Wave Optics

Newton's Corpusular Theory

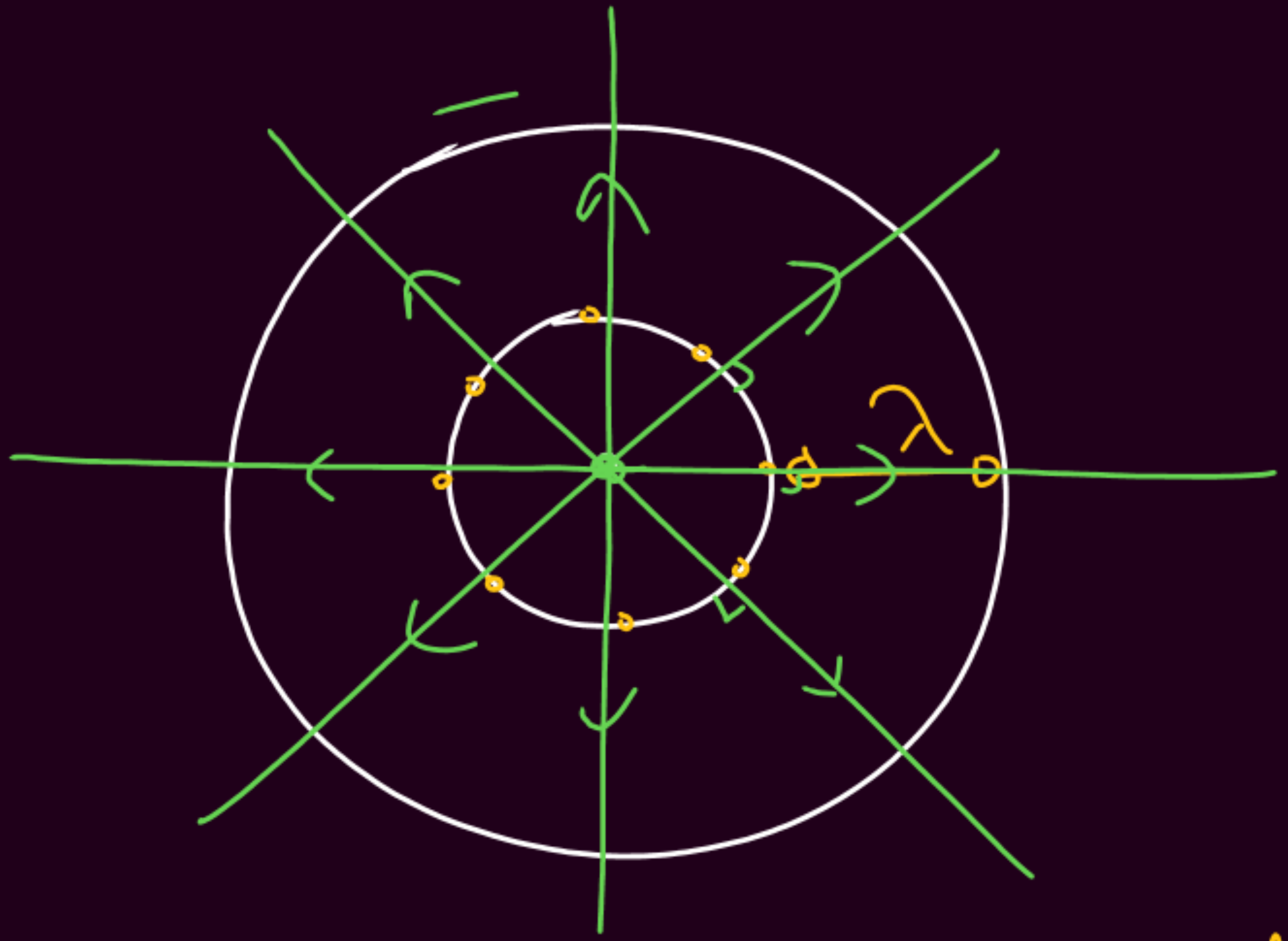
- → Corpuscles
- → Color → Diff size
- → Speed more in Denser
- → Can't Explain
  - ↳ Interference / Diffraction / polarization

→ [Huygen → "Wave theory"] → (Light → longitudinal)

"YDSE" → Light → Interference
↳ "Wave"

→ Photoelectric Effect

Huygen's Wave Theory



"Wavefronts" → locus of all the particles oscillating in same phase

→ Secondary wavelets

→ 'λ'

→ Dir of wave prop. → ⊥r to wavefront

Coherent & Incoherent Sources

$$y_1 = A_1 \sin(\omega_1 t)$$

$$y_2 = A_2 \sin(\underline{\omega_2 t + \alpha})$$

$$\Delta\phi = (\omega_2 - \omega_1)t + \alpha$$

If $(\omega_1 = \omega_2)$

$$\boxed{\Delta\phi = \alpha = \text{const.}}$$

Coherent → const. $\Delta\phi$

↳ must have same ω

→ Light → independent Sources of same freq. can not be coherent

Burst

Superposition of Waves (Coherent Sources)

$$y_1 = A_1 \sin(\omega t)$$

$$y_2 = A_2 \sin(\omega t + \phi)$$

$$y_1 + y_2 = A \sin(\omega t + \delta)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2 A_1 A_2 \cos\phi}$$

$$\boxed{I \propto A^2}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Identical Sources

$$A_1 = A_2 = A_0$$

$$I_1 = I_2 = I_0$$

$$A = 2A_0 \cos(\phi/2)$$

$$I = (4I_0)\cos^2(\phi/2) = 2I_0(1 + \cos\phi)$$

$$I = I_{max}\cos^2(\phi/2)$$

$$\left\{\begin{aligned} \cos 2\theta &= 2\cos^2\theta - 1 \\ 1 + \cos 2\theta &= 2\cos^2\theta \end{aligned}\right.$$

Q. I_0

$4I_0$

$\phi = \frac{\pi}{3}$

$$I = I_0 + 4I_0 + 2\sqrt{I_0 \, 4I_0} \cos\left(\frac{\pi}{3}\right)$$

$$\boxed{I = 7I_0}$$

Q. Identical Sources A_0

$A = A_0 \qquad \phi = ?$

① $A_0 = \sqrt{A_0^2 + A_0^2 + 2A_0A_0\cos\phi}$

$\cos\phi = -\frac{1}{2}$

$\boxed{\phi = \frac{2\pi}{3}}$ or 120°

② $A_0 = 2A_0\cos(\phi/2)$

$\phi/2 = \pi/3$

Q. $I_{max} = I_0$

Find $\Delta\phi = ?$

where $I = \frac{I_0}{2}$

$$I = I_{max} \cos^2(\phi/2)$$

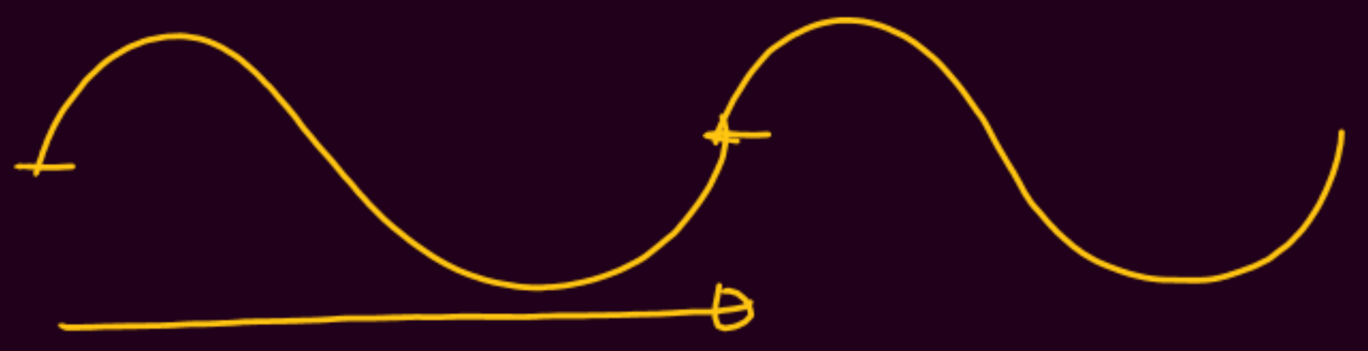
$$\frac{I_0}{2} = I_0 \cos^2(\phi/2)$$

$$\frac{1}{\sqrt{2}} = \cos(\phi/2)$$

$$\boxed{\phi = \pi/2}$$

$I_0 \quad I_0 \longrightarrow 4I_0$

$\frac{I_0}{4} \quad \frac{I_0}{4} \longrightarrow I_0$

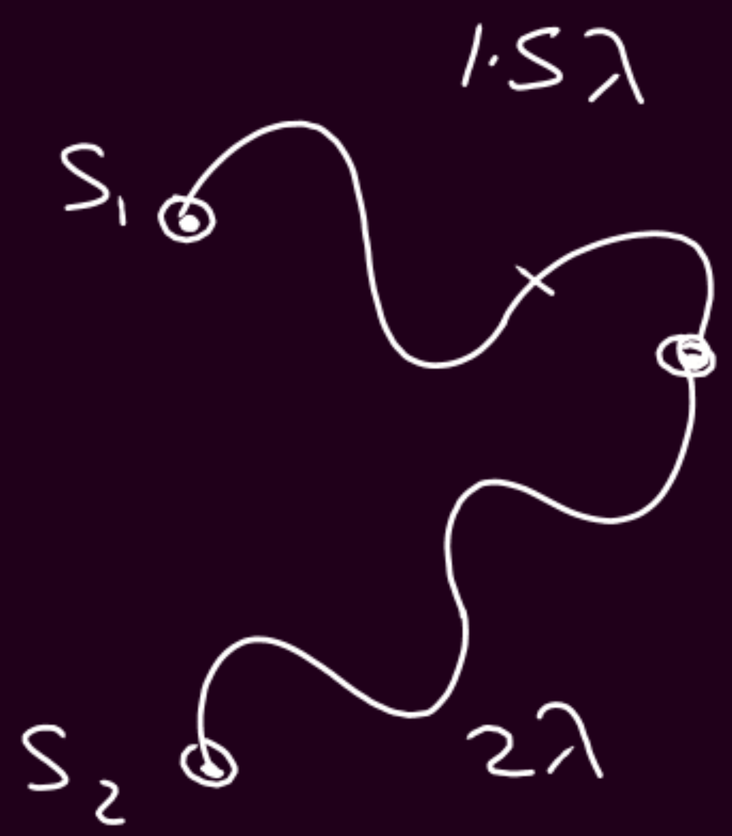


$$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{\Delta t}{T}}$$

$\Delta\phi$ = phase diff

Δx = path diff

Δt = time diff



$$\Delta x = 2\lambda - 1.5\lambda = 0.5\lambda$$

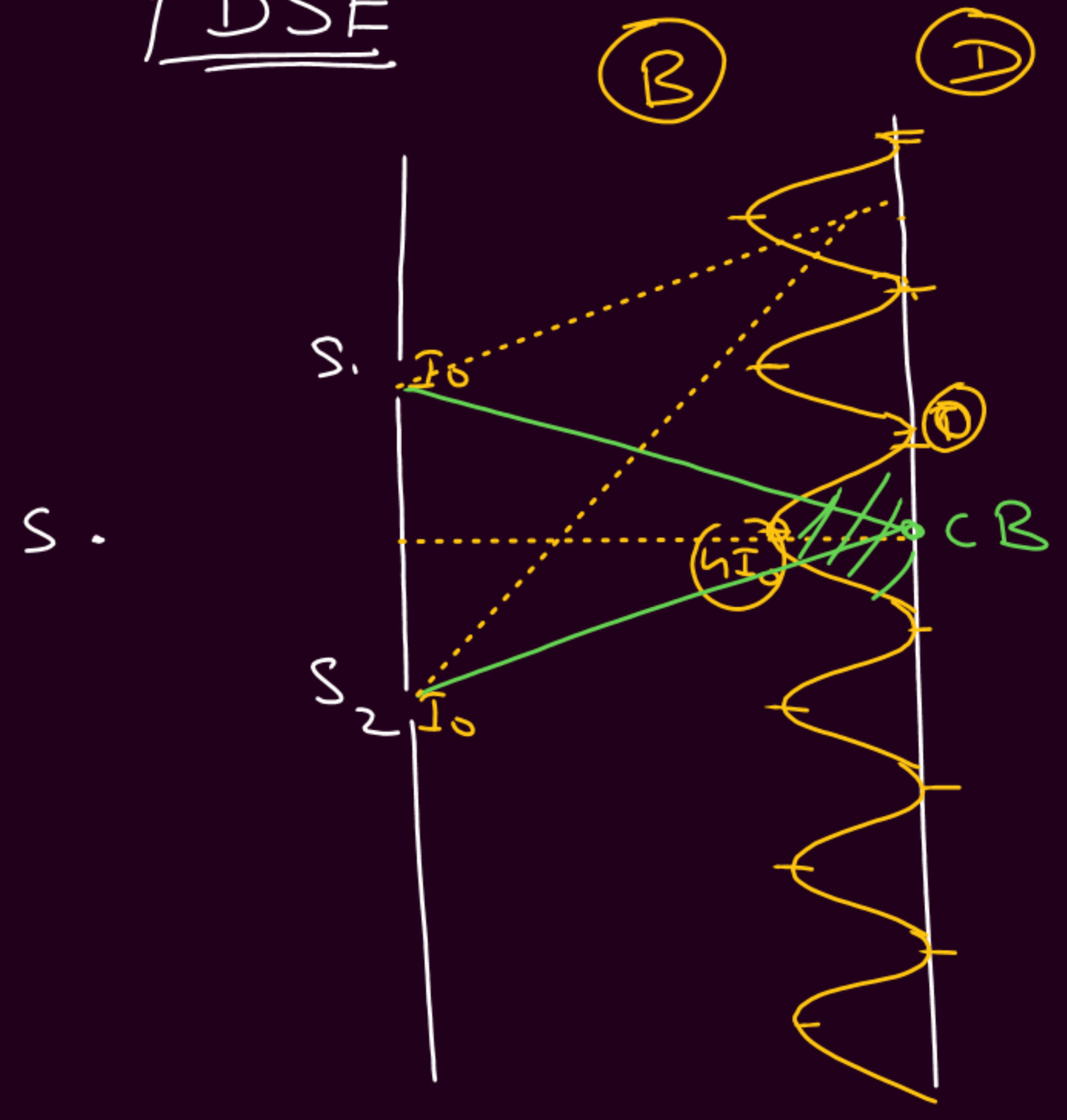
$$\frac{\Delta\phi}{2\pi} = \frac{\lambda/2}{\lambda}$$

$$\boxed{\Delta\phi = \pi}$$

Interference

- Same dir
- Coherent
- Contrast → $A_1 \approx A_2$

YDSE



Constructive Interference / Maxima / (B)

$\cos\phi = 1$

$\boxed{\phi = 2n\pi}$ $\quad \phi = 0 \quad 2\pi \quad 4\pi \quad 6\pi$

$\Delta x = n\lambda$ $\quad \Delta x = 0 \quad \lambda \quad 2\lambda \quad 3\lambda$

CB $\quad B_1 \quad B_2 \quad B_3$

$A_{max} = A_1 + A_2$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Identical Sources

$A_{max} = 2A_0$

$I_{max} = 4I_0$

Destructive Interference / Minima / (D)

$\boxed{\cos\phi = -1}$

$\boxed{\phi = (2n-1)\pi}$ $\quad \phi = \pi \quad 3\pi \quad 5\pi \quad 7\pi$

$\boxed{\Delta x = (2n-1)\frac{\lambda}{2}}$ $\quad \Delta x = \frac{\lambda}{2} \quad \frac{3\lambda}{2} \quad \frac{5\lambda}{2} \quad \frac{7\lambda}{2}$

$D_1 \quad D_2 \quad D_3 \quad D_4$

$A_{min} = |A_1 - A_2|$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Indentical Sources

$A_{min} = 0$

$I_{min} = 0$

① $$\frac{A_{min}}{A_{max}} = \frac{A_1 - A_2}{A_1 + A_2}$$

② $$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \frac{I_1 + I_2 - 2\sqrt{I_1 I_2}}{I_1 + I_2 + 2\sqrt{I_1 I_2}} = \left(\frac{A_{min}}{A_{max}}\right)^2 = \left(\frac{A_1 - A_2}{A_1 + A_2}\right)^2$$

③ Fringe Visibility $$= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

Q $\dfrac{A_1}{A_2} = \dfrac{2}{3}$

$$\frac{A_{max}}{A_{min}} = \frac{3+2}{3-2} = 5$$

Q $\dfrac{I_1}{I_2} = \dfrac{4}{9}$

$$\boxed{\frac{I_{max}}{I_{min}}} = \left(\frac{\sqrt{4}+\sqrt{9}}{\sqrt{4}-\sqrt{9}}\right)^2 = \frac{25}{1}$$

$$\frac{a-b}{a} = \frac{p-q}{p} \qquad \frac{a}{b} = \frac{p}{q}$$

$$\frac{a+b}{a-b} = \frac{p+q}{p-q}$$

Q. $\dfrac{I_{min}}{I_{max}} = \dfrac{16}{25} = \left(\dfrac{\sqrt{} - \sqrt{}}{\sqrt{} + \sqrt{}}\right)^2$

$$\frac{I_1}{I_2} = ?$$

$$\frac{4}{5} = \boxed{\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}}$$

$$\boxed{\frac{\boxed{9}}{1} = \frac{2\sqrt{I_1}}{2\sqrt{I_2}}}$$

$$\boxed{\frac{I_1}{I_2} = \frac{81}{1}}$$

Q $\dfrac{I_{min}}{I_{max}} = \dfrac{4}{9} \Rightarrow \dfrac{2}{3} = \dfrac{\sqrt{I_1}-\sqrt{I_2}}{\sqrt{I_1}+\sqrt{I_2}} = \dfrac{A_1 - A_2}{A_1 + A_2}$

$$\frac{A_1}{A_2} = \frac{5}{1} \qquad \frac{5}{1} = \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{A_1}{A_2}$$

Q. Intensity $\rightarrow n:1$

$$\text{fringe visibility} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2\sqrt{n}}{1+n}$$

Q. I_0 $\quad$ $4I_0$

$\Delta x = ?$

$$3I_0 = 5I_0 + 2(2I_0)\cos\phi$$

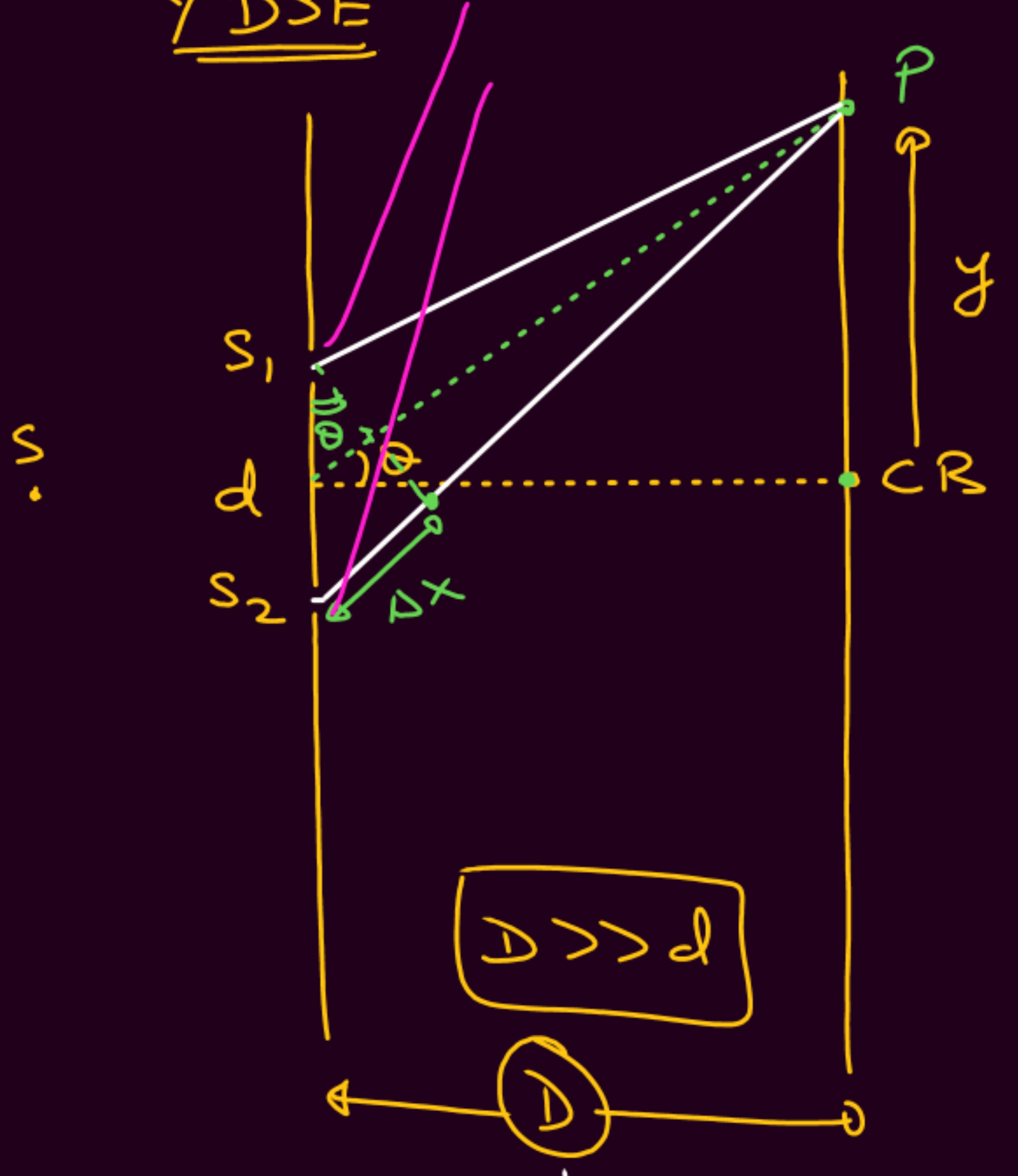
$$\cos\phi = -\frac{1}{2}$$

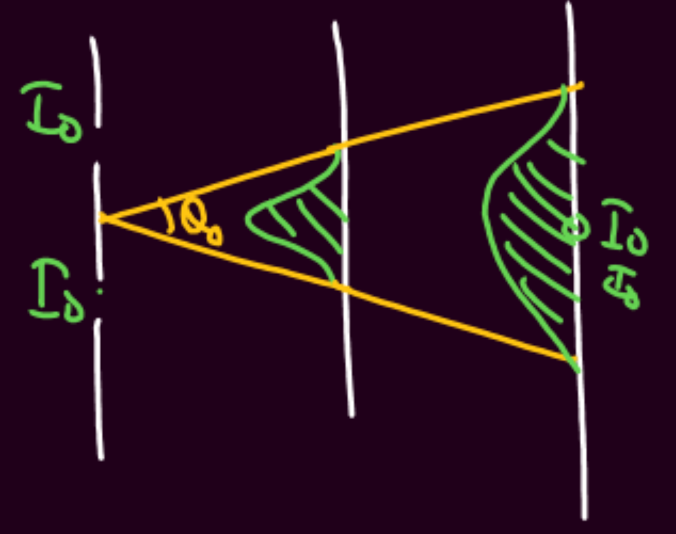
$$\phi = \frac{2\pi}{3}$$

$$\frac{2\pi/3}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta x = \frac{\lambda}{3}$$

YDSE





① CB $\Delta x = 0$ $\Delta\phi = 0$ I_{max}

② $\boxed{\Delta x = d\sin\theta}$

③ $y = \dfrac{\Delta x D}{d}$

④ $\theta = \dfrac{\Delta x}{d} = \dfrac{y}{D}$

⑤ $\boxed{\beta = \dfrac{\lambda D}{d}}$

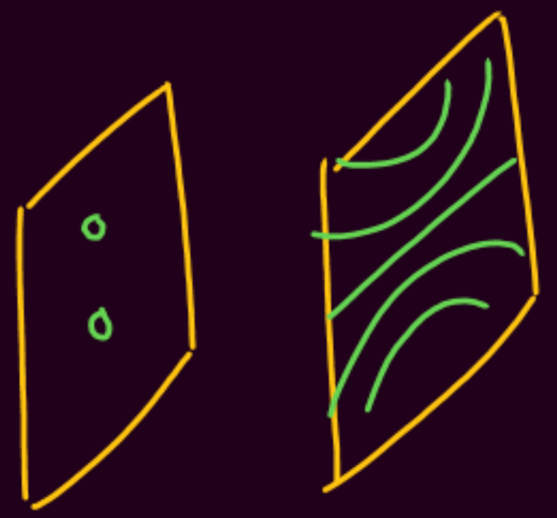
⑥ $\theta_0 = \dfrac{\lambda}{d} = \dfrac{\beta}{D}$

⑦ $\Delta x_{max} = d$ $(\theta = 90^\circ)$ at ∞

⑧ $\boxed{d = n\lambda}$ → $(2n+1)$ formed
→ $(2n-1)$ seen

Bright	CB	B_1	B_2	B_3
$\Delta\phi = 2n\pi$	0	2π	4π	6π
$\Delta x = n\lambda$	0	λ	2λ	3λ
$y = n\beta$	0	β	2β	3β

Dark	D_1	D_2	D_3	D_4
$\Delta\phi = (2n-1)\pi$	π	3π	5π	7π
$\Delta x = (2n-1)\dfrac{\lambda}{2}$	$\dfrac{\lambda}{2}$	$\dfrac{3\lambda}{2}$	$\dfrac{5\lambda}{2}$	$\dfrac{7\lambda}{2}$
$y = (2n-1)\beta/2$	$\beta/2$	$3\beta/2$	$5\beta/2$	$7\beta/2$



Hyperbola

st. line

(9) Liq $\rightarrow \mu$

$$\lambda_{Liq} = \frac{\lambda_{air}}{\mu}$$

$$\beta_{Liq} = \frac{\beta_{air}}{\mu}$$

Smaller

Q $\lambda = 6000\ \mathring{A}$

$D = 2m$

$d = 3mm$

(1) $\beta = \frac{6\times10^{-7}\times 2}{3\times10^{-3}} = 0.4mm$

(2) $\theta_0 = 2\times10^{-4}$ rad

(3) 3^{rd} B $= 3\beta = 1.2mm$

(4) 2^{nd} D $= \frac{3\beta}{2} = 0.6mm$

(5) $\boxed{5^{th}\ D\ \&\ 2^{nd}\ B}$ $\quad \frac{9\beta}{2} - 2\beta = \underline{\underline{1mm}}$

(6) [3^{rd} B above & 3^{rd} D below]

$$y = 0.1\,mm = \frac{\Delta x\, D}{d}$$

$$0.1 \times 10^{-3} = \frac{\Delta x\,(2)}{3 \times 10^{-3}}$$

$$\Delta x = \frac{3}{2} \times 10^{-7}\ m$$

$$\frac{\Delta\phi}{2\pi} = \frac{\frac{3}{2} \times 10^{-7}}{6 \times 10^{-7}} = \frac{1}{4}$$

$$\Delta\phi = \pi/2$$

$$I = I_0 \cos^2(\pi/4) = \frac{I_0}{2}$$

$$I = I_0 \cos^2(\phi/2)$$

$$\boxed{y = n\beta}$$

$$0.1 = n(0.4)$$

$$n = \frac{1}{4}$$

$$\boxed{\Delta x = \frac{1}{4}\lambda}$$

$$\frac{\Delta\phi}{2\pi} = \frac{\lambda/4}{\lambda}$$

$$2 = n(6)$$

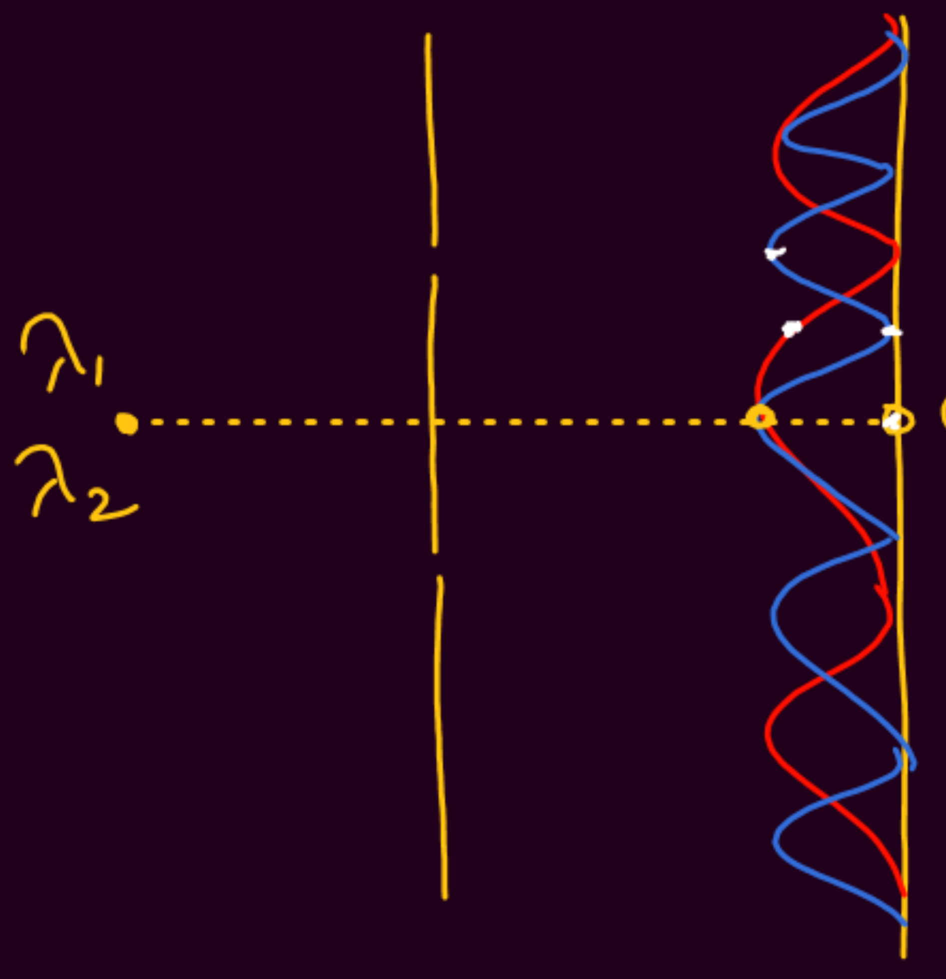
$$n = 1/3$$

$$\Delta x = \lambda/3$$

$$\Delta\phi = 2\pi/3$$

$$I = I_{max} \cos^2(\pi/3) = \frac{I_{max}}{4}$$

(10) Two wavelengths



$\lambda_R > \lambda_V$

$\beta_R > \beta_V$

Position of Common Bright

$$\boxed{y = n_1 \beta_1 = n_2 \beta_2}$$

Condition for Common B

$$\boxed{n_1 \lambda_1 = n_2 \lambda_2}$$

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(11) White light

⇒ Center → White

⇒ Red → Blue → No clear pattern

Q. $D = 2m$

$d = 5mm$

$\lambda_1 = 6000 \mathring{A}$

$\lambda_2 = 4000 \mathring{A}$

Position of 1^{st} common B?

$$n_1(6000) = n_2(4000)$$

$$\frac{n_1}{n_2} = \frac{4}{6} = \frac{2}{3}$$

$\downarrow$ 2^{nd} $\quad$ $\downarrow$ 1^{st}

$$y = n_1 \beta_1$$

$$= 2 \frac{6 \times 10^{-7}(2)}{5 \times 10^{-3}}$$

$$y = 4.8 \times 10^{-4}$$

$$\boxed{y = 0.48\ mm}$$

(11)

			Central Bright	
Coherent Sources	$(I_0 \,\&\, I_0)$	$\Rightarrow$	$4I_0$	I_{max}
Incoherent — " —	(— " —)	$\Rightarrow$	$2I_0$ (uniform)	$\frac{I_{max}}{2}$
One slit is covered (opaque)		$\Rightarrow$	I_0	$\frac{I_{max}}{4}$

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(12) Identical Sources

→ Fringe visibility (100%)

$I_{max} = 4I_0$ $\qquad \frac{4I_0 - 0}{4I_0 + 0}$

$\boxed{I_{min} = 0}$

Intensity ∝ (width of slit)

		Bright	Dark	f.V.
(I) width of one slit is ↑		↑	↓	↓
(II) — " —	↓	↓	↓	↓

$$f.V. = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$I_0 \\ I_0 \Big\} \begin{matrix} \rightarrow \text{max} & 4I_0 \\ \rightarrow \text{min} & 0 \end{matrix}$$

$$I_0 \\ 4I_0 \Big\} \begin{matrix} \rightarrow \text{max} & 9I_0 \\ \rightarrow \text{min} & I_0 \end{matrix}$$

Q.

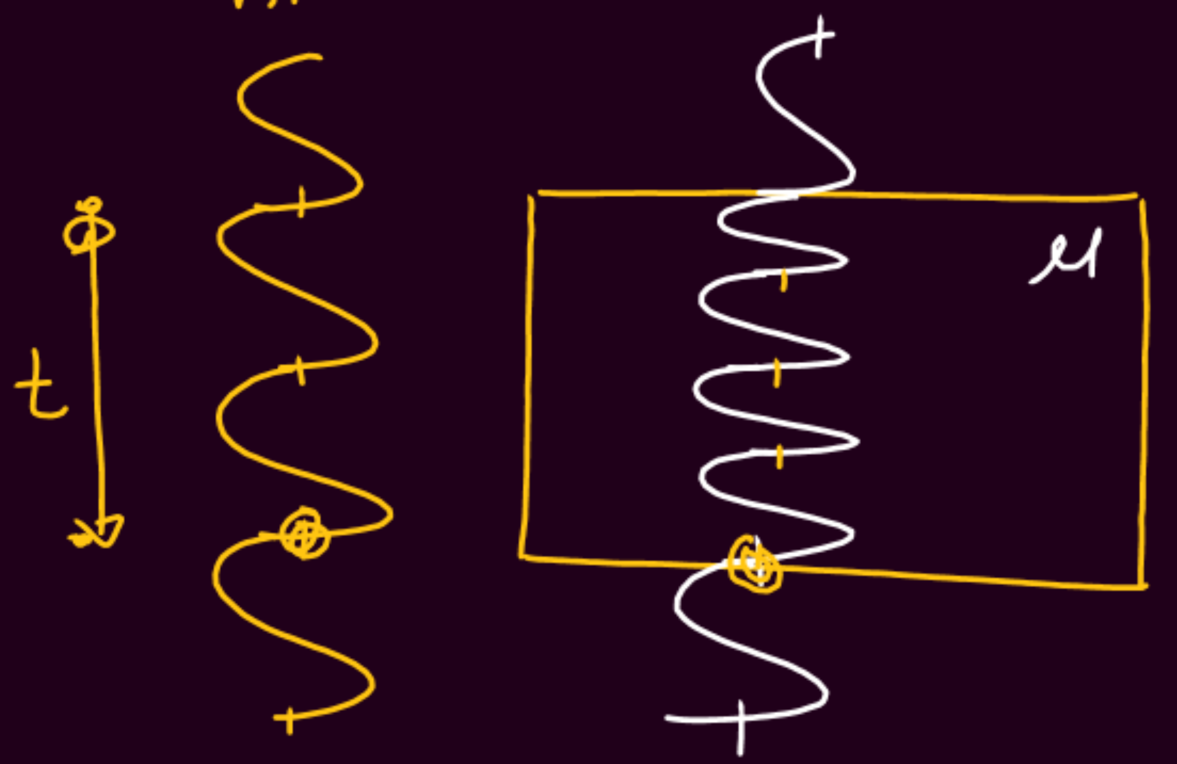
$$\left(3^{rd}\ D\right)_{liq} = \left(1^{st}\ B\right)_{air}$$

$$\frac{5}{2}\beta_{liq} = \beta_{air}$$

$$\frac{5}{2}\frac{\beta_{air}}{\mu} = \beta_{air}$$

$$\boxed{\mu = \frac{5}{2} = 2.5}$$

Optical Path

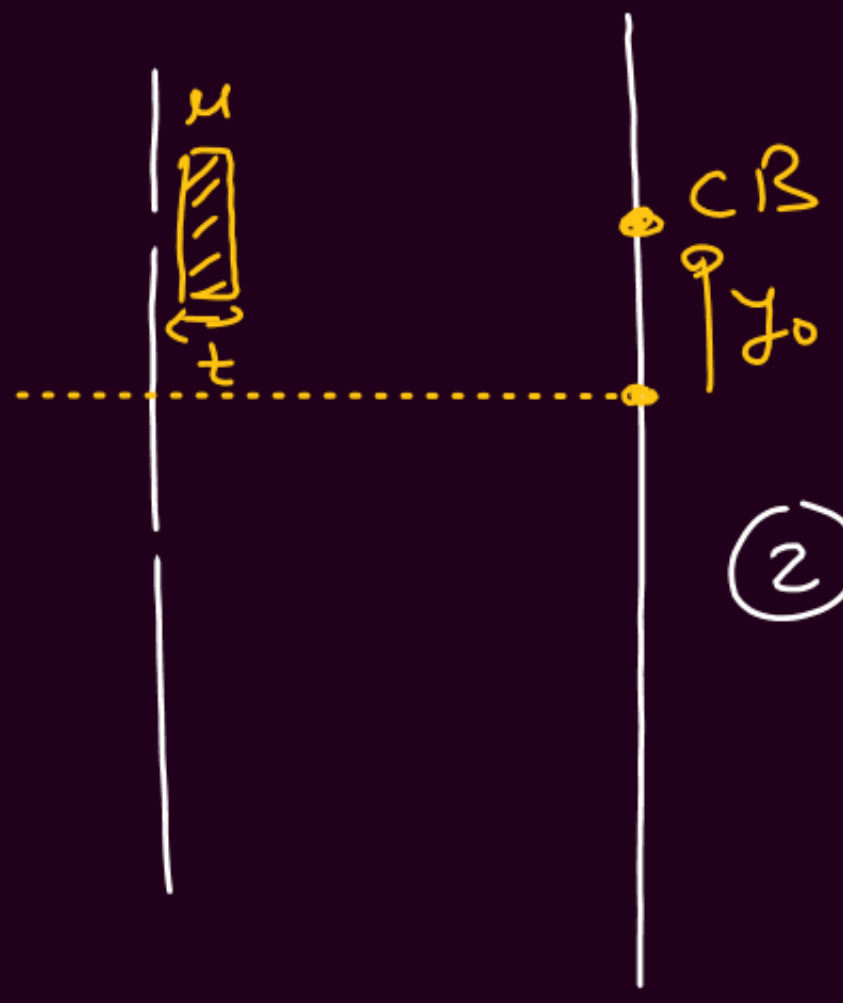


Actual Path (Air) $= t$

$$\boxed{\text{Optical Path} = \mu t}$$

Optical path Diff. $= (\mu - 1)t$

(14) Thin film is introduced in front of one of the slits.



① Path diff. at Center

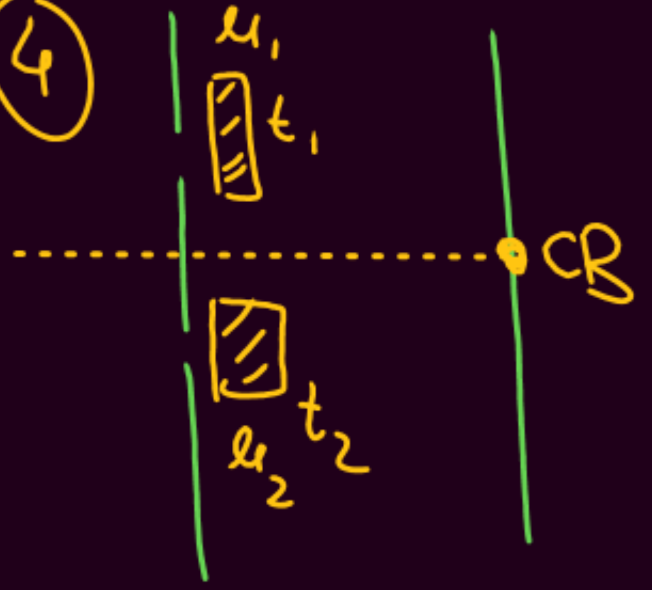
$$\Delta x = (\mu - 1)t = n\lambda$$

② No. of fringes shifted

$$\boxed{n = \frac{t(\mu - 1)}{\lambda}}$$

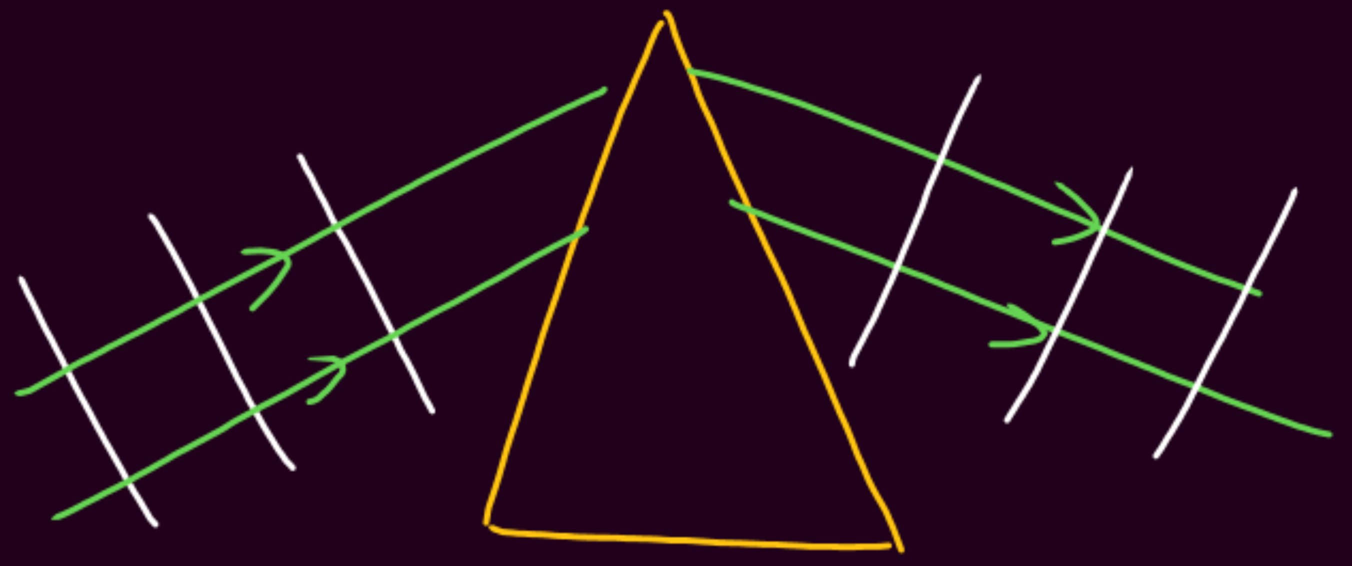
③ Position of CB $= y_0 = \dfrac{(\mu - 1)tD}{d}$

④

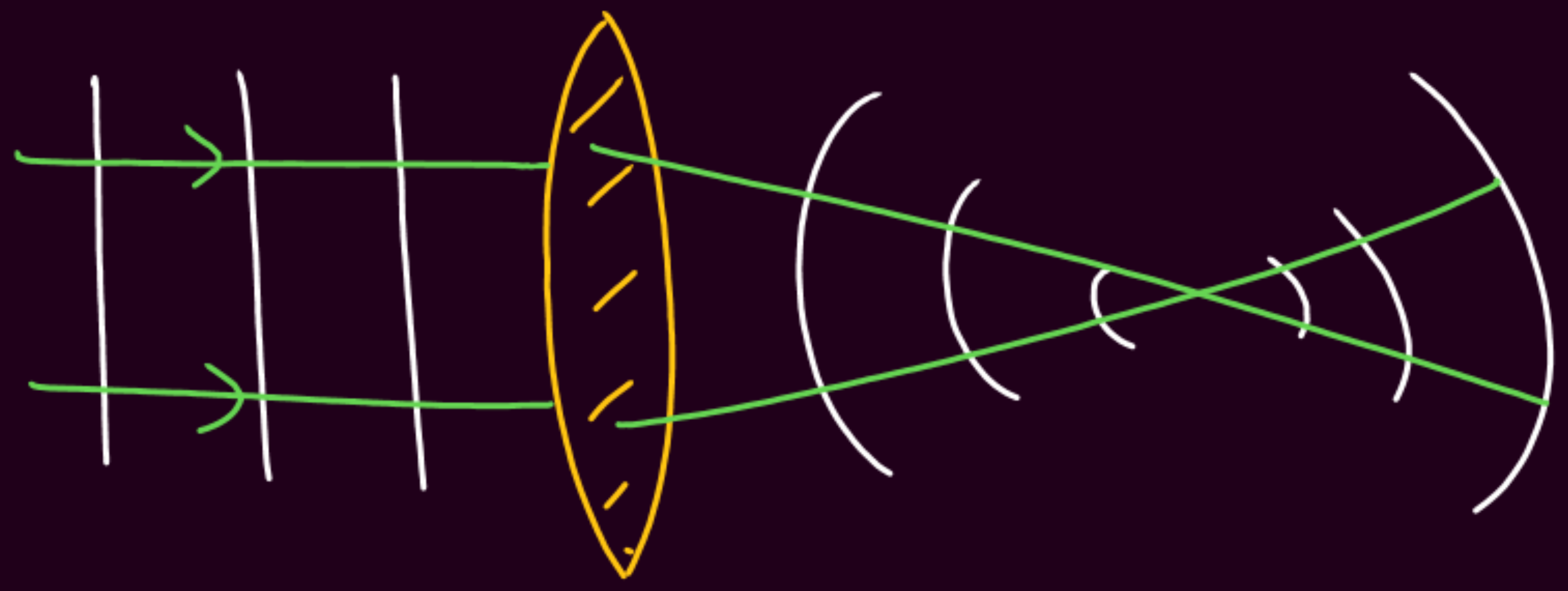


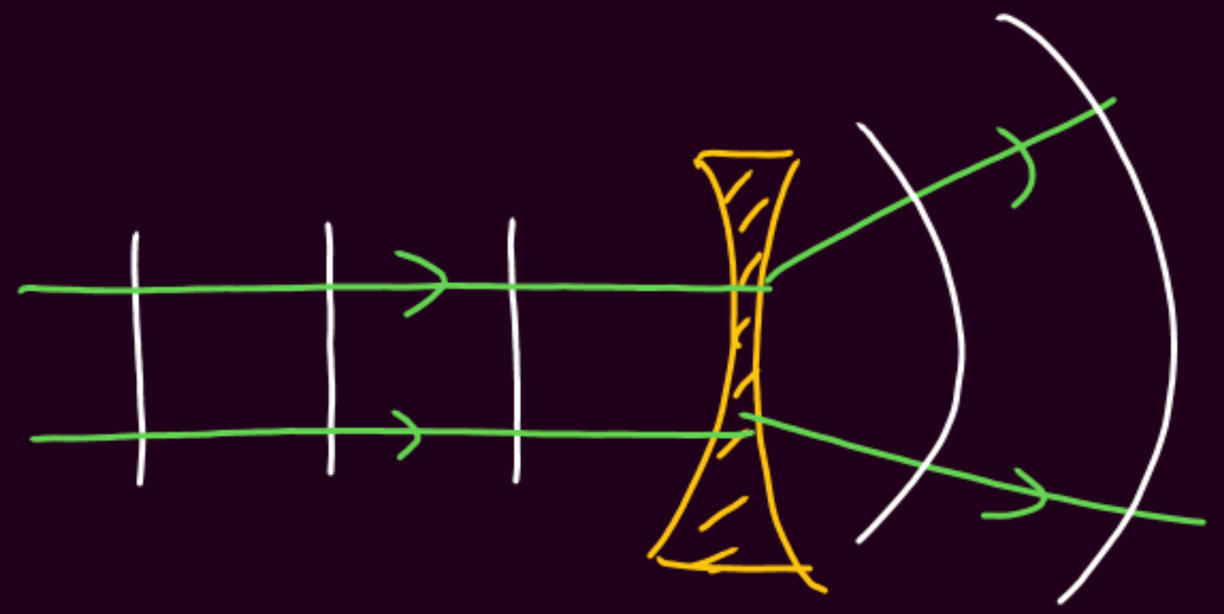
For CB to remain at Center

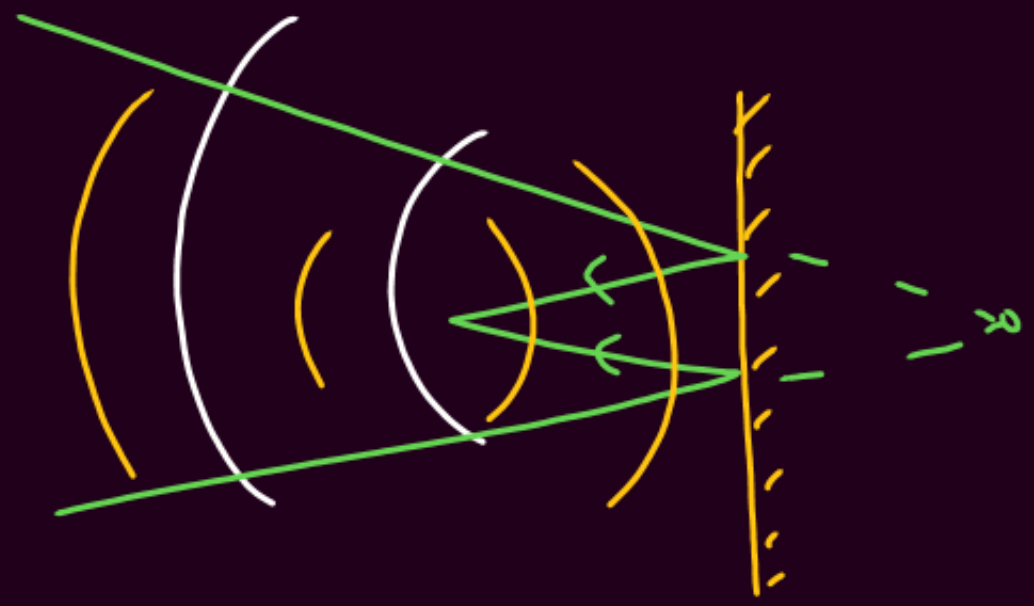
$$\boxed{(\mu_1 - 1)t_1 = (\mu_2 - 1)t_2}$$



Incident

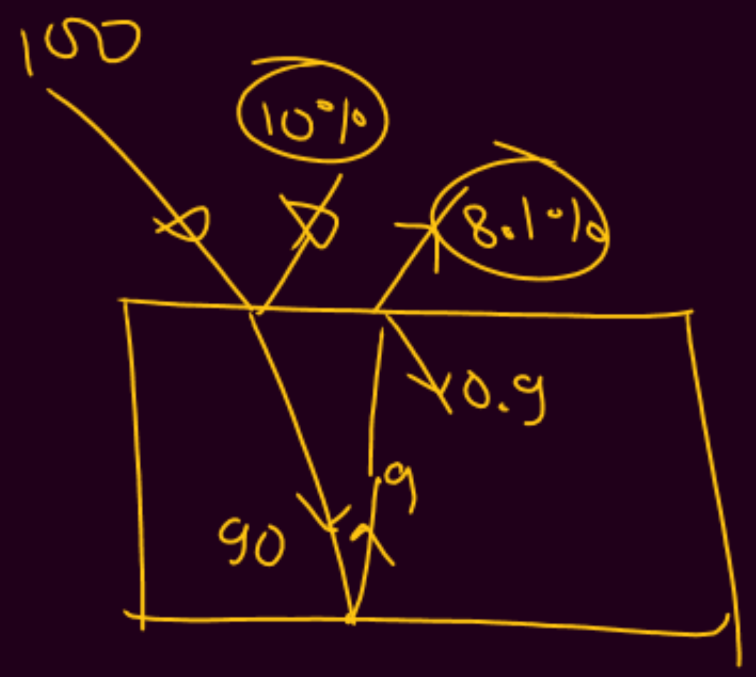




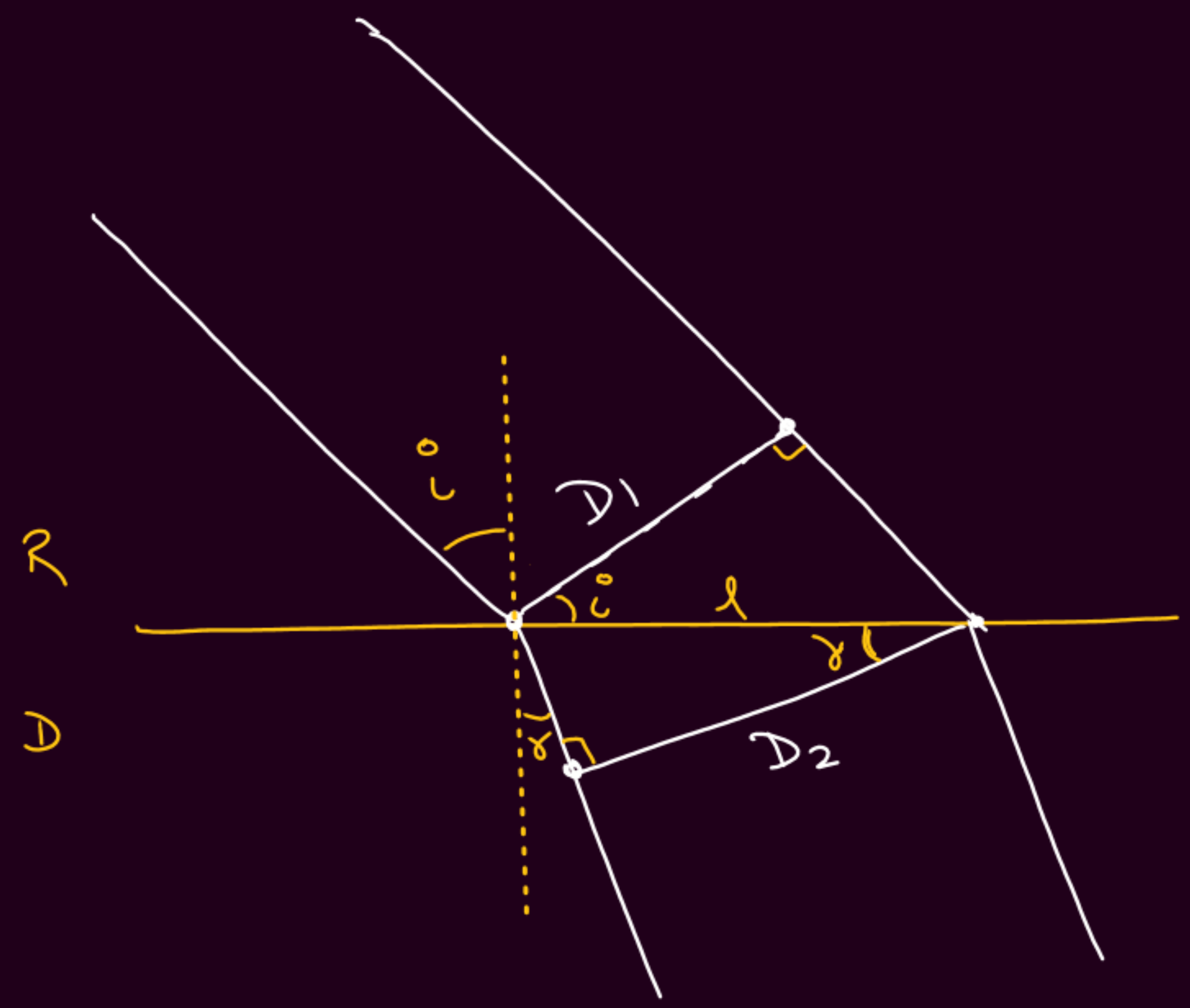


YDSE → Division of Wavefront

Thin film Interference → Division of Amplitude



Relation betⁿ Radius/Diameter of Incident & Refracted Beam



$l \cos i = D_1$

$l \cos r = D_2$

$$\boxed{\frac{D_1}{D_2} = \frac{\cos i}{\cos r}}$$

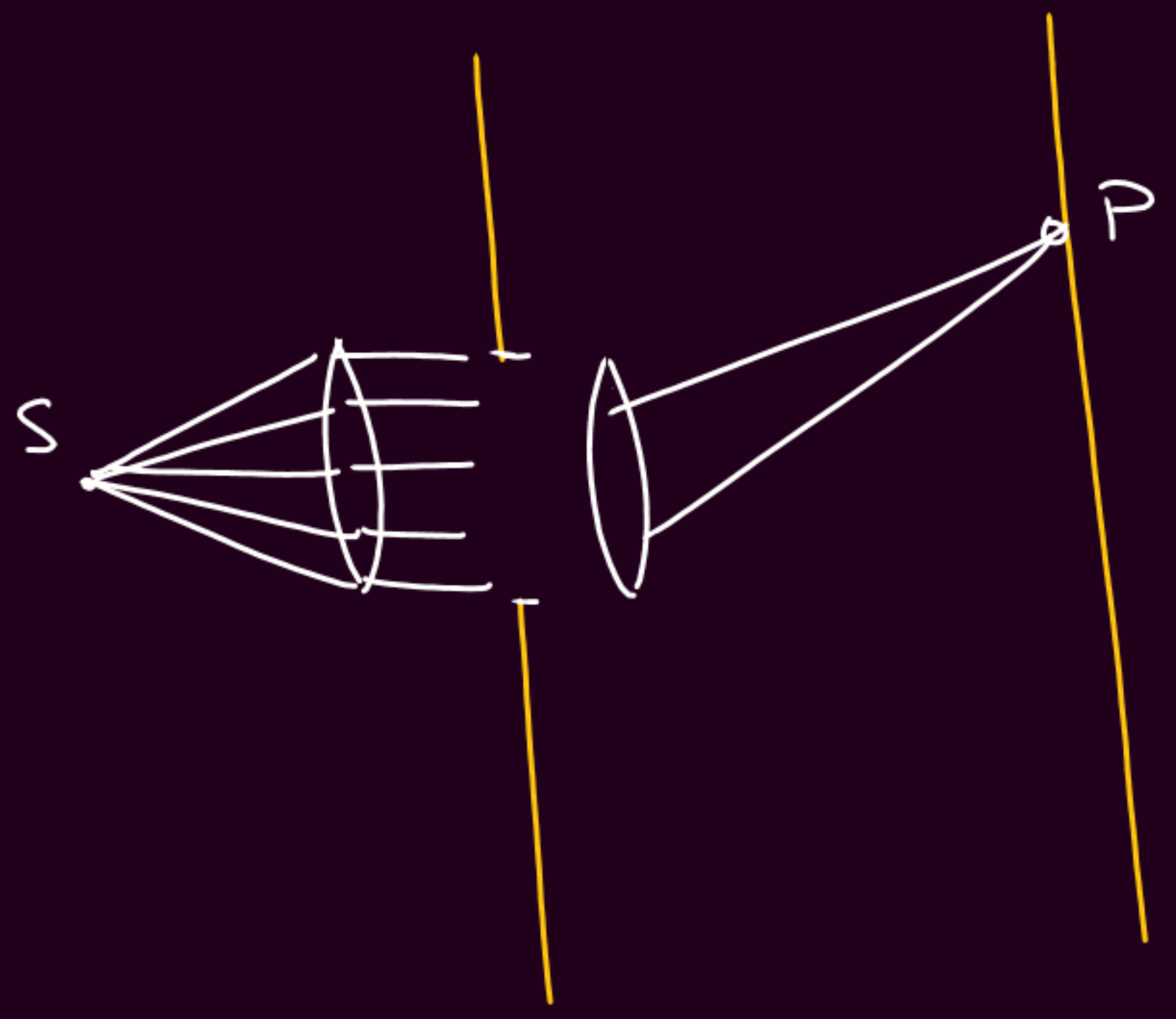
$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

Diffraction

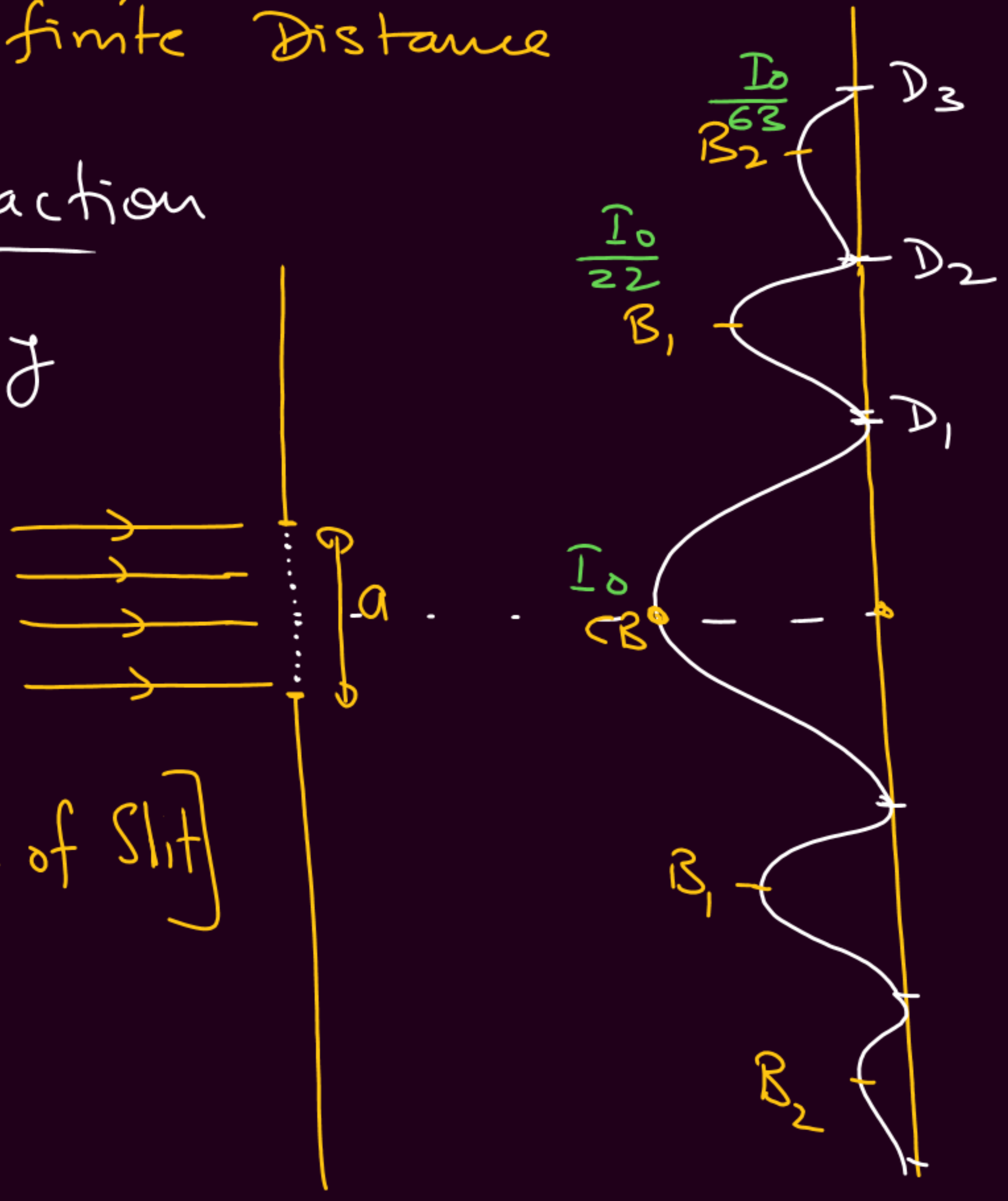
Fresnel → Source / Screen at finite Distance

Fraunhofer Single Slit Diffraction

→ Source / Screen at infinity



[a = width of Slit]



Interference	Diffraction
→ Two or few	→ many
→ $d\sin\theta = n\lambda$ (B)	→ $a\sin\theta = n\lambda$ $\boxed{\text{Dark}}$
$= (2n-1)\frac{\lambda}{2}$ (D)	$= (2n+1)\frac{\lambda}{2}$ $\boxed{\text{Bright}}$

Interference		Diffraction	
D_3	$5\lambda/2$	D_3	3λ
B_2	2λ	B_2	$5\lambda/2$
D_2	$3\lambda/2$	D_2	2λ
B_1	λ	B_1	$3\lambda/2$
D_1	$\lambda/2$	D_1	λ
CB	0	CB	0

CB B1 B2 B3

$d\sin\theta$ 0 λ 2λ 3λ

D1 D2 D3

$\frac{\lambda}{2}$ $\frac{3\lambda}{2}$ $\frac{5\lambda}{2}$

$$\theta = \frac{\lambda}{d}$$

1B

$$y_{CB} = 0$$

$$y_B = n\beta = n\left(\frac{\lambda D}{d}\right)$$

$$y_D = (2n-1)\beta/2$$

CB B1 B2 B3

$d\sin\theta$ 0 $\frac{3\lambda}{2}$ $\frac{5\lambda}{2}$ $\frac{7\lambda}{2}$

D1 D2 D3

λ 2λ 3λ

$$y_{CB} = 0$$

$$y_B = (2n+1)\beta/2$$

$$y_D = n\beta = n\left(\frac{\lambda D}{a}\right)$$

$$\theta = \frac{3\lambda}{2a}$$

1B

$$\beta_{all} = \frac{\lambda D}{d}$$

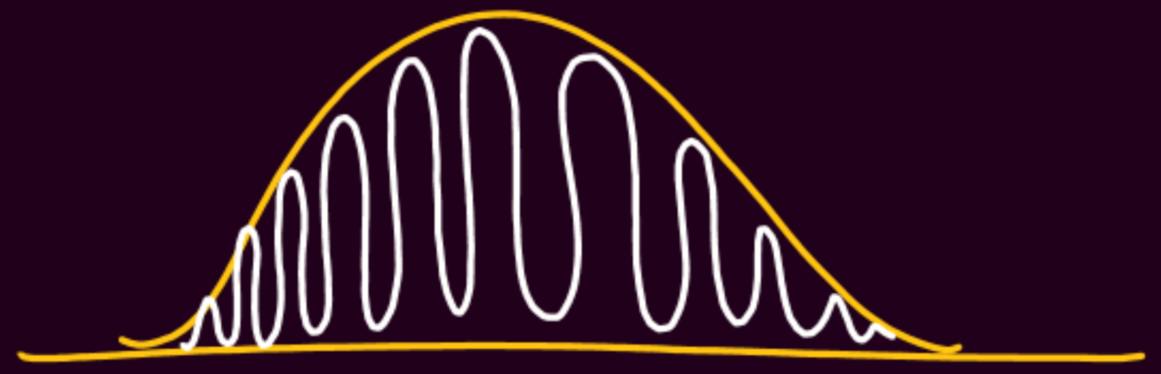
$\rightarrow$ Same I

$$\beta_{CB} = \frac{2\lambda D}{a}$$

$$\beta_{others} = \frac{\lambda D}{a}$$

$\rightarrow$

Q. no. of Interference maxima of YDSE within central maxima of Diff



Diff $\quad$ Inter

$$2\left(\frac{\lambda D}{a}\right) = n\left(\frac{\lambda D}{d}\right)$$

$$\boxed{n = \frac{2d}{a}}$$

Q. $\lambda = 6000\ \mathring{A}$

$a = 12\ \mu m$

$\theta = ?$ of D_1

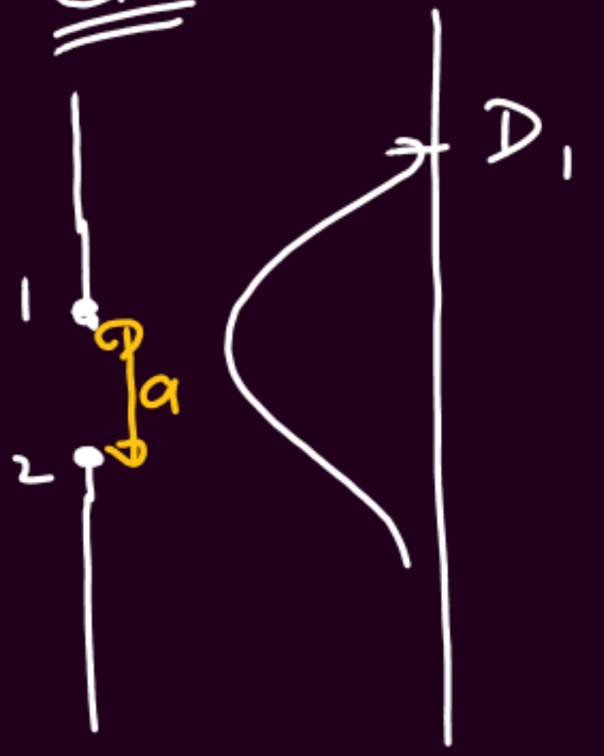
$$a \sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-7}}{12 \times 10^{-6}} = 0.05$$

$$\theta \simeq 0.05 \text{ rad}$$

$$\theta = 0.05 \times \frac{180}{\pi} \simeq 3^\circ$$

Q.



(1) D_1 $\quad a \sin\theta = \lambda$

(2) $\Delta x = a \sin\theta = \lambda$

(3) $\boxed{\Delta\phi = 2\pi}$

Q.

(1) B_1 $\quad a \sin\theta = \frac{3\lambda}{2}$

(2) $\Delta x = \left(\frac{a}{2}\right) \sin\theta = \frac{3\lambda}{4}$

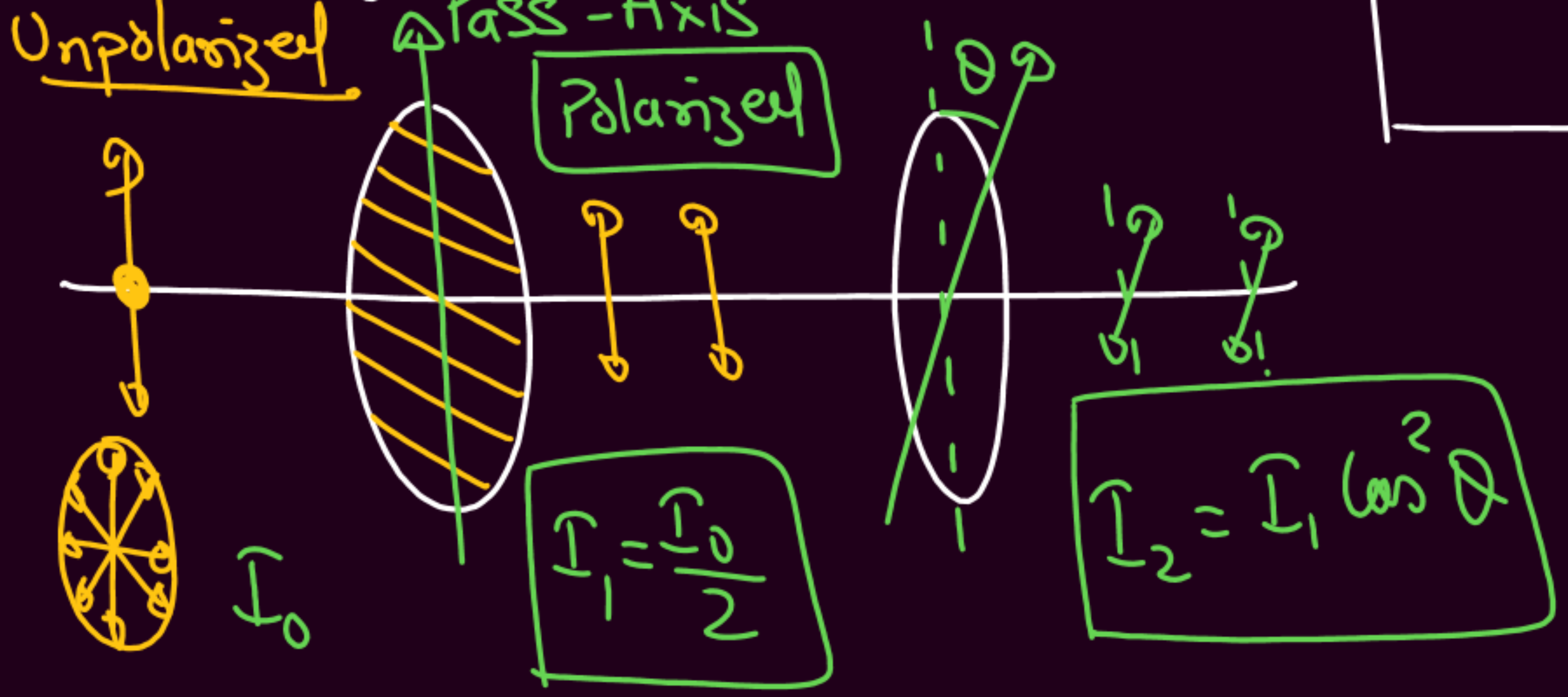
(4) $\boxed{\Delta\phi = \frac{3\pi}{2}}$

Polarization

plane polarized

Polaroid (Polarizer)

↳ long - chain molecules



Law of Malus

Q. $16 \longrightarrow 2$

$I_0 = 16$

$I_1 = \frac{16}{2} = 8$

$I_2 = 8\cos^2\theta = 2$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = 1/2$

$\boxed{\theta = 60^\circ}$

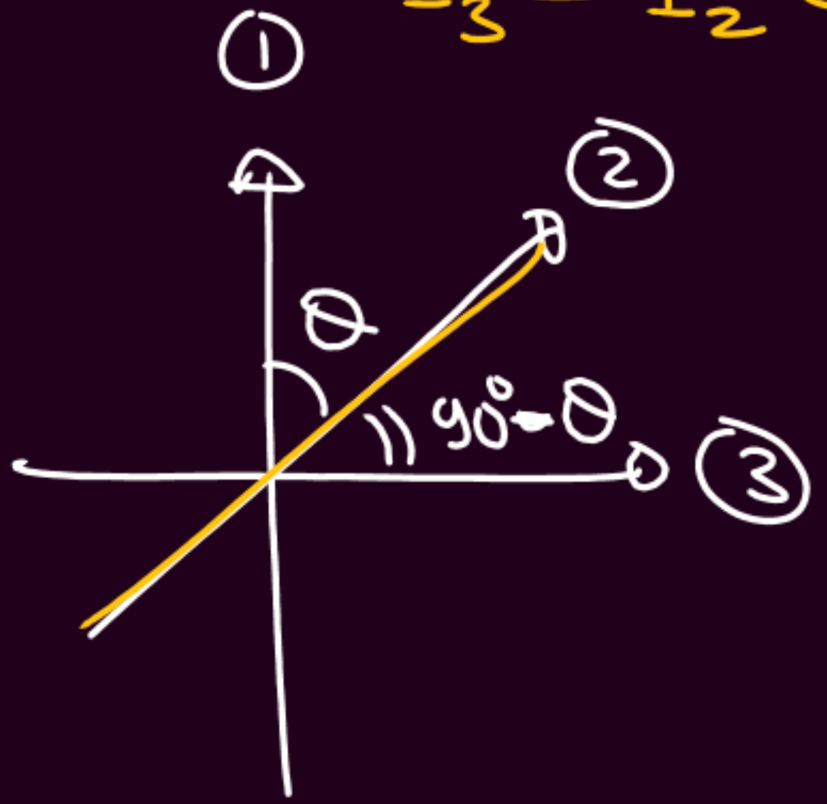
Q.

$I_0 = 32$

$\boxed{I_1 = 16}$

$\boxed{I_2 = 16\cos^2\theta}$

$$I_3 = I_2 \cos^2(90^\circ - \theta) = 16\cos^2\theta\,\sin^2\theta = 3$$



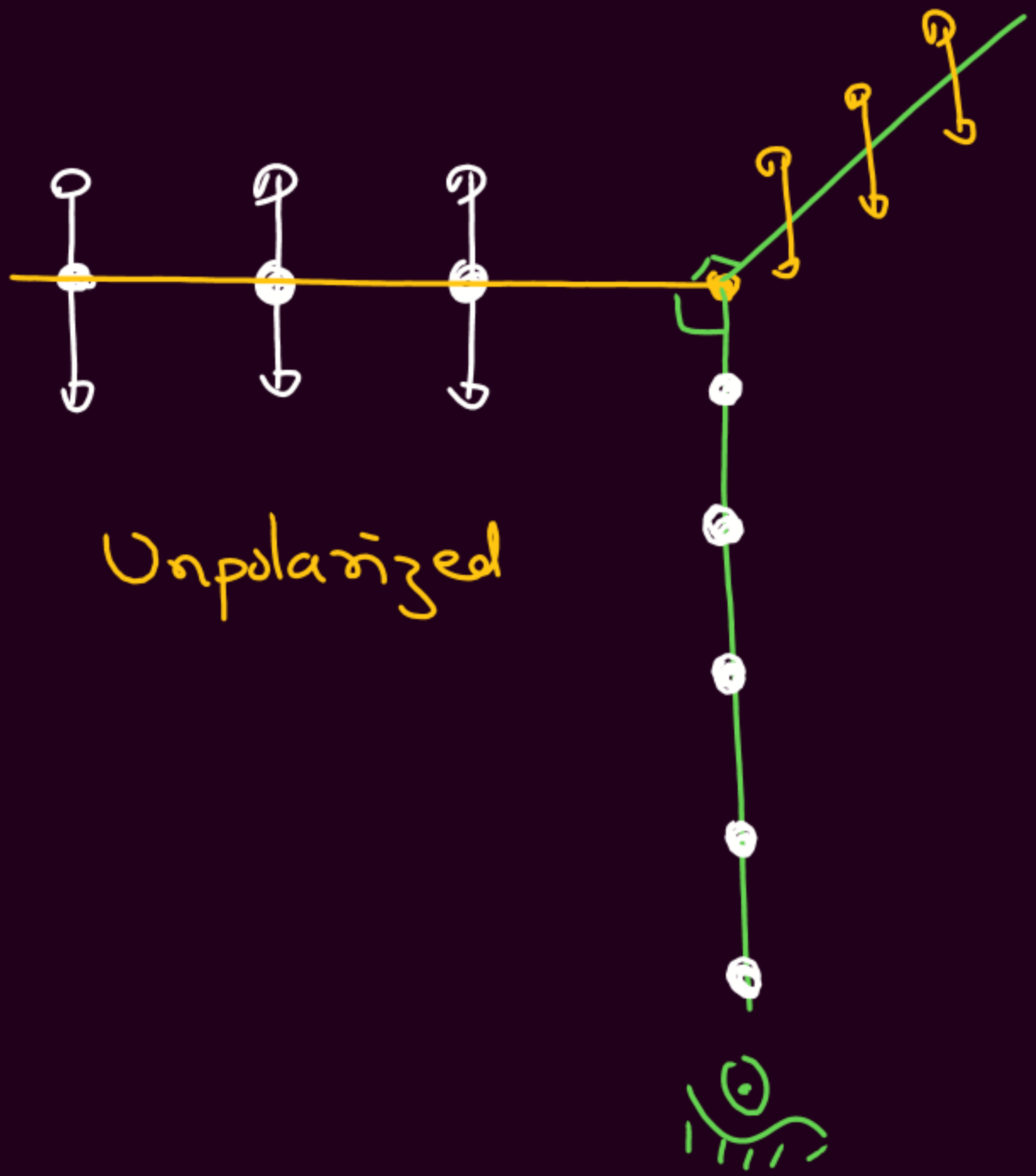
$$2\cos\theta\,\sin\theta = 2\times\frac{\sqrt{3}}{4}$$

$$\sin(2\theta) = \frac{\sqrt{3}}{2}$$

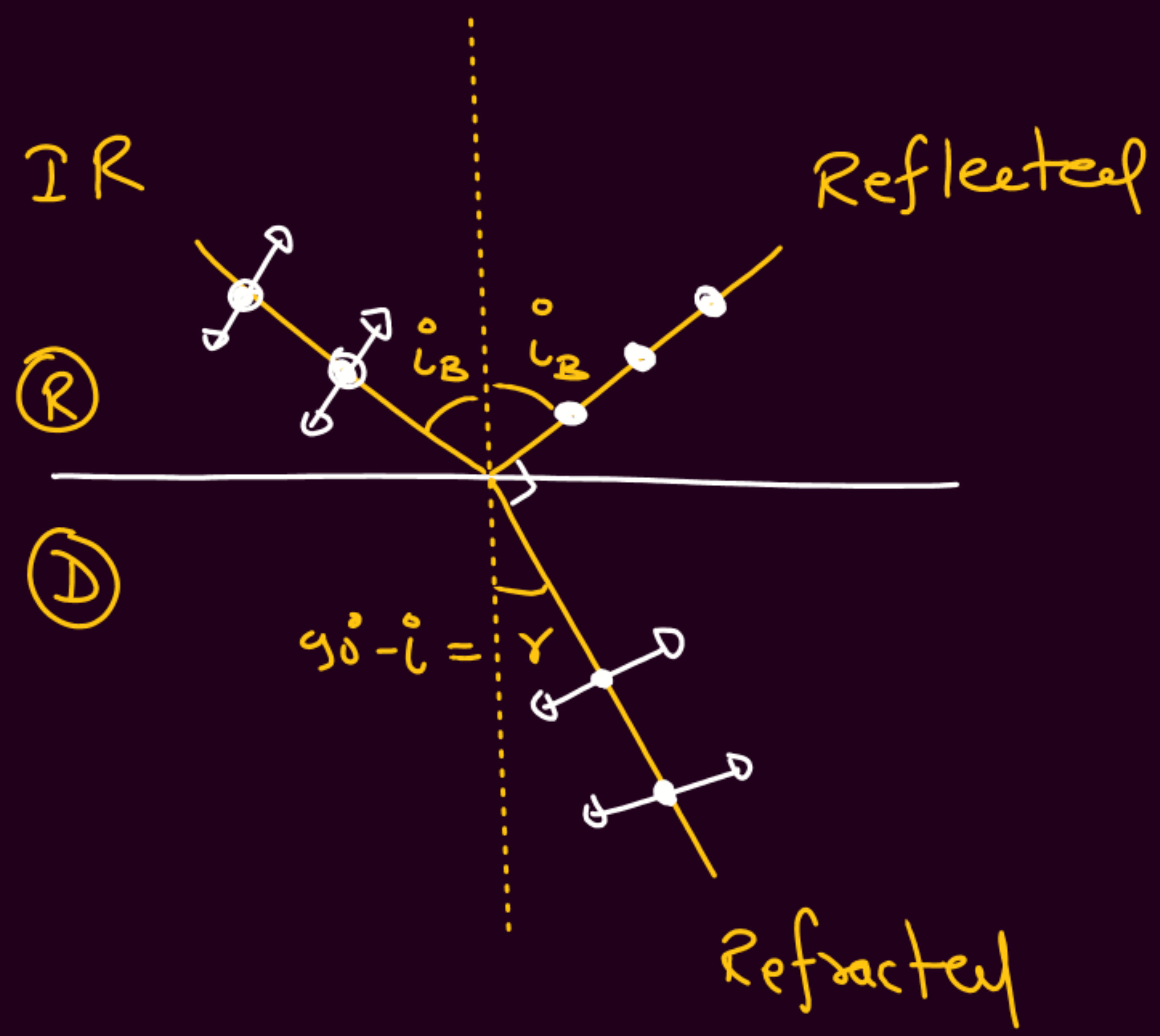
$$2\theta = 60^\circ$$

$\boxed{\theta = 30^\circ}$ or $\boxed{60^\circ}$

Polarization by Scattering → Absorb & Reradiate



Polarization by Reflection



→ R to D

→ Reflected ⊥ Refracted

↳ Reflected → plane polarized

→ $45^\circ < i_B < 90^\circ$

→ $\tan i_B = \mu = \frac{\mu_D}{\mu_R}$

→ i_B = Brewster's Angle

= Angle of polarization