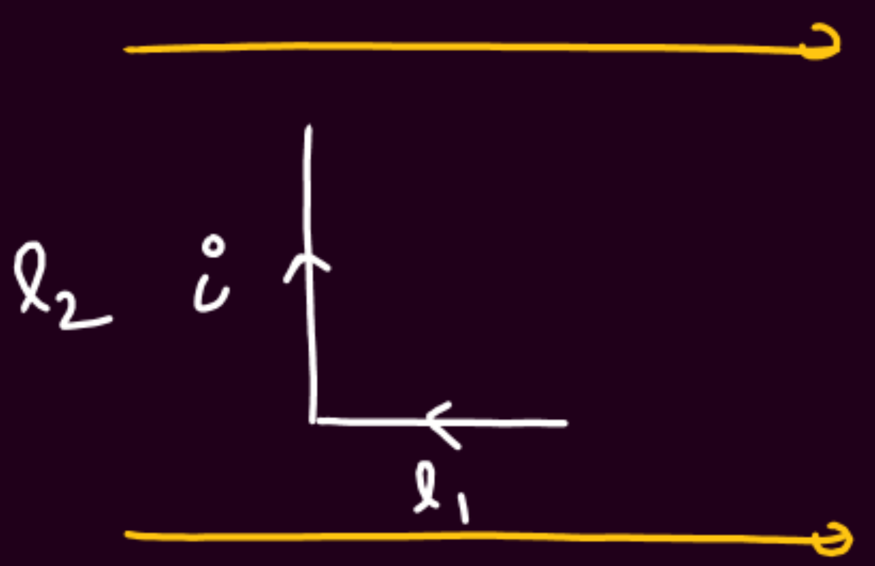
$$\boxed{F_{loop} = 0}$$

$$\boxed{\text{Tension} = BiR}$$



$$2T = Bi(2R)$$

Q

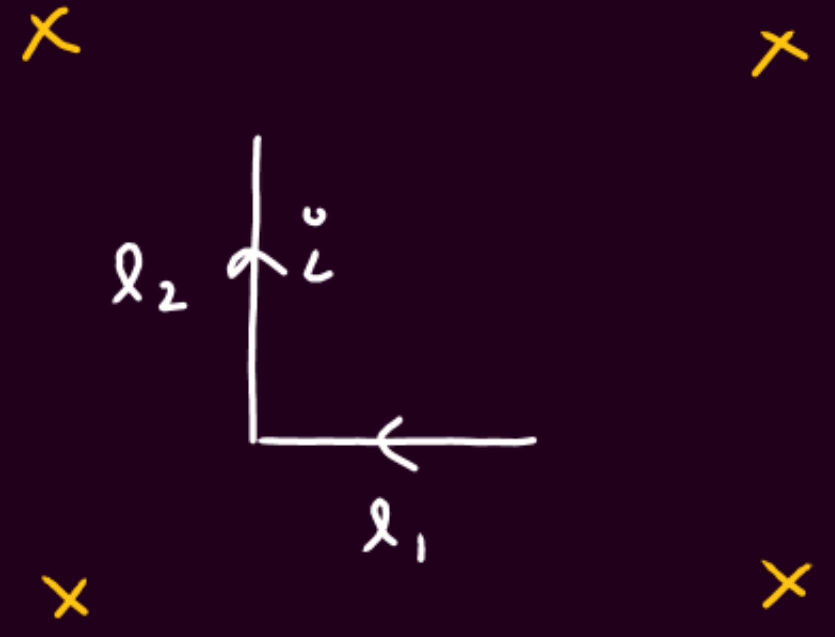


$F_1 = 0 \quad (\vec{l} \parallel \vec{B})$

$F_2 = Bil_2 \otimes$

$\boxed{\hat{j} \times \hat{i} = -\hat{k}}$

Q

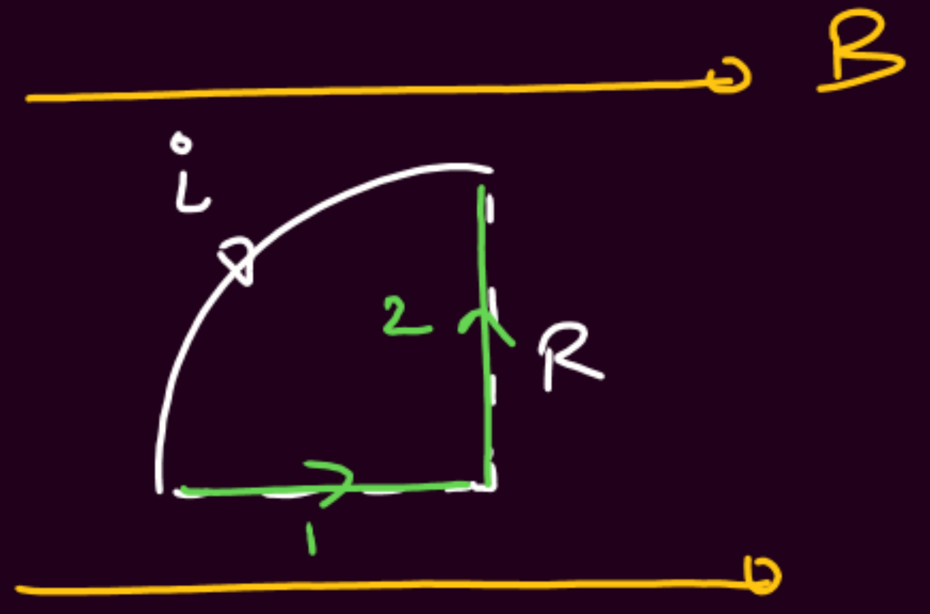


$F_1 = Bil_1(-\hat{j})$

$F_2 = Bil_2(-\hat{i})$

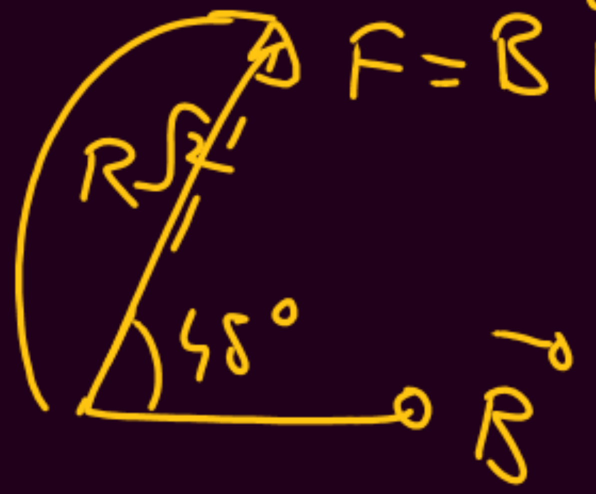
$F = Bi\sqrt{l_1^2 + l_2^2}$

Q.



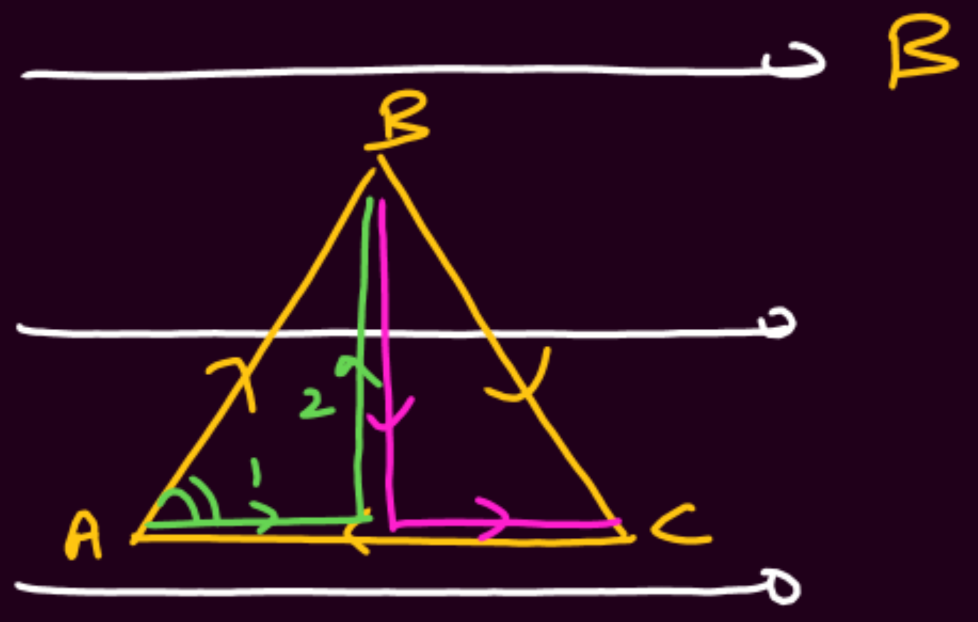
$F_1 = 0$

$$\boxed{F_2 = BiR} \quad \otimes$$



$F = Bi(R\sqrt{2})\sin 45^\circ$

Q

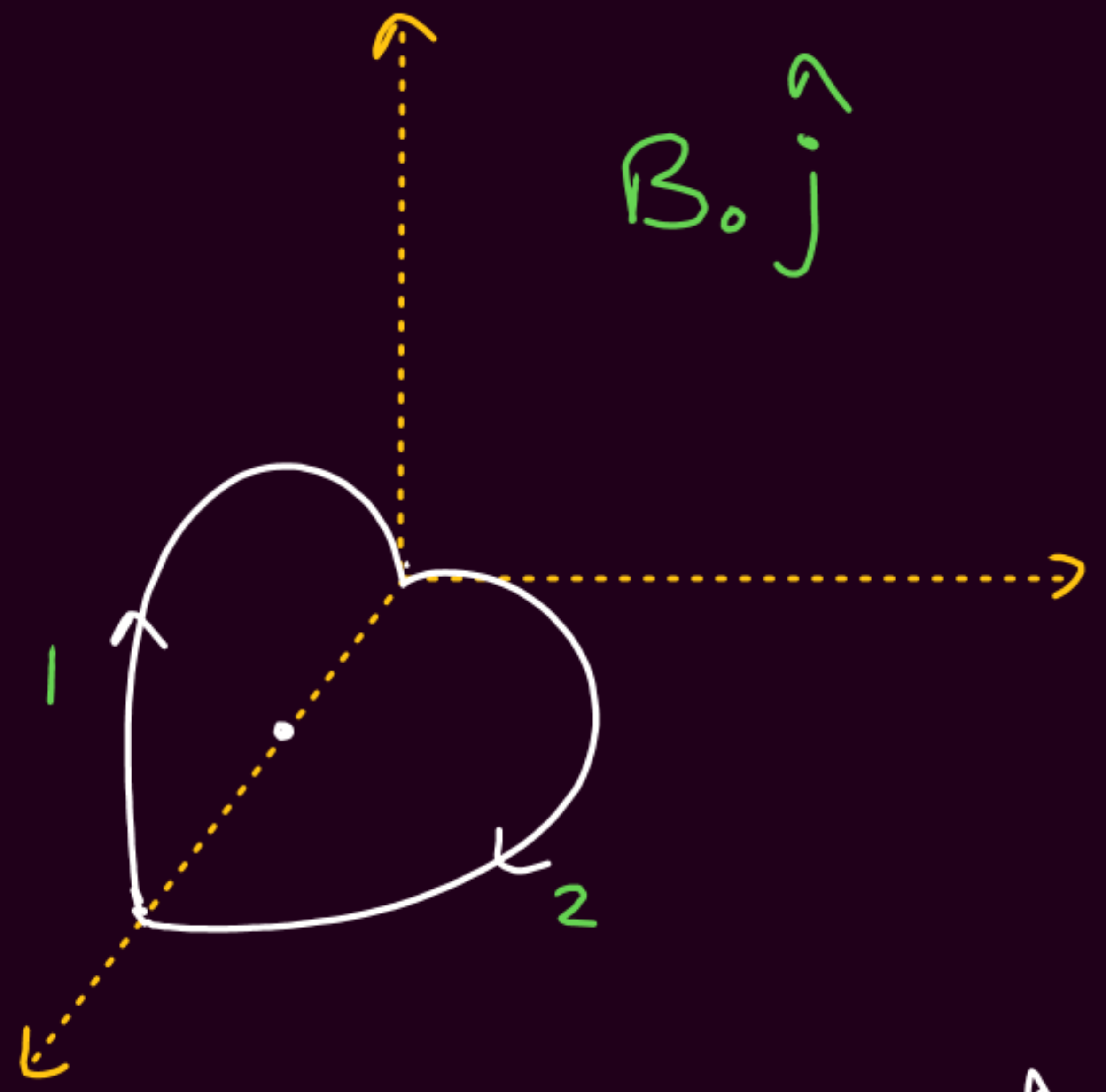


$F_{loop} = 0$

$F_{AC} = 0$

$F_{AB} = 0 + Bi\left(\frac{l\sqrt{3}}{2}\right) \quad \otimes$

$F_{BC} = Bi\left(\frac{l\sqrt{3}}{2}\right) + 0 \quad \odot$



$$F_1 = B_0 i (2R) \; \hat{i}$$

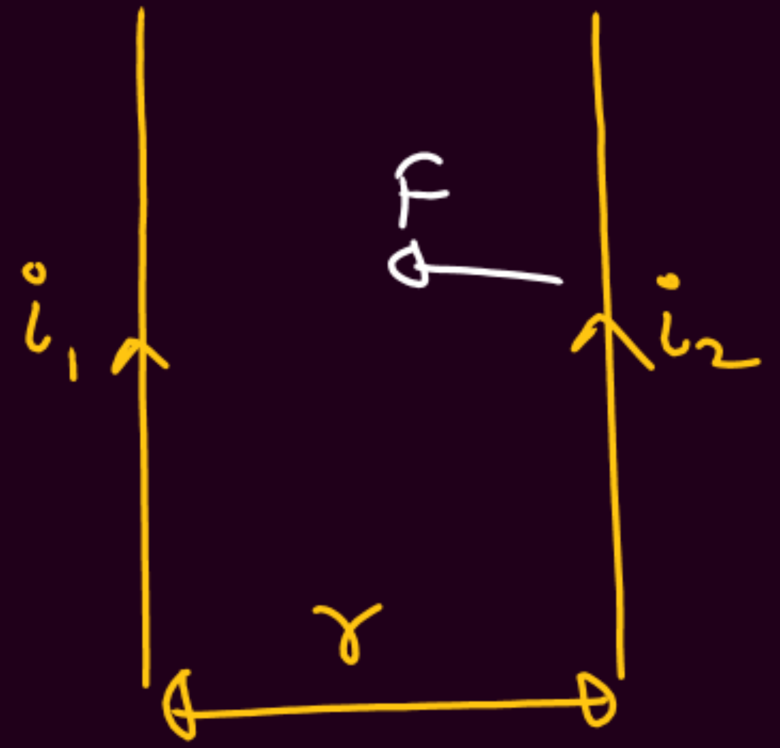
$$-\hat{k} \times \hat{j} = \hat{i}$$

$$F_2 = B_0 i (2R) \; -\hat{i}$$

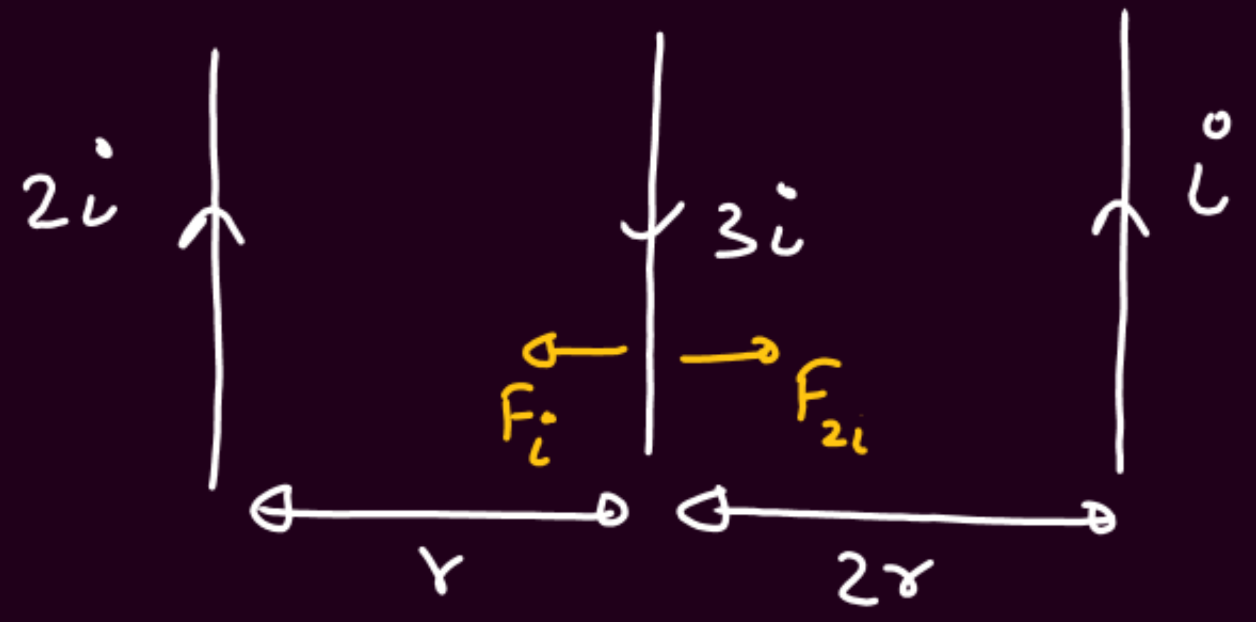
$$\hat{k} \times \hat{j} = -\hat{i}$$

→ like i → Attract
Unlike Repel

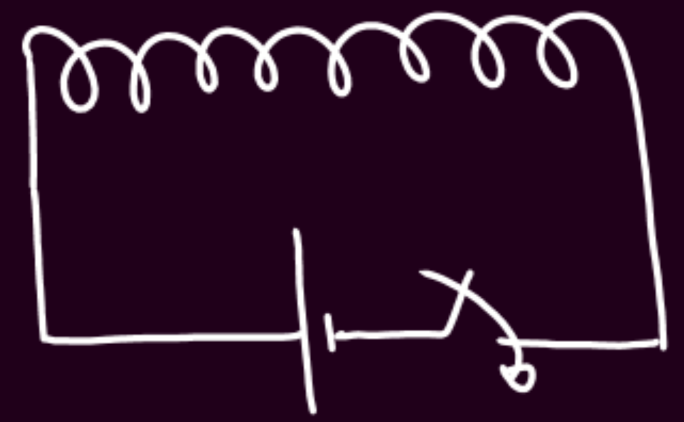
→



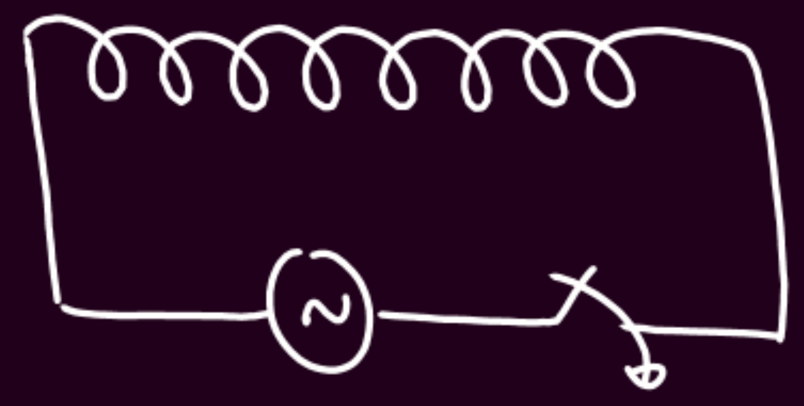
$$\boxed{\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi r}}$$



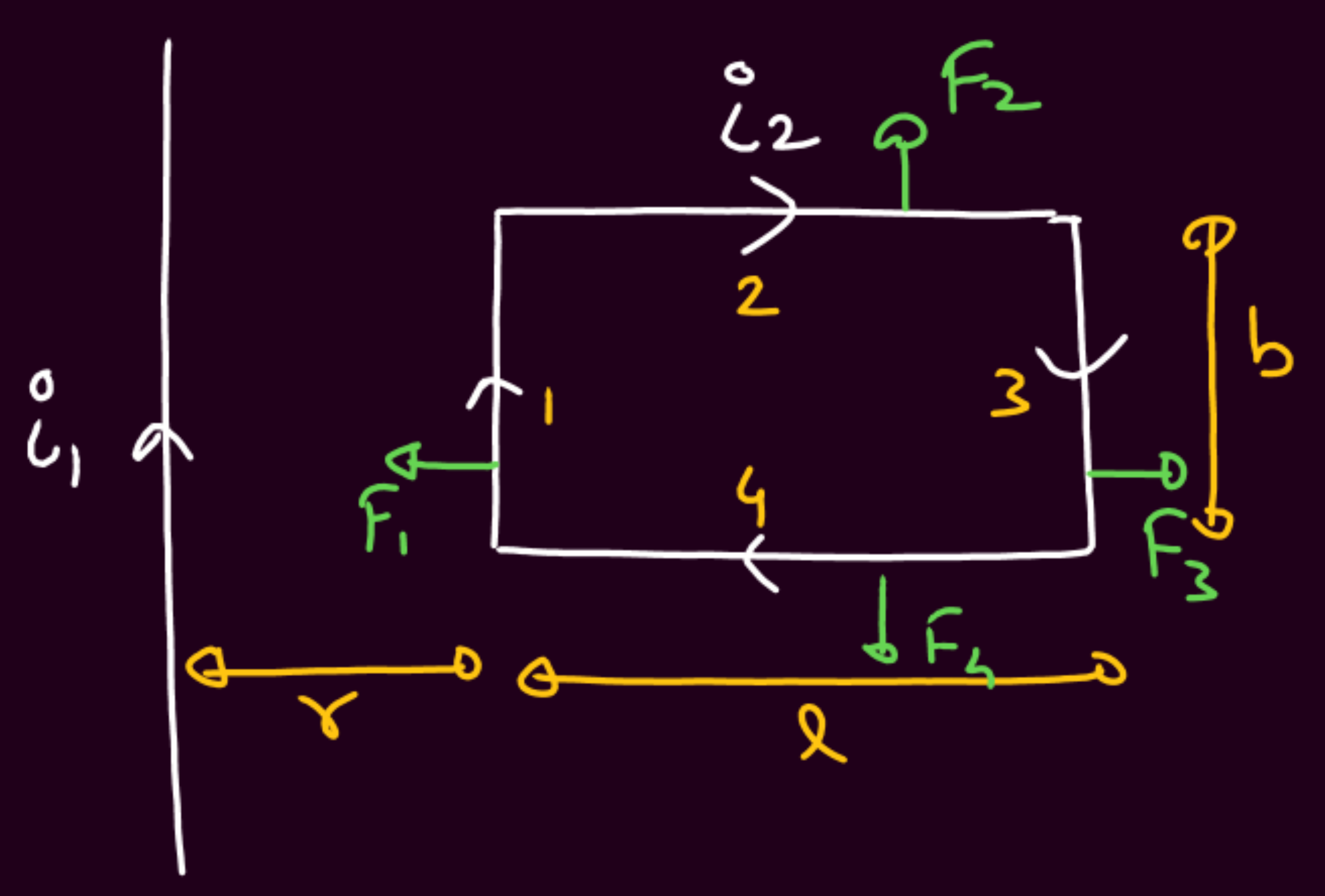
$$\frac{F_{net}}{l} = \frac{\mu_0 (2i)(3i)}{2\pi r} - \frac{\mu_0 (i)(3i)}{2\pi (2r)} = \frac{9\mu_0 i_1 i_2}{4\pi r}$$



Compress



Compress & Relax → Oscillate



$$F_{net} = F_1 - F_3$$

$$F_{net} = \frac{\mu_0 i_1 i_2}{2\pi r} b - \frac{\mu_0 i_1 i_2}{2\pi (r+l)} b$$

$$F_{net} = \frac{\mu_0 i_1 i_2 b l}{2\pi \; r(r+l)}$$

$F_{net} = ?$

$F_2 = F_4$

Stat: 1 2 cathode Rays → attract (e^-) X

Stat: 2 Like i attract ✓

Magnetic Dipole Moment

$$\boxed{\vec{M} = N i \vec{A}}$$

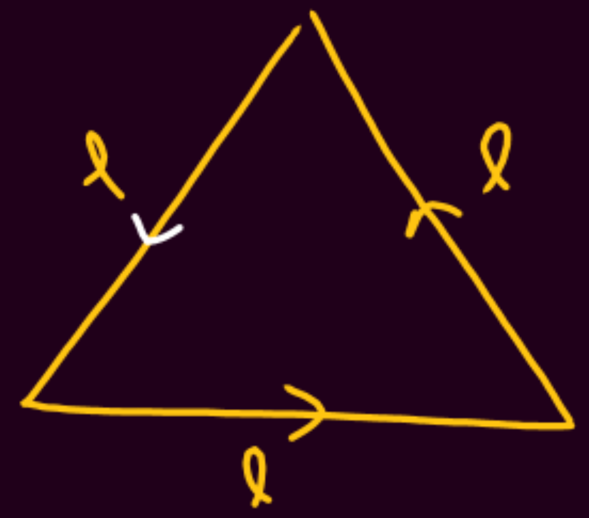
$[A\text{-}m^2]$

RHTR

ACW → N (Thumb)

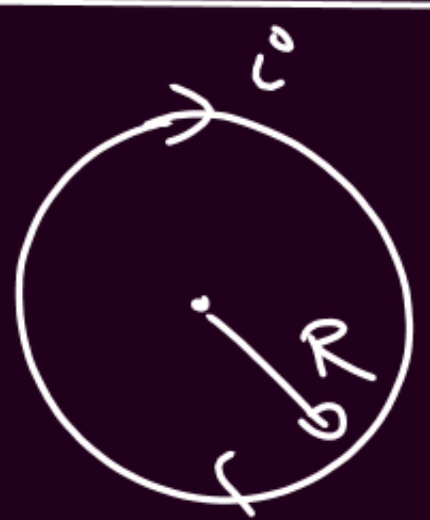
CW → S

Q

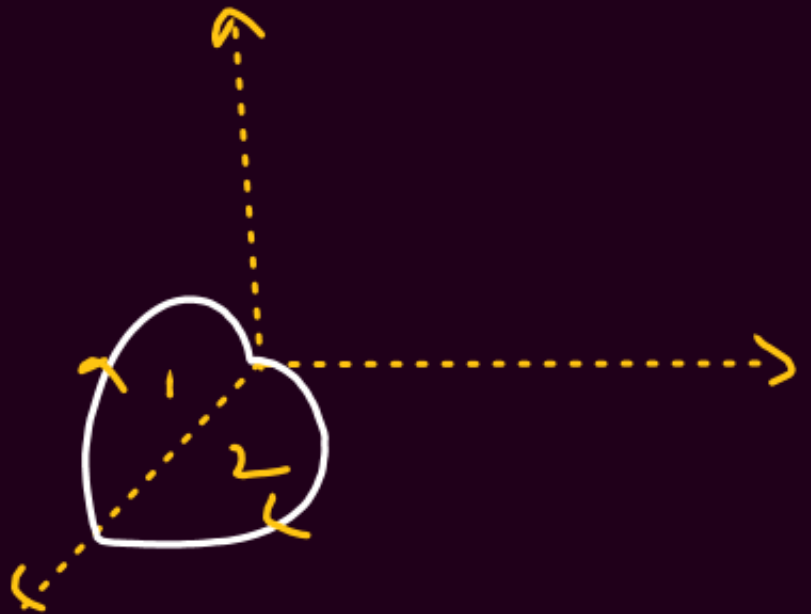


$$M = Ni\left(\frac{\sqrt{3}}{4}l^2\right)$$

Q



$$M = Ni\pi R^2$$



$$M_1 = Ni\frac{\pi R^2}{2}(-\hat{i})$$

$$M_2 = Ni\frac{\pi R^2}{2}(-\hat{j})$$

$$\vec{M} = -\frac{Ni\pi R^2}{2}(\hat{i}+\hat{j})$$

$$M = \frac{Ni\pi R^2}{2}\sqrt{2}$$

Pt. charge in UCM

$$i = qf = \frac{q}{T} = \frac{q\omega}{2\pi} = \frac{qv}{2\pi r}$$

$$M = i\pi r^2 = \frac{q\omega}{2\pi}\pi r^2$$

$$\boxed{M = \frac{q\omega r^2}{2} = \frac{qL}{2m}}$$

L = Ang. Momentum

Atomic Magnetism

① Orbital mag dipole Moment

$$M = \frac{eL}{2m} = n\left(\frac{eh}{4\pi m}\right)$$ quantised

② $\frac{eh}{4\pi m}$ = Bohr Magneton

③ $\frac{M}{L} = \frac{e}{2m}$ = Gyromagnetic ratio

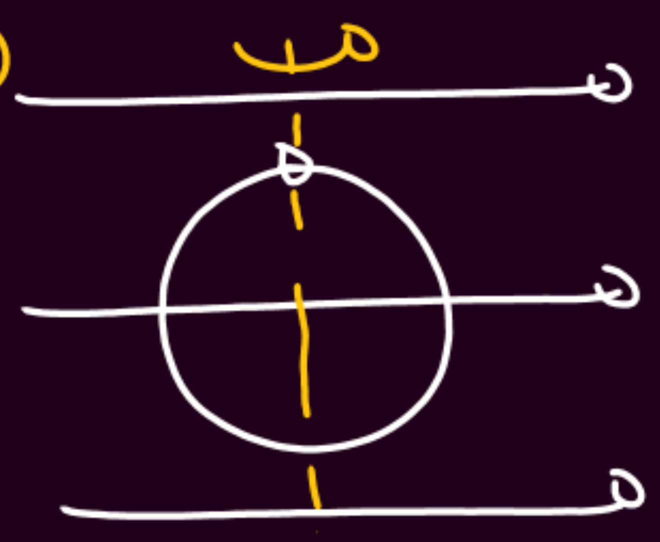
Spin magnetic Moment [Intrinsic]

Torque

$$\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$$

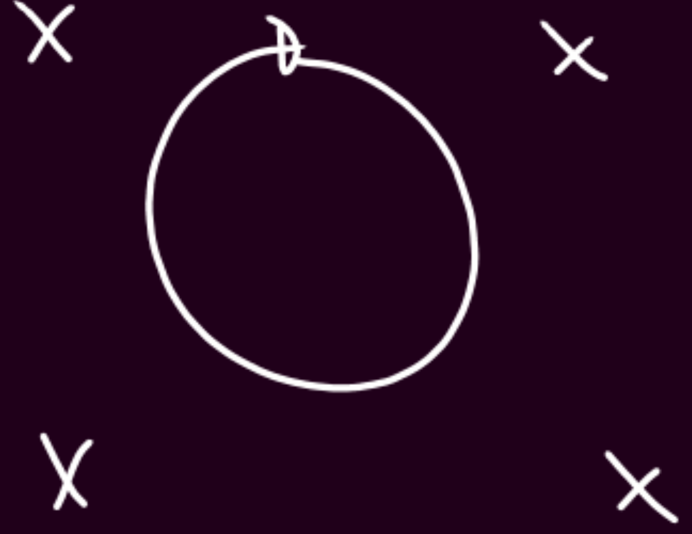
RHTR

$$\tau = MB\sin\theta$$

① 

$M \otimes \quad B \rightarrow$

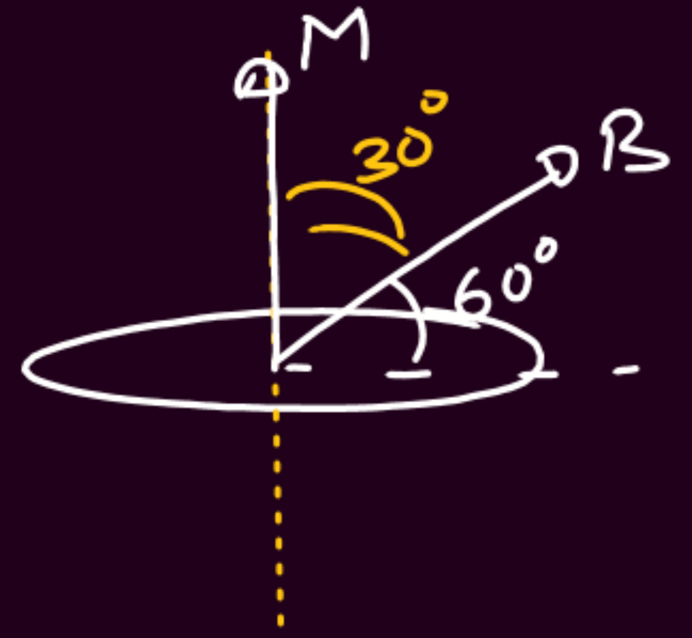
τ_{max}

② 

$$\boxed{\tau = 0}$$

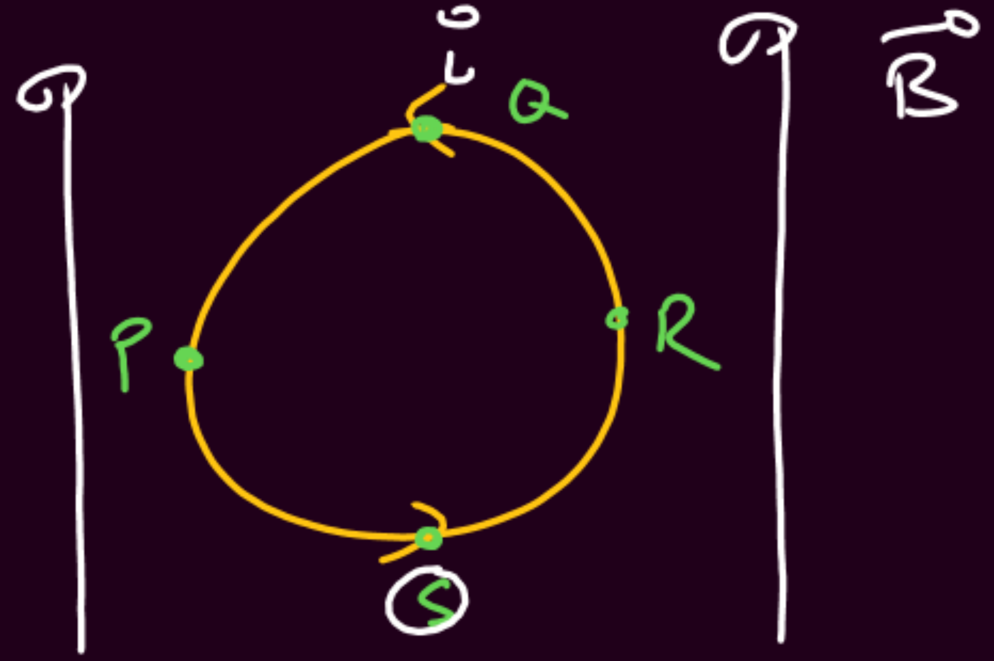
Q. $M = 0.4 \text{ A-m}^2$

$B = 10^{-3} \text{ T}$



$$\tau = MB\sin 30° = 0.4 \times 10^{-3} \times \frac{1}{2}$$

$$\boxed{\tau = 2 \times 10^{-4} \text{ N-m}}$$



$\vec{\tau} = \vec{M} \times \vec{B}$

$-\hat{i} = \hat{k} \times \hat{j}$

Axis of Rot. → (PR)

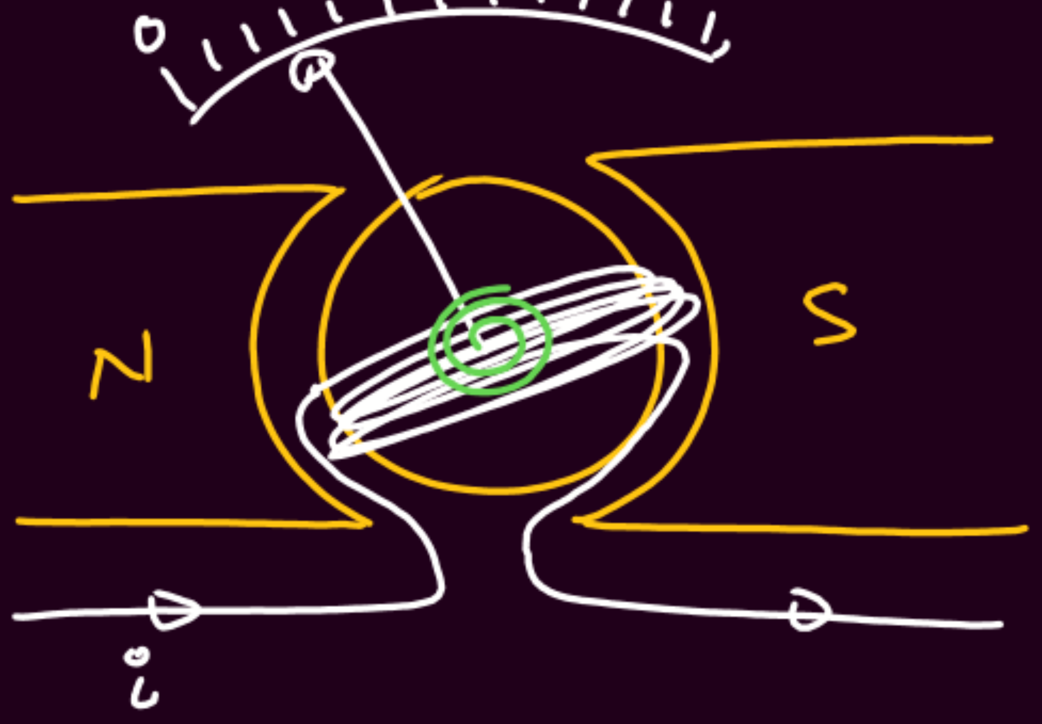
Square Rectangle Circle Ellipse

All → Same perimeter

Circle → Area → max

τ_{max}

Moving Coil Galvanometer



$(NiA)B = k\theta$

① Current Sensitivity

$$\boxed{\frac{\theta}{i} = \frac{NAB}{k}}$$

② Voltage Sensitivity

$$\boxed{\frac{\theta}{V} = \frac{NAB}{Rk}}$$

If $N \longrightarrow 2N$

C.S. $\longrightarrow$ Doubled

V.S. $\longrightarrow$ Same

Bar Magnet

$\boxed{M = m\ell}$ S to N

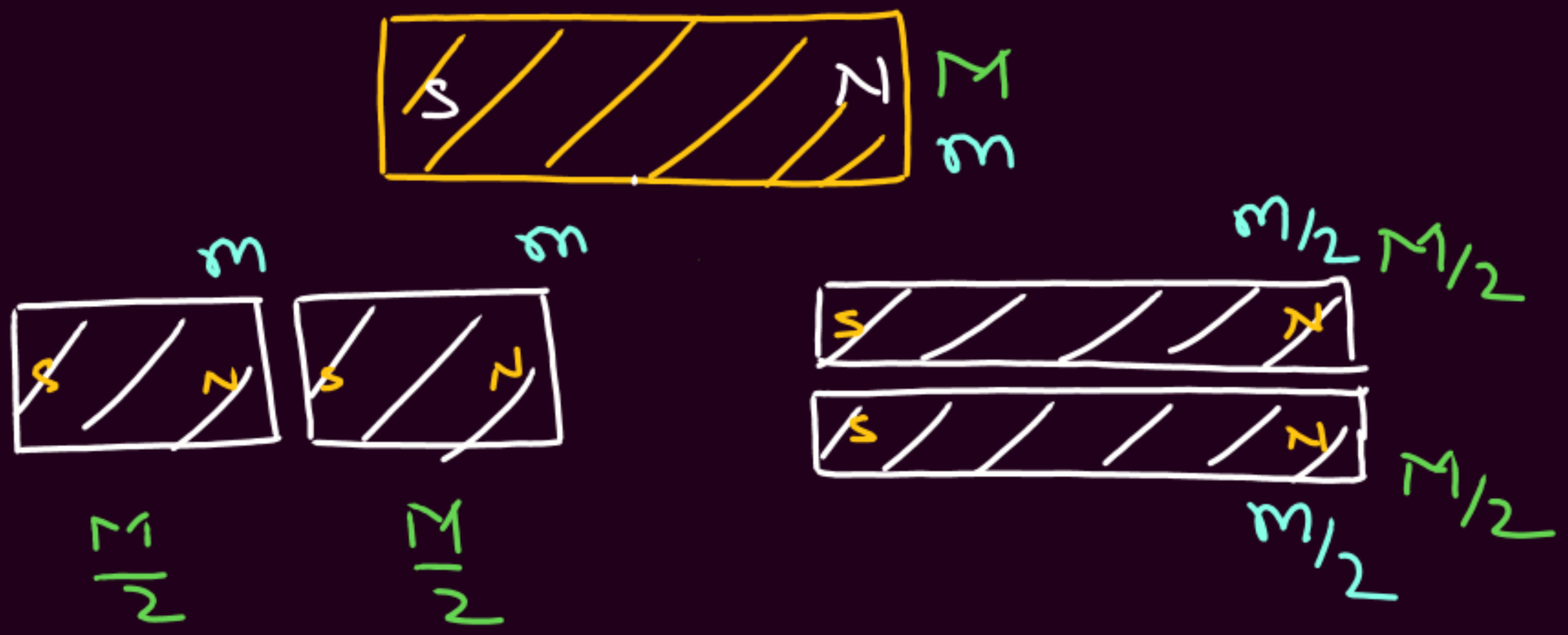
m = pole strength (A-m)

→ Donot exist in isolation

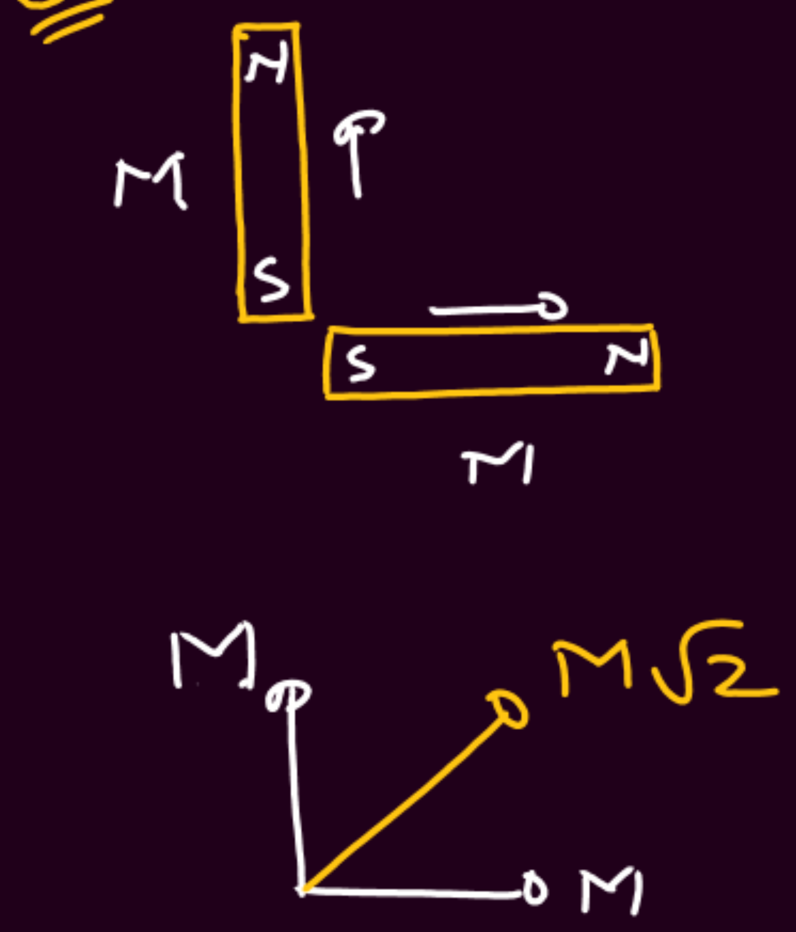
No poles

→ Repulsion → magnetisation

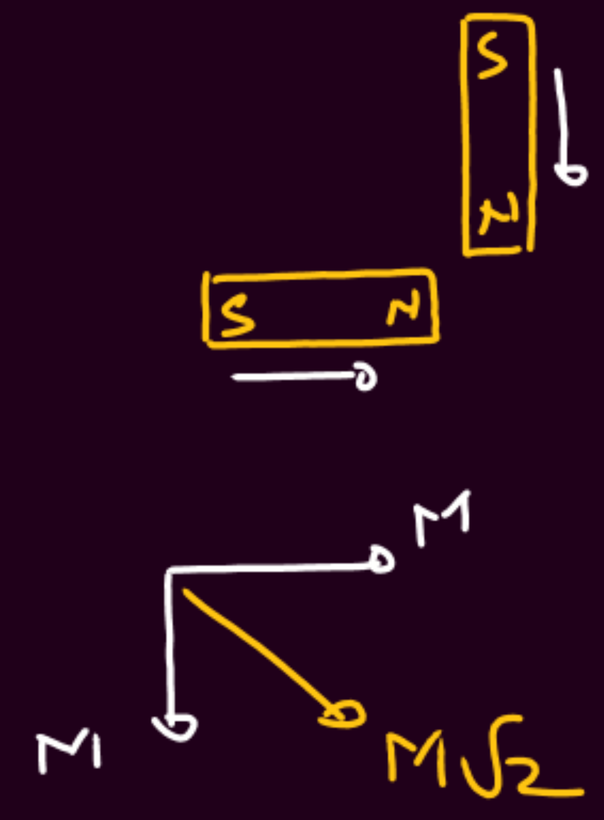
→ $m \propto$ (Cross Area)



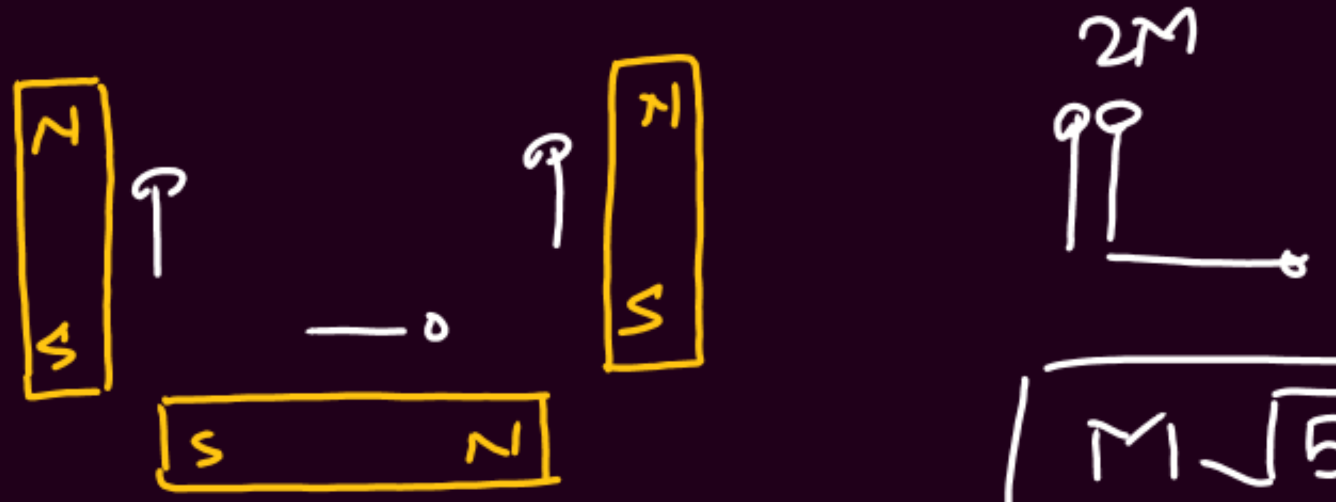
Q



Q

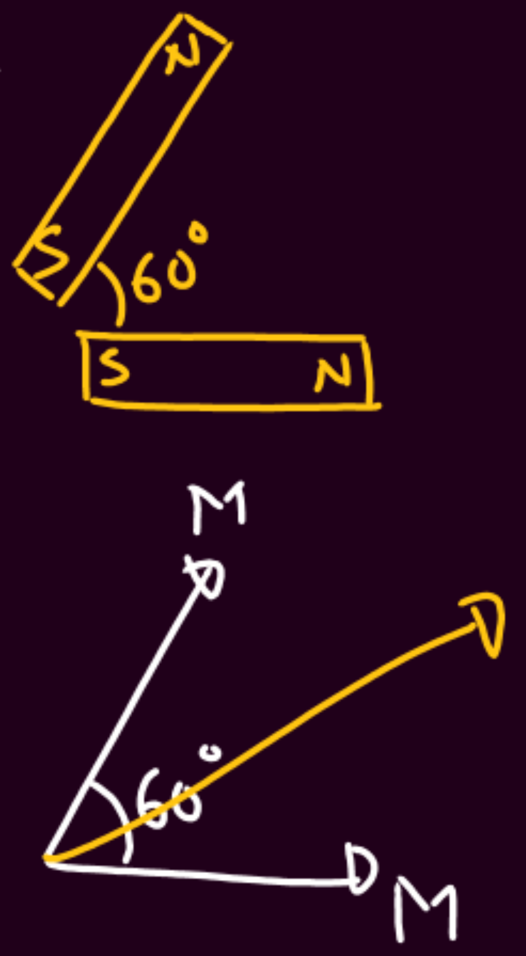


Q



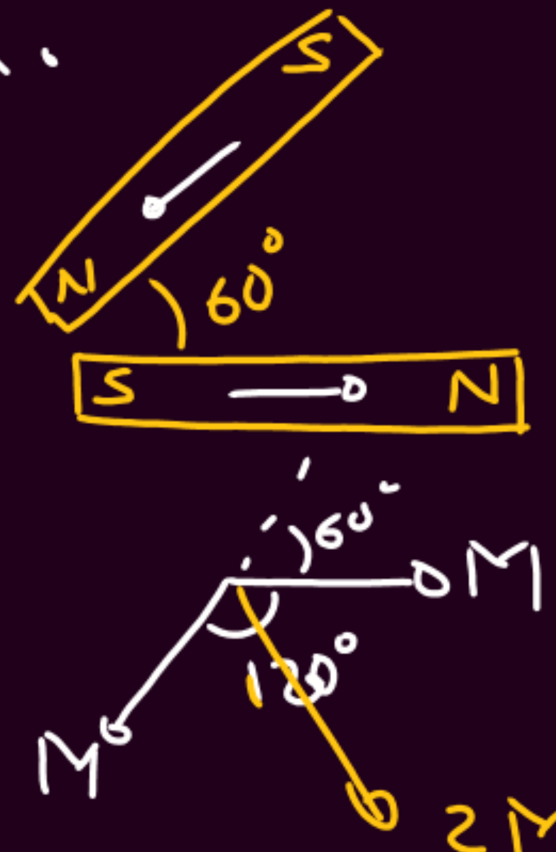
$$\boxed{M\sqrt{5}}$$

Q



$$2M\cos 30^\circ = M\sqrt{3}$$

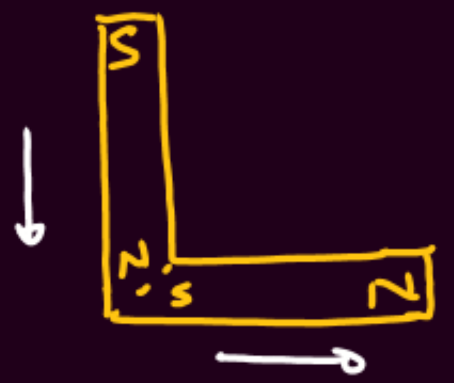
Q.

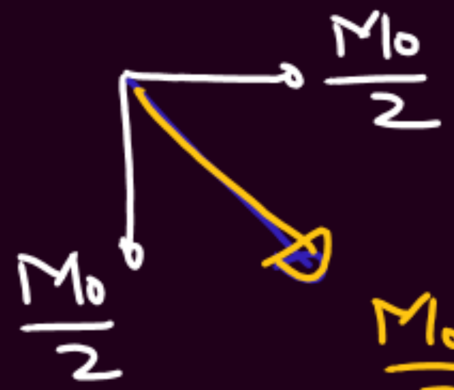


$$2M\cos\left(\frac{120^\circ}{2}\right) = M$$

Q. A Bar mag of M_0

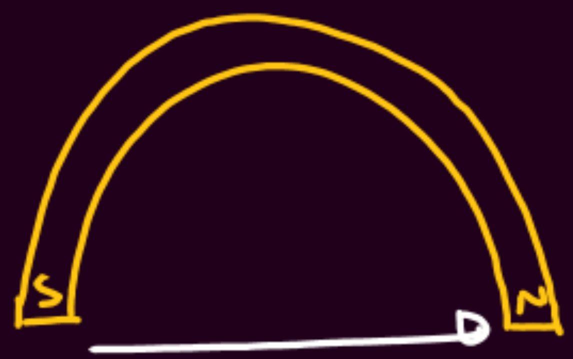
① Bent in L shape





$$\frac{M_0}{2}\sqrt{2} = \frac{M_0}{\sqrt{2}}$$

② Bent → Semicircle



① $M = m(2R)$

$$M = m\left(\frac{2L}{\pi}\right)$$

$$M = \frac{2}{\pi}(mL)$$

$$\boxed{M = \frac{2M_0}{\pi}}$$

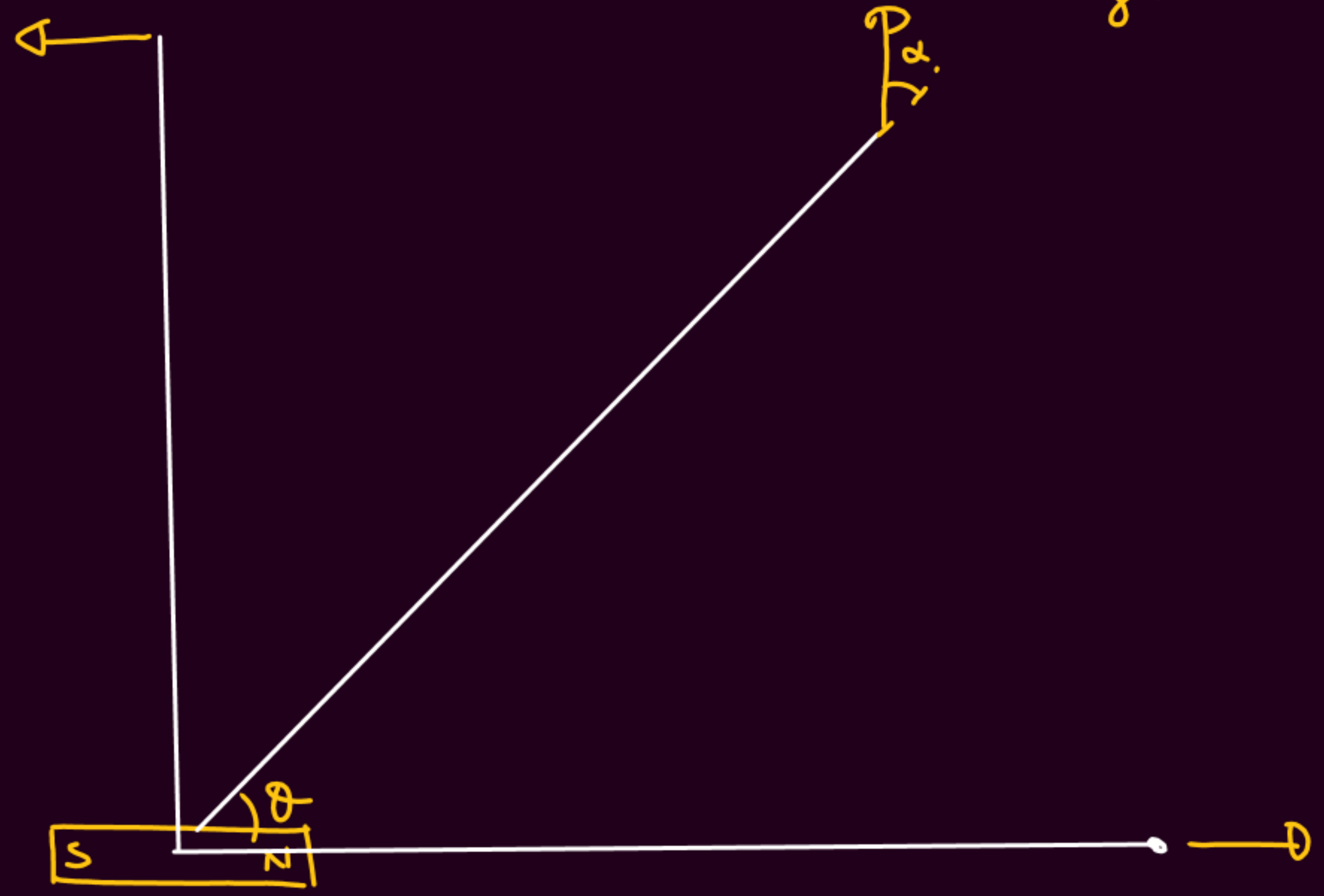
② $L = \pi R$

Field due to Dipole

$$B_{eq} = -\frac{kM}{r^3}$$

Equatorial Position

$$B = \frac{kM}{r^3}\sqrt{1 + 3\cos^2\theta}$$



Axial Position

$$B_a = \frac{2kM}{r^3}$$

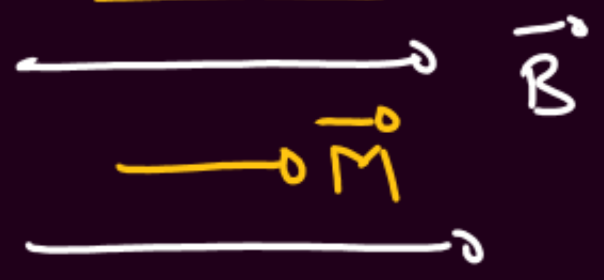
$$\boxed{k = \frac{\mu_0}{4\pi} = 10^{-7}\ \frac{T\text{-}m}{A}}$$

Torque $\vec{\tau} = \vec{M} \times \vec{B} = MB\sin\theta$

P.E. $U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$

W.D. / ΔU $\Delta U = -MB(\cos\theta_2 - \cos\theta_1)$

Stable

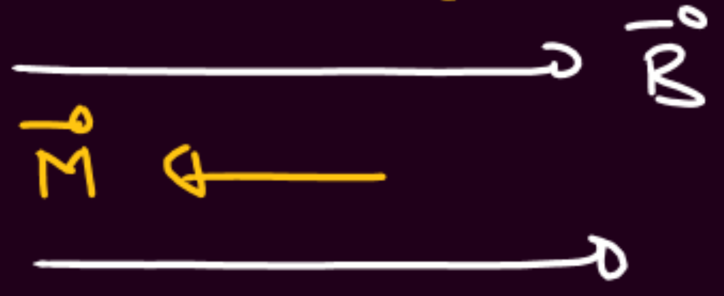


$\theta = 0^\circ$

$\tau = 0$

$U_{min} = -MB$

Unstable



$\theta = 180^\circ$

$\tau = 0$

$U_{max} = MB$

Disturb → Oscillate

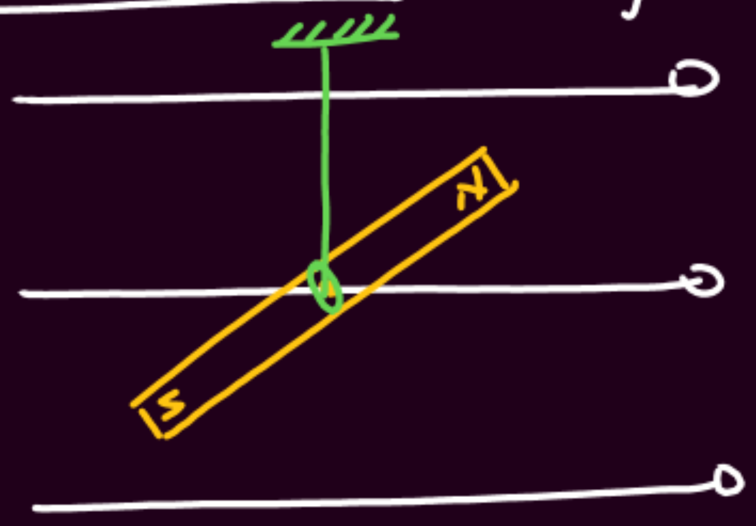
↓ for small θ

(SHM) $\boxed{T = 2\pi\sqrt{\frac{I}{MB}}}$

Q. W.D. in rotating from st. to unst.

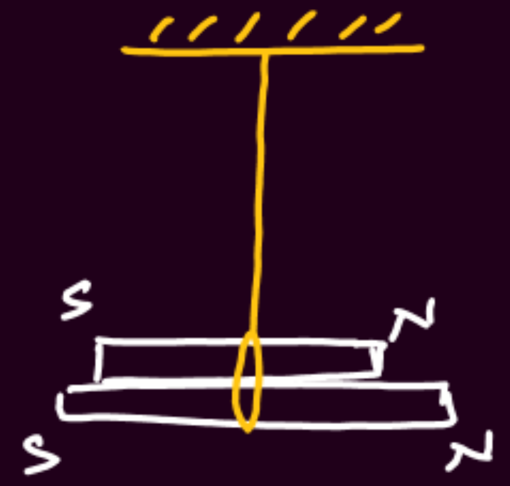
$$W = MB - (-MB) = 2MB$$

Vibration Magnetometer



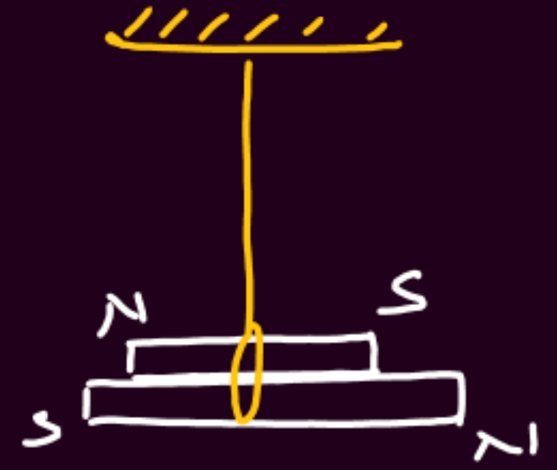
$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{12}ml^2}{MB}}$$



Like poles together

$$T = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B}}$$



Unlike poles together

$$T = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}}$$

Q. $T_0 \to$ 3 equal parts

① T of any one part

$$T_0 = 2\pi\sqrt{\frac{\frac{1}{12} m l^2}{MB}}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2}{\left(\frac{M}{3}\right) B}} = \frac{T_0}{3}$$

② 3 parts $\to$ joined

$$T = 2\pi\sqrt{\frac{3\times\frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2}{3\times\left(\frac{M}{3}\right) B}}$$

$$\boxed{T = \frac{T_0}{3}}$$

Q. M_0 & $3M_0$

Like poles together $T_1 = 3s$

Unlike —— " ——— $T_2 = ?$

$$\frac{T_2}{T_1} = \sqrt{\frac{M_1 + M_2}{M_1 - M_2}} = \sqrt{\frac{3M_0 + M_0}{3M_0 - M_0}}$$

$$\frac{T_2}{3} = \sqrt{2}$$

$$\boxed{T_2 = 3\sqrt{2}\,s}$$

Q Like & Unlike $\frac{T_1}{T_2} = \frac{2}{3}$

$\frac{M_1}{M_2} = ?$

$$\frac{T_{Like}}{T_{Unlike}} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} = \frac{2}{3}$$

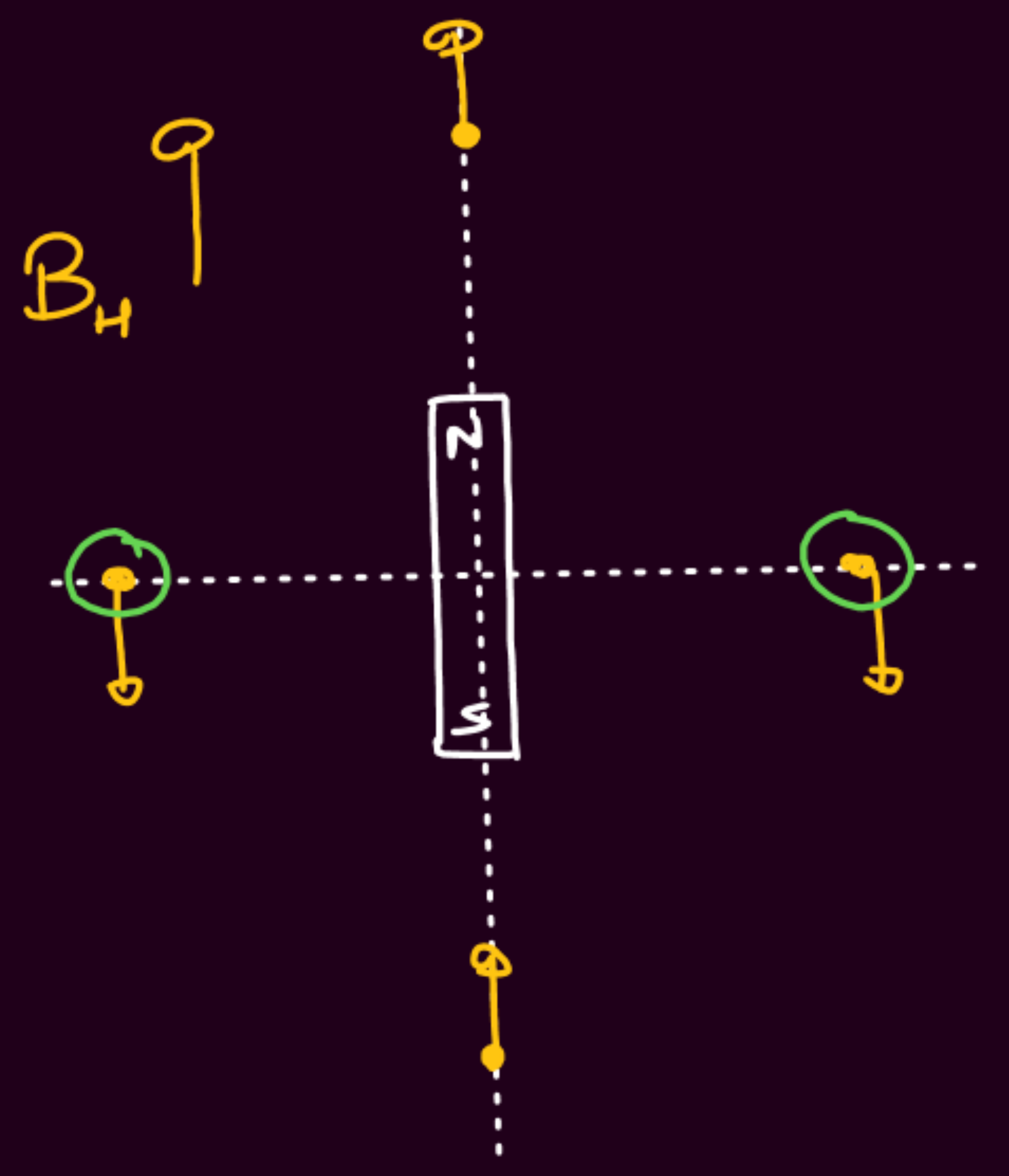
$$\frac{M_1 - M_2}{M_1 + M_2} = \frac{4}{9}$$

$$9M_1 - 9M_2 = 4M_1 + 4M_2$$

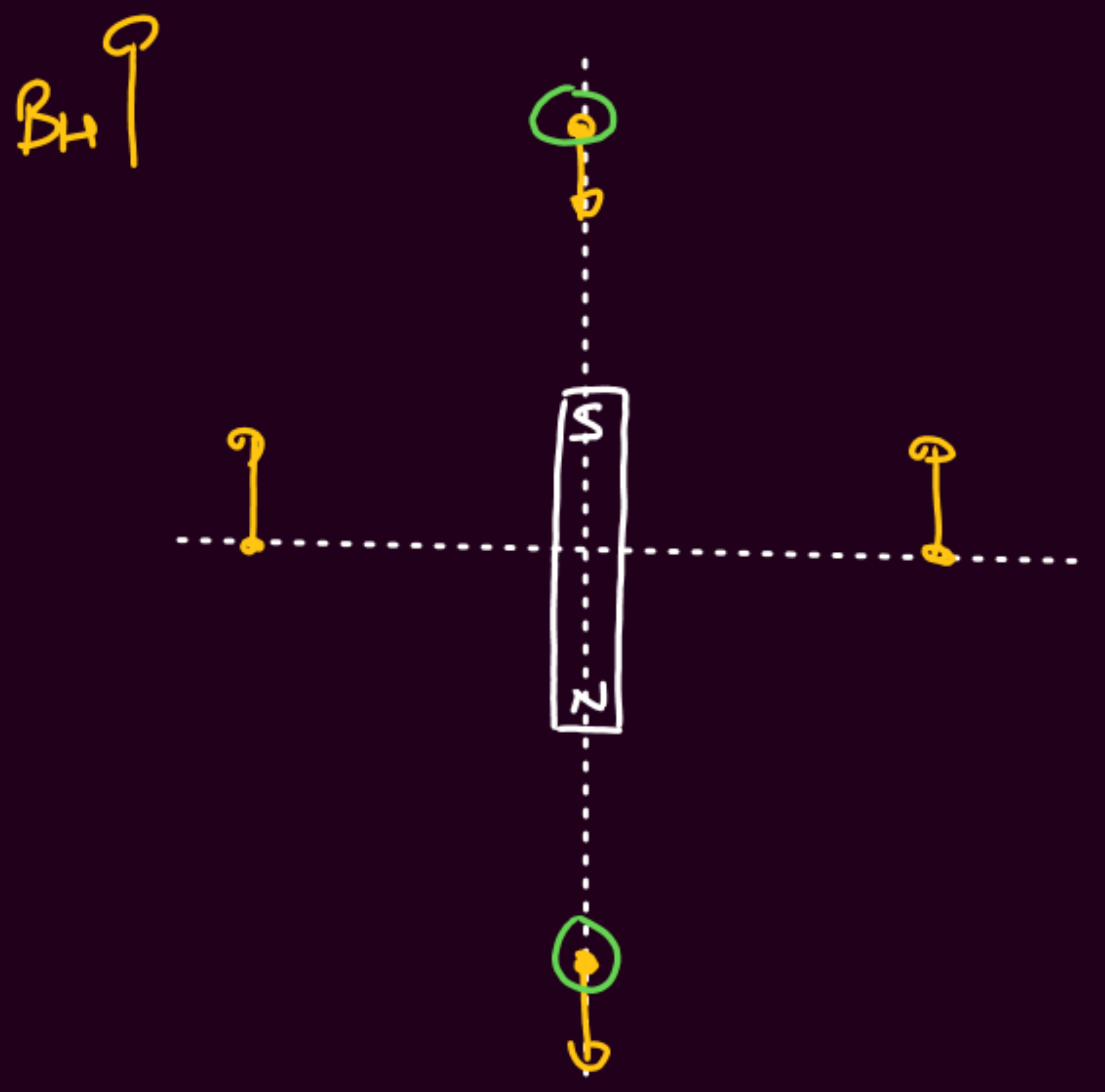
$$5M_1 = 13M_2$$

$$\boxed{\frac{M_1}{M_2} = \frac{13}{5}}$$

Neutral point $\boxed{B_{net} = 0}$



[N of mag in North dir]



[S of mag. in North dir]

Magnetism & Matter

B = Intensity of magnetic field

H = —— " —— Magnetising field

I = —— " —— Magnetisation

χ_m = Susceptibility

$$H = \frac{B}{\mu} = \frac{B_0}{\mu_0} \qquad A/m$$

$$I = \frac{M}{V} = \frac{m}{A} \qquad A/m$$

$$\chi_m = \frac{I}{H}$$ (unitless / Dimensionless)

$B_0 = \mu_0 H$

$B = \mu H = \mu_0 \mu_r H = \mu_r B_0$

$I = \chi_m H$

$B = B_0 + B_{induced}$

$\mu H = \mu_0 H + \mu_0 I$

$\mu H = \mu_0 H + \mu_0 \chi_m H$

$$\boxed{\mu = \mu_0 (1 + \chi_m)}$$

$\mu_r = 1 + \chi_m$

$\mu_r \geq 0$

$\chi_m \geq -1$

$\frac{1}{4\pi \epsilon_0 K}$ $\frac{\mu_0 \mu_r}{4\pi}$

$\frac{E}{K}$ $B_0 \mu_r$

Diamagnetic

- → Lenz's law
- → Weakly

Meissner Effect

- Superconductivity
- Perfect Diamagnetism

$$B_{net} = 0$$

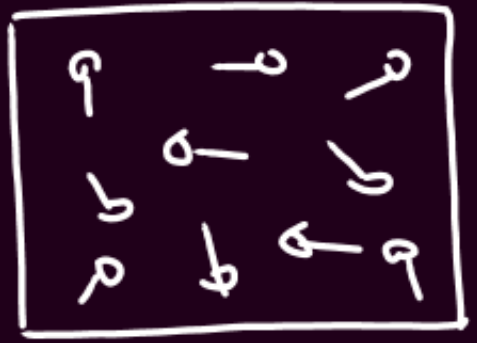
$$\mu_r = 0$$

$$\chi_m = -1$$

Paramagnetic

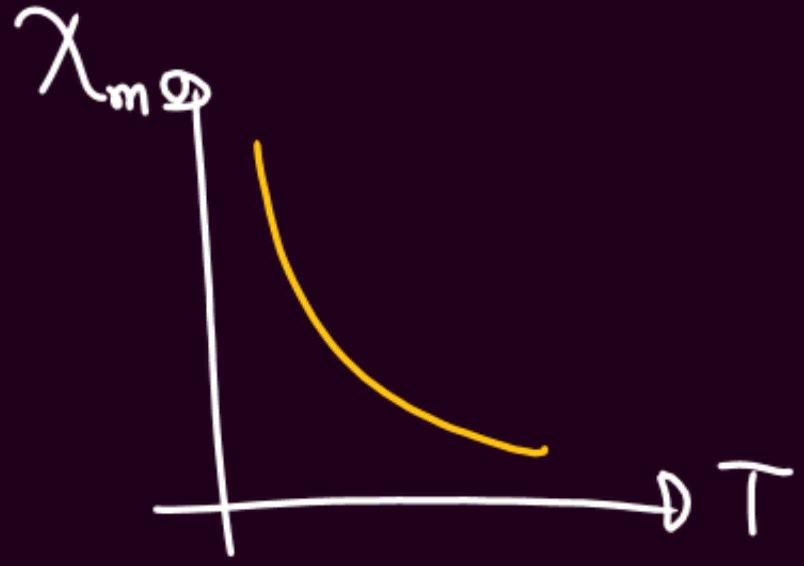
- Unpaired e^-
- weakly attract

Curie's law



$$\chi_m = \frac{C}{T}$$

$$\frac{\chi_{m_2}}{\chi_{m_1}} = \frac{T_1}{T_2}$$



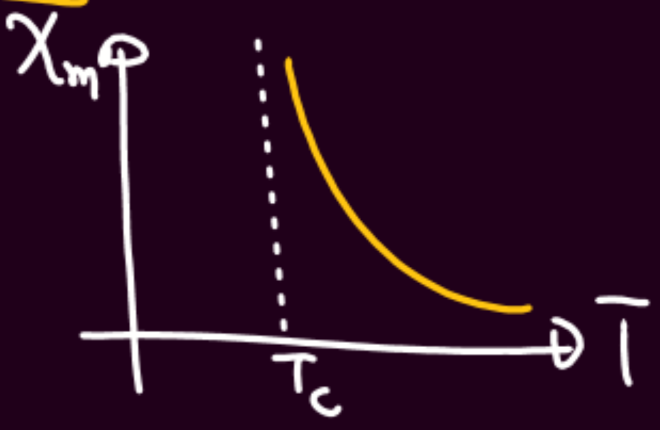
Ferromagnetic

Fe Ni Co

- → unpaired e^- & Domain formation

- → Strongly Attracted
- → Curie's Temp. (T_c) [Ferro → Para]
- → Curie-Weiss Law

$$\chi_m = \frac{C'}{T - T_c}$$



Dia

→ orbital motion of e^-

lenz's law

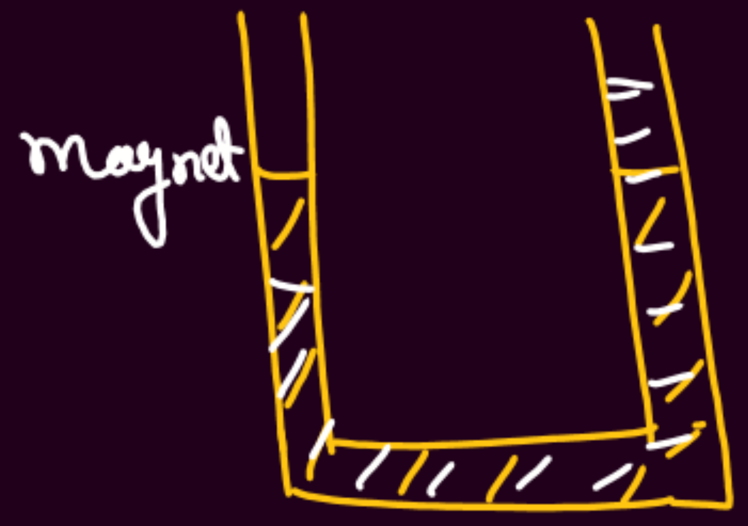
→ weakly repelled

→ S/L/G

$0 \leq \mu_r < 1$

$-1 \leq \chi_m < 0$

$B_{net} < B_0$



Para

→ upaired e^-

→ weakly attracted

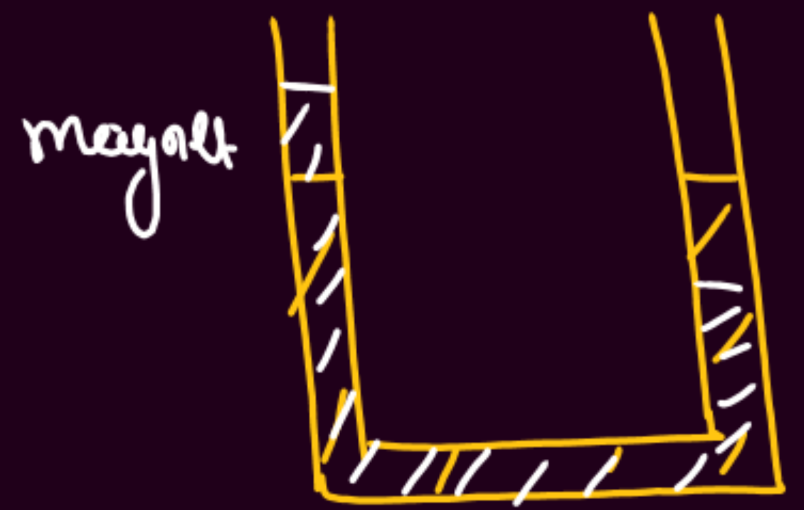
→ S/L/G

$1 < \mu_r$

$0 < \chi_m$

$B_{net} > B_0$

slightly



Ferro

→ unpaired e^- & Domain

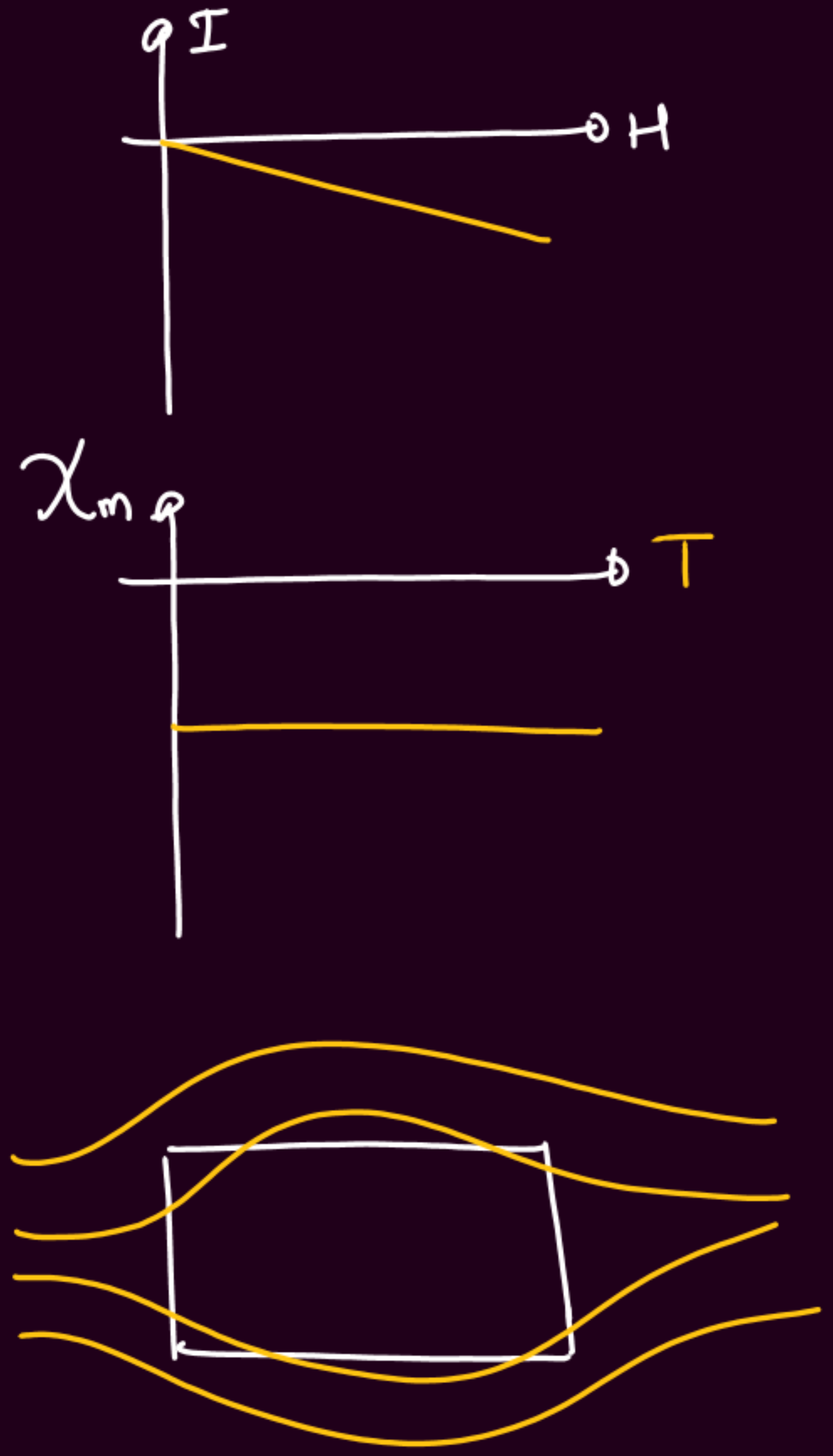
→ Strongly attracted

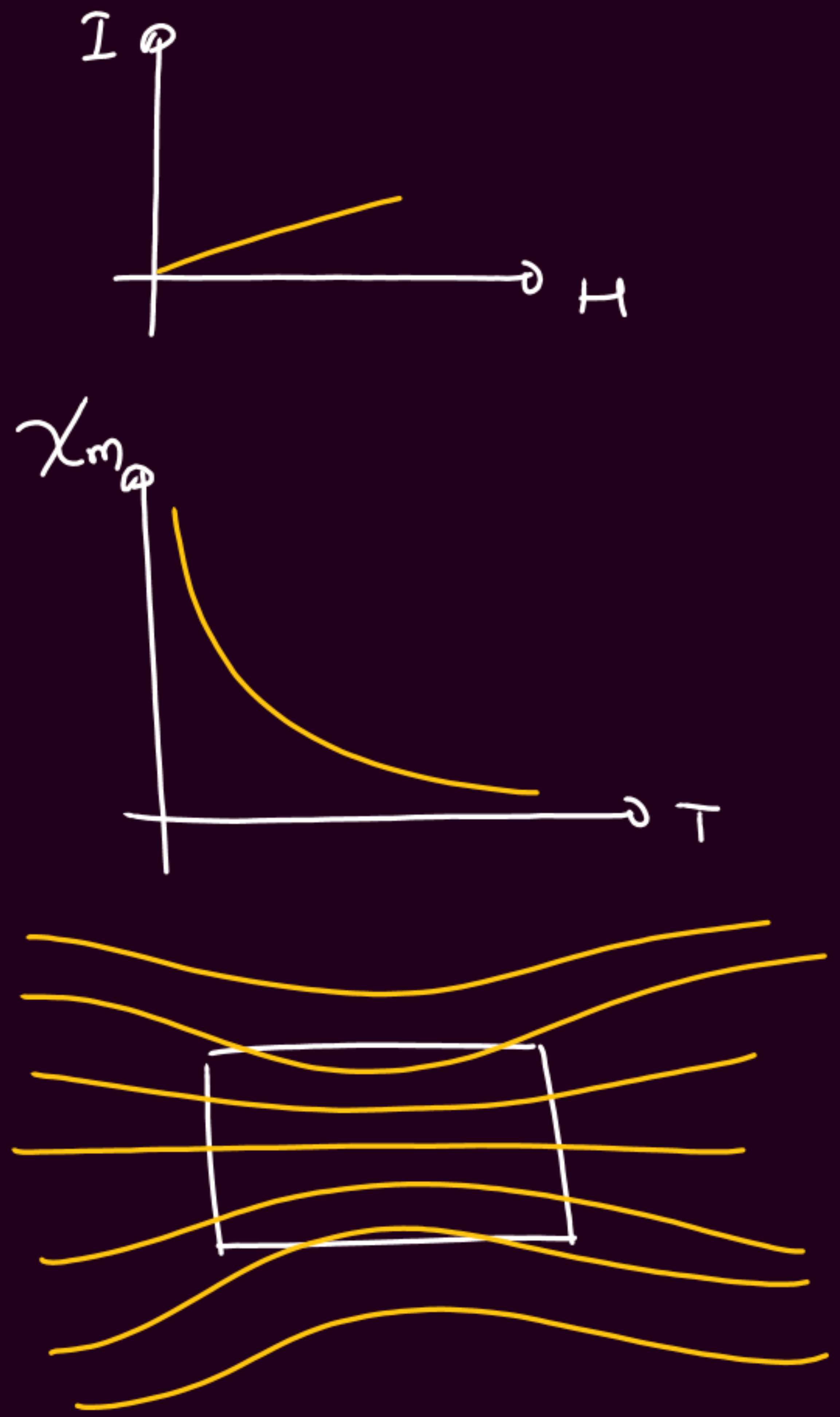
→ only solids

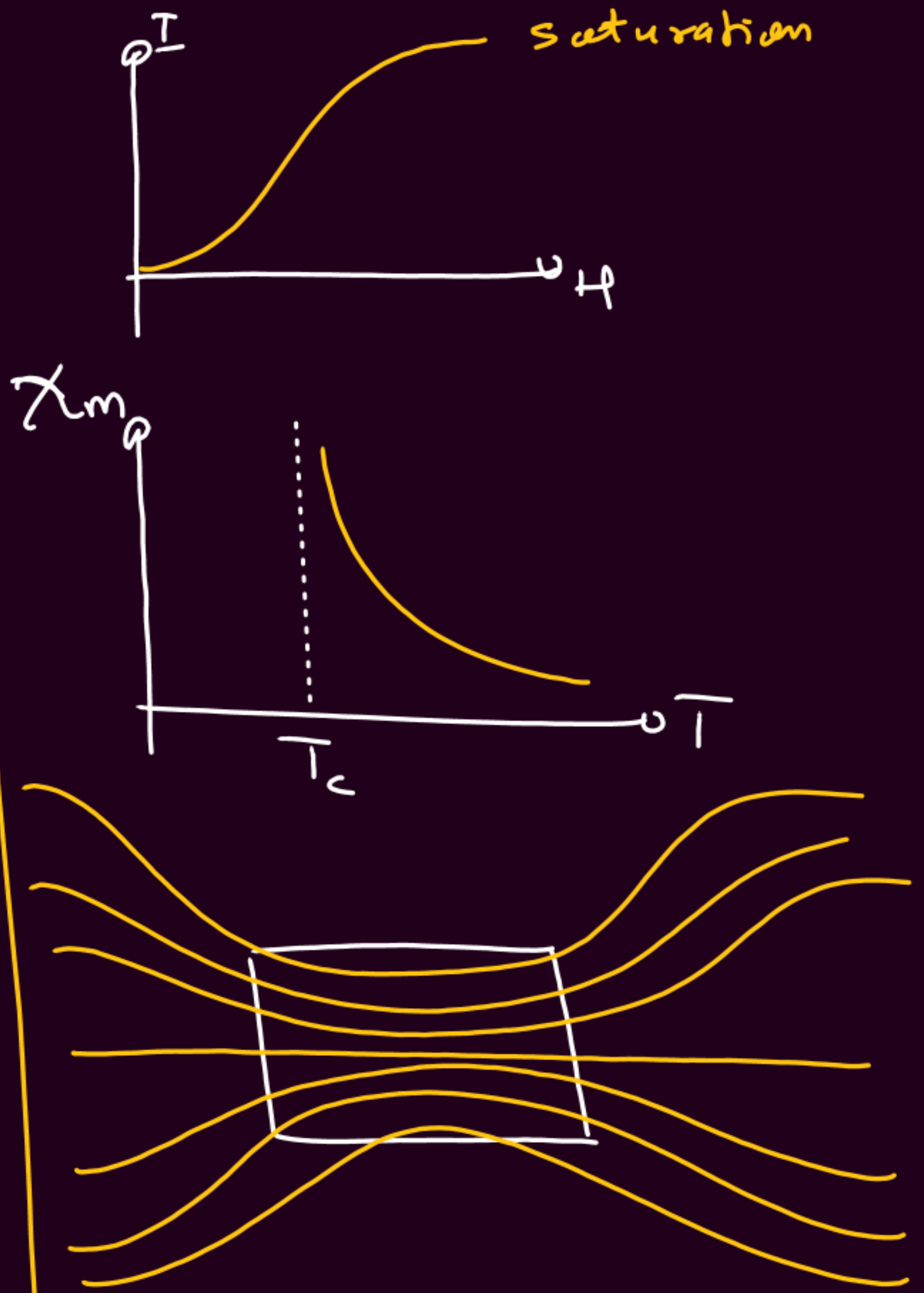
$\mu_r >> 1$

$\chi_m >> 1$

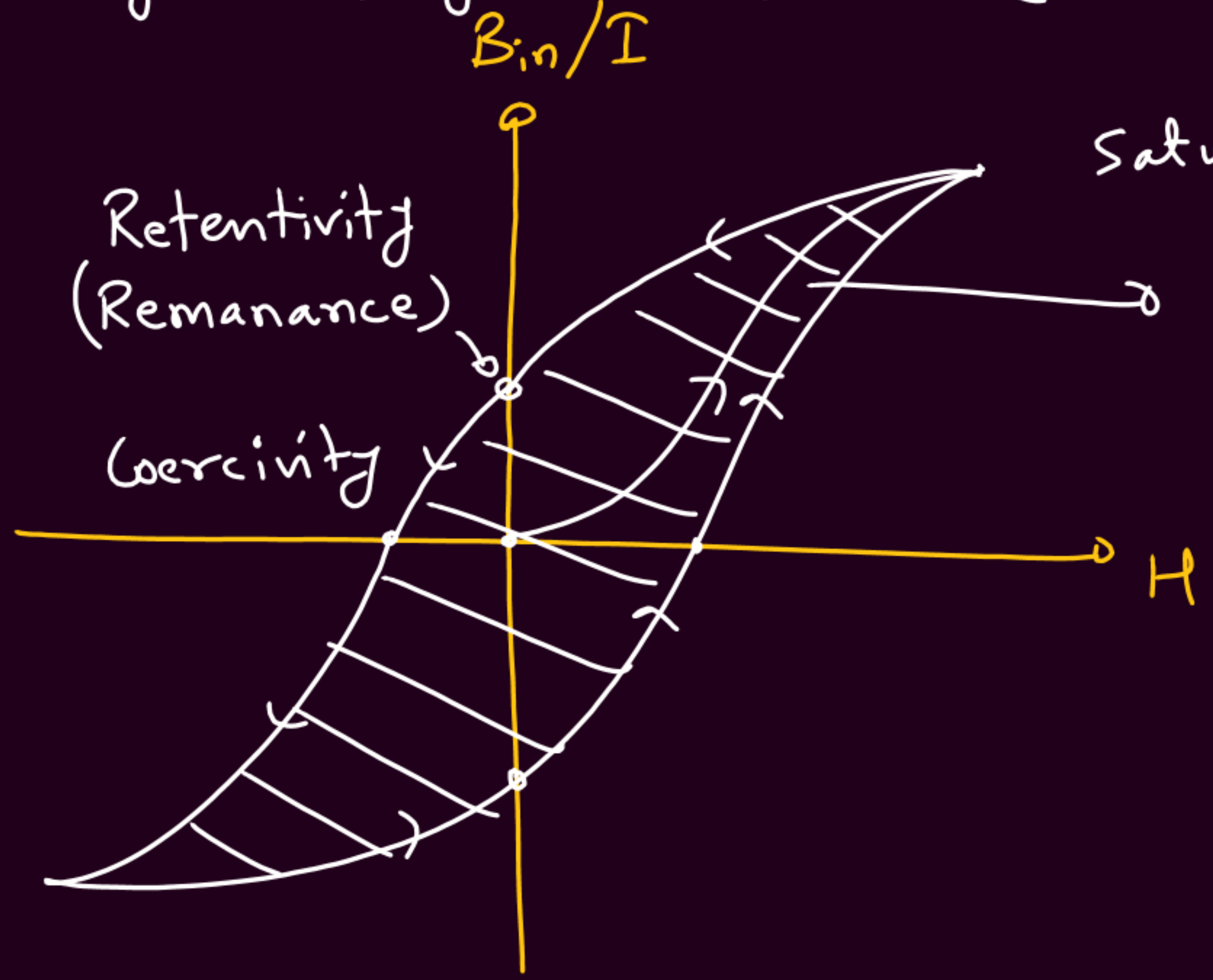
$B_{net} >> B_0$


I
H
χm
T


I
H
χm
T


I
saturation
H
χm
T
Tc

Magnetic Hysteresis → [lagging behind]



Area → Energy loss/cycle/volume

Soft magnetic	Hard magnetic

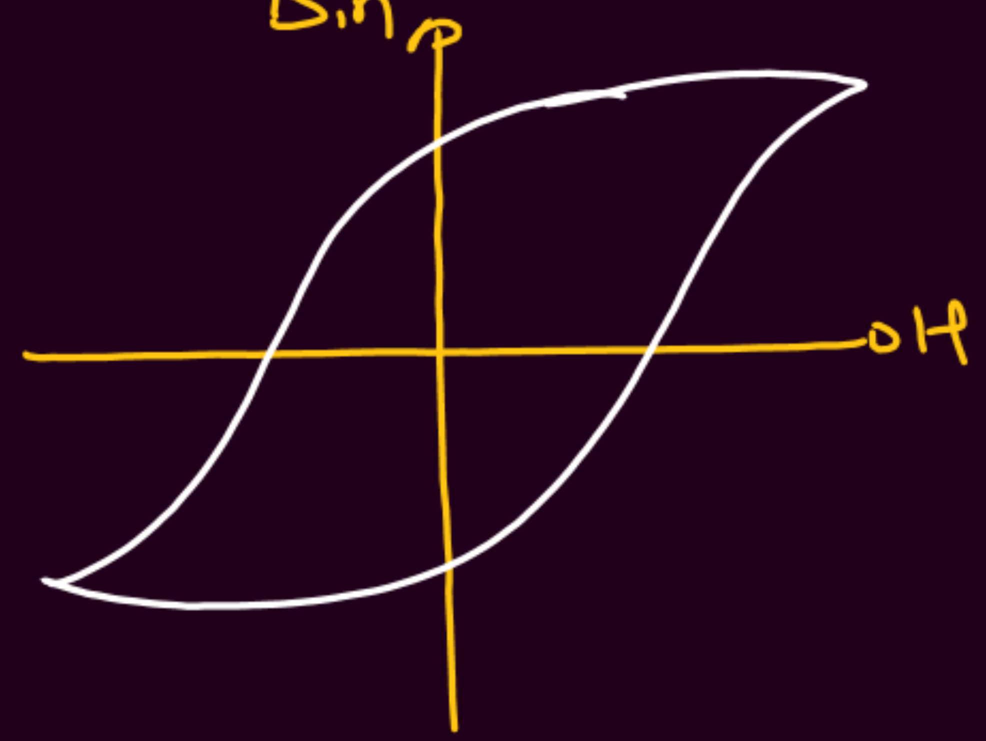
Soft magnetic

Low retentivity
Low coercivity
→ Low losses

Electromagnets

→ Soft iron core

Hard magnetic



High Retentivity
High coercivity
High losses

Permanent magnets

→ Steel / Alnico

Magnetic Sheilding

① Ferromagnetic

② Meisser Effect