Alternating Current

→ Generator / Dynamo

$\phi = NBA\cos(\omega t)$

$\mathcal{E} = NBA\omega \sin(\omega t)$

$i = \frac{NBA\omega}{R} \sin(\omega t)$

$$i = i_0 \sin(\omega t)$$

Sinusoidal

i_0 → peak value

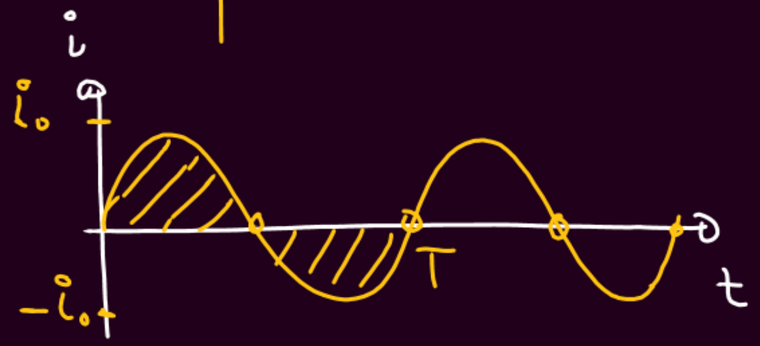
$2i_0$ → peak to peak

ω → Angular freq

$$\omega = 2\pi f = \frac{2\pi}{T}$$

India - { 220V, 50Hz

Avg over cycle → 0



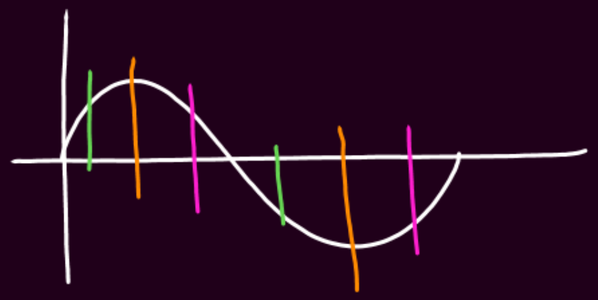
Avg over ⊕ Half cycle } $i_{av} = \frac{2i_0}{\pi}$

RMS (Effective) $i_{rms} = \frac{i_0}{\sqrt{2}}$

Q. Bulb Supply 220V, 50Hz

off in a sec → 100 times

Q. Avg value of i in Half cycle may be



① +

② -

③ 0

④ All of these

Q. $i = 10\sin\left(50\pi t + \frac{\pi}{3}\right)$ A

→ peak = 10 A

→ RMS = $\frac{10}{\sqrt{2}} = 5\sqrt{2}$ A

→ Avg of ⊕ Half = $\frac{20}{\pi}$ A

→ initial current ($t=0$) $i = 10\sin\left(\frac{\pi}{3}\right)$

$i = 10\left(\sqrt{3}/2\right) = 5\sqrt{3}$ A

→ $t = \frac{1}{100}$ s $i = 10\sin\left(\frac{\pi}{2} + \pi/3\right) = 10\cos\left(\pi/3\right)$

$= 5$ A

Q. $i = 10 \sin\left(100\pi t + \frac{\pi}{6}\right)$ A

$t = ?$ when i is max

$$100\pi t + \frac{\pi}{6} = \frac{\pi}{2}$$

$$100\pi t = \frac{\pi}{3}$$

$$t = \frac{1}{300} \text{ s}$$

Q. 100 V, 50 Hz

Avg. value of ⊕ Half =

$$\frac{2}{\pi}(100\sqrt{2}) = \frac{200\sqrt{2}}{\pi}$$

	Peak		freq
$i = i_0 \sin(\omega t + \phi)$ $i = i_0 \cos(\omega t + \phi)$	i_0	ω	$\frac{\omega}{2\pi}$
$i = i_0 \sin(\omega t)\cos(\omega t)$ $= \frac{i_0}{2}\sin(2\omega t)$	$\frac{i_0}{2}$	2ω	$\frac{2\omega}{2\pi}$
$i = i_0 \sin^2(\omega t)$ $i = i_0 \cos^2(\omega t)$	i_0	2ω	$\frac{2\omega}{2\pi}$

Peak

$i = a\sin(\omega t) + b\cos(\omega t)$ $\quad \sqrt{a^2+b^2}$ $\quad w$ $\quad i_{rms} = \frac{i_p}{\sqrt{2}}$

DC + AC

$i = a + b\sin(\omega t)$ $\quad (a+b)$ $\quad w$ $\quad i_{rms} = \sqrt{a^2 + \frac{b^2}{2}}$

Q. $i = 8\sin(\omega t) + 6\cos(\omega t)$

RMS ?

$$\text{peak} = \sqrt{8^2 + 6^2} = 10$$

$$\text{RMS} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

Q. $i = 10\sin(100\pi t)\cos(100\pi t)$

$$\text{peak} = \frac{10}{2} = 5$$

$$\omega = 2(100\pi)$$

$$f = \frac{200\pi}{2\pi} = 100\ \text{Hz}$$

Q. $i = 3 + 4\sin(100\pi t)$

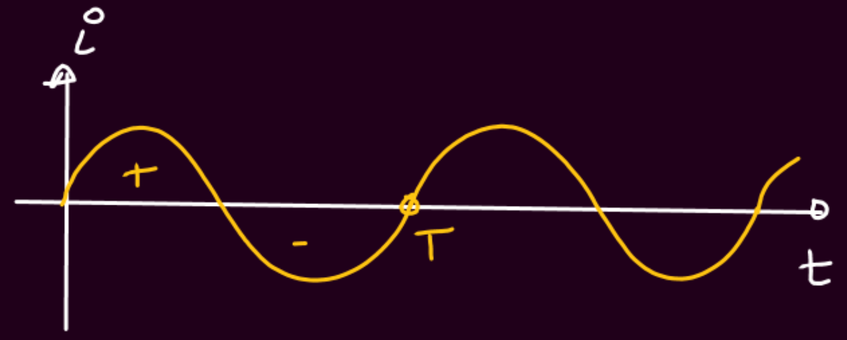
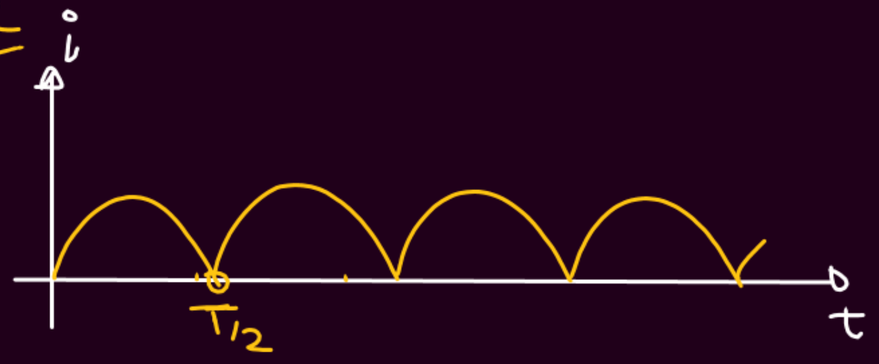
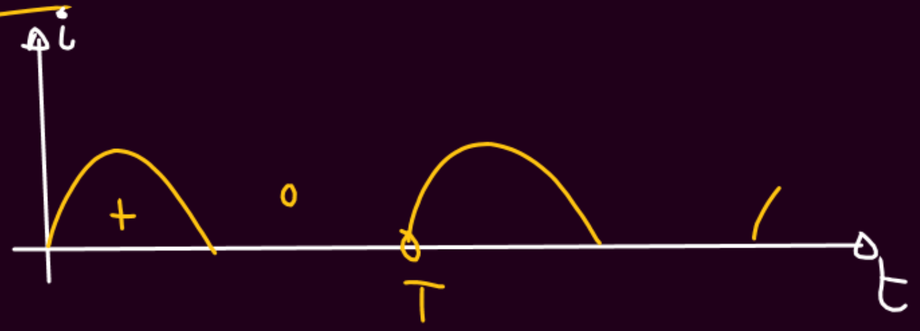
$$\text{RMS} = \sqrt{3^2 + \frac{4^2}{2}} = \sqrt{17}$$

$$\left[\sin(\omega t)\right]_{av} = 0$$

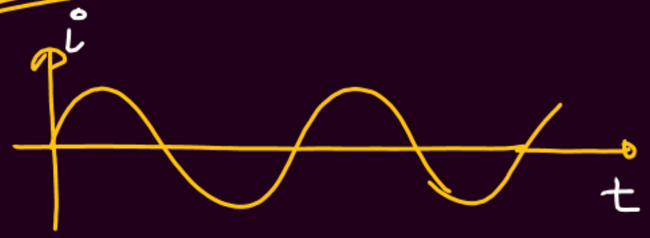
$$\left[\sin(\omega t)\right]_{av,\,\oplus\,\text{Half}} = \frac{2}{\pi}$$

$$\left[\sin^2(\omega t)\right]_{av} = \frac{1}{2}$$

$$\sqrt{\left[\sin^2(\omega t)\right]_{av}} = \left[\sin(\omega t)\right]_{rms} = \frac{1}{\sqrt{2}}$$

	Avg.	RMS	Time Period	freq
AC 	0	$\frac{i_0}{\sqrt{2}}$	T	f
FWR 	$\frac{2i_0}{\pi}$	$\frac{i_0}{\sqrt{2}}$	$\frac{T}{2}$	$2f$
HWR 	$\frac{i_0}{\pi}$	$\frac{i_0}{2}$	T	f

Extra



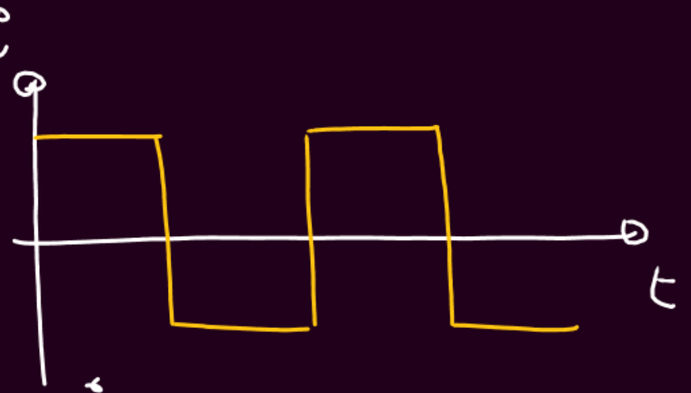
RMS

$\frac{i_0}{\sqrt{2}}$

Avg of ⊕ Half

$\frac{2i_0}{\pi}$

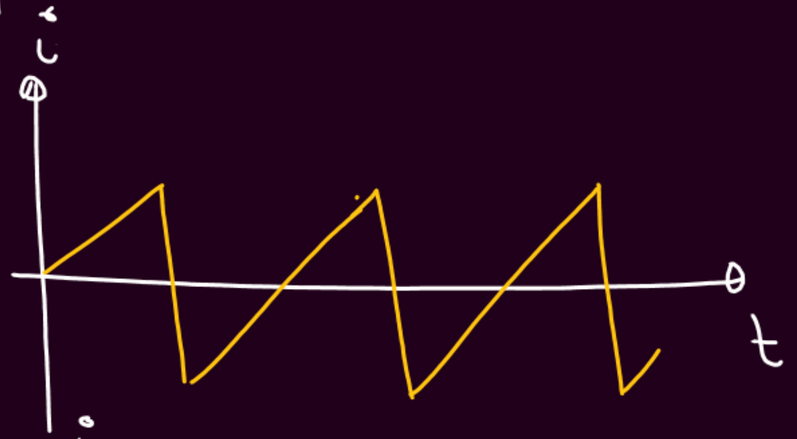
Square



i_0

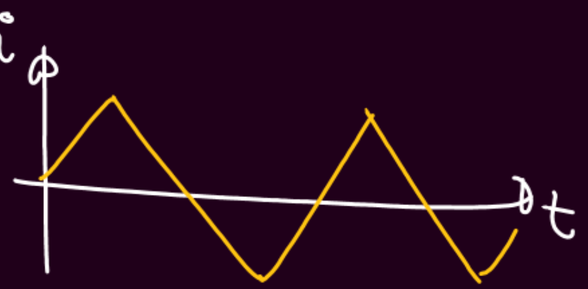
i_0

Triangular
Saw tooth



$\frac{i_0}{\sqrt{3}}$

$\frac{i_0}{2}$



Pure R

① Instantaneous

$$V = iR$$

② Phasor

③ if $i = i_0 \sin(\omega t)$

$V = V_0 \sin(\omega t)$

④ $V_0 = i_0 R$

⑤ $P_{inst} = V_0 i_0 \sin^2(\omega t)$

if V & i → ω

then power → 2ω

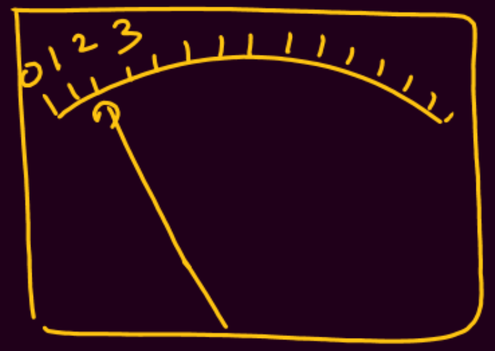
⑥ $P_{av} = \frac{V_0 i_0}{2} = V_{rms} i_{rms}$

$$P_{av} = i_{rms}^2 R$$

DC meters

e.g. → moving coil galvanometer

Deflection $\theta \propto i$

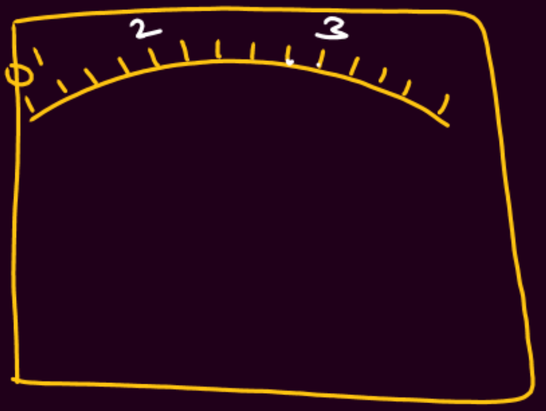


Linear Scale

if DC → useful in AC → '0'

AC meter

Hot wire Instruments → $P_w \propto i^2$



$[\theta \propto i^2]$

Non-linear Scale

it can be used → AC
→ DC

AC meter → "RMS"

R

V & i → in same phase

Impedance

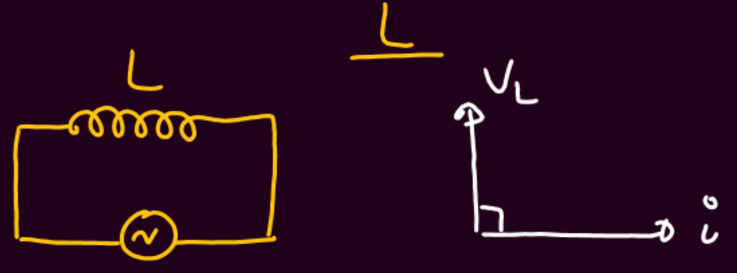
$$Z = R$$

$$V_0 = i_0 R$$

$$V = V_0 \sin(\omega t)$$

$$i = i_0 \sin(\omega t)$$

L



V_L leads i by $\frac{\pi}{2}$

Inductive Reactance

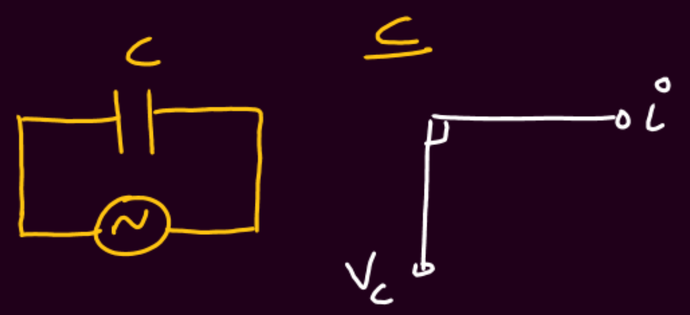
$$Z = X_L = \omega L$$

$$V_0 = i_0 X_L$$

$$V = V_0 \sin(\omega t)$$

$$i = i_0 \sin(\omega t - \pi/2)$$

C



V_C lags i by $\pi/2$

Capacitive Reactance

$$Z = X_C = \frac{1}{\omega C}$$

$$V_0 = i X_C$$

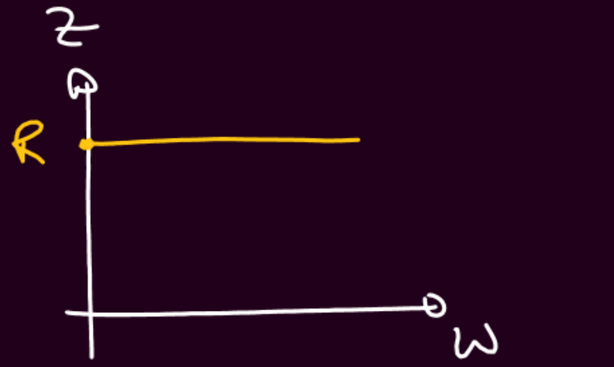
$$V = V_0 \sin(\omega t)$$

$$i = i_0 \sin(\omega t + \pi/2)$$

R

$P_{inst} \longrightarrow 2\omega$

$$P_{av} = i_{rms}^2 R$$



$$i_0 = \frac{V_0}{R}$$

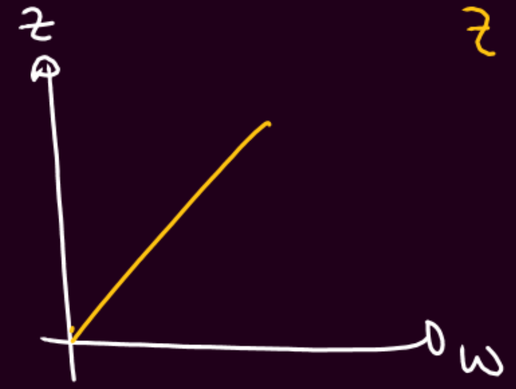
L

$P_{inst} \longrightarrow 2\omega$

$$P_{av} = 0$$

Inductor & Capacitor are non-dissipative elements

$Z = \omega L$



$$i_0 = \frac{V_0}{\omega L}$$

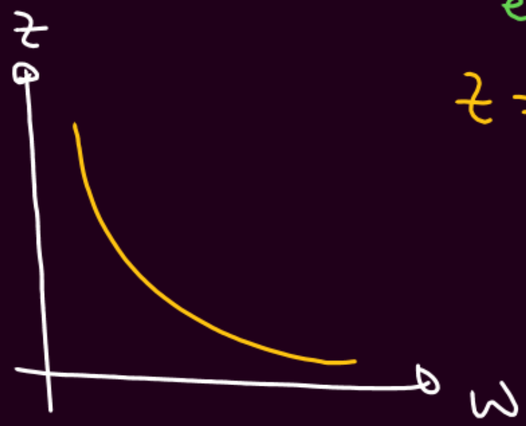
L → low pass filter

C

$P_{inst} \longrightarrow 2\omega$

$$P_{av} = 0$$

$Z = \frac{1}{\omega C}$



$$i_0 = V_0 \omega C$$

C → High pass filter

Q.

200V, 50Hz

① $i_{peak} = ?$

$$V_0 = i_0 R$$

$$200 = i_0 (10)$$

$$i_0 = 20 \text{ A (rms)}$$

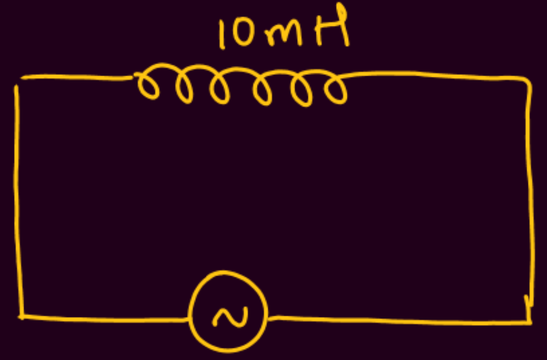
$$i_{peak} = 20\sqrt{2} \text{ A}$$

② $P_{av} = i_{rms}^2 R$

$$= (20)^2 (10)$$

$$= 4000 \text{ W}$$

Q



$V = 100 \sin(200t)$

① $X_L = \omega L$

$= (200)(10 \times 10^{-3})$

$= 2\,\Omega$

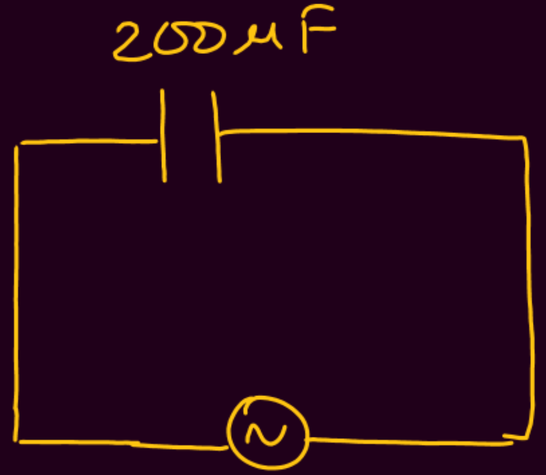
② $V_0 = i_0 X_L$

$100 = i_0 (2)$

$i_0 = 50\ A$ (peak)

③ $P_{av} = 0$

Q.



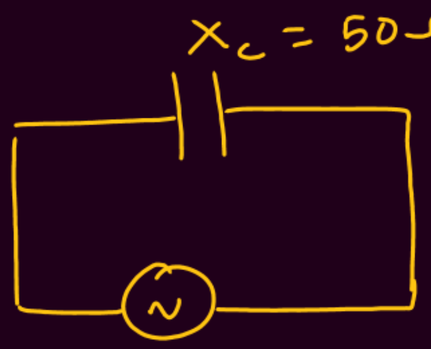
① $\omega = 2\pi\left(\frac{150}{\pi}\right) = 200 \text{ rad/s}$

② $X_C = \frac{1}{\omega C} = \frac{1}{200 \times 200\mu} = 25\,\Omega$

③ $V = i X_C$

$200 = i\,(25)$

$i = 8 \text{ A} \quad (\text{rms})$



$$V = 100 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$i = \left(\frac{100}{50}\right) \sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right)$$

$$i = 2 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$i = 2 \cos\left(\omega t + \frac{\pi}{6}\right)$$

Q. $V = 100 \sin(50\pi t)$ V

$i = 2 \cos(50\pi t)$ A

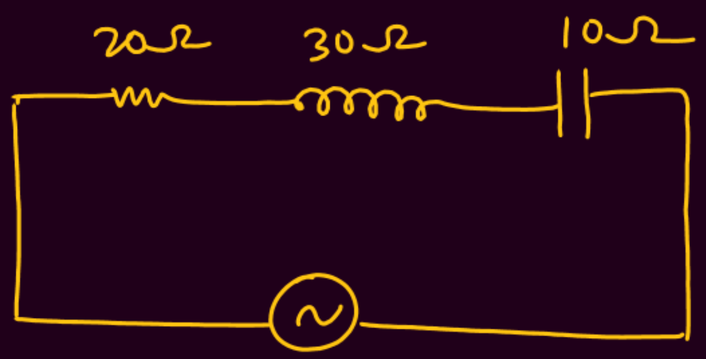
$P_{av} = ?$

$\phi = \pi/2$ $\quad P_{av} = 0$

$\therefore$ i leads V by $\pi/2$

$\therefore$ C

Q



$V = 100 \sin(200\pi t)$

① $Z = \sqrt{20^2 + (30-10)^2} = 20\sqrt{2}\ \Omega$

② $i_0 = \frac{V_0}{Z} = \frac{100}{20\sqrt{2}} = \frac{5}{\sqrt{2}}\ A$

$i_{rms} = \frac{5}{\sqrt{2}\sqrt{2}} = \underline{\underline{2.5\,A}}$

③ $\cos\phi = \frac{20}{20\sqrt{2}} = \frac{1}{\sqrt{2}}$

④ 45°

⑤ Wattless $= 2.5 \sin 45^\circ$

Q.

$V = 100\sin(200t)$ Volt

$X_L = \omega L = 200(100\times10^{-3}) = 20\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{200\times200\times10^{-6}} = 25\ \Omega$

$Z = \sqrt{10^2 + (20-25)^2} = 5\sqrt{5}$

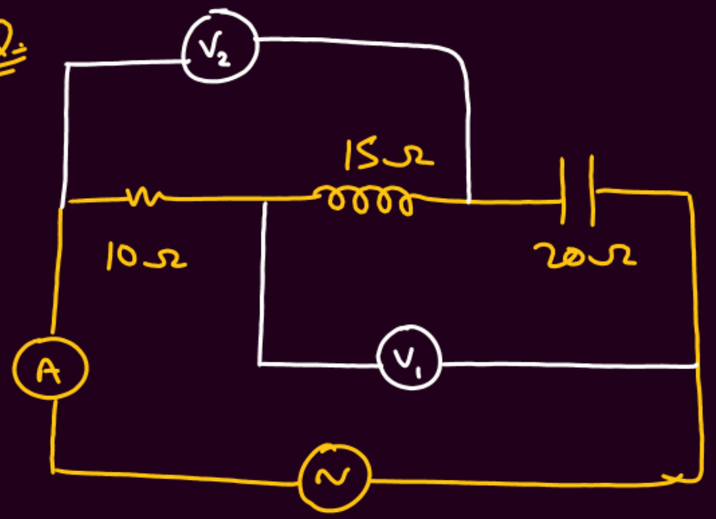
$i_{rms} = \frac{100}{\sqrt{2}(5\sqrt{5})} = \frac{20}{\sqrt{10}}$ A

$P_{av} = i_{rms}^2 R$

$= \left(\frac{20}{\sqrt{10}}\right)^2 (10)$

$= 400$ W

Q:

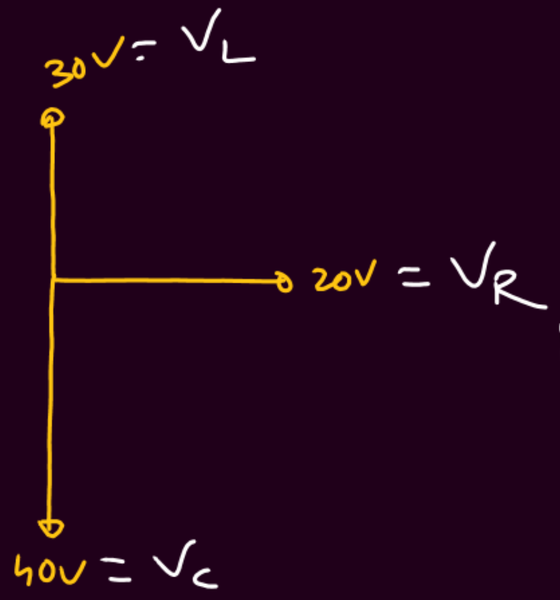


(A) = 2A

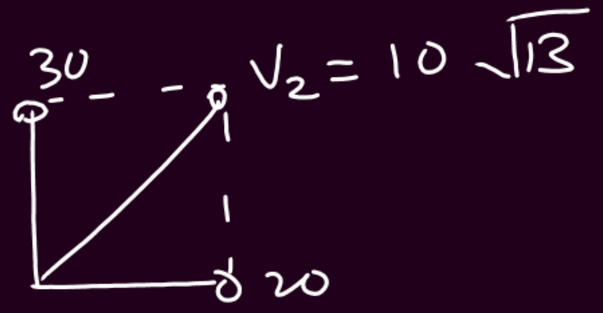
$V_L = 2(15) = 30V$

$V_C = 2(20) = 40V$

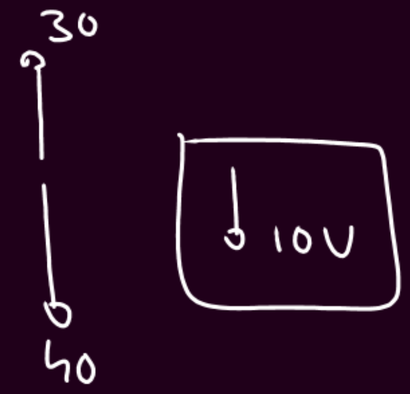
$V_R = 2(10) = 20V$

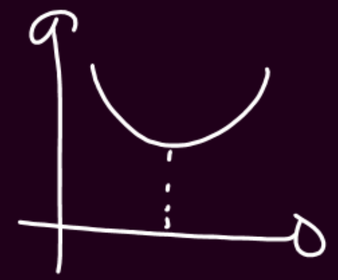


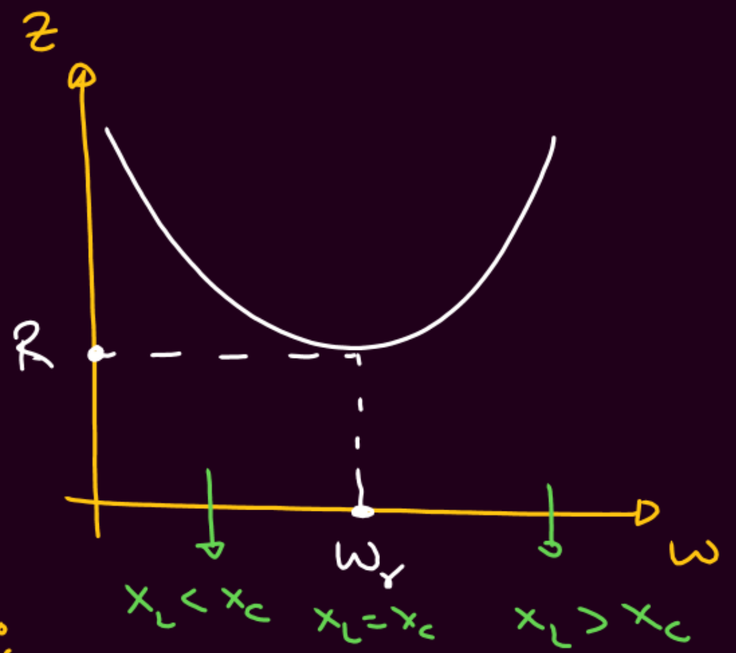
V_2

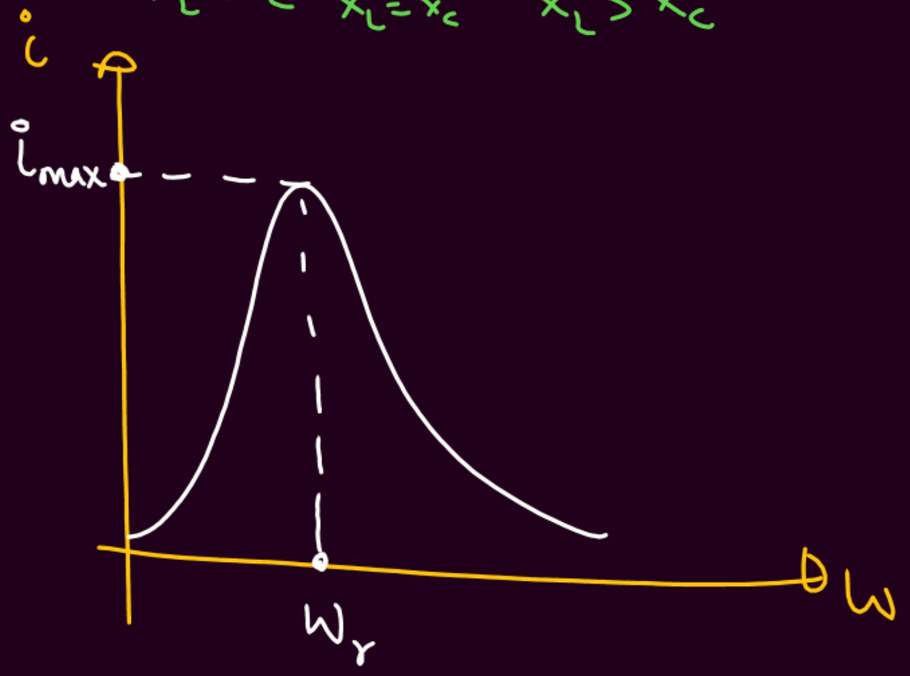


V_1









$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad i_0 = \frac{V_0}{Z}$$

① if $\omega \to 0$ $\quad Z \to \infty$ $\quad i \to 0$

② if $\omega \to \infty$ $\quad Z \to \infty$ $\quad i \to 0$

③ $Z_{min} = R$ $\qquad i_{max} = \frac{V_0}{R}$

when $X_L - X_C = 0$

$$X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C}$$

Resonant freq.

$$\omega_r = \frac{1}{\sqrt{LC}}$$

① $\omega = \omega_r$

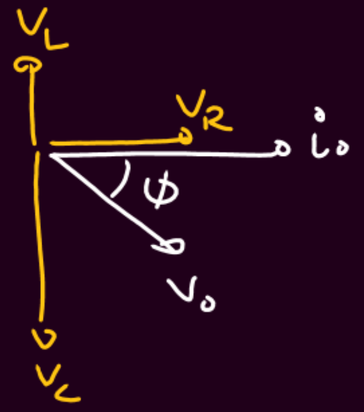
$X_L = X_C$

$\phi = 0$

Same phase

② $\omega < \omega_r$

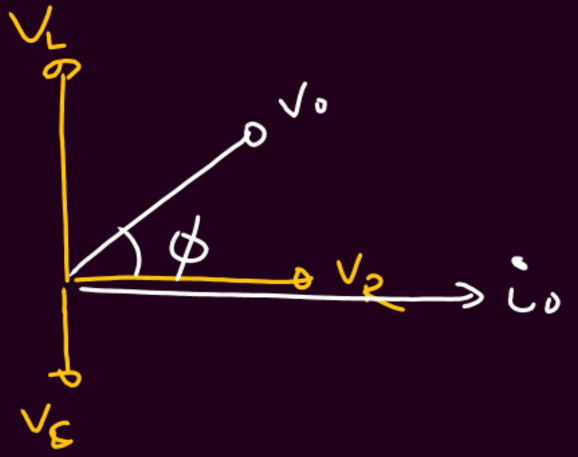
$X_L < X_C$



i_0 leads V_0

③ $\omega > \omega_r$

$X_L > X_C$



V_0 leads i_0

Resonance

① i_{max}

② $Z_{min} = R$

③ $X_{L_r} = X_{C_r} = \sqrt{\frac{L}{C}}$

④ $\omega_r = \frac{1}{\sqrt{LC}}$

⑤ $f_r = \frac{1}{2\pi\sqrt{LC}}$

⑥ $T = 2\pi\sqrt{LC}$

⑦ $\tan\phi = 0$

⑧ $\phi = 0$ V & i same in same phase

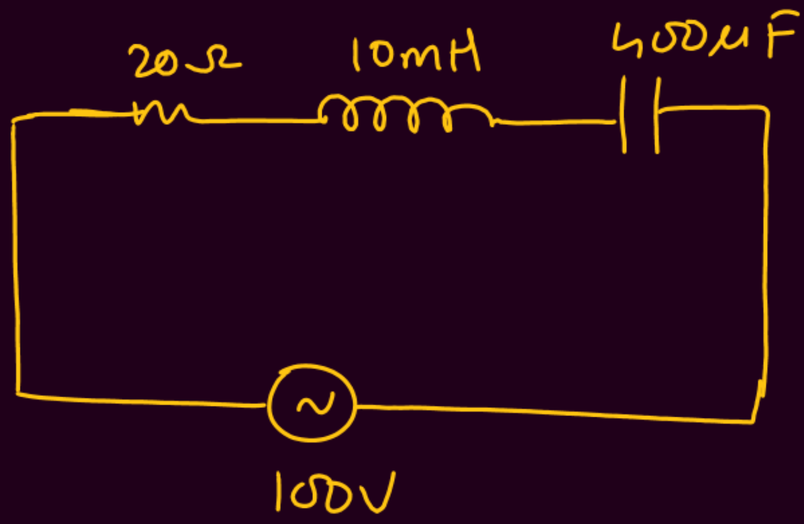
⑨ $\cos\phi = 1$

⑩ $P_{av} \to$ max

ωL

$\frac{1}{\sqrt{LC}}$ $L = \sqrt{\frac{L}{C}}$

Q.



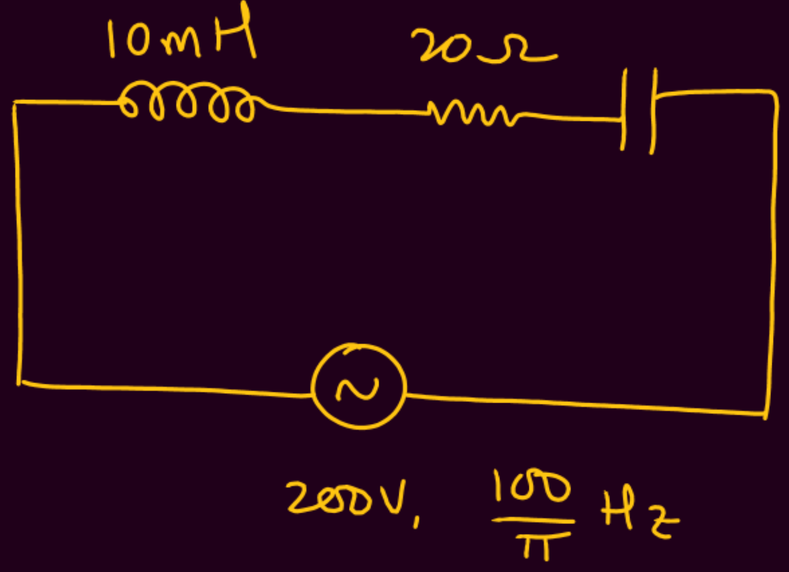
① $f_r = \dfrac{1}{2\pi\sqrt{10\times10^{-3}\times400\times10^{-6}}}$

$= \dfrac{1}{2\pi(2)\times10^{-3}} = \dfrac{250}{\pi}$ Hz

② $P_{av} = i_{rms}^2 R$

$= \left(\dfrac{100}{20}\right)^2 (20) = \underline{500\ W}$

Q



$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

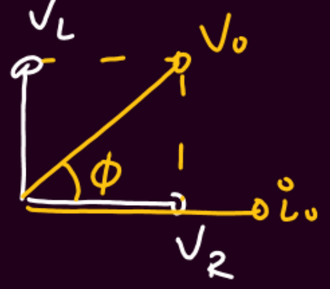
$$\frac{100}{\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$LC = \frac{1}{200 \times 200}$$

$$C = \frac{1}{200 \times 200 \times 10 \times 10^{-3}} = \frac{1}{400}$$

$$C = \frac{10^6}{400}\ \mu F = 2500\ \mu F$$

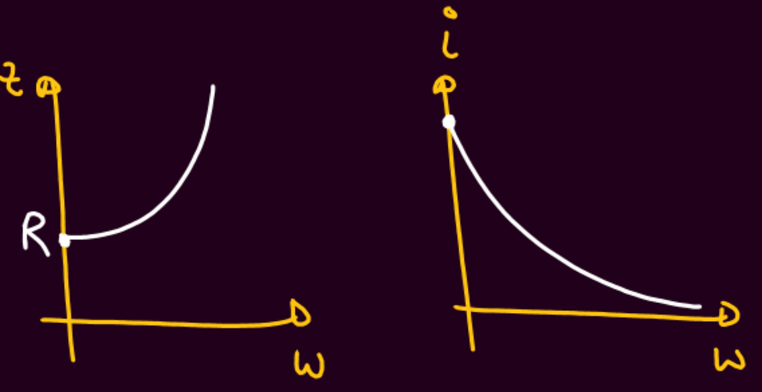
R – L



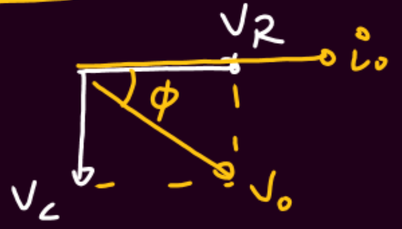
$$Z = \sqrt{R^2 + X_L^2}$$

$$V_0 = \sqrt{V_R^2 + V_L^2}$$

V leads i



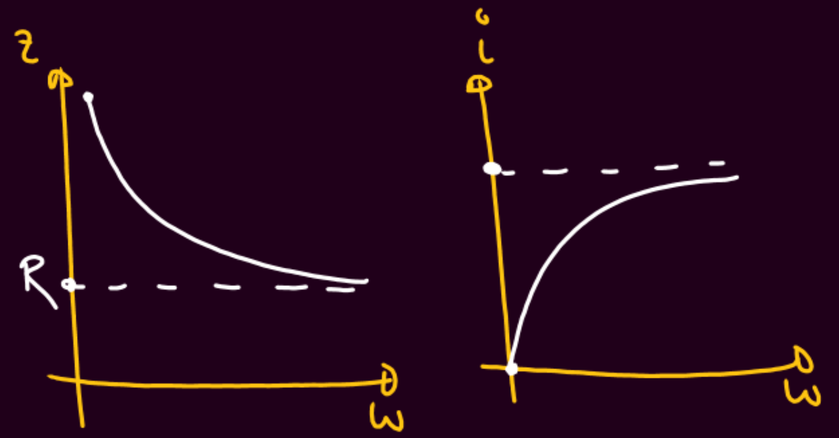
R – C



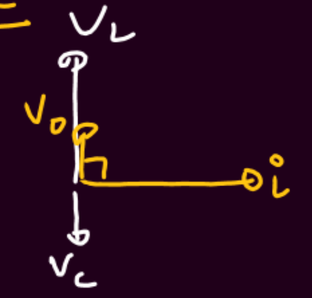
$$Z = \sqrt{R^2 + X_C^2}$$

$$V_0 = \sqrt{V_R^2 + V_C^2}$$

i leads V



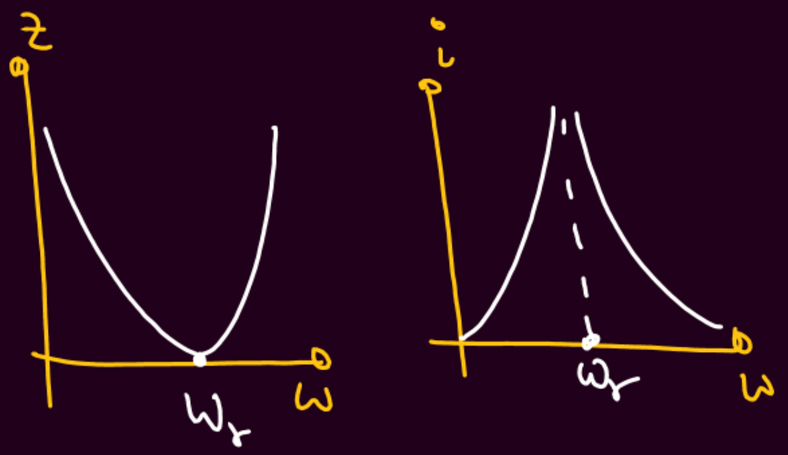
L – C



$$Z = |X_L - X_C|$$

$$V_0 = |V_L - V_C|$$

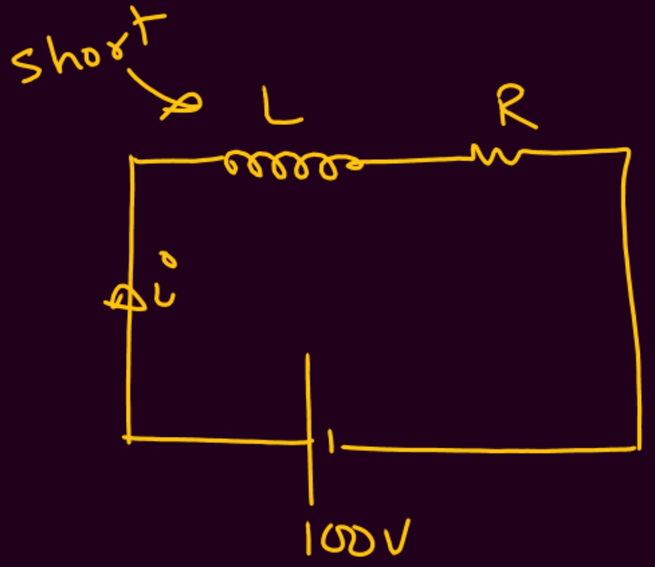
$$\phi = \pm \pi/2$$



Q. Choke Coil

100 V DC → 5A

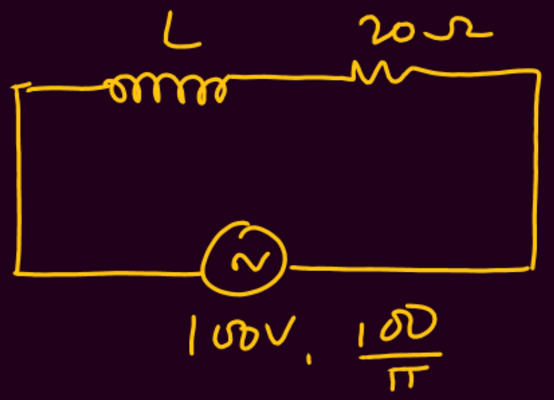
100V, $\frac{100}{\pi}$ Hz → 2A



$i = \frac{V}{R}$

$5 = \frac{100}{R}$

$R = 20\Omega$



$$\frac{V_0}{i_0} = Z = \sqrt{R^2 + (\omega L)^2}$$

$$\frac{100}{2} = \sqrt{20^2 + (200L)^2}$$

$$50 = 20\sqrt{1^2 + (10L)^2}$$

$$\frac{25}{4} = 1 + 100L^2$$

$$\frac{21}{4} = 100L^2$$

$$L^2 = \frac{21}{400}$$

$$L = \frac{\sqrt{21}}{20}\ H$$

Q. If i lags V (possibility)

✓ 1) L 

2) C

✓ 3) LC 

✓ 4) RL 

5) RC

✓ 6) RLC

L ↑

X_L ↑

Z ↑

i ↓ Brightness ↓

Skin Effect

→ AC

→ multiple strands

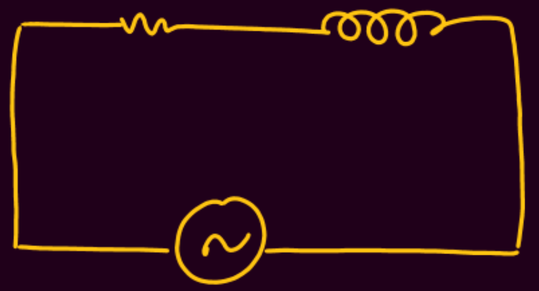
$$i = 3\sin(\omega t) + 4\sin\left(\omega t + \frac{\pi}{3}\right)$$

rms = ?

$$i_{peak} = \sqrt{3^2 + 4^2 + 2(3)(4)\cos\frac{\pi}{3}}$$

$$= \sqrt{37}$$

$$rms = \sqrt{\frac{37}{2}}$$



200V, $\frac{100}{\pi}$ Hz

if $V_R = 100\,V$

① $\phi = ?$

② $V_L = ?$

① $\cos\phi = \frac{V_R}{V_0} = \frac{100}{200} = \frac{1}{2}$

$\phi = \pi/3$

② $V_0^2 = V_R^2 + V_L^2$

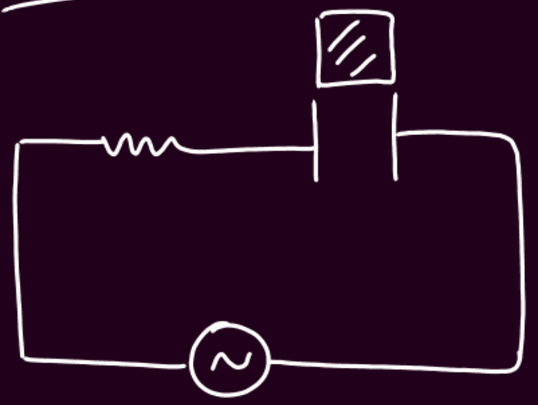
$(200)^2 = 100^2 + V_L^2$

$V_L^2 = 200^2 - 100^2$

$V_L = 100\sqrt{2^2 - 1^2}$

$= \underline{100\sqrt{3}\,V}$

PYQ



① Air filled ② Mica filled

1) $i_1 < i_2$

2) $V_{C_1} > V_{C_2}$

Mica $C \rightarrow KC$ ↑

X_C ↓ $\quad X_C = \frac{1}{\omega C}$

Z ↓ $\quad Z = \sqrt{R^2 + X_C^2}$

i_0 ↑ $\quad i_0 = \frac{V_0}{Z}$

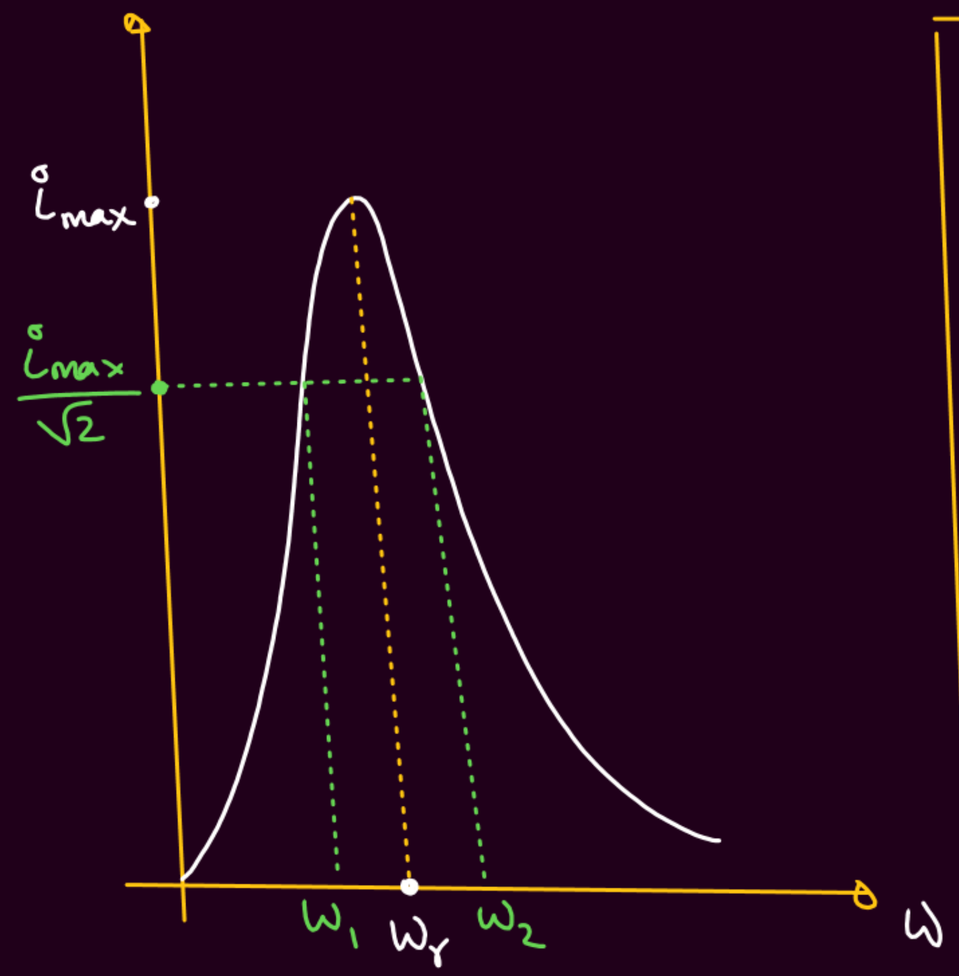
$V_0^2 = V_R^2 + V_C^2$

Const. $\quad (i_0 R)^2$ ↑ + ↓

Quality factor (Sharpness)

$$Q = \frac{\omega_r}{\Delta\omega} = \frac{\omega_r L}{R} = \frac{X_{Lr}}{R} = \frac{X_{Cr}}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$



"Half Power"

$$P_{max} = i_{max}^2 R$$

$$P_{Half} \rightarrow \frac{1}{2} P_{max}$$

① $i = \frac{i_{max}}{\sqrt{2}}$

$$Z_{min} = R$$

② $Z = R\sqrt{2}$

③ $X_L - X_C = R$

④ $\phi = 45°$

⑤ P.F. $= \cos\phi = \frac{1}{\sqrt{2}}$

⑤ Bandwidth

$$\Delta f = f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi}$$

⑥ $\Delta\omega = \frac{R}{L}$

⑦ Sharpness / Quality ↑

R ↓ L ↑ C ↓

(26) $L = 5H$

$C = 80\mu f$

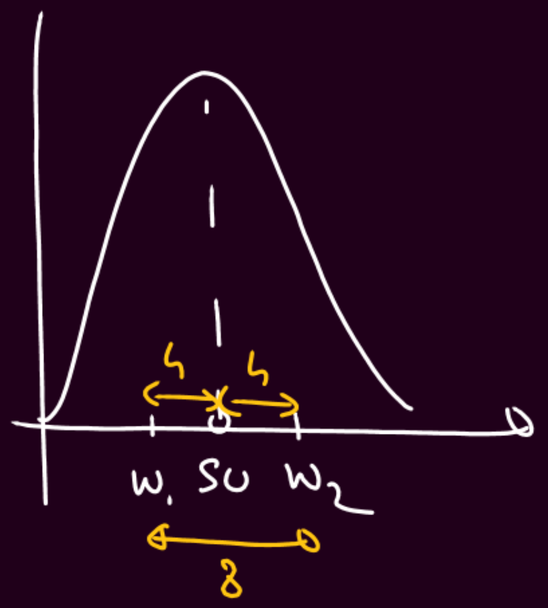
$R = 40\Omega$

$V = 230V$

ω_1 / ω_2

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{1}{\sqrt{400 \times 10^{-6}}}$$

$$= \frac{1}{2 \times 10 \times 10^{-3}} = \frac{100}{2} = 50 \text{ rad/s}$$



$$\Delta\omega = \frac{R}{L} = \frac{40}{5} = 8 \text{ rad/s}$$