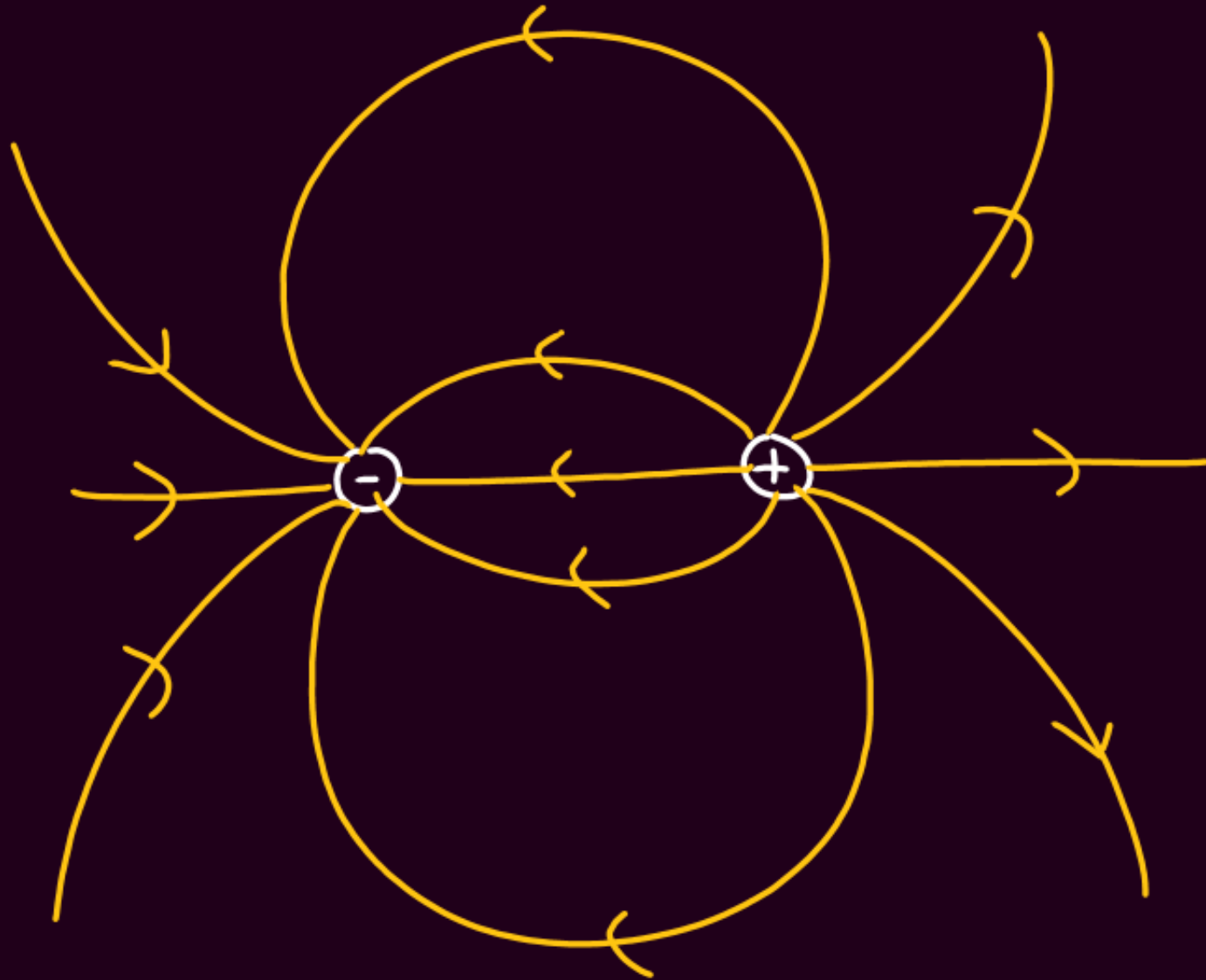
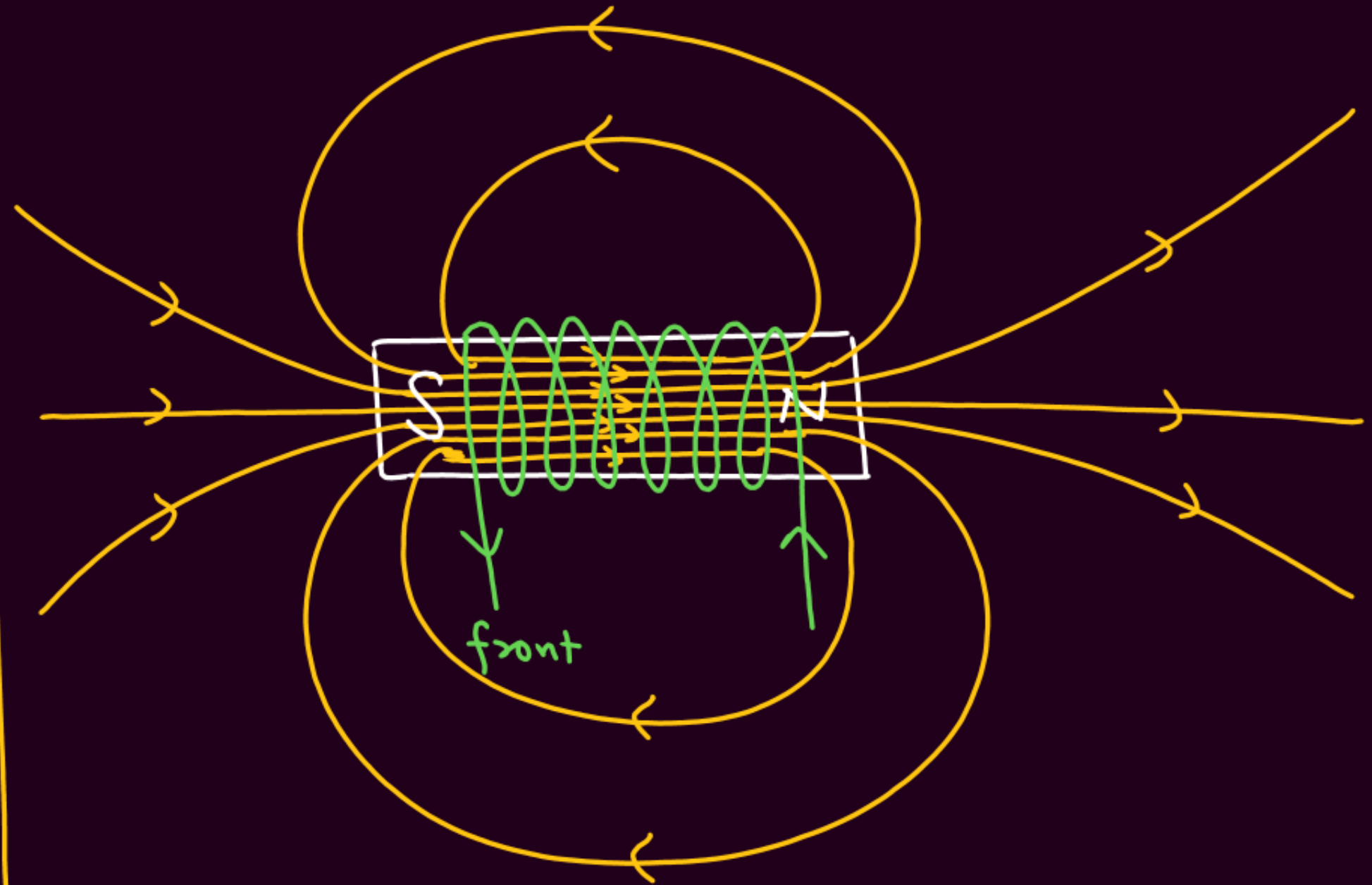


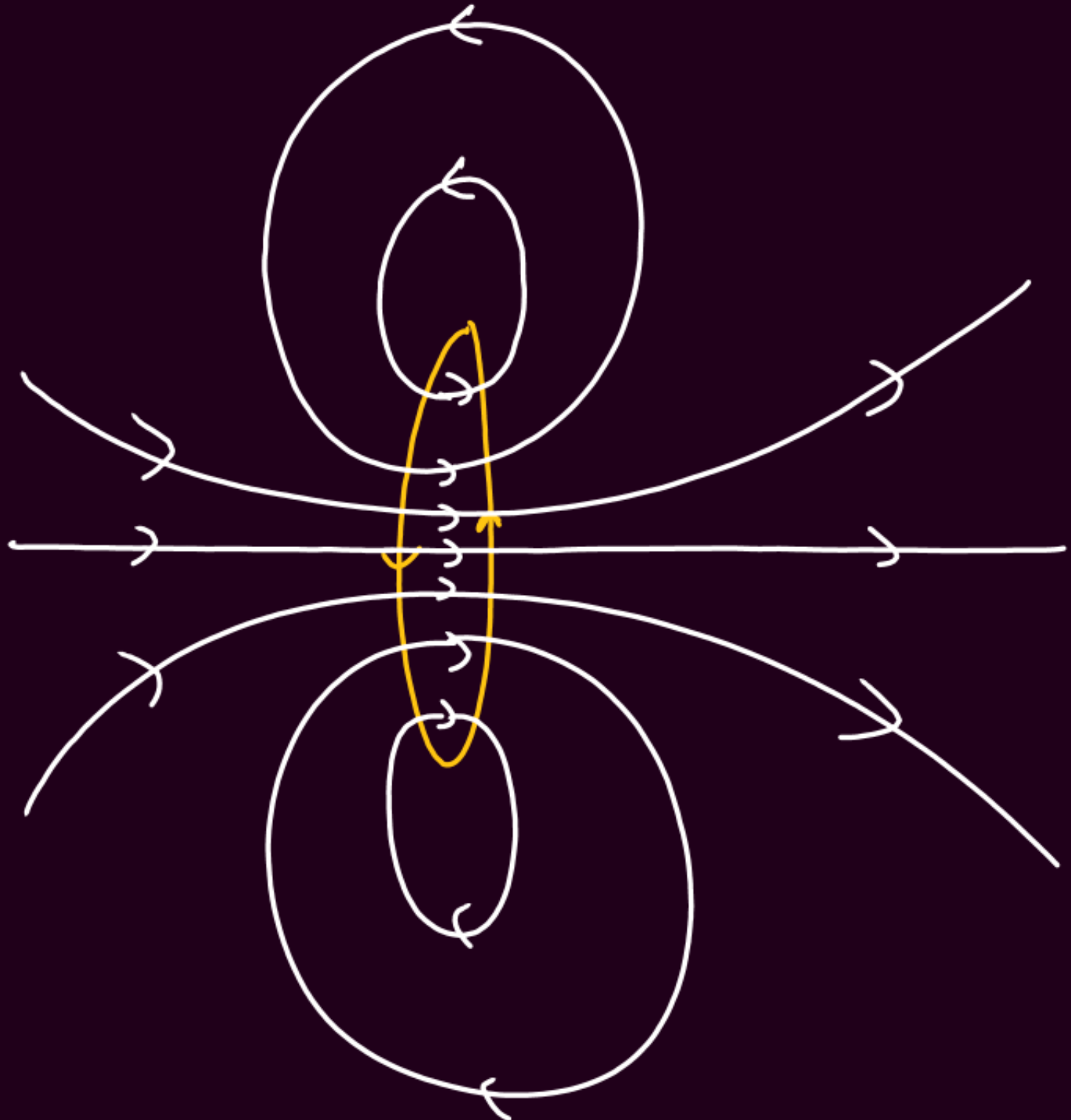
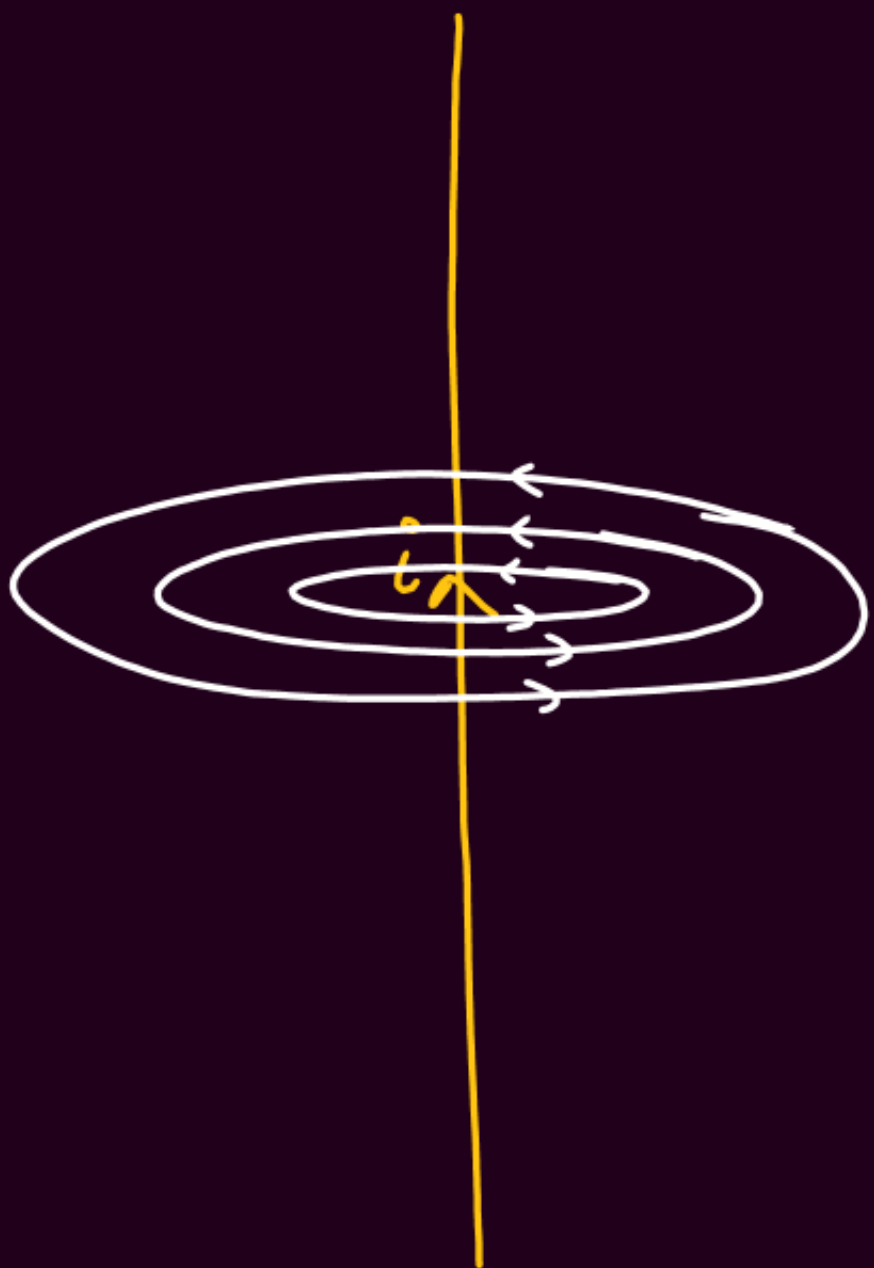
Magnetic field lines

Electric dipole



Bar Mag. / Solenoid





Magnetic flux

$$\boxed{\int \vec{B} \cdot d\vec{A} = \phi_B} \text{ Wb (weber)}$$

$$B \quad T = \frac{\text{Wb}}{\text{m}^2}$$

Case Uniform $\vec{B}$

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta = BA \sin \alpha$$



$$F = B i l$$

$$[B] = \left[\frac{F}{i l} \right] = \left[\frac{\text{MLT}^{-2}}{\text{AL}} \right]$$

$$[B] = [\text{MT}^{-2} \text{A}^{-1}]$$

$$[\mu] = \left[\frac{F}{i^2} \right] = [\text{MLT}^{-2} \text{A}^{-2}]$$

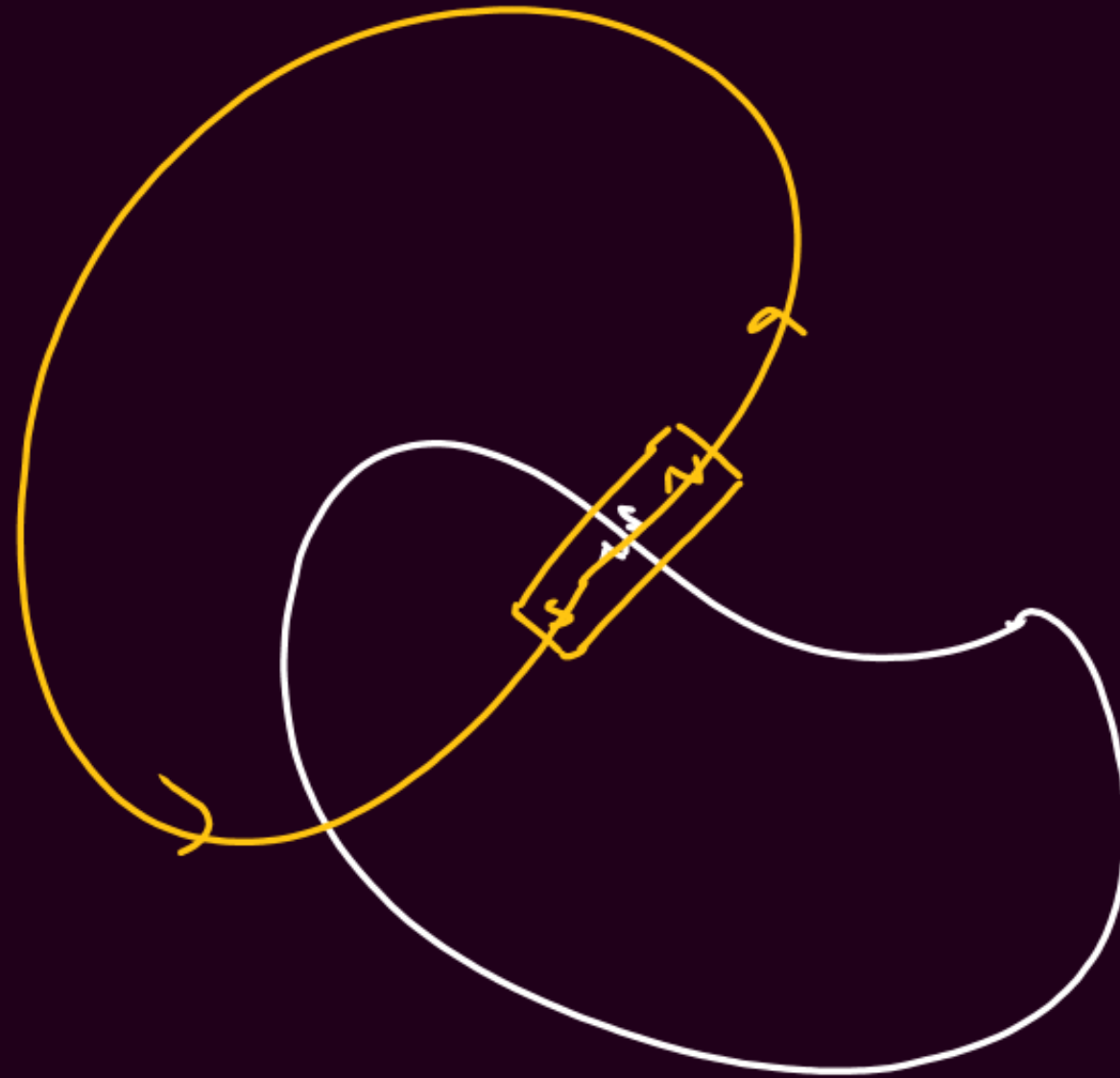
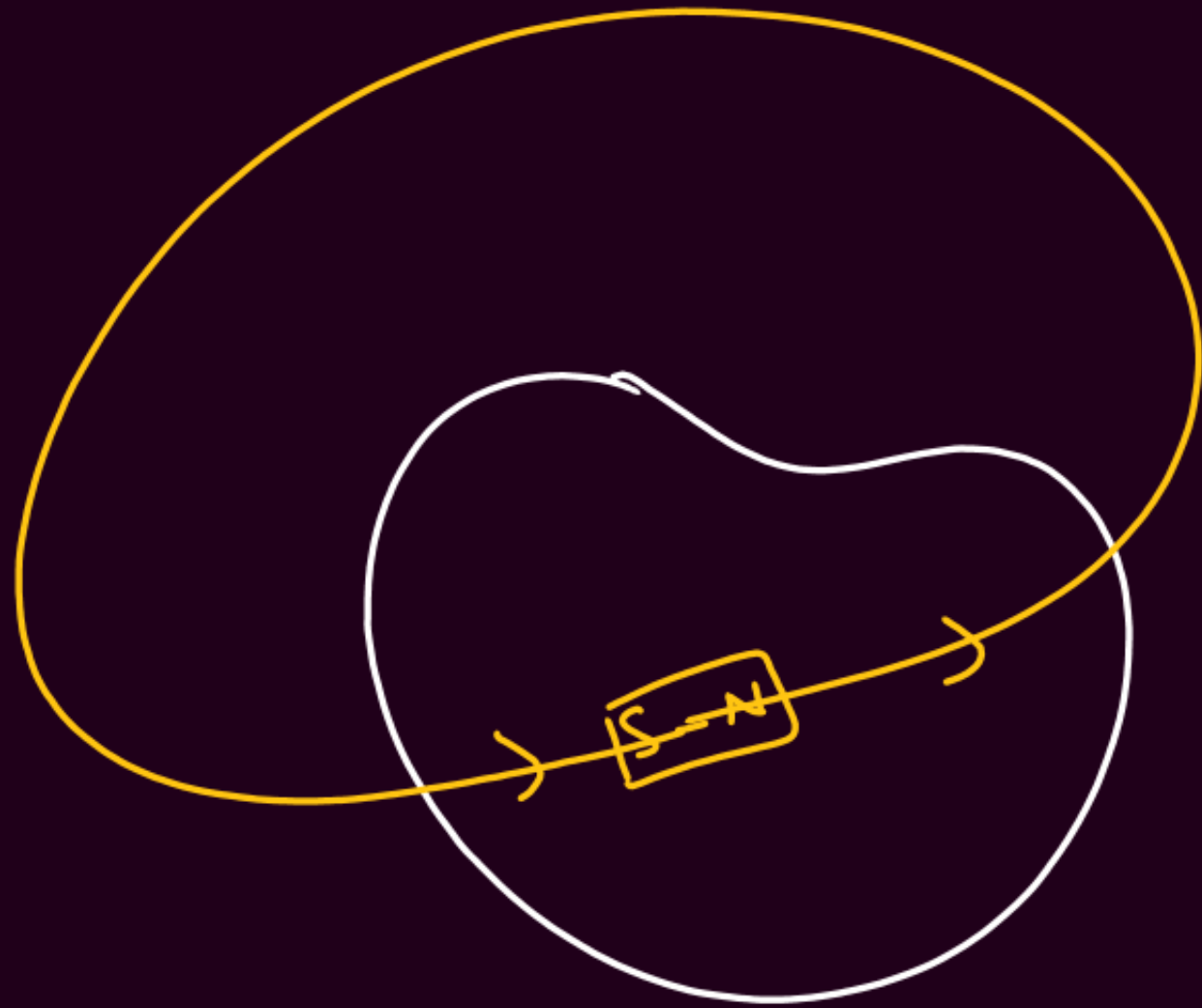
$$F = B i l$$

$$F = \frac{\mu N i}{l} i l$$

Gauss's Law for Magnetism

$$\oint_{\mathcal{B}} \vec{B} \cdot d\vec{A} = 0$$

→ MFL always form CL



Q Bar may along x-axis

$$\phi_{yz} = 0$$

$$\phi_{xz} = 0$$

Q A Solenoid

$$\left\{ \begin{array}{l} n = 100 \text{ per cm} \\ i = 2 \text{ A} \\ r_0 = 4 \text{ cm} \end{array} \right.$$

① $r = 2 \text{ cm}$



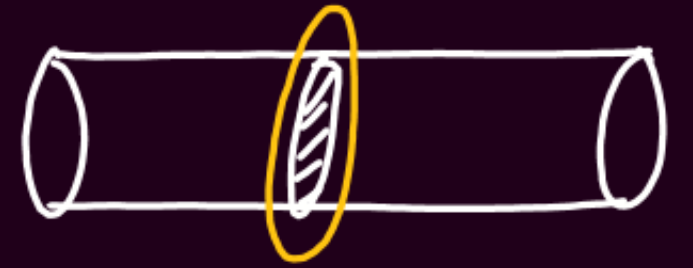
$$\phi = BA_c = (\mu_0 n i) (\pi r^2)$$

$$= (4\pi \times 10^{-7}) \left(\frac{100}{10^{-2}} \right) (2) \pi (2 \times 10^{-2})^2$$

② $r = 5 \text{ cm}$

$$\phi = BA_c$$

$$\phi = (\mu_0 n i) (\pi r_0^2)$$



$$\phi = (4\pi \times 10^{-7}) \left(\frac{100}{10^{-2}} \right) (2) \pi (4 \times 10^{-2})^2$$

Faraday's law

Induced EMF

$$\mathcal{E} = - \frac{d\phi}{dt}$$

$$[\phi = NBA \cos \theta]$$

Lenz's law

S N



N S



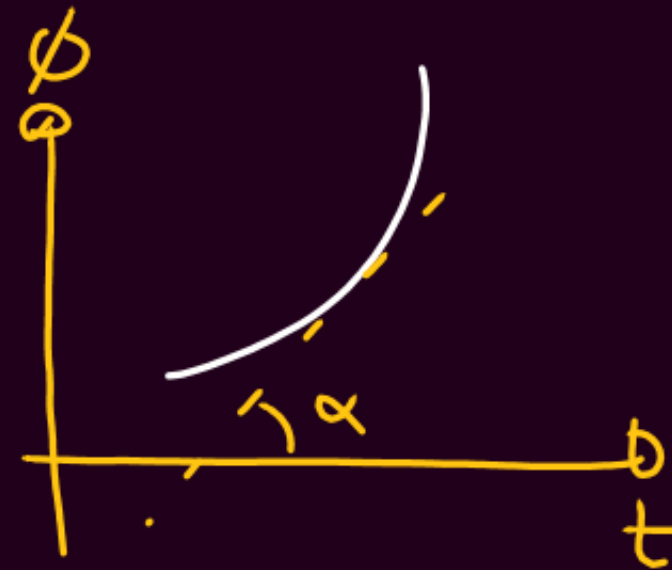
$\phi \uparrow$

$$i = \frac{\mathcal{E}}{R}$$

induced current

$$\Delta Q = - \frac{\Delta \phi}{R}$$

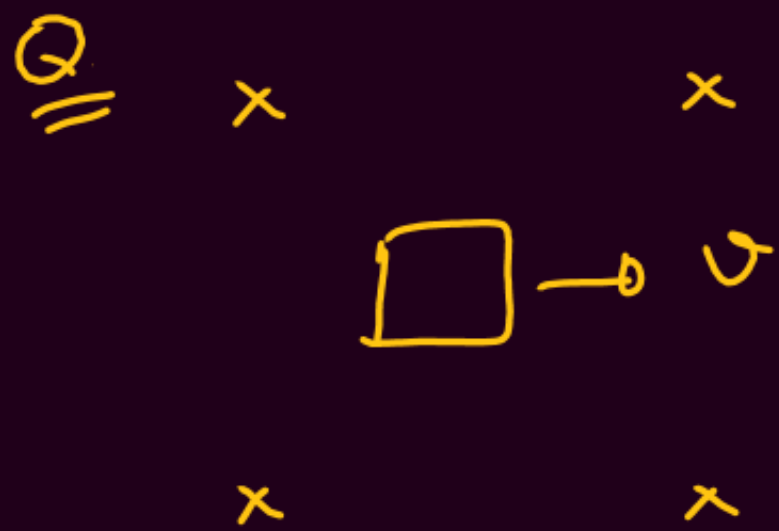
charge flown



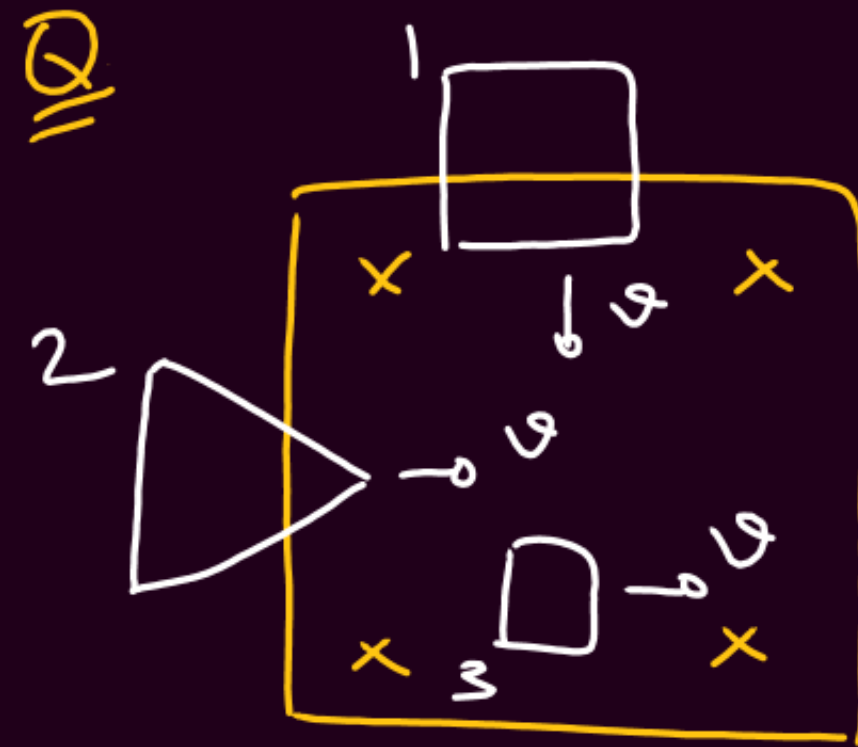
$$\mathcal{E} = - (\text{slope})$$

$$i = - \frac{1}{R} \frac{d\phi}{dt}$$

$$\Delta Q = \int i dt = - \frac{1}{R} \int d\phi$$



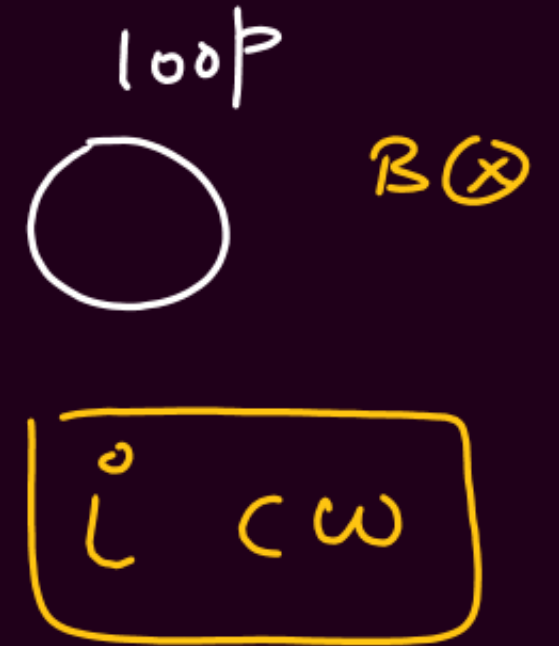
$\phi = \text{lowst.}$
 $\varepsilon = 0$
 $i = 0$



① $\phi \uparrow$ (ACW i)

② — — — — —

③ $\phi = \text{lowst.}$ $i = 0$



11Q



circle



ellipse

A ↓

∅ ↓

i cw

11Q



x

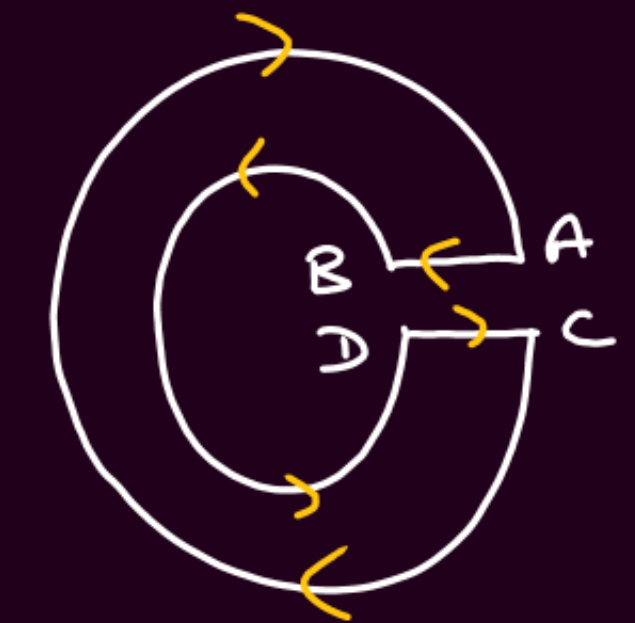
x

B?

∅?

2 → ACW

11Q



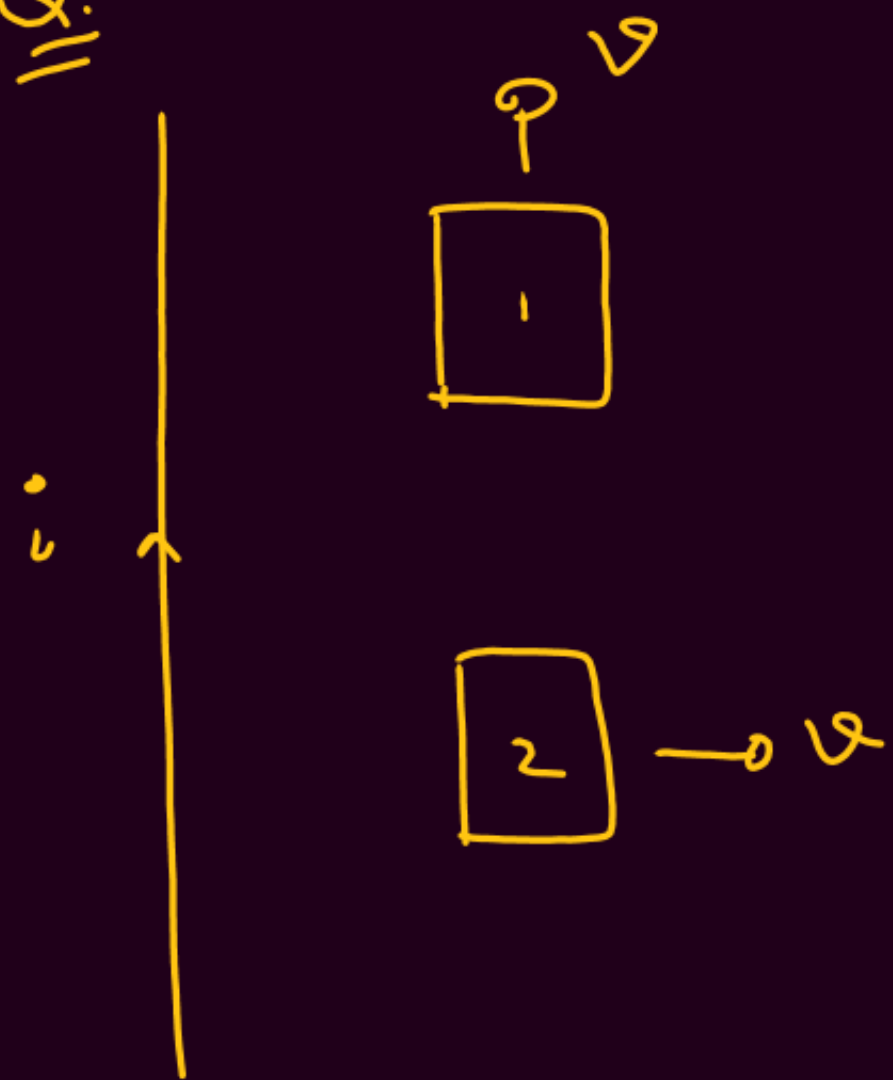
B?

∅?

A → B

D → C

Q.

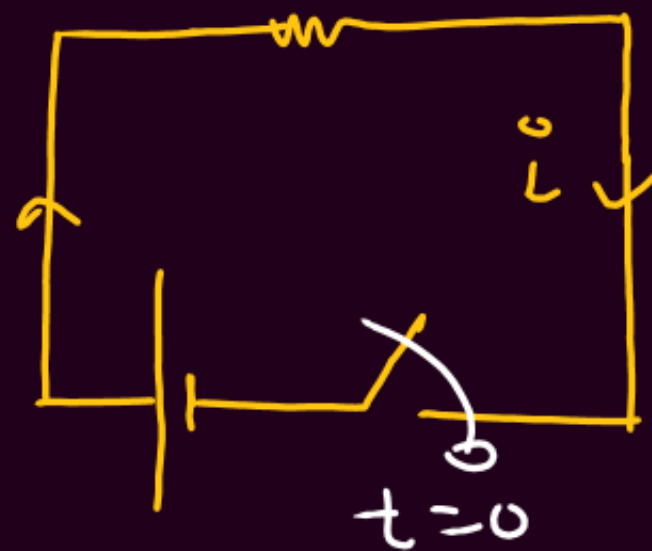


① $\phi = \text{const}$ $i = 0$

② $\phi \downarrow$

i^o cw

Q.



Just after the switch is closed

i^o
 ϕ^o

i cw

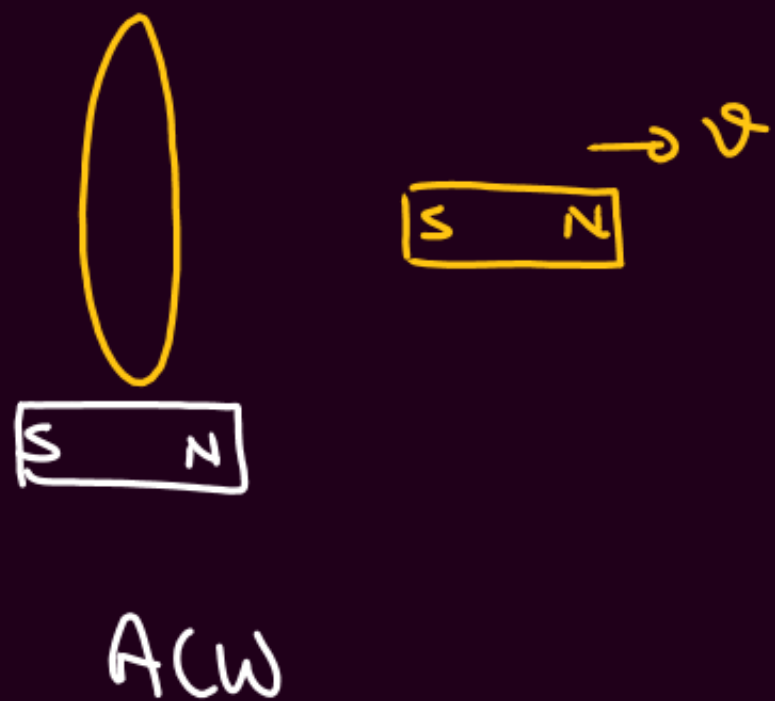
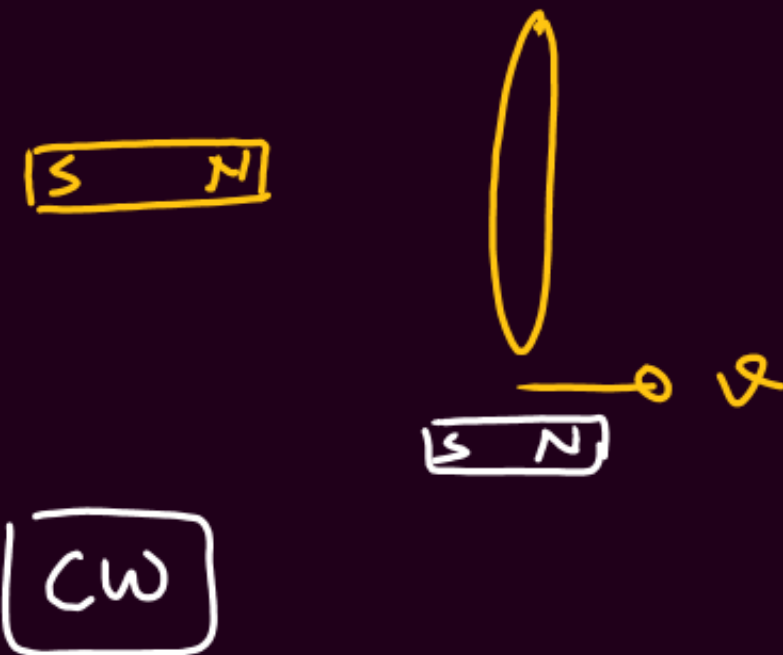
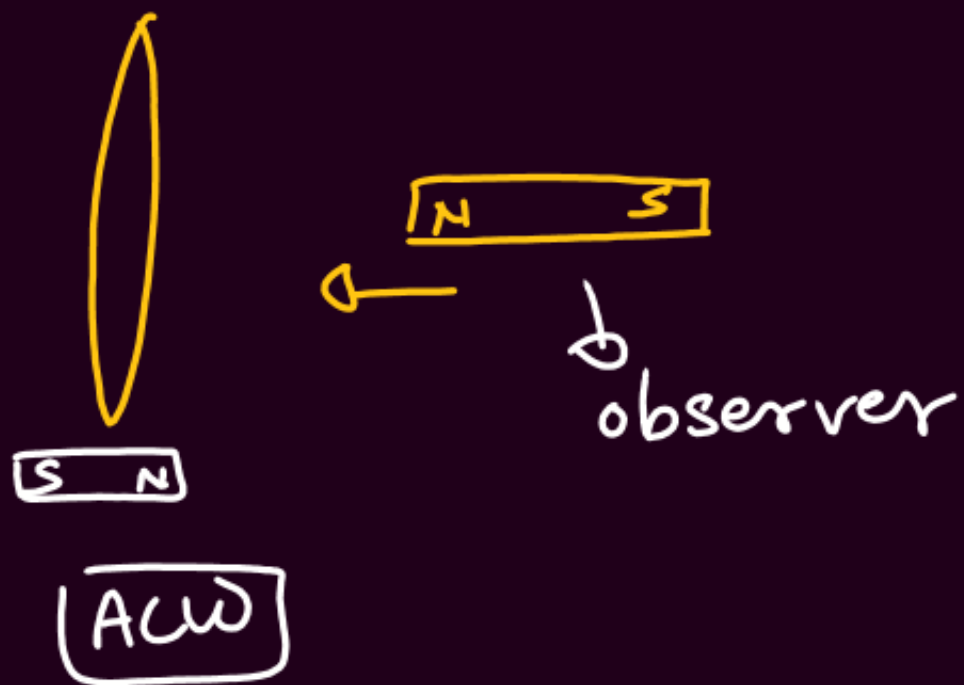


(A) cw

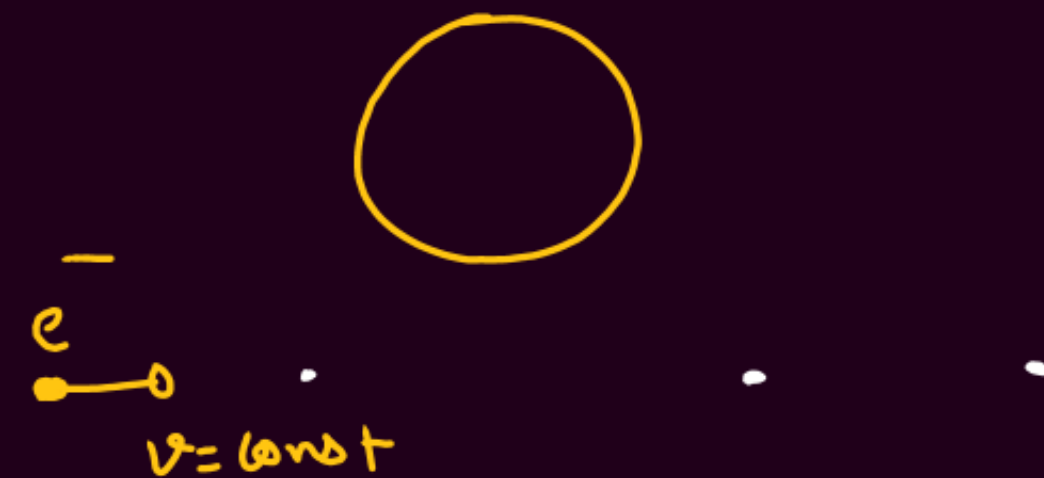
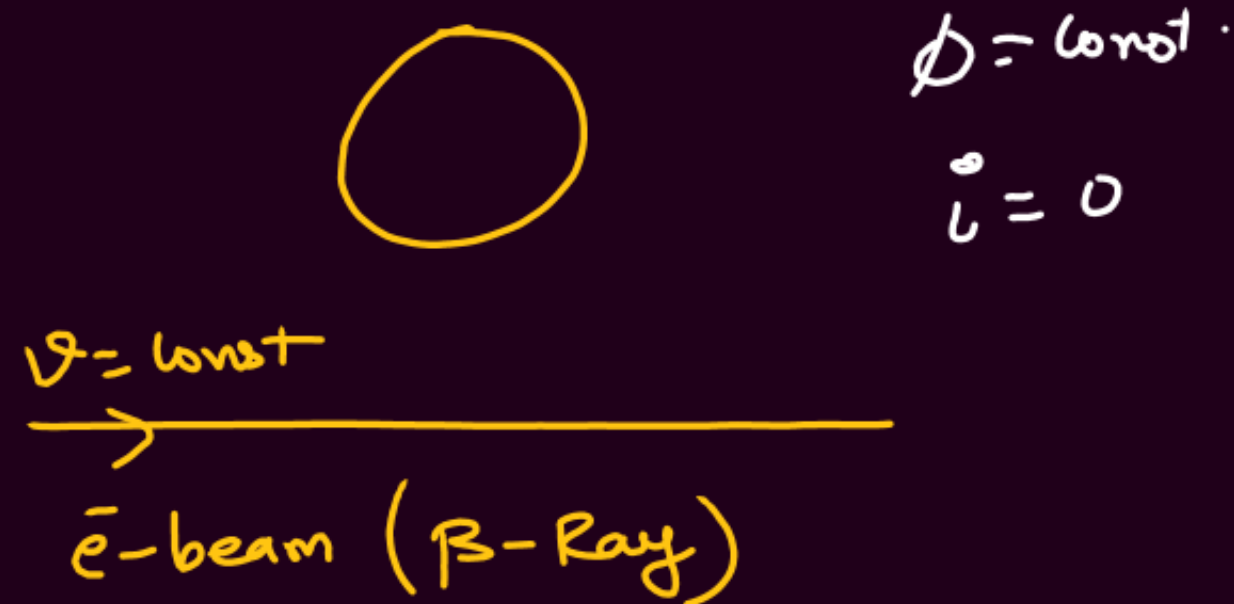
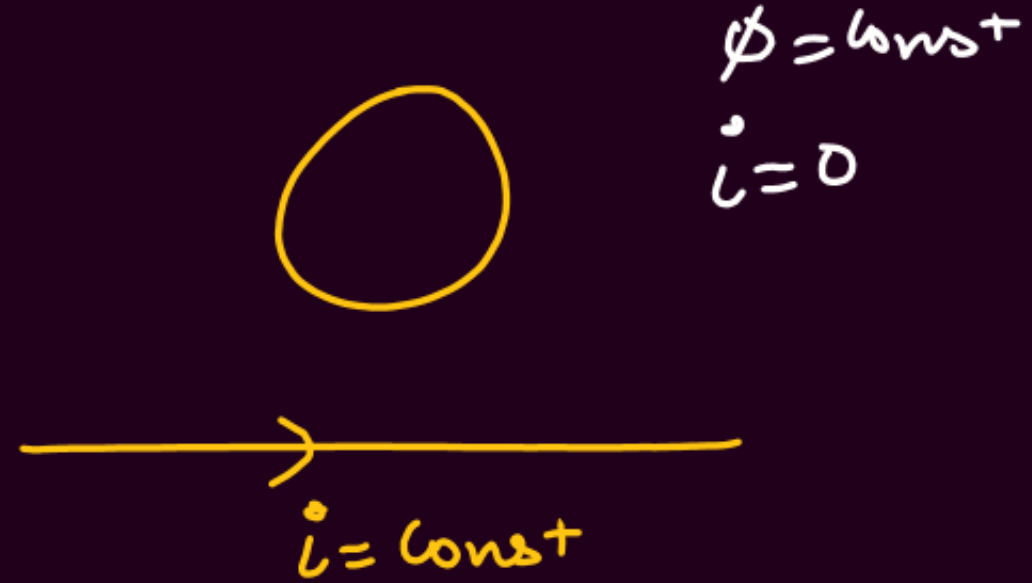
(B) ACW

(C) $\phi = 0$
 $i = 0$





EMF is induced
But induced $i = 0$



① Approach

$B \otimes$

$B \uparrow$

$\phi \uparrow$

$\boxed{i \text{ ACW}}$

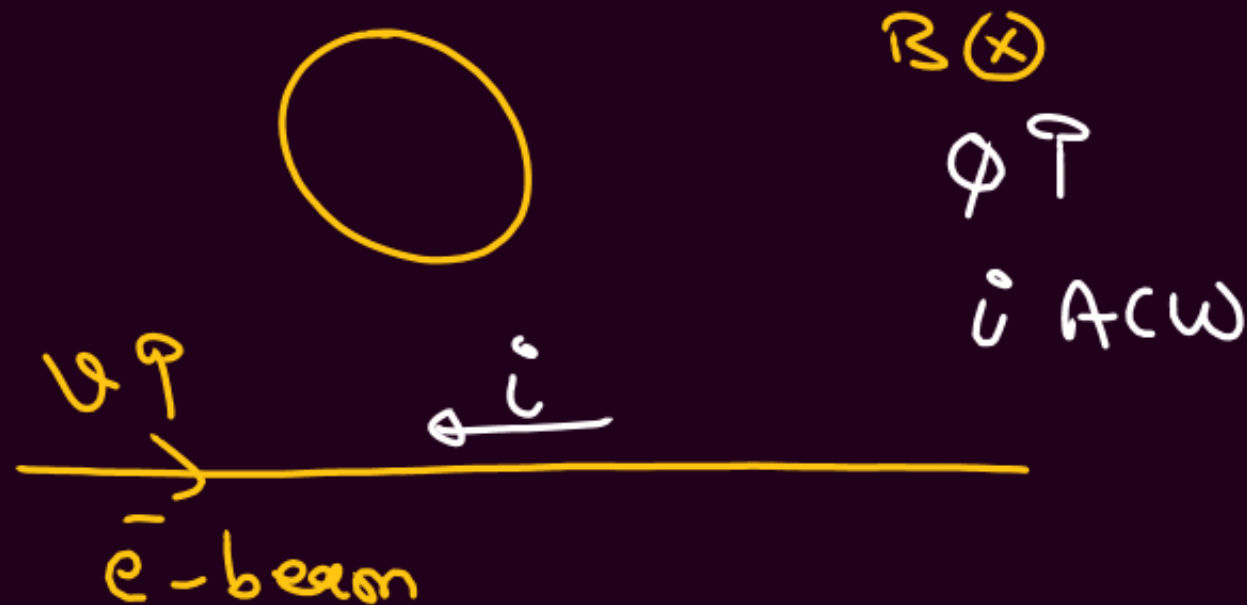
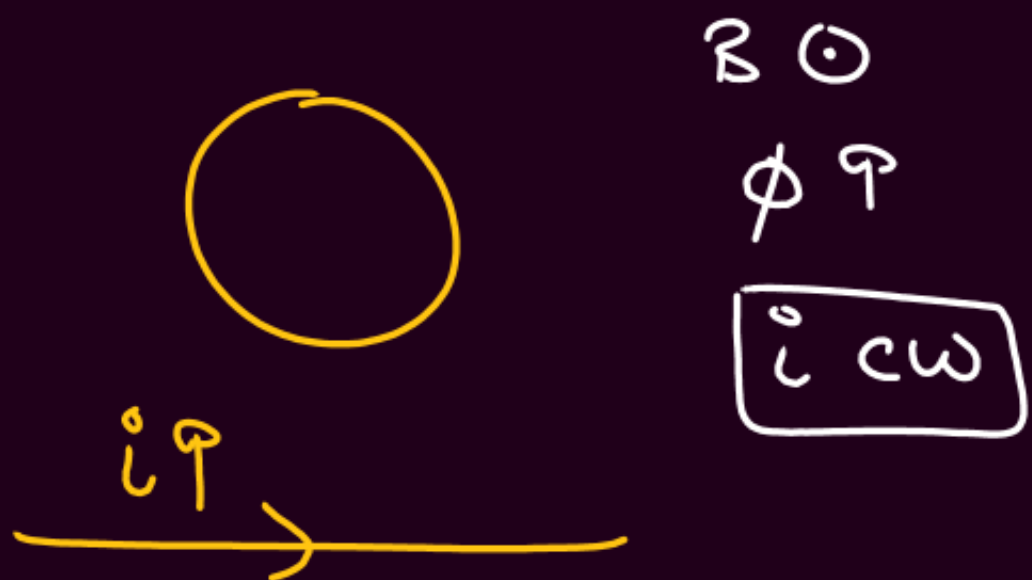
② going Away

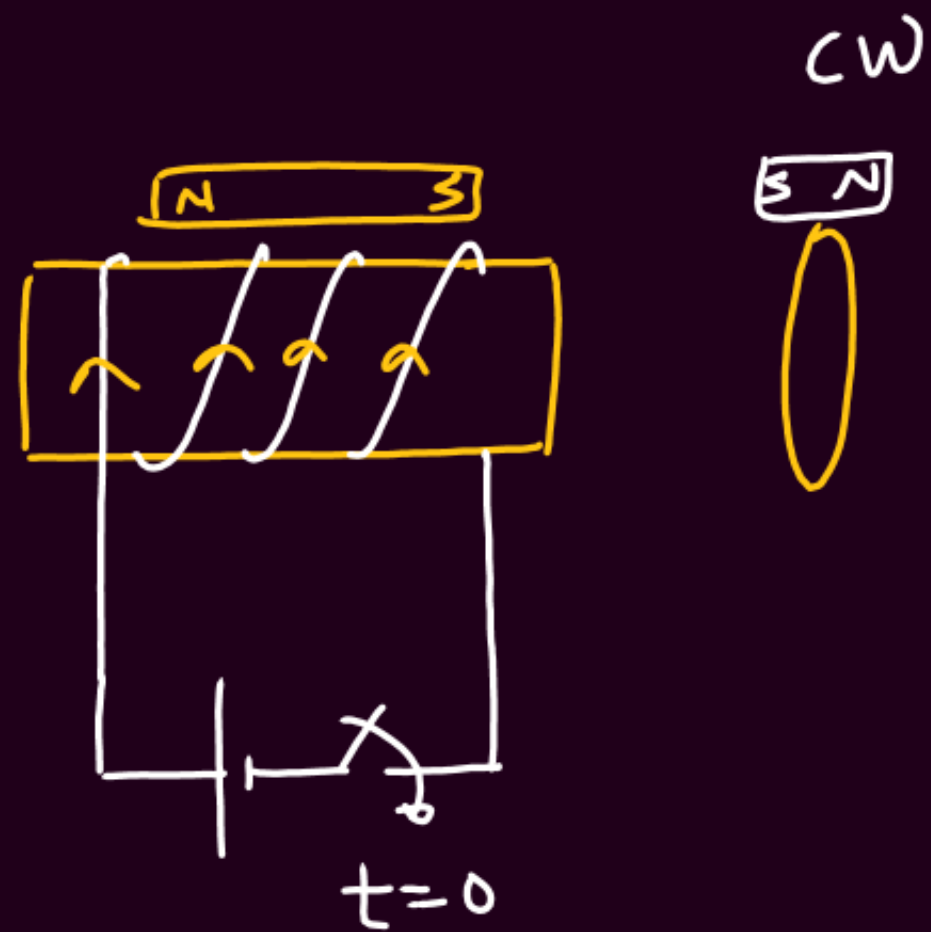
$B \otimes$

$B \downarrow$

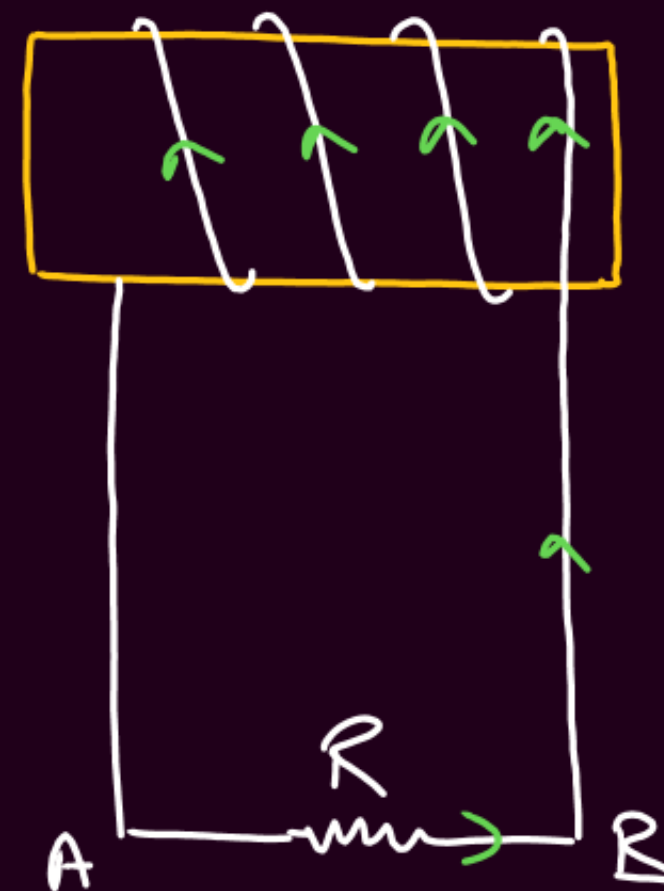
$\phi \downarrow$

$\boxed{i \text{ CW}}$

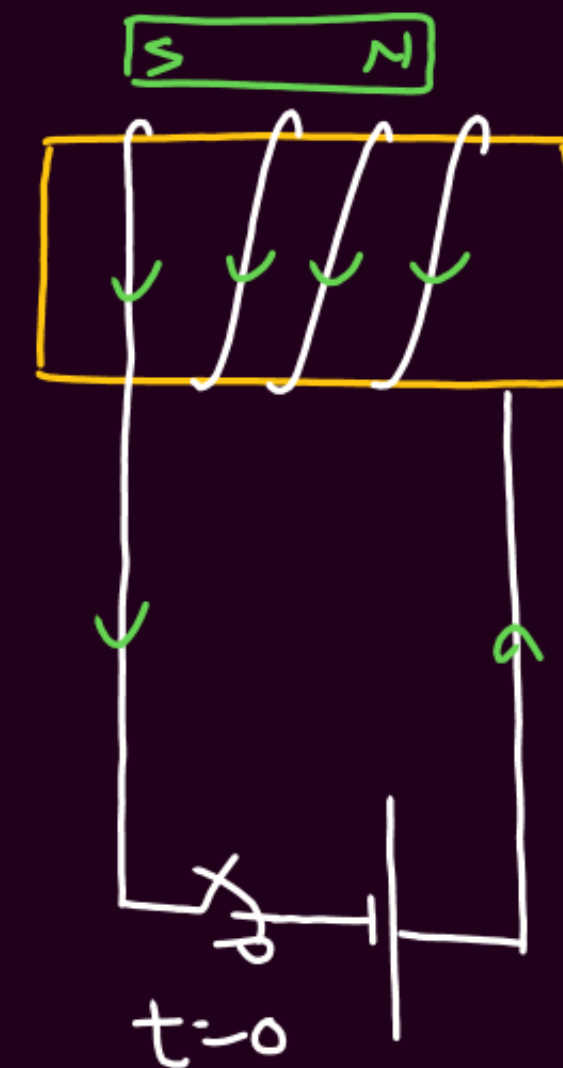




Just after the switch is closed



A to B



Just after the Switch is closed

Q $\phi = 3t^2 - 2t + 4$

find $\mathcal{E}_{\text{ind}}$ at $t = 2\text{s}$

$$\frac{d\phi}{dt} = 6t - 2$$

$$\mathcal{E} = -\frac{d\phi}{dt} = -6t + 2$$

$$\mathcal{E} = -(6 \times 2) + 2$$

$$\underline{\mathcal{E} = -10\text{V}}$$

Q $\phi = (t^2 - 4t + 3)$

find ϕ when $\mathcal{E} = 0$

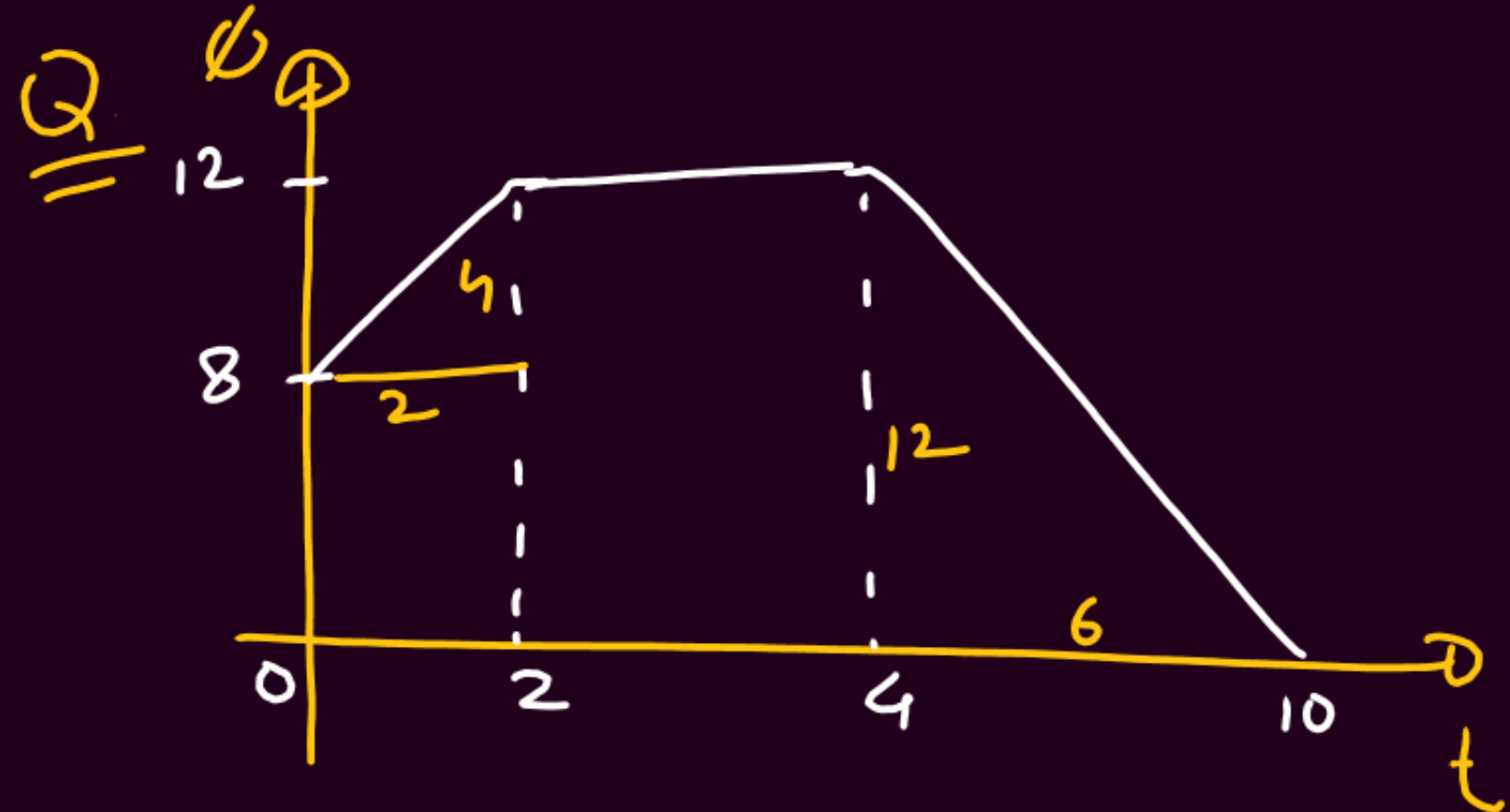
$$\mathcal{E} = 0$$

$$\frac{d\phi}{dt} = 0$$

$$2t - 4 = 0$$

$$t = 2\text{s}$$

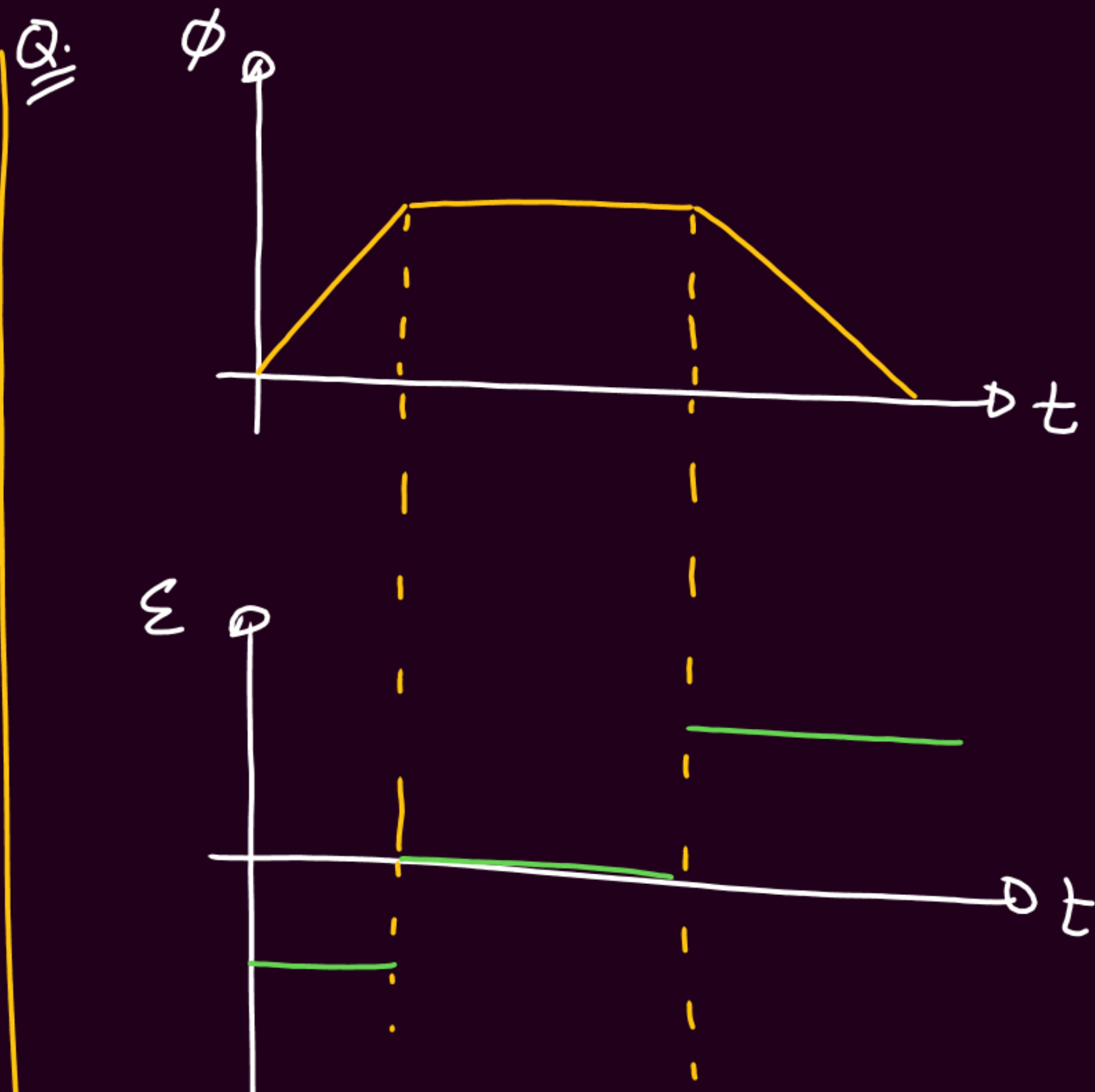
$$\phi = (2)^2 - 4(2) + 3 = -1 \text{ Wb}$$



$t = 1s$ $\mathcal{E} = -\left(\frac{4}{2}\right) = -2V$

$t = 3s$ $\mathcal{E} = -\left(0\right) = 0$

$t = 8s$ $\mathcal{E} = -\left(-\frac{12}{6}\right) = +2V$



$$\boxed{\phi = NBA \cos \theta}$$



$$N = 100$$

$$B = 10^{-3} \text{ T}$$

$$A = 5 \text{ cm}^2$$

Transverse B_0

$$\textcircled{1} \quad B \rightarrow 0 \text{ in } 2 \text{ s}$$

$$\mathcal{E} = - \left(\frac{0 - NBA}{\Delta t} \right)$$

$$\mathcal{E} = \frac{100 \times 10^{-3} \times (5 \times 10^{-4})}{2}$$

$$\mathcal{E} = 2.5 \times 10^{-5} \text{ V}$$

$\textcircled{2}$ Dir of B is reversed in 2 s
or [Coil is rotated by 180° about dia]

$$\mathcal{E} = - \frac{(-NBA) - (NBA)}{\Delta t} = \frac{2NBA}{\Delta t}$$

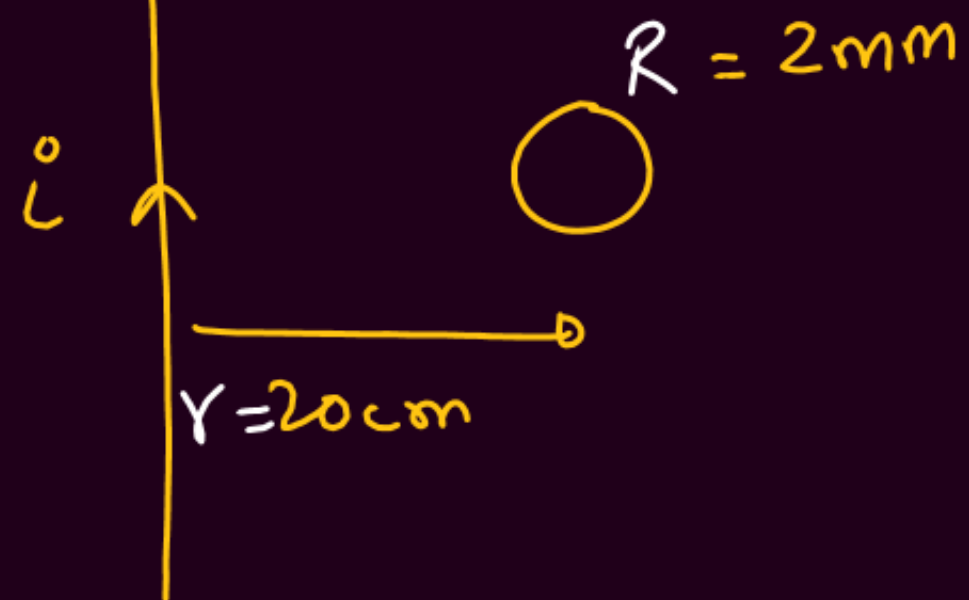
$$\mathcal{E} = 5 \times 10^{-5} \text{ V}$$

$$\textcircled{3} \quad B \text{ } (10^{-3} \text{ T}) \rightarrow (0.2 \times 10^{-3} \text{ T}) \text{ in } 2 \text{ ms}$$

$$\mathcal{E} = - \frac{NB_2 A - NB_1 A}{\Delta t} = \frac{NA}{\Delta t} (B_1 - B_2) = \frac{100 \times (5 \times 10^{-4}) (10^{-3} - 0.2 \times 10^{-3})}{2 \times 10^{-3}}$$

$$= \underline{0.02 \text{ V}}$$

Q



given i 4A to 1A
in 0.2s

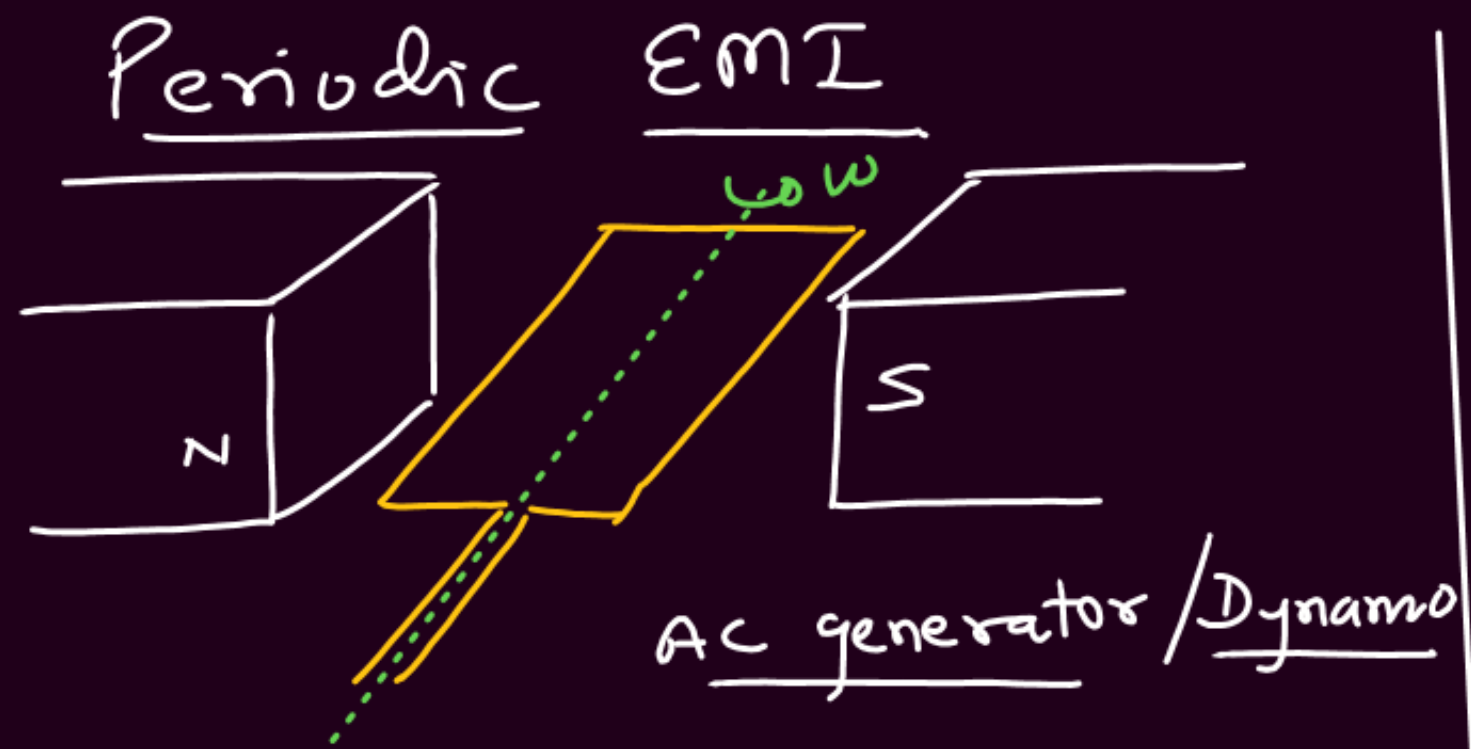
$\mathcal{E} = ?$

$$\phi = NBA = \frac{\mu_0 i}{2\pi r} (\pi R^2)$$

$$\mathcal{E} = \frac{\mu_0 R^2}{2r} \frac{(i_2 - i_1)}{\Delta t}$$

$$\mathcal{E} = \frac{(2 \times 10^{-7}) (4 \times 10^{-6}) (3)}{2 \times 0.2 \times 0.2}$$

$$\mathcal{E} = 6\pi \times 10^{-11} \text{ V}$$



① $\theta = \omega t$

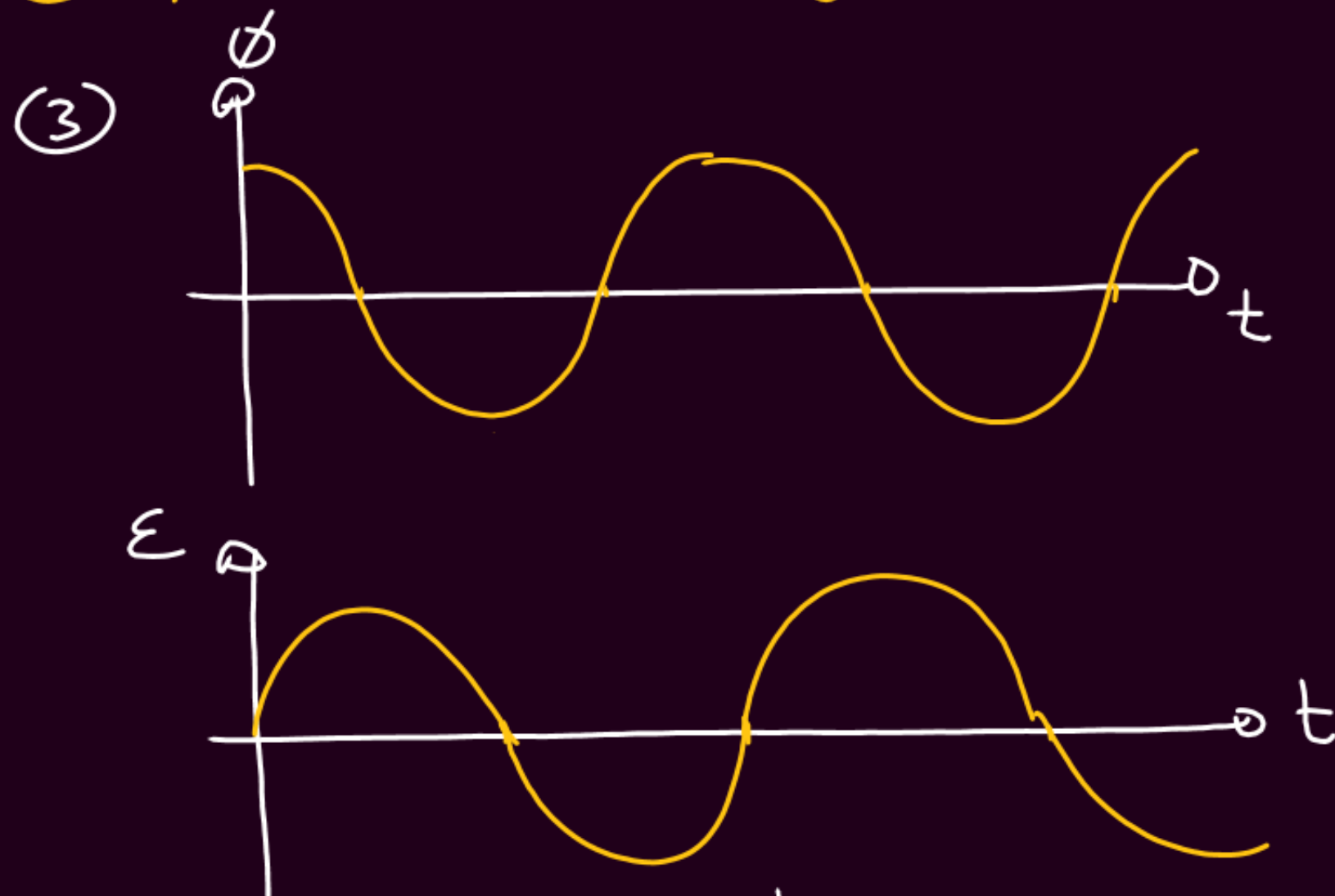
② $\phi = (NBA) \cos(\omega t)$

③ $\mathcal{E} = (NBA\omega) \sin(\omega t)$

④ $i = \left(\frac{NBA\omega}{R} \right) \sin(\omega t)$
Peak/Max./Amplitude

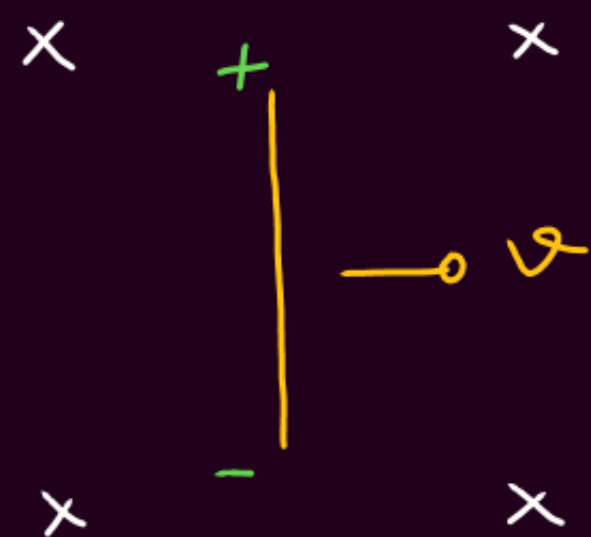
① $\mathcal{E}_p = \phi_p \omega$

② ϕ leads $\mathcal{E}$ by $\pi/2$



④ $i_{\text{inst.}}$ $\phi = 0$ $\mathcal{E}_{\text{max}} = NBA\omega$
 $\phi_{\text{max}} = NBA$ $\mathcal{E} = 0$

Motional EMF [only for conductors]



$$\boxed{\mathcal{E} = Blv} \quad B \perp l \perp v$$

⊕ polarity $\Rightarrow (\vec{v} \times \vec{B})$

$\Rightarrow \mathcal{E} = 0$ if any two of $B/l/v$ are parallel



$$B = 10^{-2} \text{ T}$$

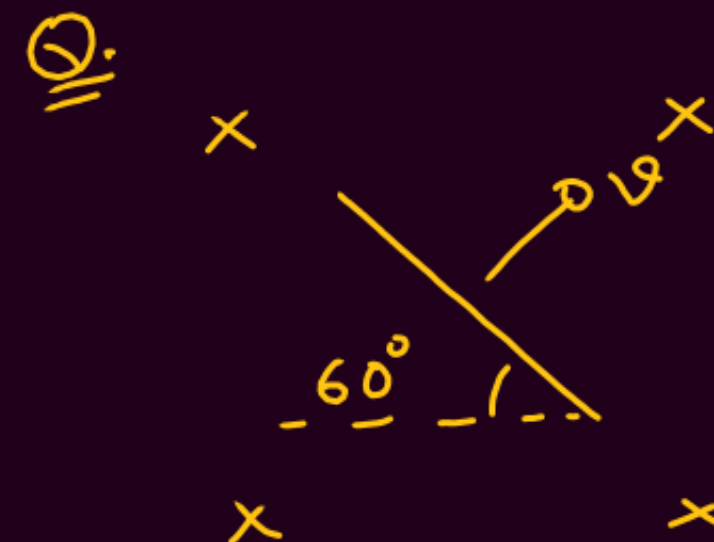
$$l = 10 \text{ cm}$$

$$v = 10 \text{ m/s}$$

$$\mathcal{E} = ?$$

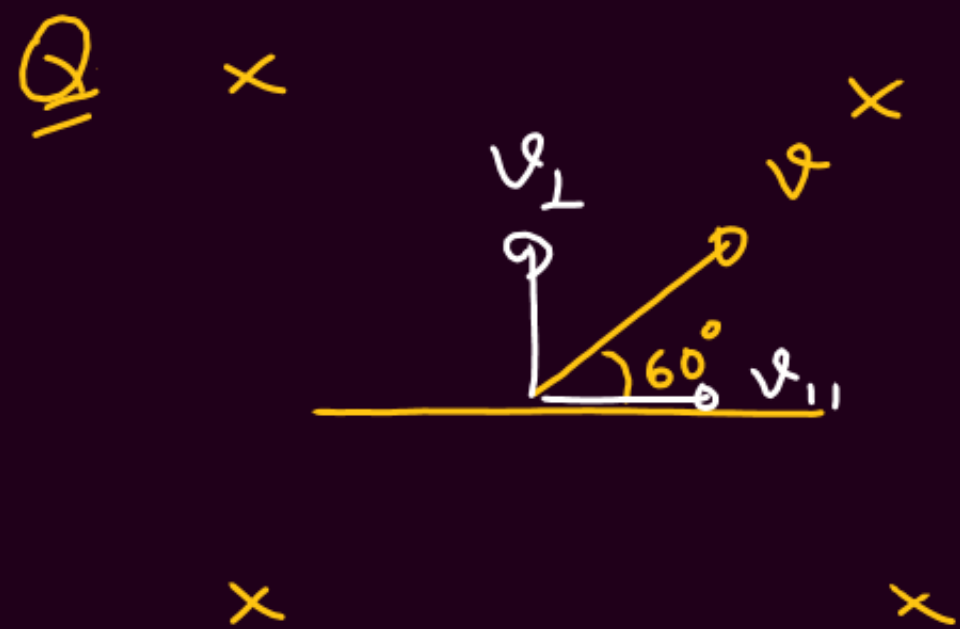
$$\mathcal{E} = 10^{-2} (0.1) (10)$$

$$\mathcal{E} = 10^{-2} \text{ V}$$

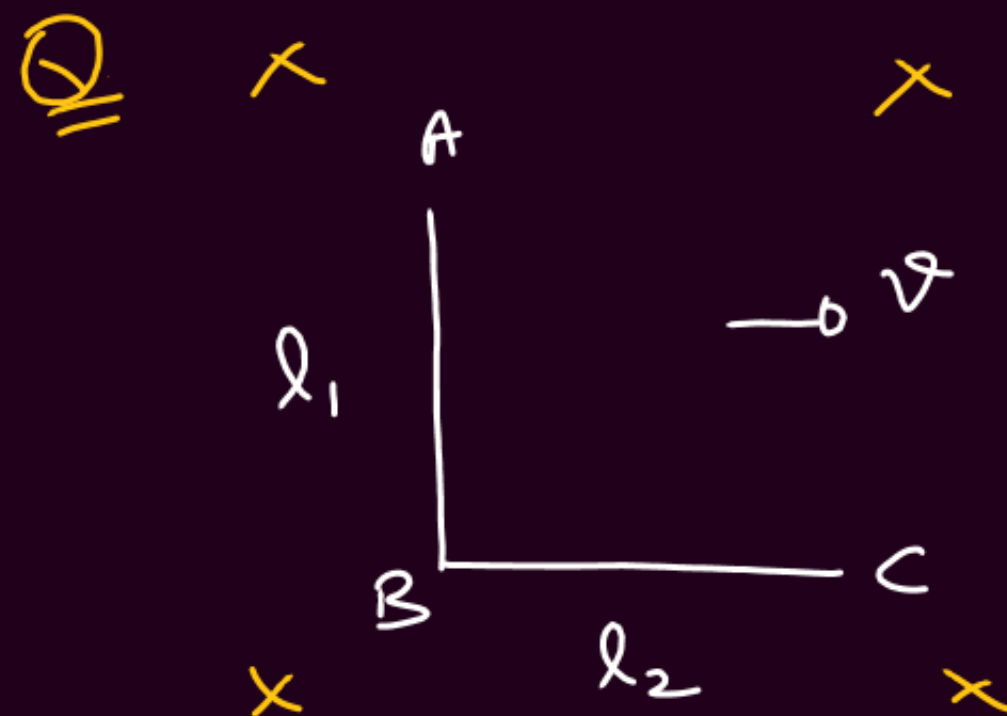


$$B \perp l \perp v$$

Same



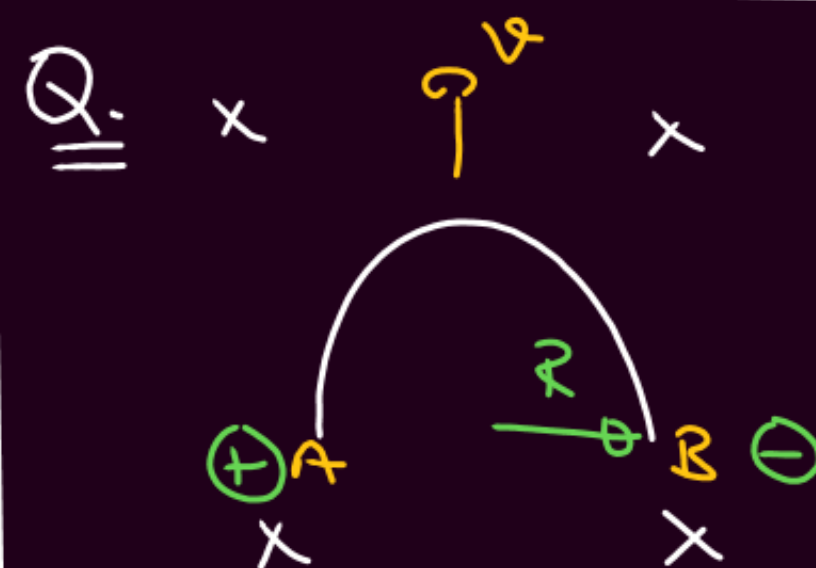
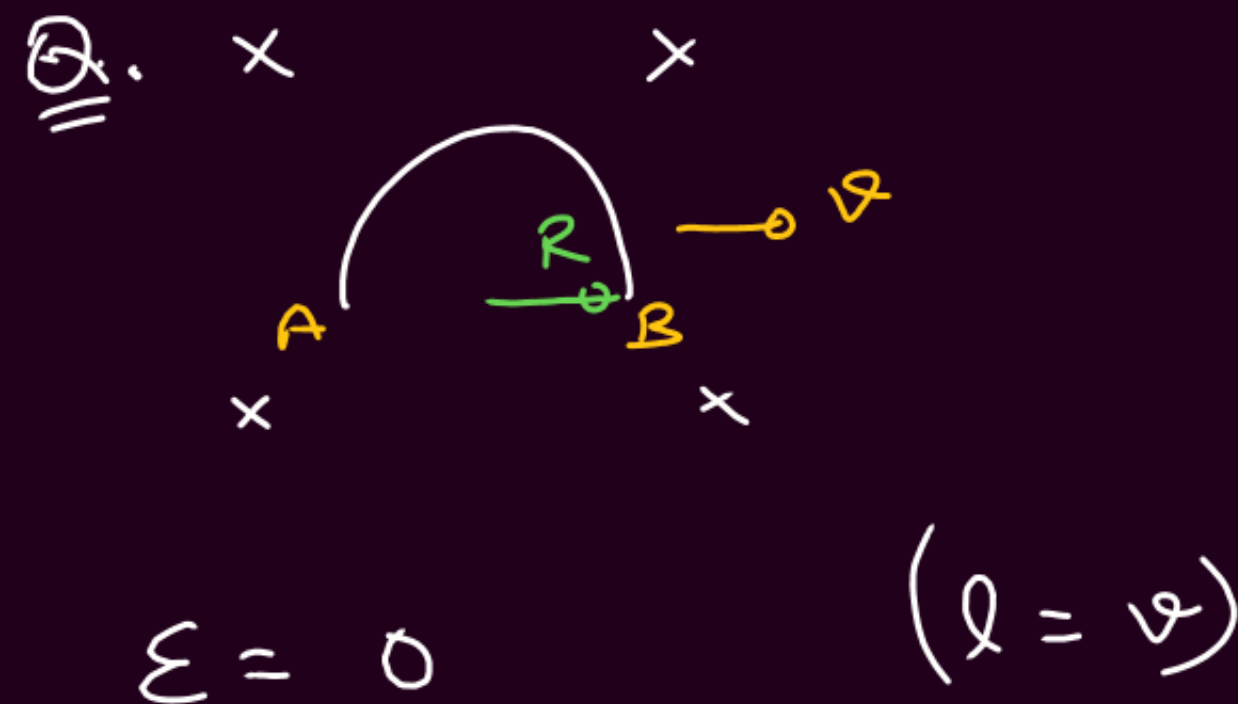
$$\begin{aligned} \mathcal{E} &= B l v_\perp \\ &= B l v \sin 60^\circ \end{aligned}$$



$$\mathcal{E}_{AB} = B l_1 v$$

$$\mathcal{E}_{BC} = 0$$

$$\mathcal{E}_{AC} = B l_1 v$$

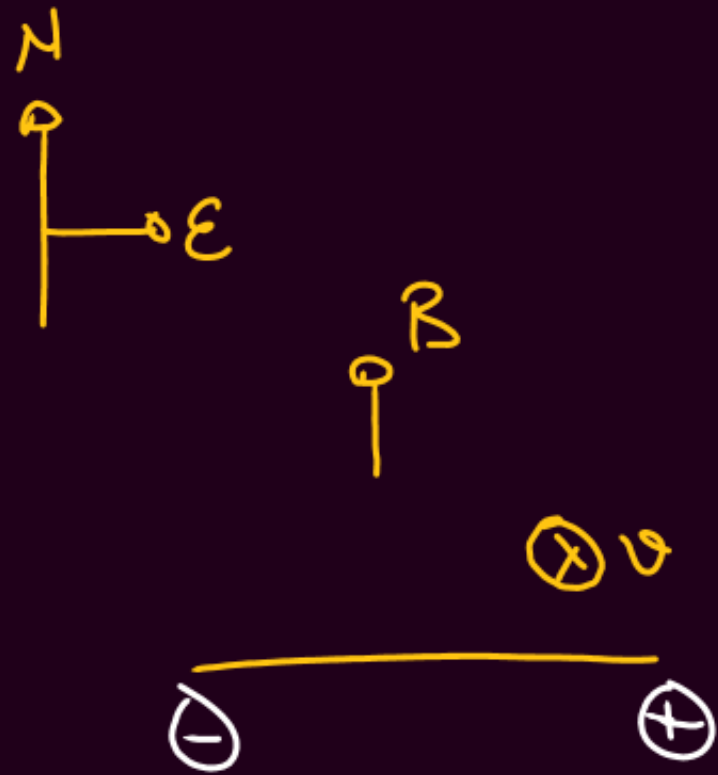


$$\mathcal{E} = B(2R)v$$

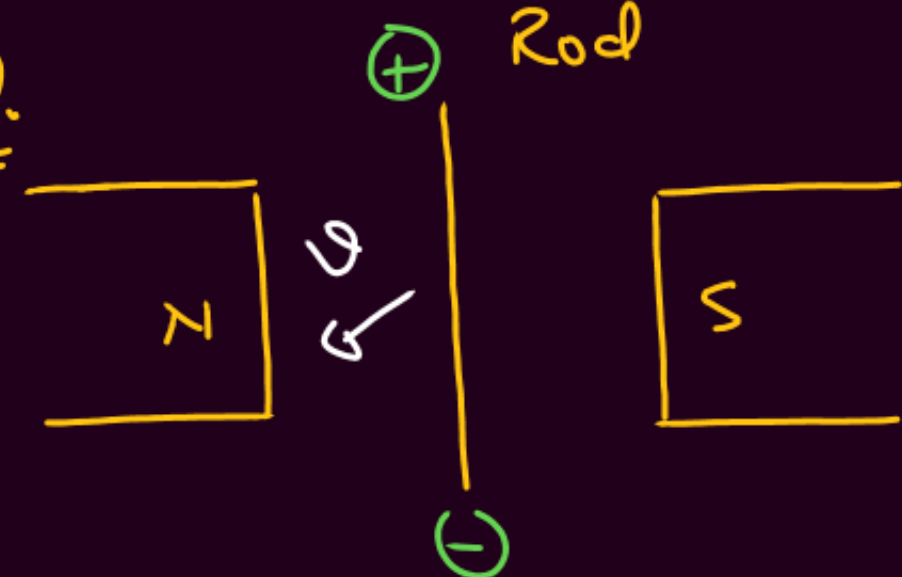
Q. Horizontal along E-W
falling vertically

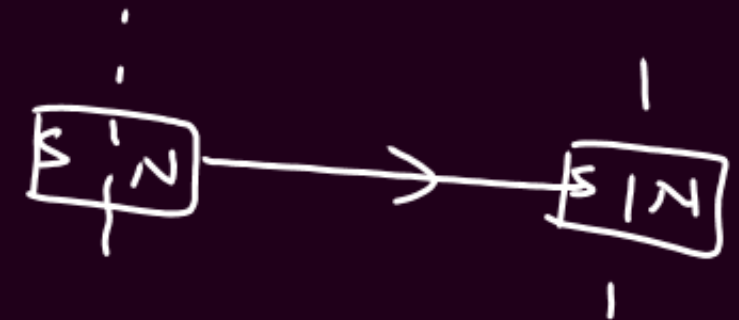
Which end is at H.P.?

↘ East



Q. 
Vertical Rod

Q. 



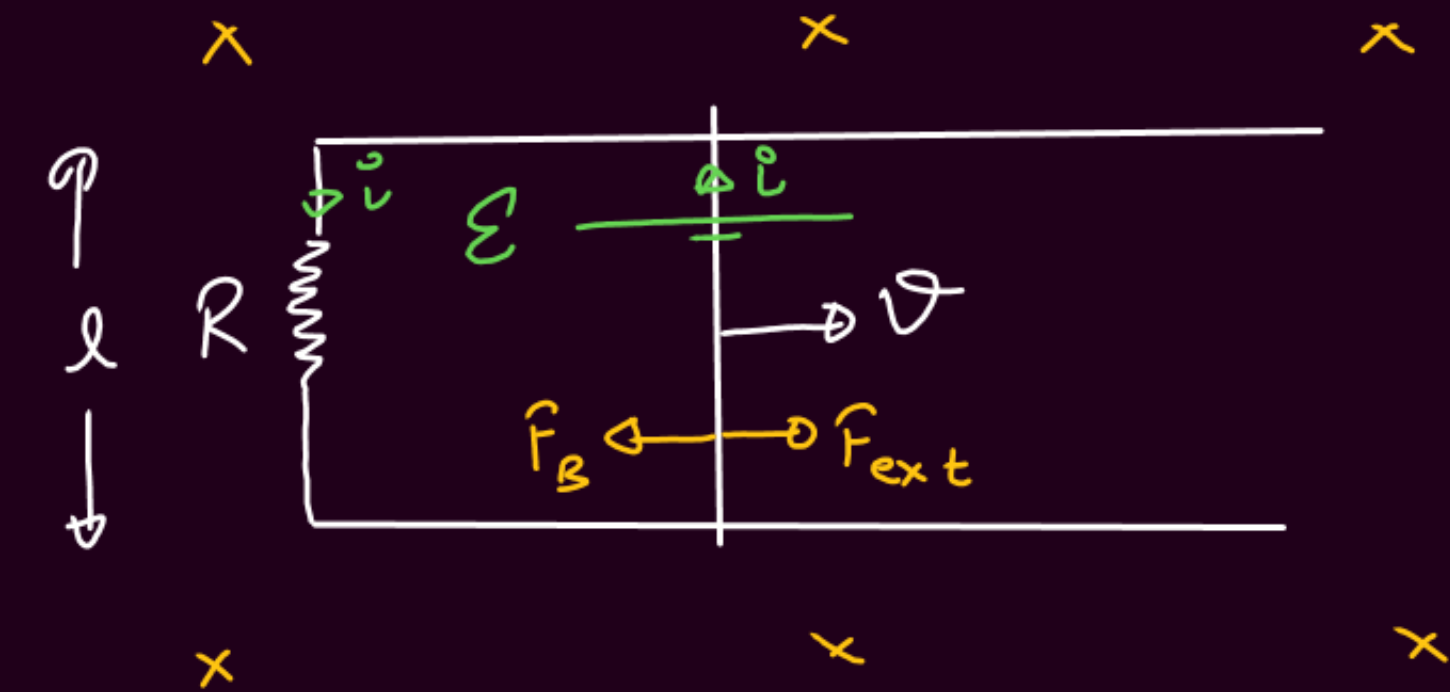
$$\Sigma_{loop} = 0$$

$$\sum PQ = 0$$

$$\varepsilon_{QR} = -Bl_2 \dot{\varphi}$$

$$\xi_{RS} = 0$$

$$\epsilon_{sp} = +1312 \text{ } ^\circ$$



① $\epsilon = Blv$

② $i = \frac{\epsilon}{R} = \frac{Blv}{R}$

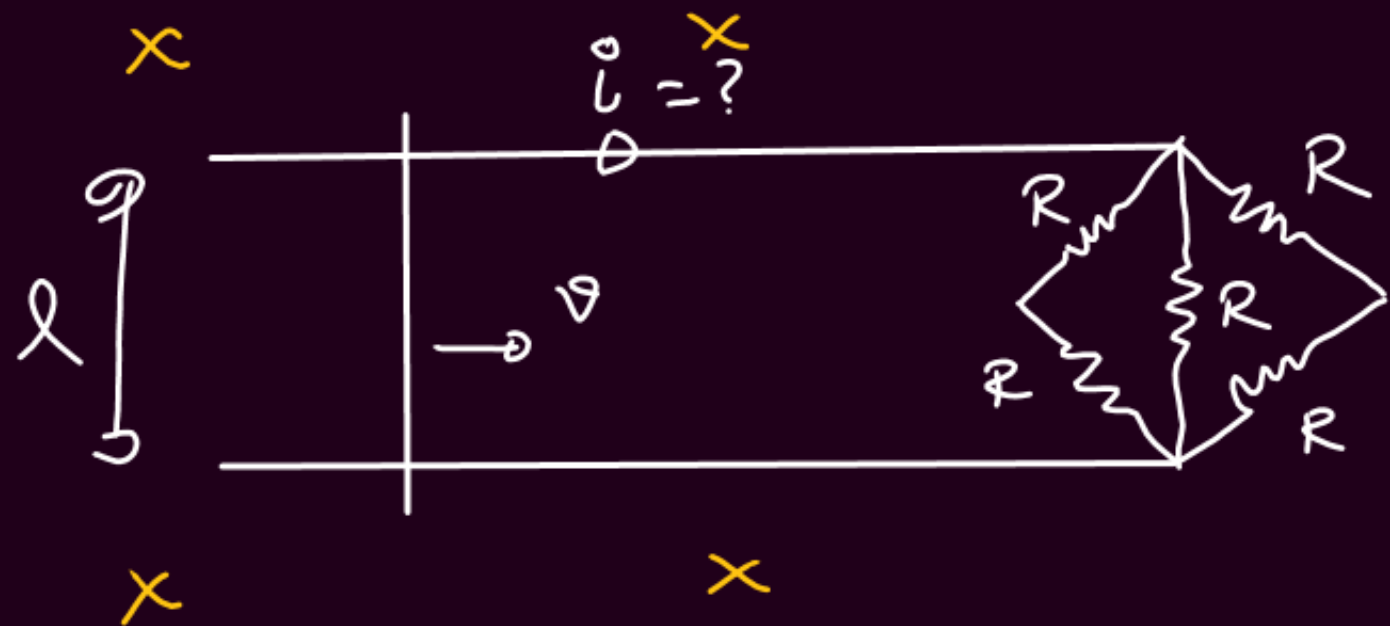
③ Power = $\frac{\epsilon^2}{R} = \frac{B^2 l^2 v^2}{R}$
Dissipated

④ Force req to move with $v = \text{const}$

$$F_{ext} = F_B = Bil = B \left(\frac{Blv}{R} \right) l = \frac{B^2 l^2 v}{R}$$

⑤ Power req to move with const. vel

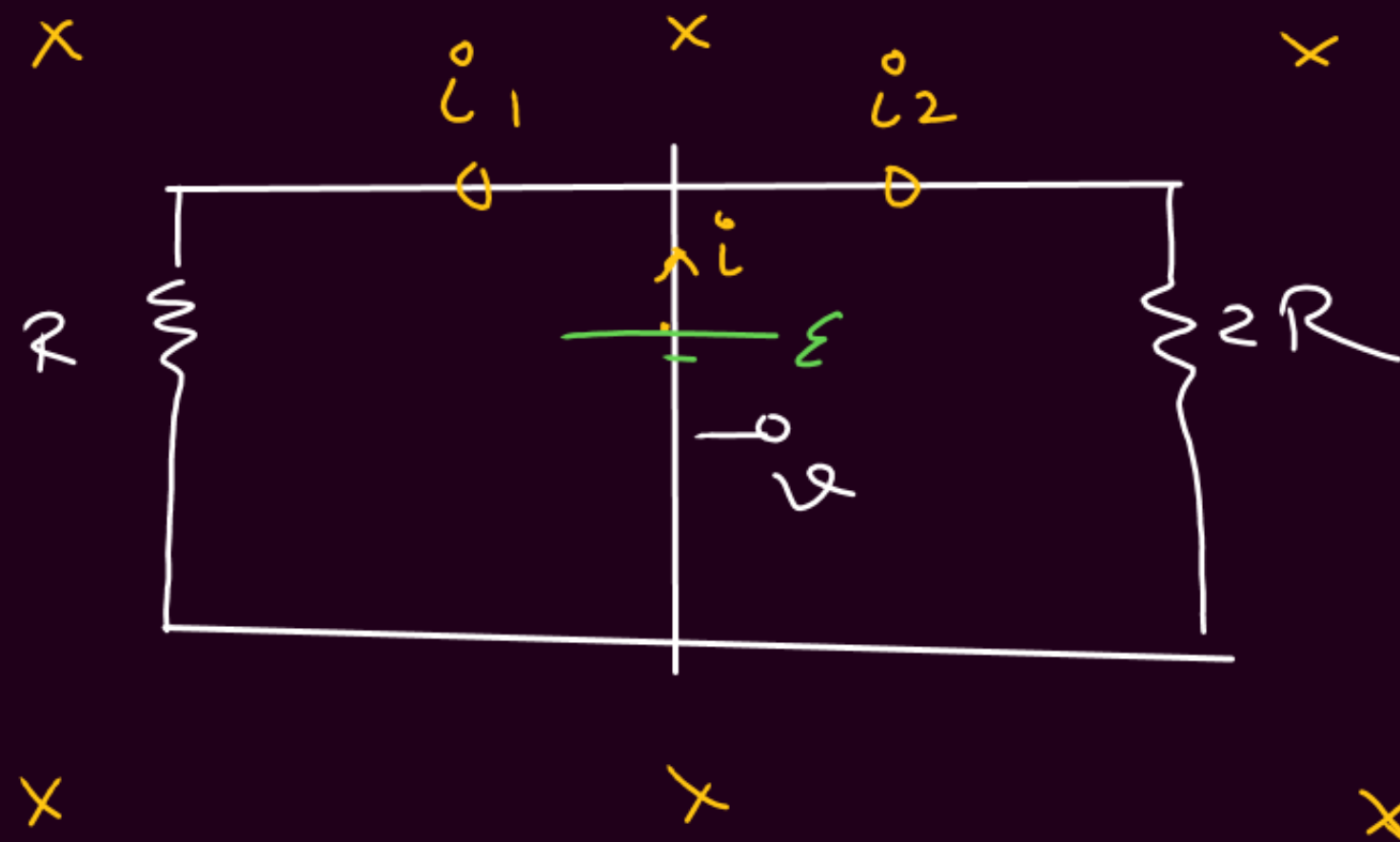
$$P = (\text{Force})(\text{vel.})$$



$$\mathcal{E} = Blv$$

$$R_{eq} = \frac{R}{2}$$

$$i = \frac{2Blv}{R}$$

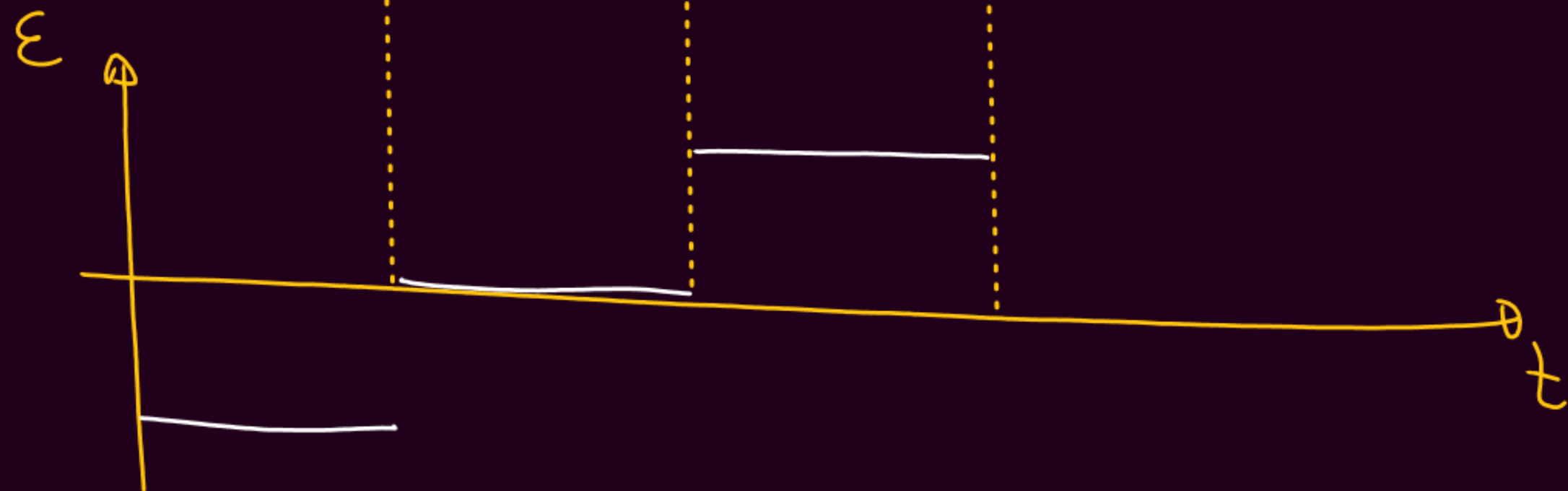
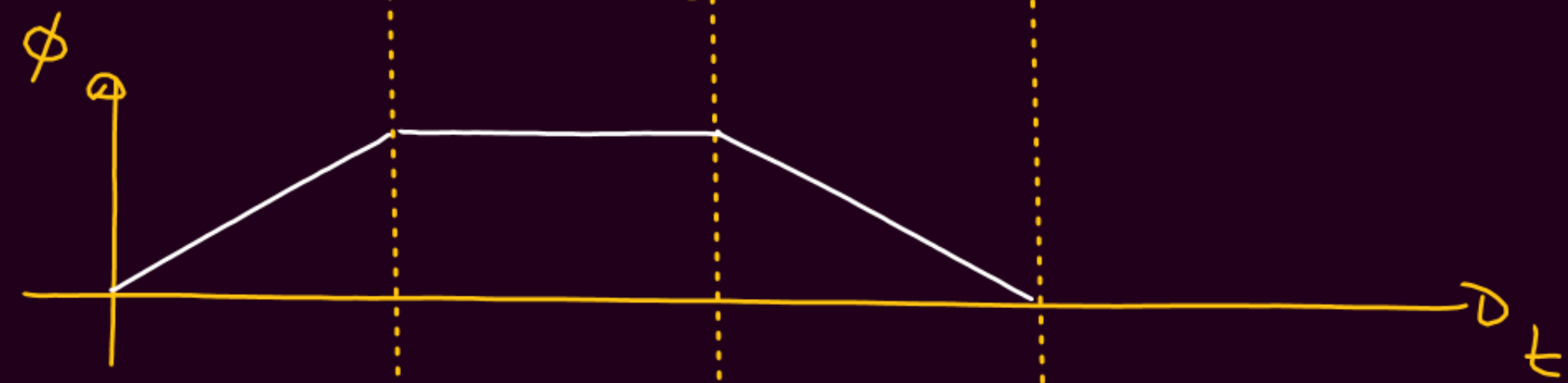
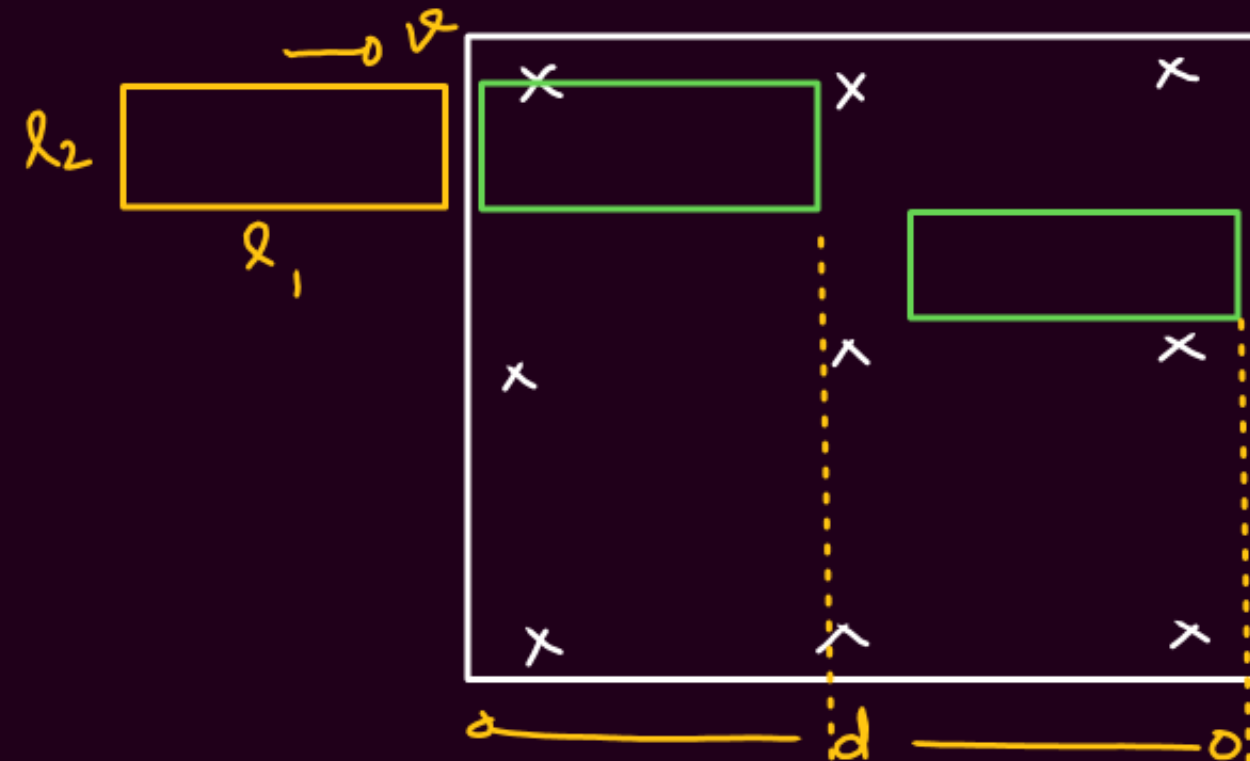


$$\mathcal{E} = Blv$$

$$i_1 = \frac{Blv}{R}$$

$$i_2 = \frac{Blv}{2R}$$

$$i = \frac{3Blv}{2R}$$

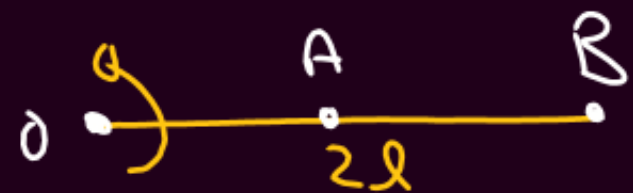


Rotating Rod



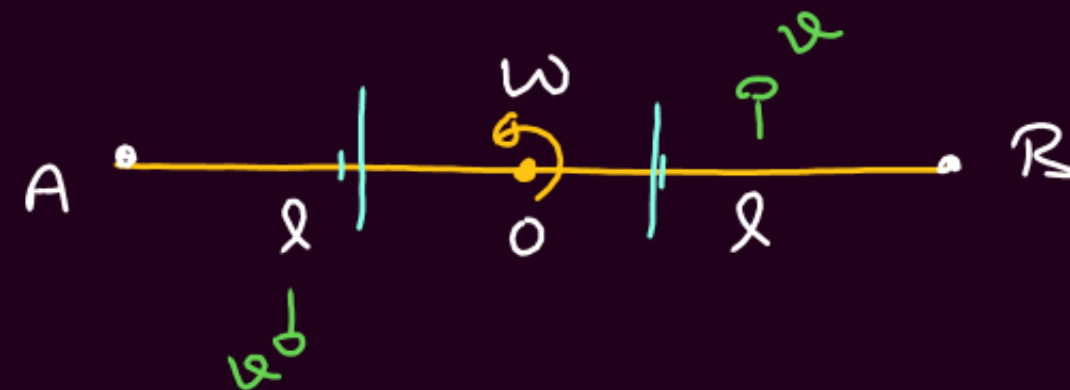
$$\boxed{\mathcal{E} = \frac{B\omega R^2}{2}}$$

$$\mathcal{E}_{OB} = \frac{B\omega(2l)^2}{2}$$



$$\mathcal{E}_{OA} = \frac{B\omega l^2}{2}$$

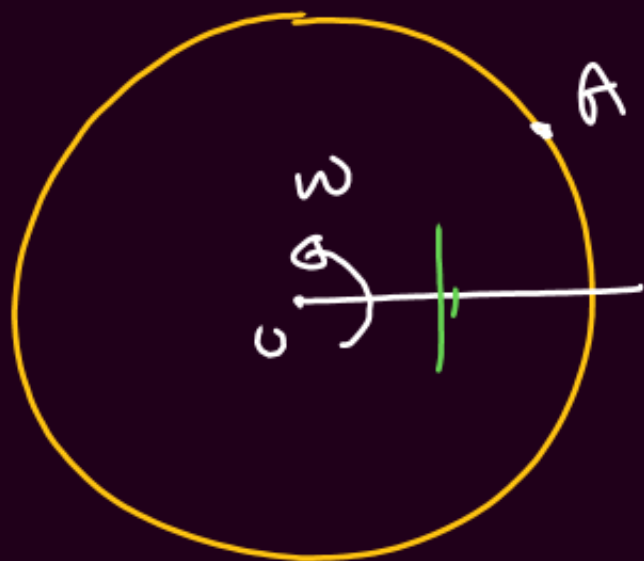
$$\mathcal{E}_{AB} = \frac{B\omega[(2l)^2 - l^2]}{2}$$



$$\mathcal{E}_{OB} = \frac{B\omega l^2}{2}$$

$$\mathcal{E}_{AO} = -\frac{B\omega l^2}{2}$$

$$\mathcal{E}_{AB} = 0$$



$$\epsilon = \frac{B\omega R^2}{2}$$



$$\epsilon_{ey} = \frac{B\omega R^2}{2}$$



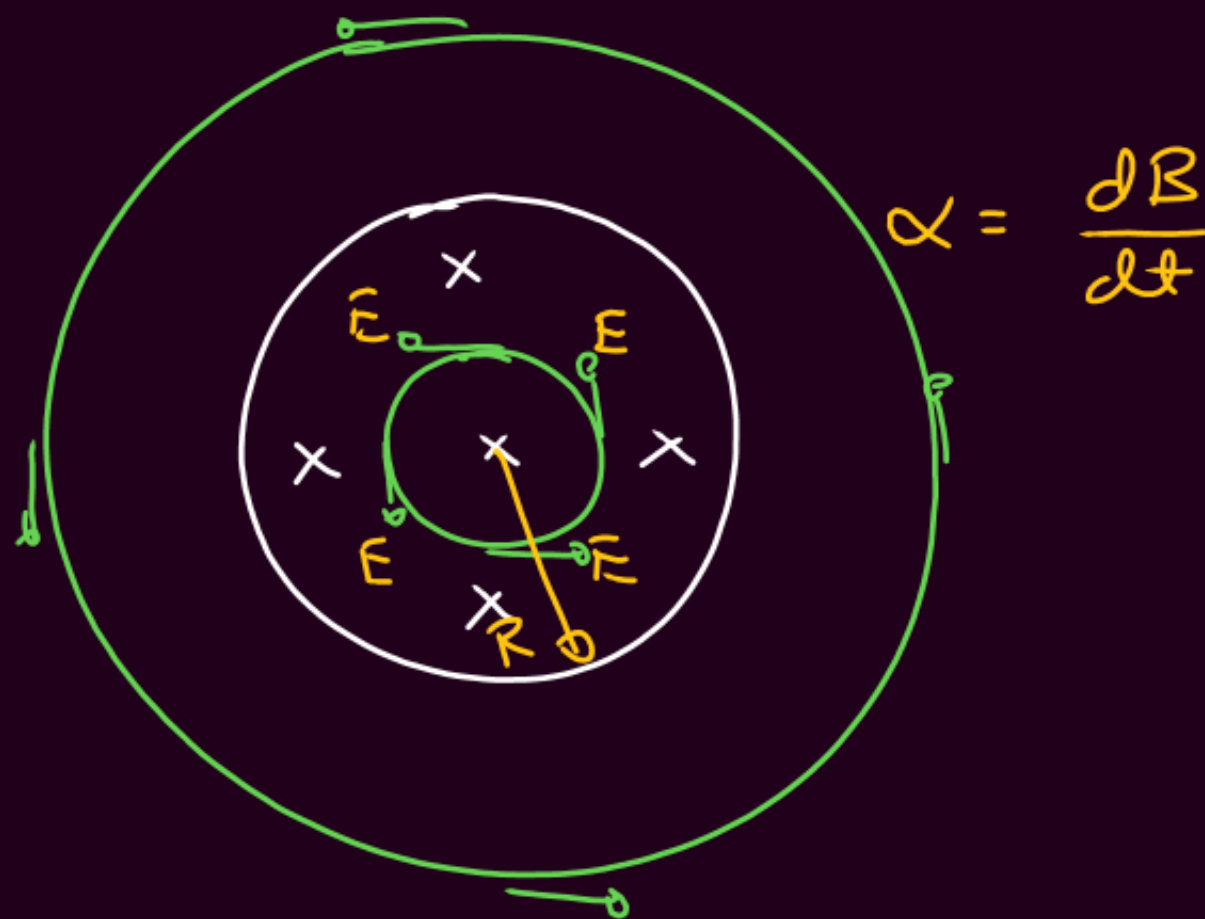
$$\epsilon = \frac{B\omega R^2}{2}$$



$$\epsilon = 0$$

Induced $\vec{E}$

$$\vec{F} = q \vec{E}$$



$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = - \frac{d\phi}{dt}$$

① Inside

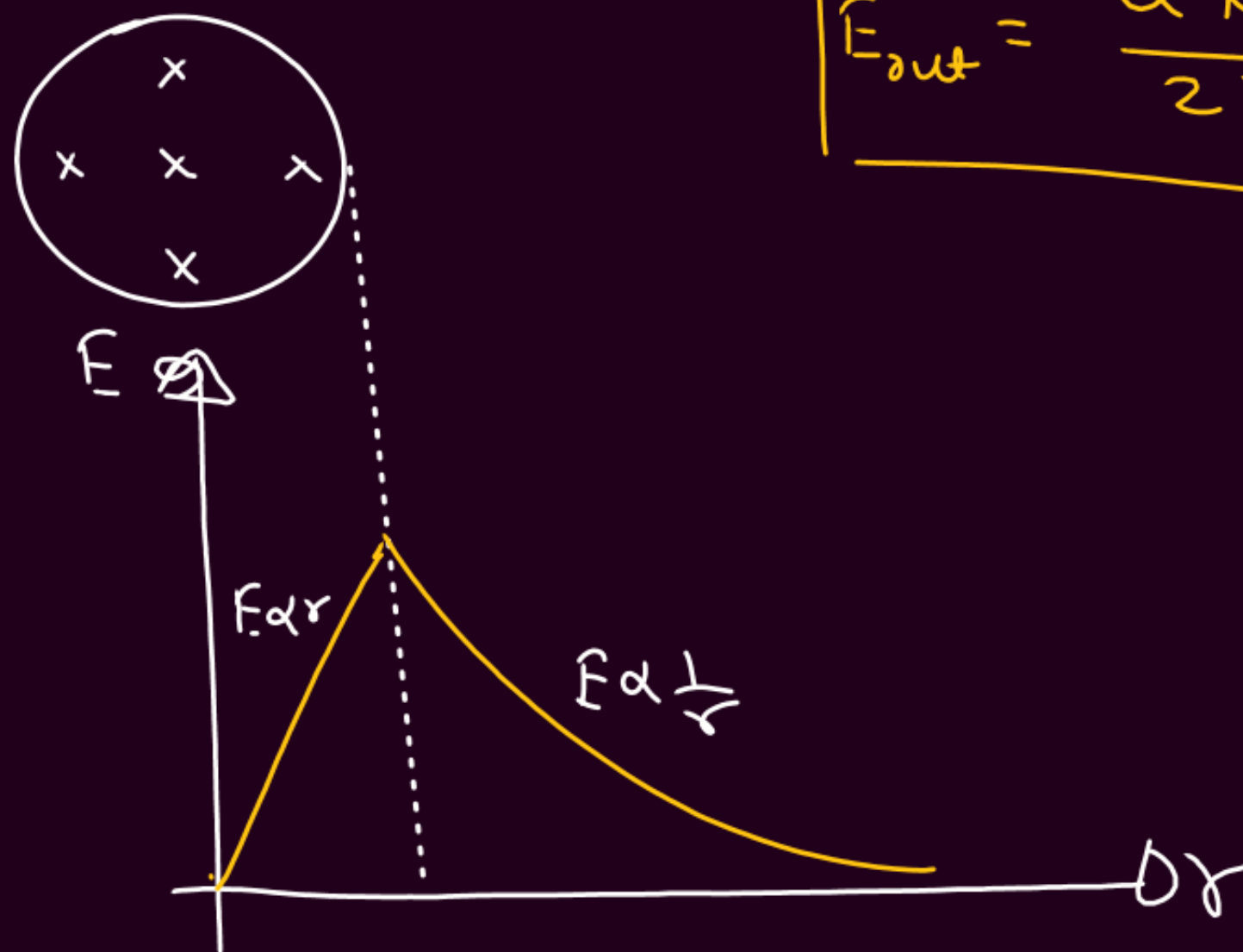
$$E(2\pi r) = \pi r^2 \left(\frac{dB}{dt} \right)$$

$$E_{in} = \frac{\alpha r}{2}$$

② Outside

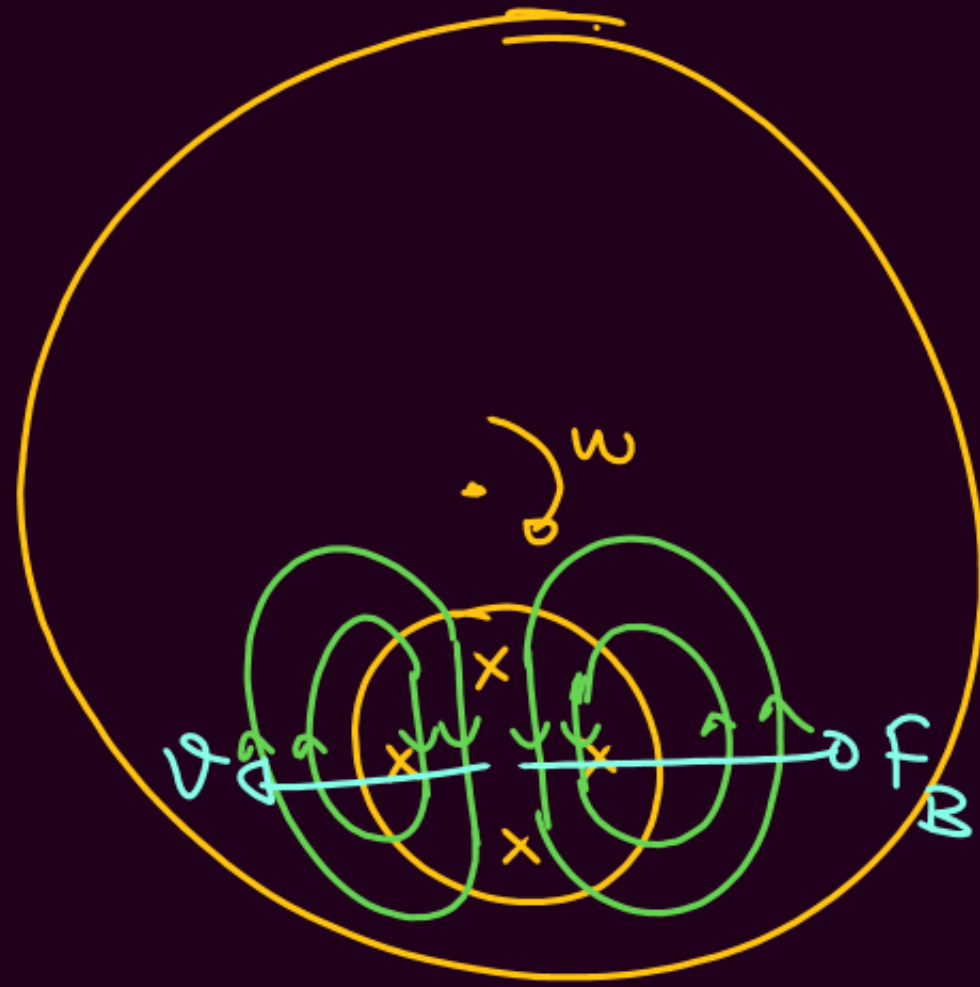
$$E(2\pi r) = \pi R^2 \left(\frac{dB}{dt} \right)$$

$$E_{out} = \frac{\alpha R^2}{2r}$$



Eddy Currents

$i \rightarrow$ swirling Eddies



- ① Braking of Train
- ② Electromagnetic Damping
- ③ Induction furnace
- ④ Metal Detectors
- ⑤ Moving coil galvanometer



To reduce damping $\rightarrow$ slots

Self Inductance (L)

$$\phi \propto i$$

$$\boxed{\phi = L i}$$

$$- \frac{d\phi}{dt} = -L \frac{di}{dt}$$

$$\boxed{\mathcal{E} = -L \frac{di}{dt}}$$

Unit $(H) = \frac{Wb}{A} = \Omega \cdot s$
(Henry)

Solenoid

$$N \left(\frac{\mu_0 N i}{l} \right) A = L i$$

Medium μ_r

N
 A & l

$$\boxed{L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 (Al)}$$

$$\boxed{L = \mu_0 \mu_r n^2 (Al)}$$

$$\boxed{L \propto N^2}$$

 L_0

 $L = 2L_0$

Q iron core $\mu_r = 100$
coil $\rightarrow N = 200$
 $l = 10\text{cm}$
 $r = 2\text{mm}$

$L = ?$

$$L = (4\pi \times 10^{-7}) (100) \frac{(200)^2}{(0.1)} \pi (4 \times 10^{-6})$$

$$= 64 \times 10^{-5} \text{ H}$$

$$L = 0.64 \text{ mH}$$

Q $L = 10\text{mH}$
 1A to 3A in 0.2s

$$\mathcal{E} = -(10 \times 10^{-3}) \left(\frac{2}{0.2} \right)$$

$$\boxed{\mathcal{E} = -0.1 \text{ V}}$$

Energy stored in L

$$U = \frac{1}{2} L i^2$$

$$U = \frac{1}{2} \phi i = \frac{\phi^2}{2L}$$

$$\text{Energy Density} = \frac{B^2}{2\mu_0}$$

Inductor / Solenoid / choke coil

Dimension

$$[L] = \frac{\text{Energy}}{i^2} = \frac{ML^2T^{-2}}{A^2}$$

$$[L] = [ML^2T^{-2}A^2]$$

Q.

$$L = 10 \text{ mH}$$

$$i = 2 \text{ A}$$

$$U = \frac{1}{2} (10 \times 10^{-3}) (2)^2$$

$$\underline{U = 0.02 \text{ J}}$$

Mutual Inductance (M)

$$\phi_2 \propto i_1$$

$$\phi_2 = M_{21} i_1$$

unit $\Rightarrow$ H

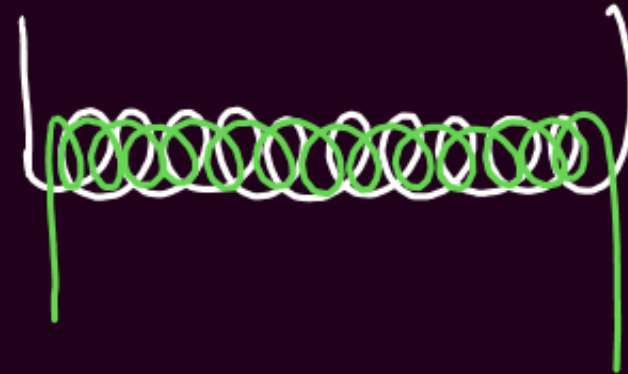
$$M_{21} = M_{12}$$

$$\phi_2 = M i_1$$

$$N_2 B_{1A_c} = M i_1$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

Two solenoids tightly wound over each other



$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l$$

$$M = \sqrt{L_1 L_2}$$

$$L_1 = \frac{\mu_0 N_1^2 A}{l}$$

$$L_2 = \frac{\mu_0 N_2^2 A}{l}$$

$$M = k \sqrt{L_1 L_2}$$

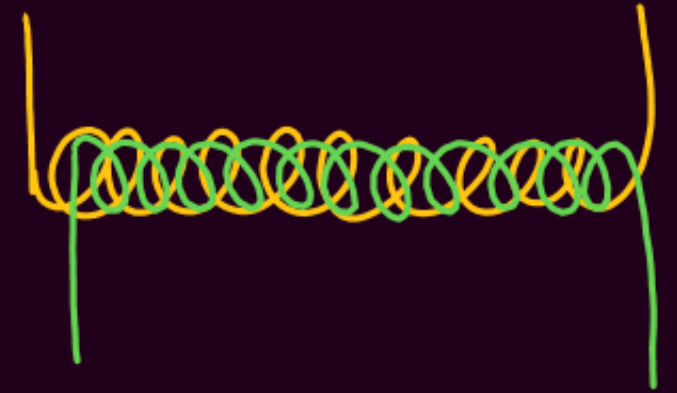
k = coeff of coupling / coupling factor

$$0 \leq k \leq 1$$

① $k=1$

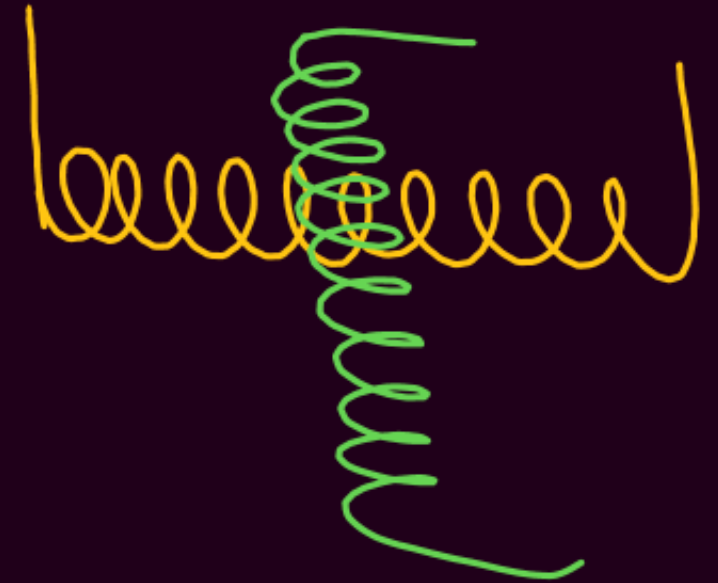
$$M = \sqrt{L_1 L_2}$$

'Max Linkage / No leakage'
(Tightly wound over each other)



② $k=0$

$$M = 0$$



① Series Combination (Extra)

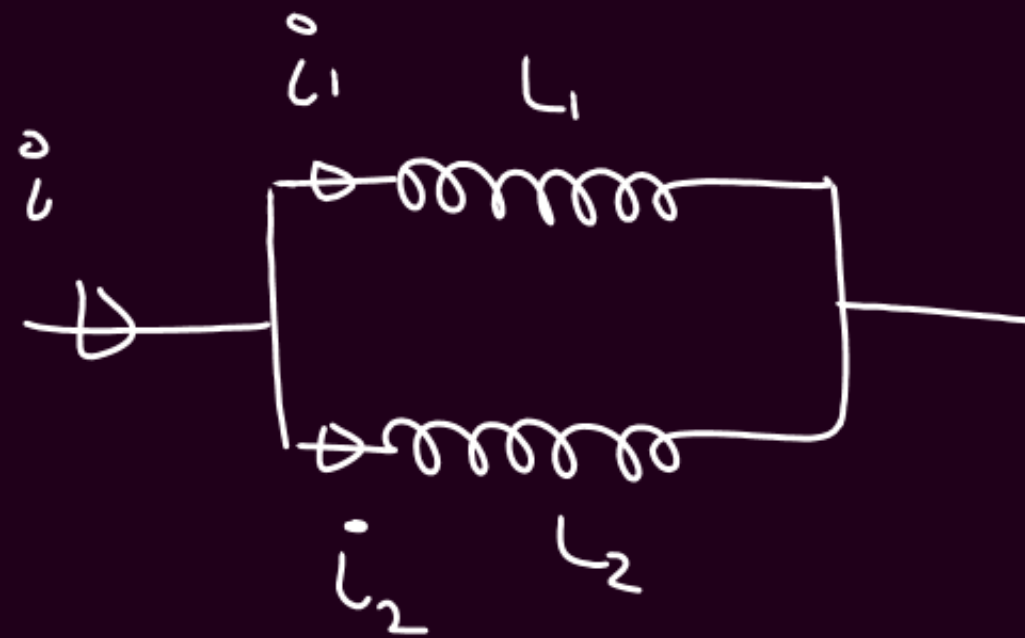


$$L_{eq} = L_1 + L_2 + 2M$$



$$L_{eq} = L_1 + L_2 - 2M$$

② Parallel Combination



①
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

②
$$i_1 L_1 = i_2 L_2$$

③
$$\mathcal{E} = -L_1 \frac{di_1}{dt} = -L_2 \frac{di_2}{dt}$$

$$i_1 = \frac{i L_2}{L_1 + L_2}$$

Q $i_1 [0 \rightarrow 2A \text{ in } 0.5\text{sec}]$

$\& \quad \mathcal{E}_2 = 0.02 \text{ V}$

$$\mathcal{E} = -M \frac{di}{dt}$$

$$0.02 = M \left(\frac{2}{0.5} \right)$$

$$M = 0.005 \text{ H} = 5\text{mH}$$

Q.

$$i_1 = i_0 \cos(\omega t)$$

M

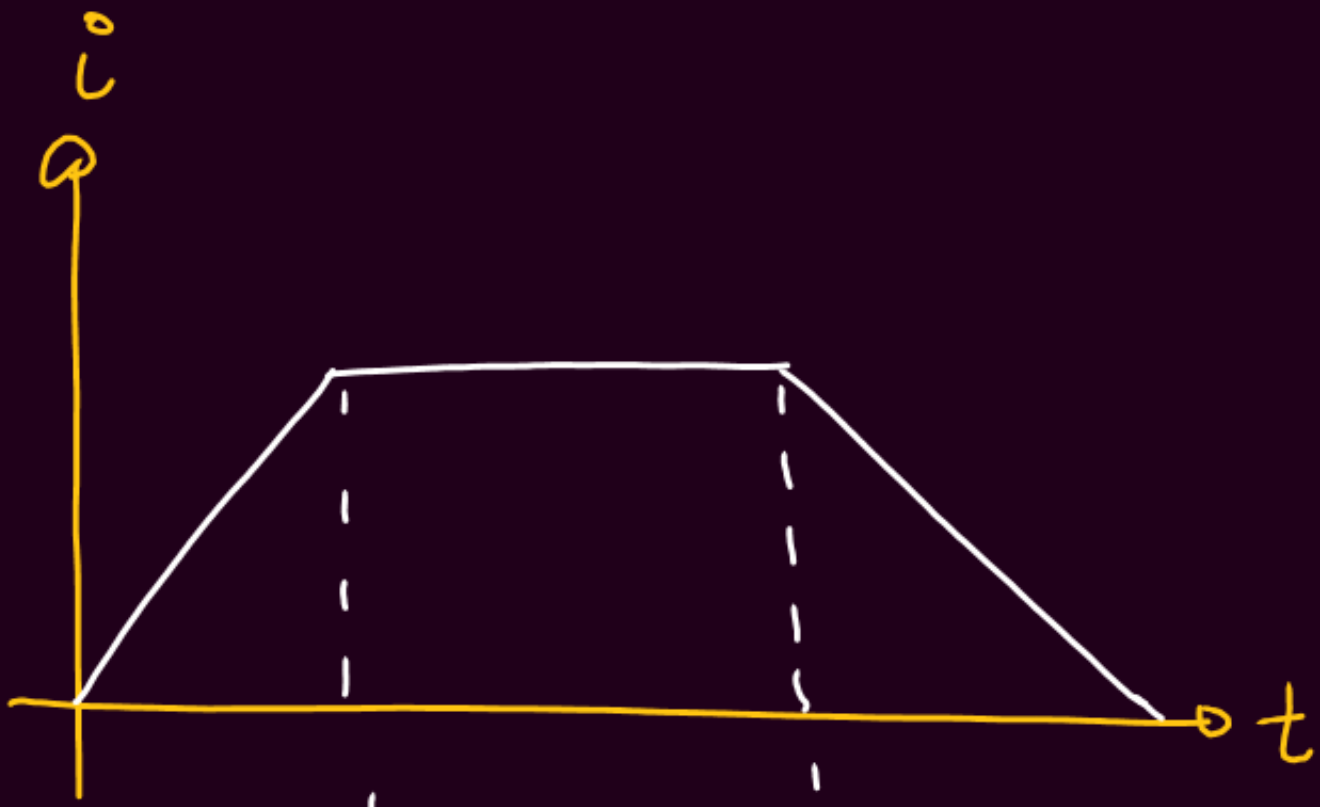
$$\mathcal{E}_{\text{peak}, 2} = ?$$

$$\mathcal{E} = -M \left(\frac{di}{dt} \right)$$

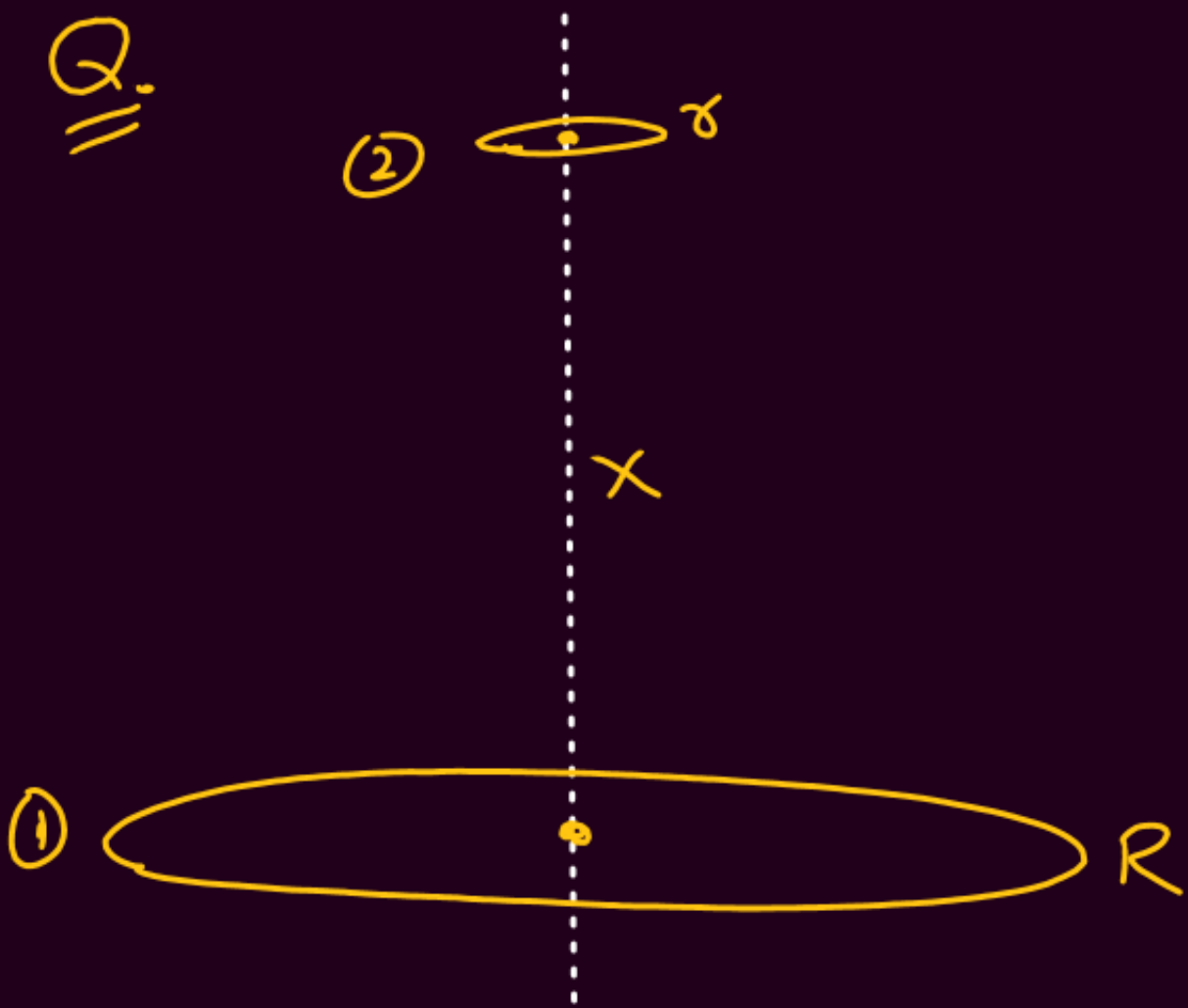
$$\mathcal{E} = + (M i_0 \omega) \sin(\omega t)$$

$$\mathcal{E}_{\text{peak}} = M i_0 \omega$$

Q11



$$\underline{\epsilon = -L \left(\frac{di}{dt} \right)}$$



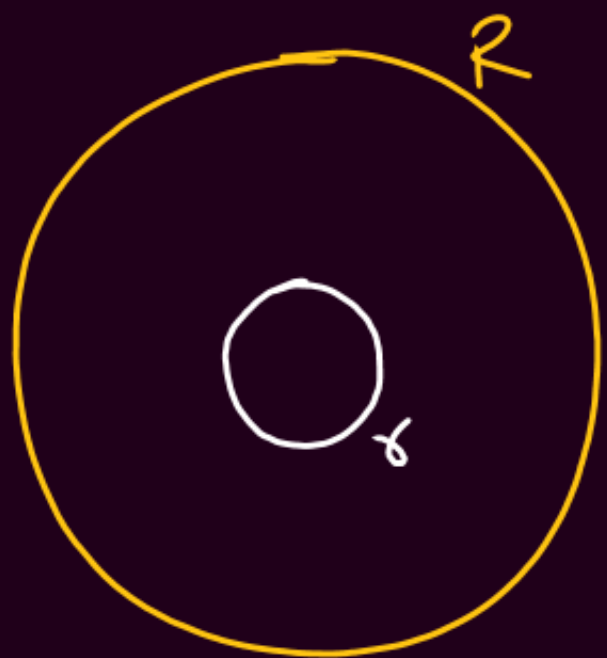
$[R \gg r]$ $M = ?$

$$\phi_2 = M \vec{L}_1$$

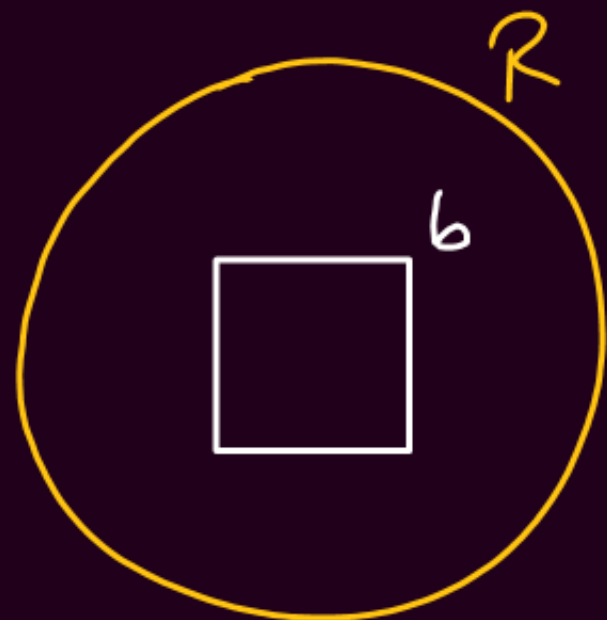
$$N_2 B_1 A_2 = M \vec{L}_1$$

$$N_2 \frac{\mu_0 N_1 \vec{L}_1 R^2}{2(x^2 + R^2)^{3/2}} \pi r^2 = M \vec{L}_1$$

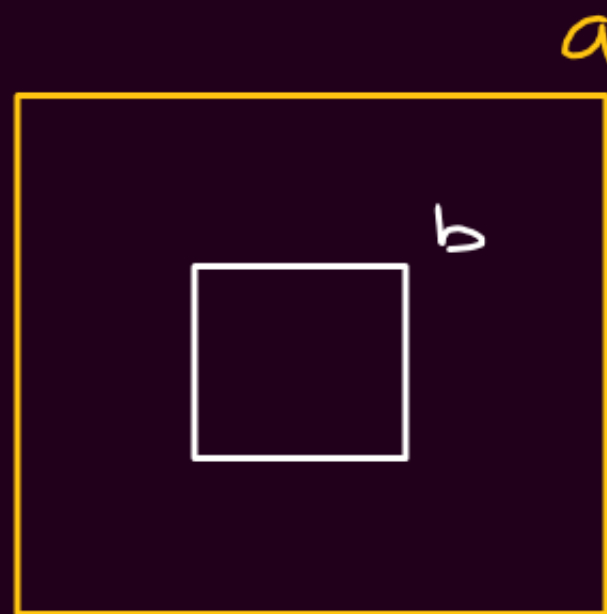
$$M = \frac{\mu_0 N_1 N_2 \pi r^2 R^2}{2(x^2 + R^2)^{3/2}}$$



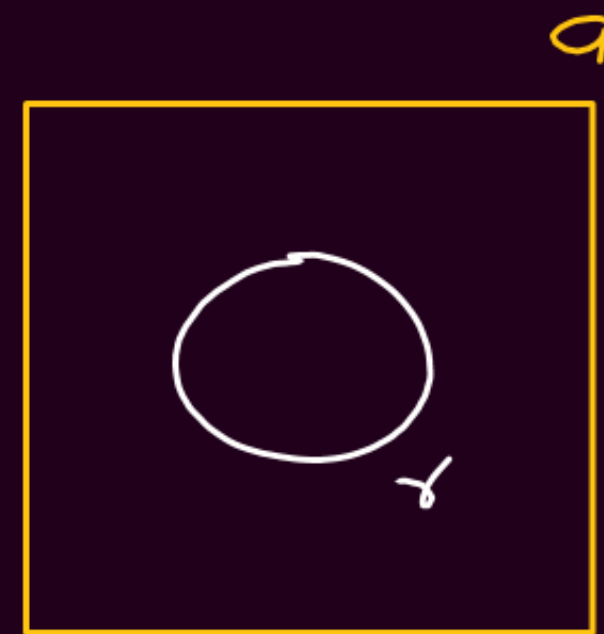
$$M \propto \frac{N_1 N_2 r^2}{R}$$



$$M \propto \frac{N_1 N_2 b^2}{R}$$



$$M \propto \frac{N_1 N_2 b^2}{a}$$



$$M \propto \frac{N_1 N_2 r^2}{a}$$

Sign convention



$$\mathcal{E}_{AB} = L \frac{di}{dt}$$

① in the dir of i $\mathcal{E} = -L \frac{di}{dt}$

② opp to dir of i $\mathcal{E} = L \frac{di}{dt}$



$$i = 2A$$

$$L = 10mH$$

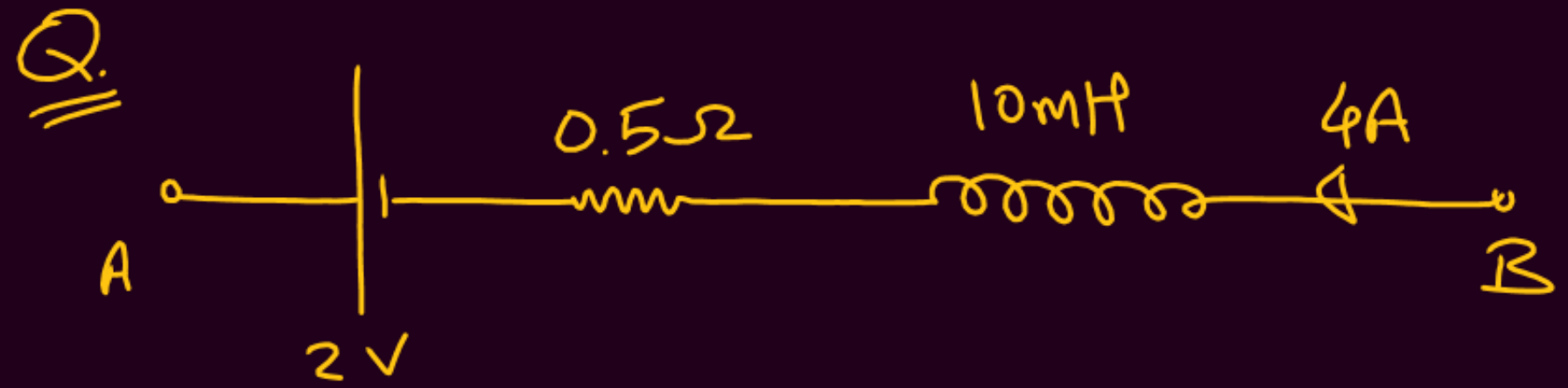
i ↓ at the rate $2A/s$

$$\mathcal{E}_{AB} = ?$$

$$\mathcal{E} = L \frac{di}{dt}$$

$$\mathcal{E} = (10 \times 10^{-3}) (-2)$$

$$\underline{\mathcal{E} = -0.02V}$$



i is increasing at rate of $200A/s$

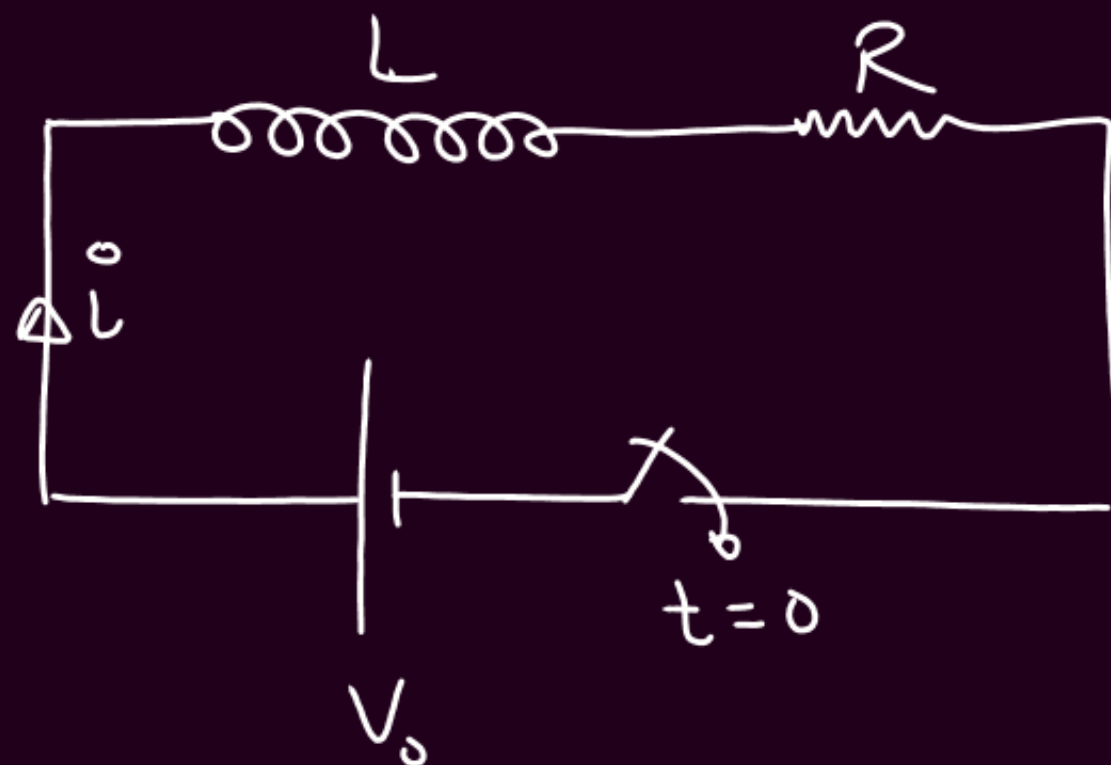
$$V_{AB} = -L \frac{di}{dt} - iR + \mathcal{E}$$

$$V_{AB} = -(10^{-2})(200) - 4(0.5) + 2$$

$$\boxed{V_{AB} = -2V}$$

L-R Circuit

① Growth of i



$$① \quad V_0 = V_L + V_R$$

$$V_0 = L \left(\frac{di}{dt} \right) + iR$$

$$④ \quad \text{Time constant } \tau = \frac{L}{R}$$

$$⑤ \quad t_{1/2} = \tau \ln(2) \quad \left| \quad \begin{array}{l} i \rightarrow \frac{i_0}{2} \\ V \rightarrow \frac{V_0}{4} \end{array} \right.$$

② $t=0$ (initially)

$$i = 0$$

$$V_R = 0$$

$$V_L = V_0 = L \left(\frac{di}{dt} \right)$$

[initially $L \rightarrow$ open circuit]

③ $t \rightarrow \infty$ (finally)

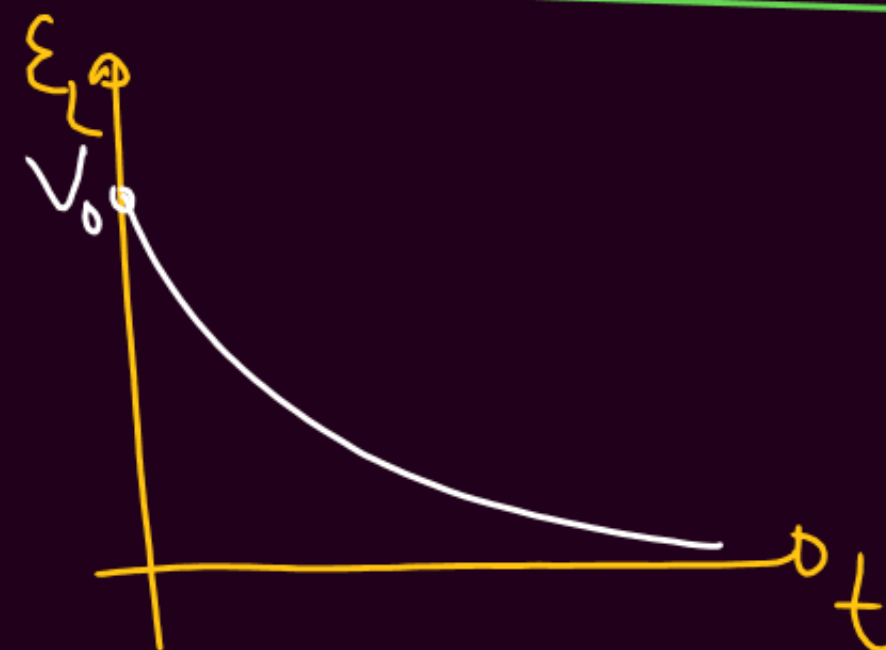
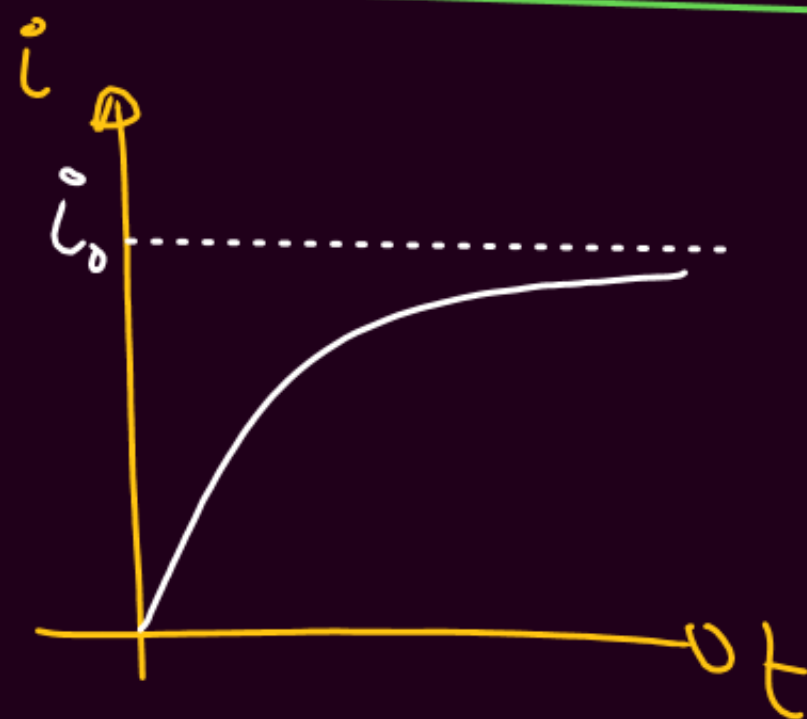
$$i_{\max} = \frac{V_0}{R}$$

$$V_R = V_0$$

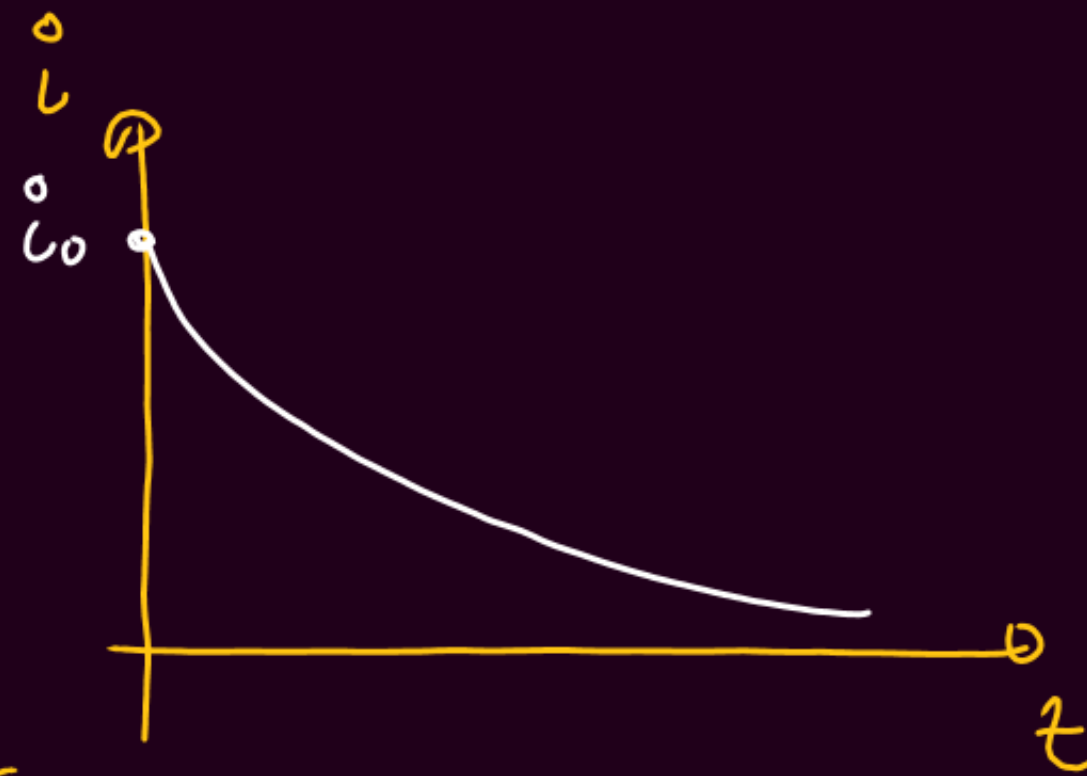
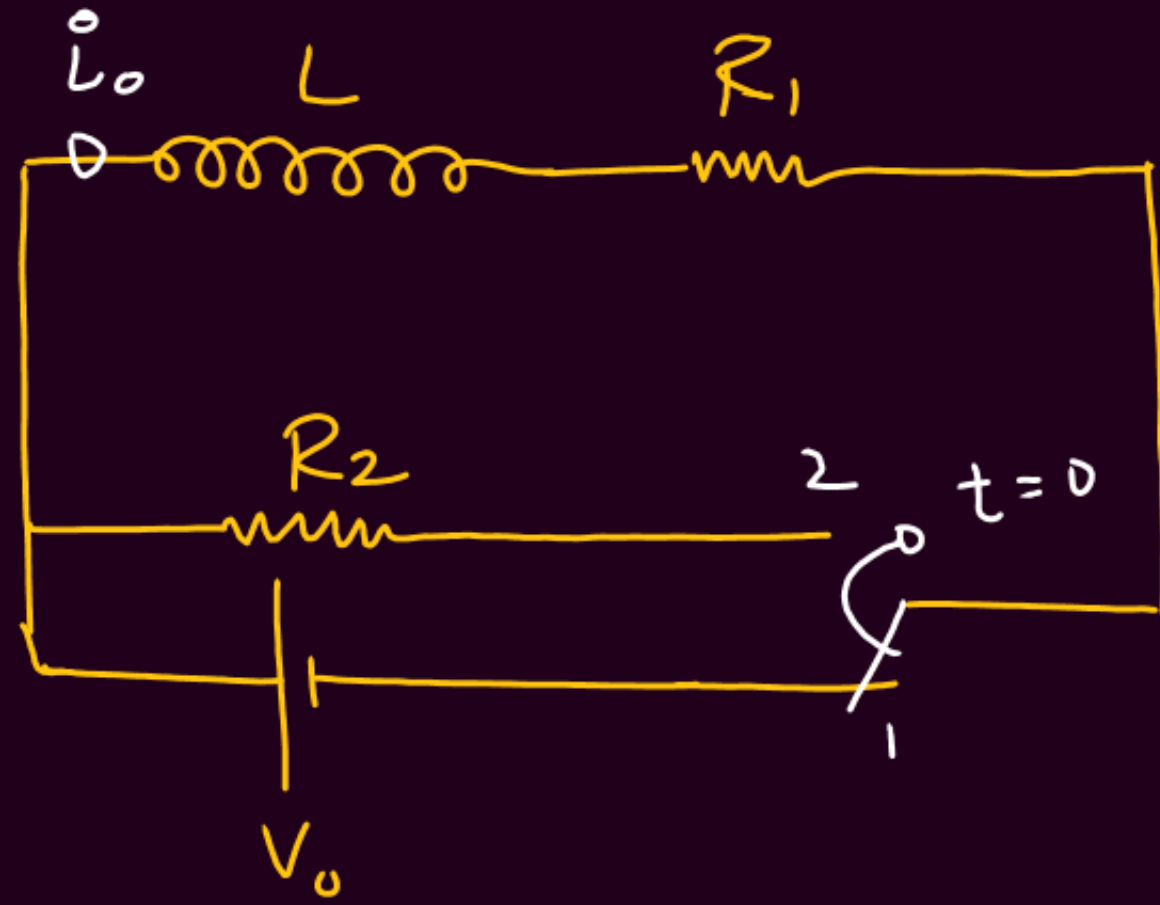
$$\frac{di}{dt} = 0$$

$$V_L = 0$$

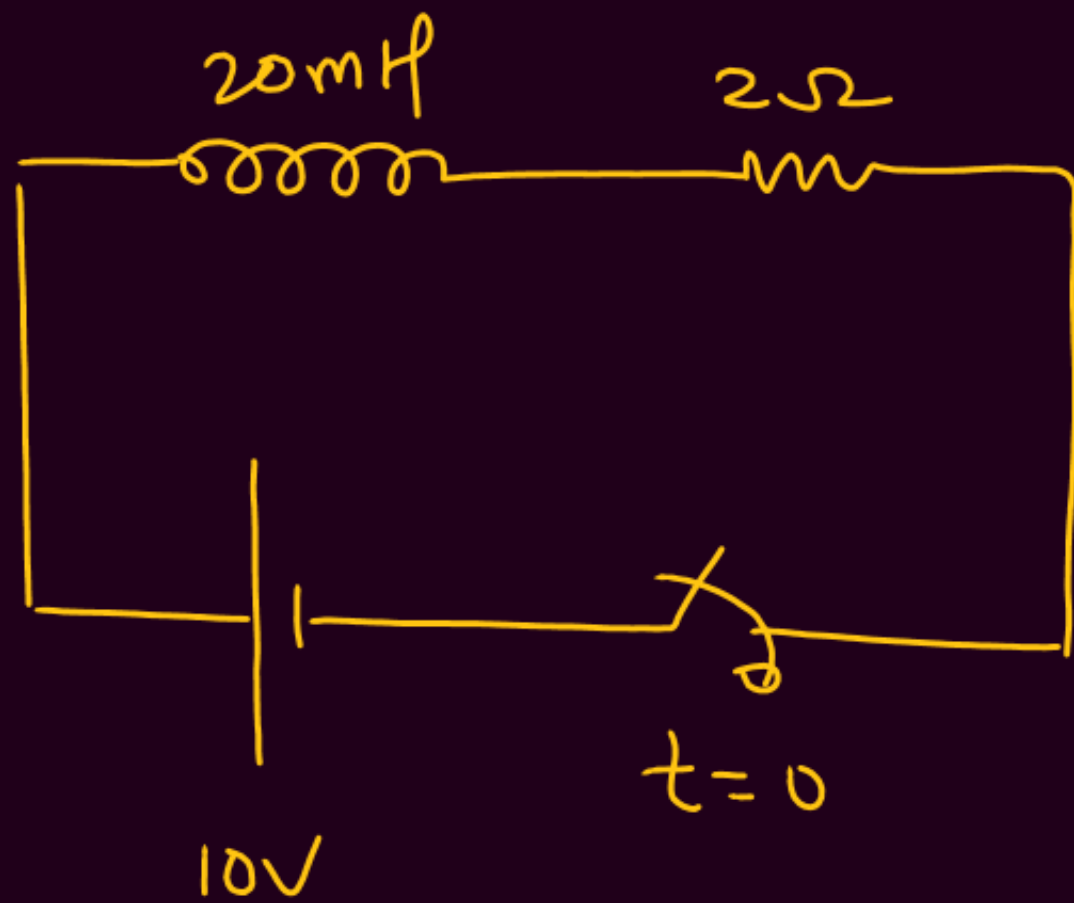
[finally
 $L \rightarrow$ short circuit]



② Decay of i



①



① Find initial Rate of change of i

$$V_s = V_L + V_R$$

$$10 = (20 \times 10^{-3}) \left(\frac{di}{dt} \right) + 0$$

$$\frac{di}{dt} = 500 \text{ A/s}$$

② Find rate of change of i when $i = 2 \text{ A}$

$$10 = (20 \times 10^{-3}) \left(\frac{di}{dt} \right) + (2 \times 2)$$

$$\frac{6}{20 \times 10^{-3}} = \left(\frac{di}{dt} \right)$$

$$\boxed{\frac{di}{dt} = 300 \text{ A/s}}$$

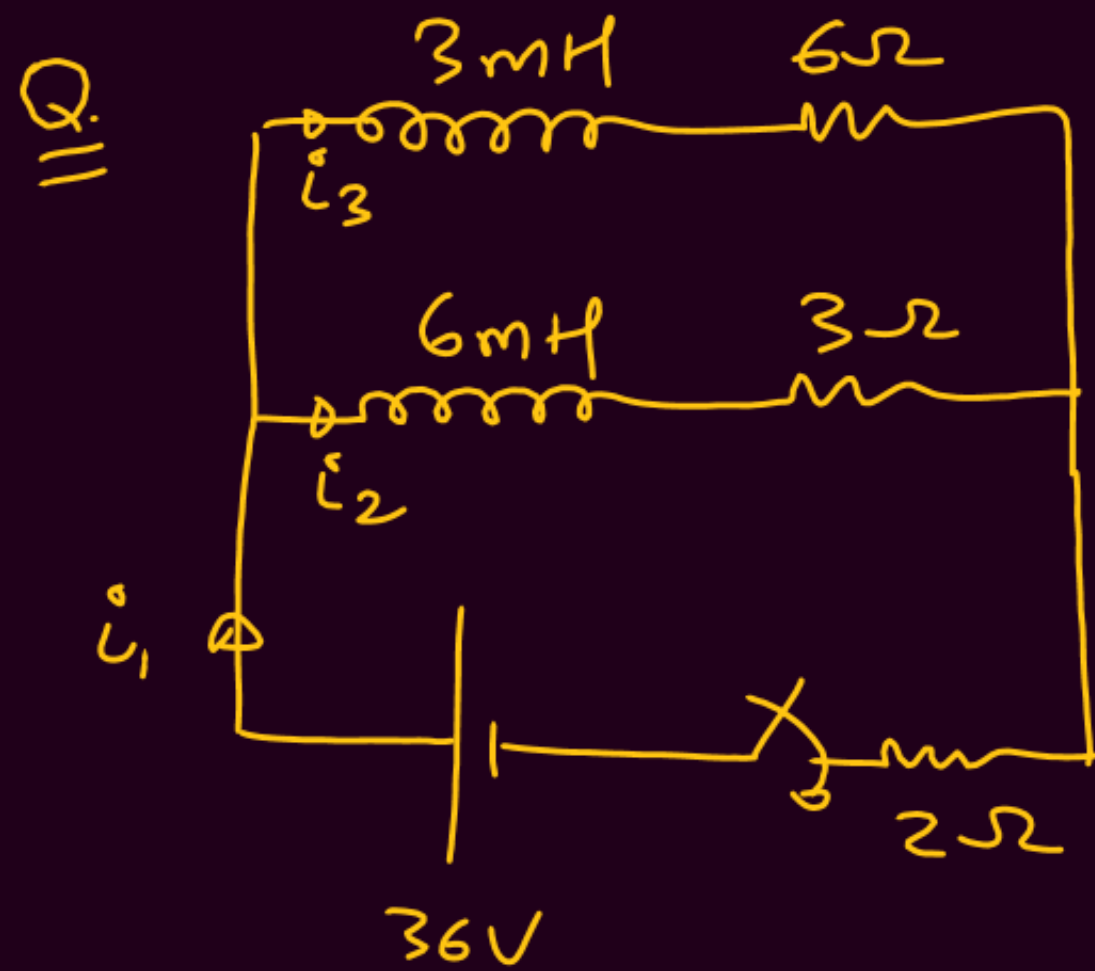
③ Find $i = ?$ when it is 100 A/s

$$10 = (20 \times 10^{-3}) (100) + i(2)$$

$$10 = 2 + 2i$$

$$8 = 2i$$

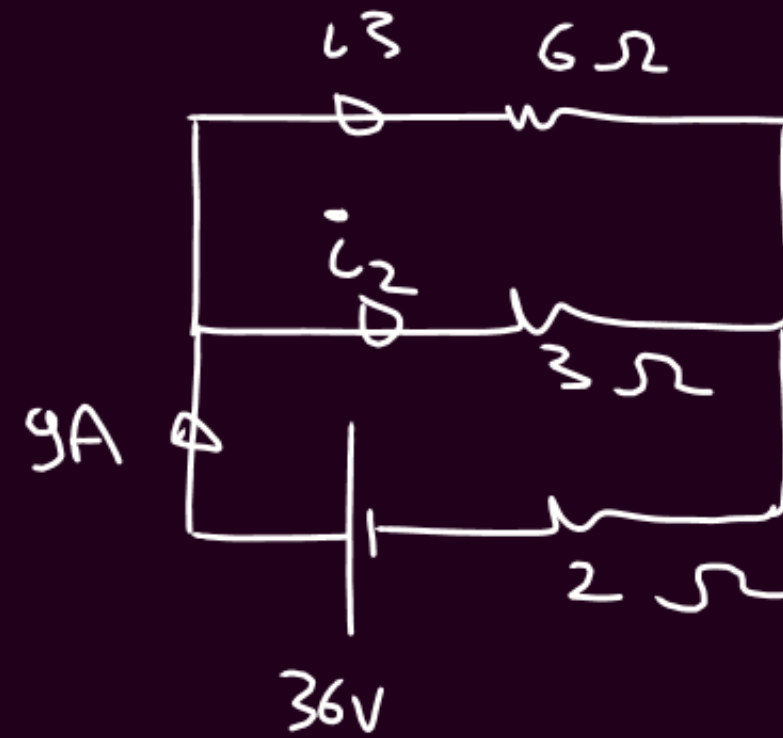
$$\boxed{i = 4 \text{ A}}$$



① Initial current

$$i_1 = i_2 = i_3 = 0$$

② Final current.

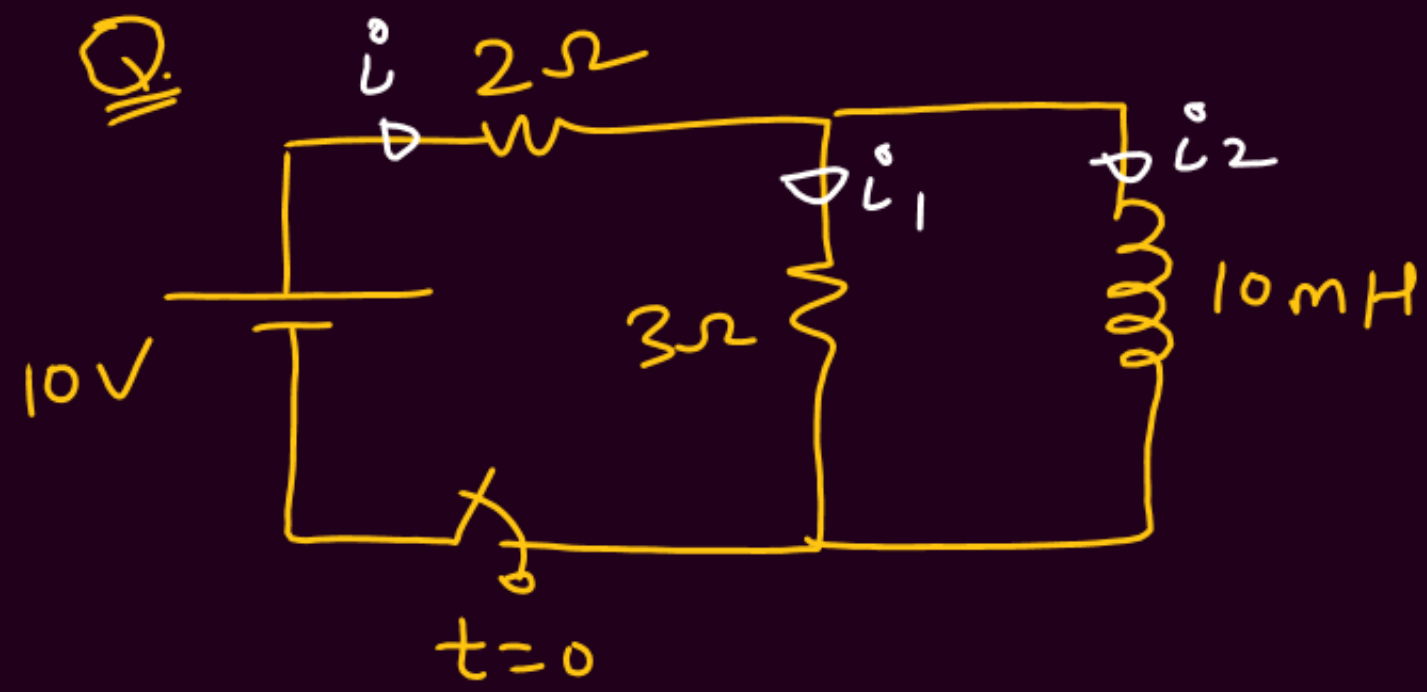


$$R_{eq} = 4\Omega$$

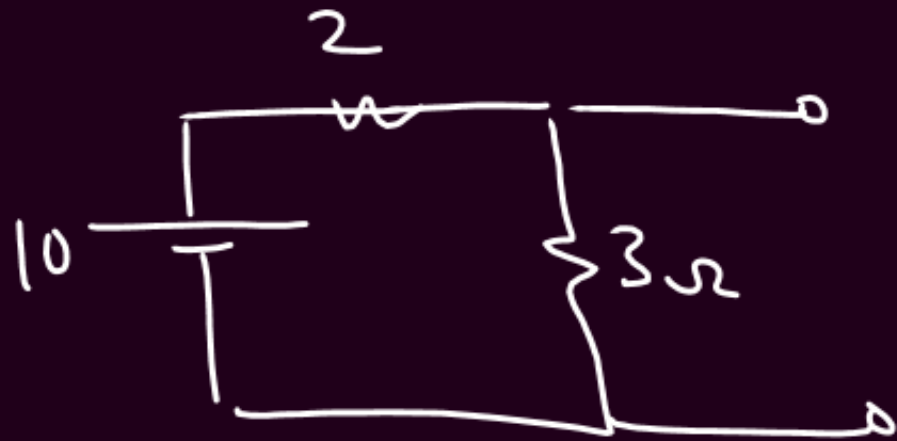
$$i_1 = 9A$$

$$i_2 = \frac{9 \times 6}{6 + 3} = 6A$$

$$i_3 = 3A$$



① Initial current ($t=0$)



$$i_1 = i = \frac{10}{5} = 2\text{A}$$

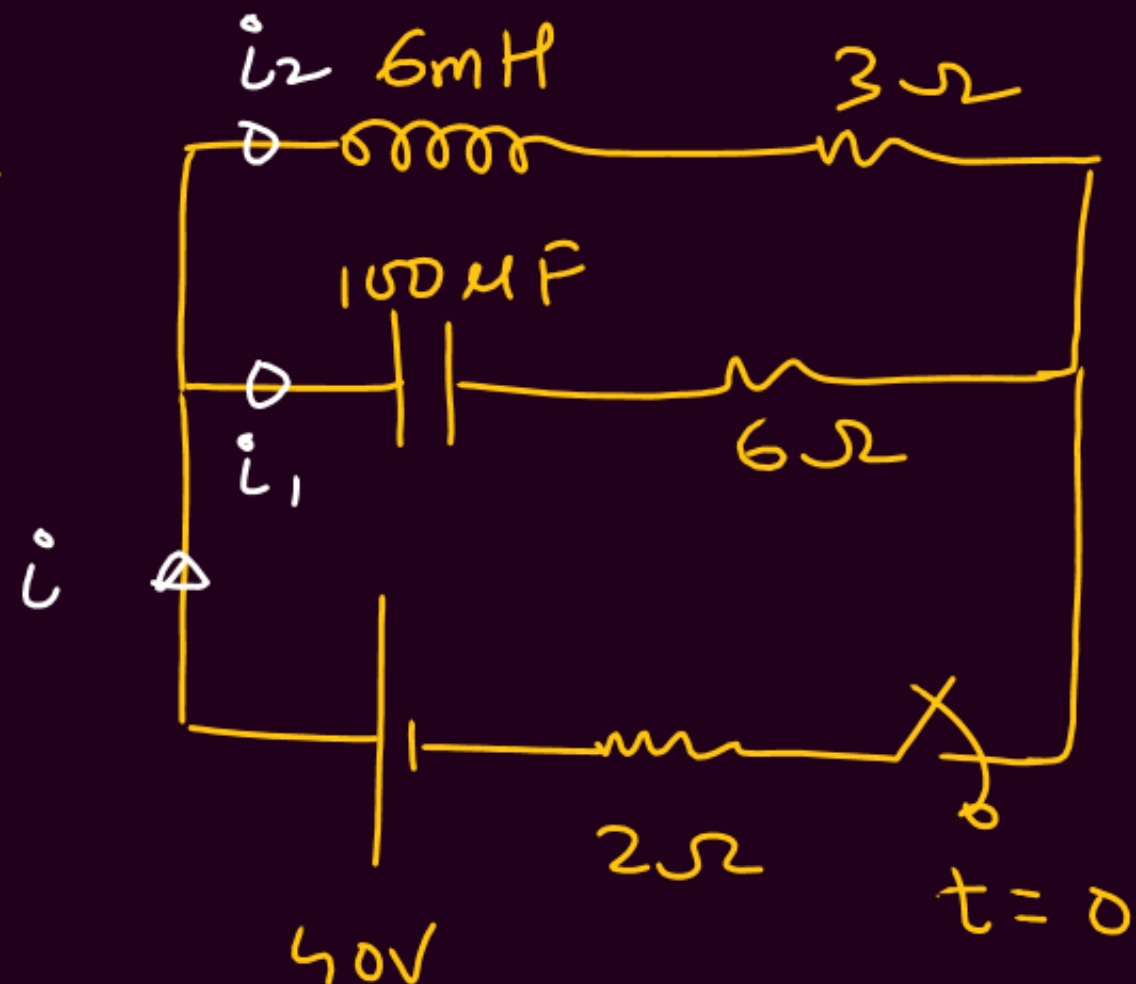
$$i_2 = 0$$

② Final current

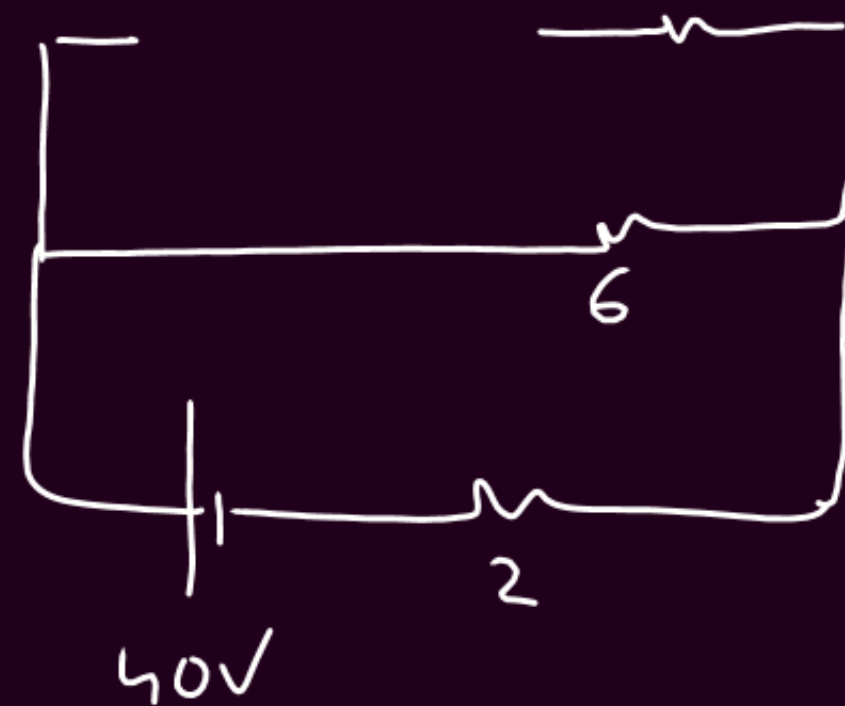


$$R_{eq} = 2\Omega$$

Q.1



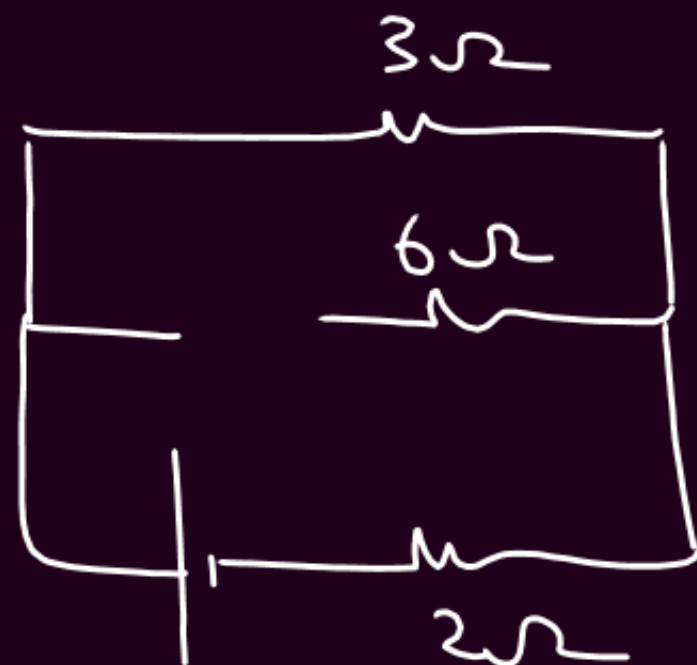
① Initial Current



$$i_2 = 0$$

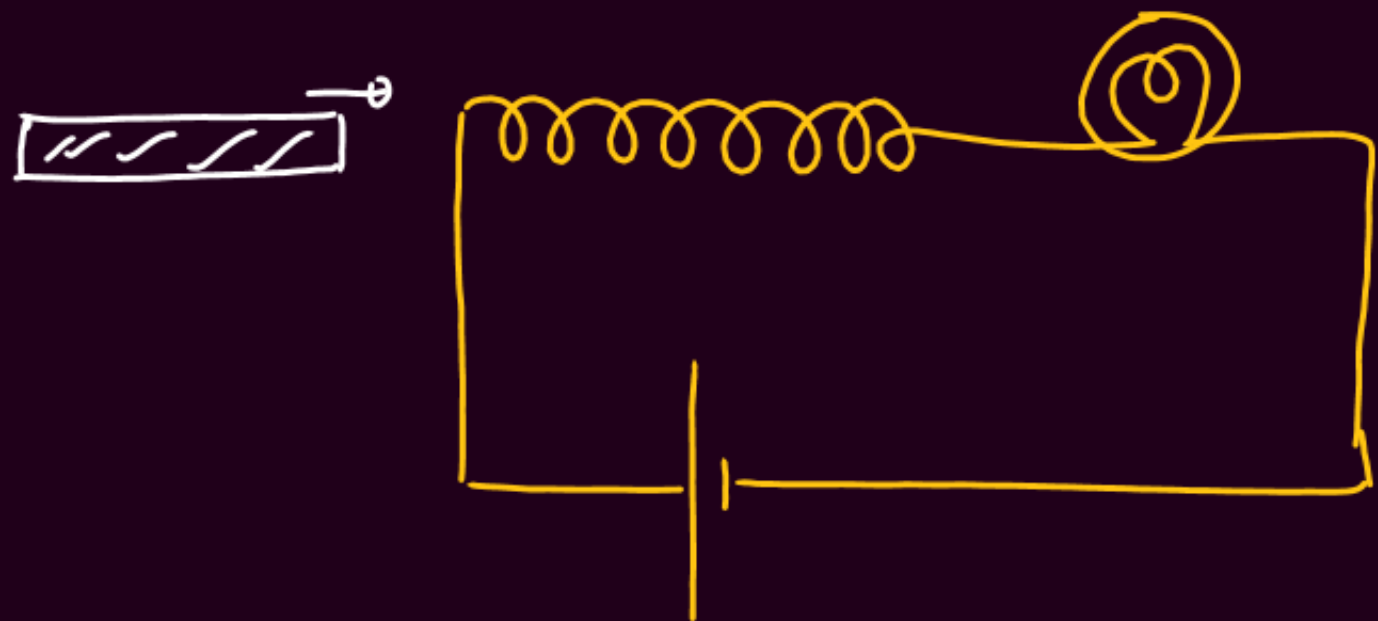
$$i = i_1 = \frac{40}{8} = 5A$$

② Final Current



$$i_1 = 0$$

$$i = i_2 = \frac{40}{5} = 8A$$



① μ

Dia mag.

$\Phi \downarrow$

$i \uparrow$

Brightness $\uparrow$

then same as initial

② μ

Ferro mag.

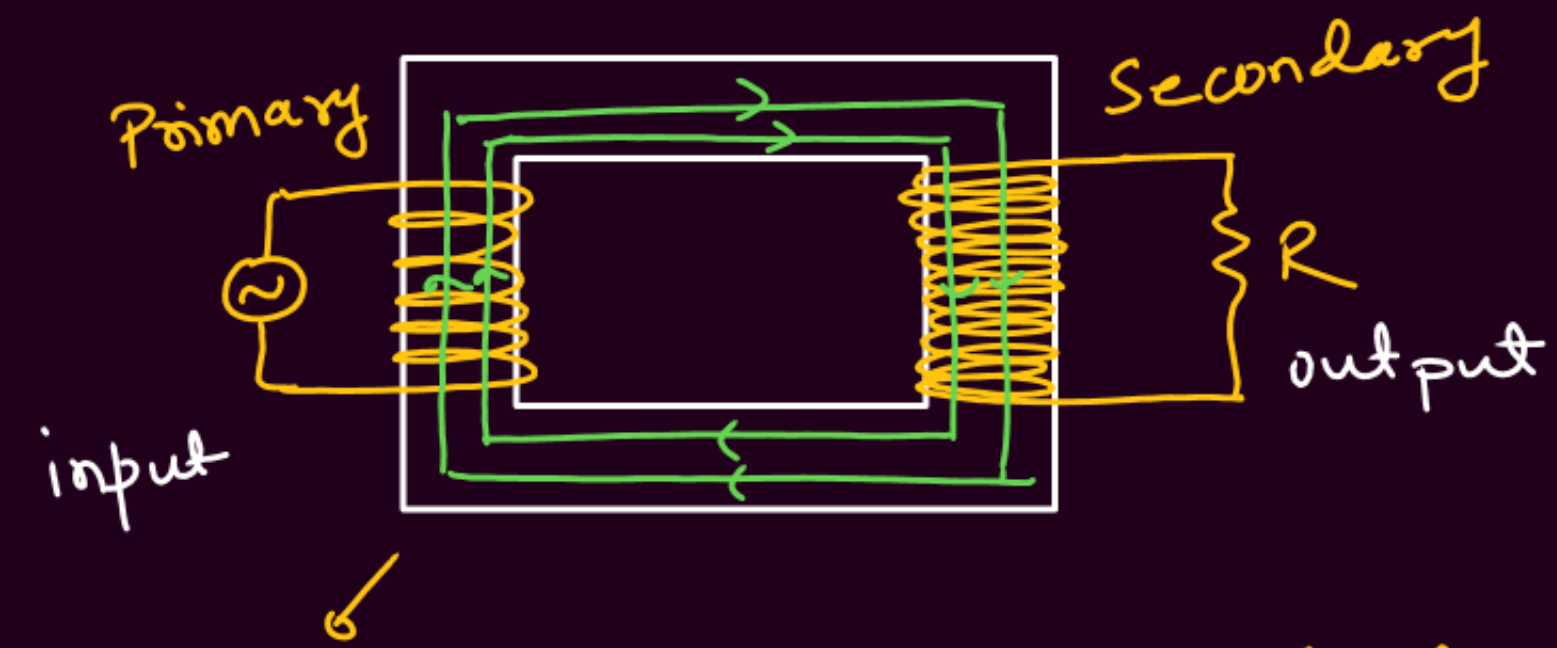
$\Phi \uparrow \uparrow$

$i \downarrow \downarrow$

Brightness $\downarrow$

then same as initial

Transformer.



Soft iron core $\rightarrow$ Laminated sheets



Concept

ϕ per turn is same

$$\frac{E_s}{N_s} = \frac{E_p}{N_p}$$

[EMF per turn]

$$\boxed{\frac{E_s}{E_p} = \frac{N_s}{N_p}}$$

= turn ratio
(Transformer)

Step-up

$$N_s > N_p$$

$$E_s > E_p$$

$$I_s < I_p$$

Step-Down

$$N_s < N_p$$

$$E_s < E_p$$

$$I_s > I_p$$

Efficiency $\eta = \frac{\text{output Power}}{\text{Input Power}} = \frac{E_s i_s}{E_p i_p}$

Assuming no power loss

$$E_s i_s = E_p i_p$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \rightarrow \text{Always}$$

$$\frac{i_s}{i_p} = \frac{N_p}{N_s} \rightarrow \text{only when no Power loss}$$

Losses in Transformer

① Cu loss ($i^2 R_t$)

② Flux leakage

③ Iron Losses

→ Hysteresis (soft Fe core)
→ Eddy (laminated)

④ Line loss

[High tension line] $i \downarrow$
Power generate Town

Step-Up

Step down

Q. $N_p = 100$ turns
 $N_s = 500$

① 50V battery

$$\mathcal{E}_s = 0$$

② 200V mains (RMS)

$$\frac{\mathcal{E}_s}{200} = \frac{500}{100}$$

$$\mathcal{E}_s = 1000 \text{ V (RMS)}$$

Q. Transformer

input (100V, 2A)

output (500V, 0.2A)

$$\eta = \frac{500 \times 0.2}{100 \times 2} = \frac{1}{2} = 50\%$$

Q. Input (100W, 2A)

primary $\rightarrow$ 200

Secondary $\rightarrow$ 50

① $\mathcal{E}_s$

$$\frac{\mathcal{E}_s}{50} = \frac{50}{200}$$

$$\boxed{\mathcal{E}_s = 12.5 \text{ V}}$$

$$100 = \mathcal{E}_p i_p$$

$$100 = \mathcal{E}_p (2)$$

$$\mathcal{E}_p = 50 \text{ V}$$